2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

A General Gaussian-Mixture Approach for Range-only Mapping using Multiple Hypotheses

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***Abstract*— Radio signal-based localization and mapping is becoming more interesting as applications involving the collaboration between robots and static wireless devices are more common. Under certain assumptions, the problem is basically equivalent to the range-only localization and mapping problem. The paper presents a method for mapping with a mobile robot the position of a set of nodes using radio signal measurements. It uses Gaussian Mixtures for undelayed initialization of the position of the wireless nodes. The paper shows how the approach can be integrated within a Kalman Filter. This way, information can be used in the filter since the first measurement. The paper describes simulations to verify the feasibility of the approach, and presents results obtained with experimental data involving one mobile robot and a wireless sensor network.**

I. INTRODUCTION

The explosion of the wireless communication systems and networks in the last decade has boost the research in range-only mapping techniques. The main objective of these applications is to estimate the position of wireless communications devices by means of the power of the signal from these devices, computed by the communication circuitry of the receiver. The main assumption is that this power is directly related to the distance between emitter and receiver. Fig. 1 shows a simple range-only localization example.

Early research work is presented in [1], in the field of simultaneous localization and mapping (SLAM), where radio-frequency (RF) transponders are used to build a range-only approach. An estimation of the time of flight is used to compute the distance between transmitter and receiver. Extended Kalman filtering is used to formulate the approach with good localization and mapping results. The partial directivity of the transponders simplifies the problem and allows undelayed gaussian initialization of the position. The integration of pure range-only measurements provided by radio beacons into a SLAM approach is considered in [2], where the position of the beacons is initialized by means of a robust outliers rejection method and a delayed trilateration method.

This work is partially supported by the ROBAIR Project funded by the Spanish Research and Development Program (DPI2008-03847), and the CONET Network of Excellence (ICT-224053) and URUS project (IST-2006-045062) funded by the European Commission

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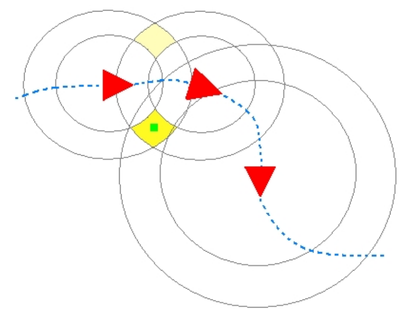


Fig. 1. An example of range-only localization. The robot (red triangle) receives range data from the beacon (green square) at three different positions. The yellow area becomes the most likely after the range information is integrated

Several approaches have been also proposed for range-only localization in the field of wireless sensor networks (WSN). Thus, [3] proposes a grid-based approach that makes use of the received signal strength information (RSSI) in the nodes of a sensor network to localize them, while [4] considers Particle Filtering to solve the same problem. In [5], time of arrival and direction of arrival are used to successfully localize a wireless sensor network, but a number of requirements such as sensor node processor clock synchronization, special signal source devices and direction of arrival are needed. The models used for signal propagation in these papers are usually empirical and consider the antenna geometry simple enough to be approximately isotropic; and the effects of the the environment are usually incorporated through an increase in variance with distance, deemed too complex to be explicitly modeled. Recently, [6] presented a Extended Kalman Filtering approach for signal-based SLAM. Distance is estimated from the signal strength, and a pre-filtering procedure is used to cope with the noise in indoor environments due to multi-path propagation and interferences. Again, delayed initialization through trilateration is used.

A technique for WSN localization using mobile aerial robots has been also presented recently in [7]. The authors detail a technique based on potential fields that exploits the position information of a team of Unmanned Aerial Vehicles (UAVs) to localize the nodes. The main drawback of this approach is that an accurate range sensor is required for computing the distance from the UAVs to the nodes.

The authors proposed in [4] a solution for range-only mapping based on particle filtering over the received signal strength. The paper shows good results in the localization of a small wireless sensor network but the approach has two main disadvantages: first, estimating when the filter has converged to a single hypothesis is computationally demanding and, second, it is not possible to integrate the estimation of the filter into more complex localization architectures such a SLAM until the particle filter has converged to a single solution, i.e., a delayed feature initialization.

The problems related with multiple hypotheses in the early steps of the estimation in range-only localization approaches have been recently addressed in [8]. The paper describes an algorithm that allows delayed initialization of the node position by means of tracking the most probable two hypotheses. Sequential range-only measures provide enough information to discard the wrong hypothesis and then including the correct one into the SLAM filter.

This paper proposes generalizing the approach presented in [8] improving the following two aspects: undelayed node position initialization and extension to n-hypotheses. Thus, it will be possible to integrate range-only measurements into the estimations from the very first time and it will not be necessary to wait for position triangulation to incorporate the measurement. For this purpose, the paper proposes using a weighted Gaussian Mixture to represent the non-Gaussian prior distribution of the node position in a way similar to the approach presented in [9]. This mixture is integrated into a Kalman Filter to sequentially estimating the prior distribution by means of the range-only measurements.

The paper is structured as follows. First, Section II introduces the concept and benefits of Gaussian Mixture Models. Then, Section III describes the multiple hypotheses localization approach. Later, Sections IV and V show experimental results in simulation and with real data respectively, followed by the conclusions and future work in Section VI.

II. Gaussian Mixture Models in a nutshell

A probability mixture model is a probability distribution that is a convex combination of other distributions. Mixture models are a semi-parametric alternative to non-parametric probability distribution (as Particle Filters) and provide greater flexibility and precision in modelling the underlying statistics of sample data.

Gaussian Mixture Models are a type of density model which comprises a number of Gaussian functions. These component functions are combined to provide a multimodal density. Thus, if we assume that the discrete random variable X is a mixture of k component discrete Gaussian variables $\mathcal{N}(\mu_i, \sigma_i)$, then, the probability mass function of X, $f_X(x)$, is a weighted sum of its component distributions:

$$f_X(x) = \sum_{i=1}^{k} w_i \mathcal{N}(\mu_i, \sigma_i) \quad (1)$$

where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{k} w_i = 1$. All in all, the more complex is the probability mass function $f_X(x)$, the larger will be the number of components k of the mixture.

The ability of representing arbitrary Non-Gaussian distributions as linear combination of Gaussian distributions allows simplifying the integration of such distributions into Gaussian filters like the Kalman Filter and, hence, into the classic approaches for SLAM. Particularly, this characteristic is very valuable in range-only localization approaches where no bearing information is provided by the sensor. This process is depicted in Fig. 1: when the robot receives the first range information from the beacon to be localized, the initial position of the beacon is uniformly distributed around the robot position at the given distance. This distribution will evolve into a multi-modal distribution depending on the robot triangulation, and it can eventually converge to a single Gaussian solution if the triangulation is good enough.

Nevertheless, it is easy to see in the example of Fig. 1 that the convergence of the beacon localization to a single solution depends on the triangulation made by the robot. This limitation usually leads to having a bimodal distribution of the beacon position for a long period of time (even for ever). If a delayed approach has been selected to initialize the beacon position into the SLAM filter (like in [10], [8] or [2]), the integration of the measurements provided by such beacon will be also delayed until the probability distribution converges to a single solution. If including the beacon position into the SLAM filter from the very first measurement was possible, significant information could be saved and used to better localize the robot.

The approach presented in this paper will make use of a Gaussian Mixture Model to represent the position of a range-only beacon. The paper will show how this model can be included into the SLAM filter with the first measurement and how this model can be easily updated with new measurements from the beacon.

III. Range-only mapping approach

A. The state vector

The state vector of the filter will be composed by the estimated position and orientation of the robot, and the estimated position of all the beacons. These positions will be considered as static. Thus, the state vector can be described as follows:

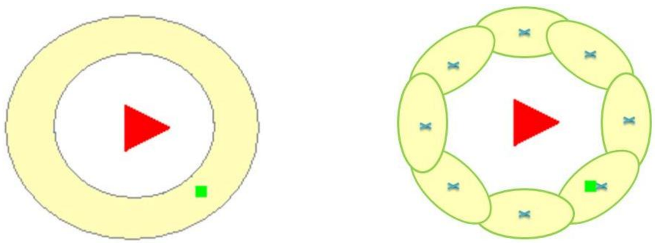
$$\mathbf{x} = [x_r, y_r, \theta_r, \mathbf{b}_1^t, \mathbf{b}_2^t, \dots, \mathbf{b}_n^t]^t \quad (2)$$

where (x_r, y_r, θ_r) represent the Euclidean position and orientation of the robot and $\mathbf{b}_i$ represents the position of all the n beacons considered in the filter.

The beacon position $\mathbf{b}_i$ will be expressed in polar coordinates with respect to the position from which the robot received the very first range information (x_i, y_i). Thus, if the beacon position was completely known, it would be expressed as $\mathbf{b}_i = [x_i, y_i, \rho_i, \theta_i]^t$, where ρ_i is the distance between the robot and the beacon position and θ_i is the angle.

However, this paper assumes non prior information about the angle of arrival of the beacon information, so the value of θ_i is unknown. We propose quantizing the space of possible values of θ_i into k possible hypotheses. Thus, each beacon will be expressed as follows:

$$\mathbf{b}_i = [x_i, y_i, \rho_i, \theta_{i0}, \theta_{i1}, \dots, \theta_{i(k-1)}]^t \quad (3)$$



(a) (b)

Fig. 2. An example of a Gaussian Mixture Model used to approximate the probability mass function of the position a beacon based on range-only measurements. On (a) is presented the original probability function and on (b) the Gaussian Mixture approximation can be seen. (Red triangle): robot position, (Green square): actual beacon position

All the hypotheses θ_{ij}, together with their weights w_{ij}, will compose a Gaussian Mixture Model such as the probability mass functions of θ_i, $f_{\theta_i}(x)$, will be uniformly distributed from 0 to 2π, that is:

$$f_{\theta_i}(x) = U(0, 2\pi) \simeq \sum_{j=1}^{k} w_{ij}\mathcal{N}(\theta_{ij}, \sigma_{ij}) \quad (4)$$

Then, it can be seen how the state vector presented in (2) will consist of the robot position/orientation estimation and the hypotheses of every beacon considered into the filter. Of course, the number of hypotheses will evolve, reducing its number as range information is integrated from different robot positions.

The questions that follow are how to optimally initialize the beacons into the filter if the number of hypotheses k is given, how to integrate new measurements into the filter and how to prune low probable hypotheses from the filter. Next sections will deal with these issues.

B. Beacon initialization

As previously introduced, after the first range information of a beacon is considered, the probability mass function of its position will be uniformly distributed around the robot location, as shown in Fig. 2.a. This probability function will be approximated by a Gaussian Mixture Model using (4), Fig. 2.b shows an example. Each of these Gaussians will be considered as an independent hypothesis into the localization filter.

The number k of hypotheses will be manually setup depending on the computational resources, because the length of the state vector depends on the number of beacons n and the number of hypotheses per beacon k through the expression $L = 3 + 3n + nk$. Using a very large number of hypotheses could be overkill, while reducing this number too much could lead to inconsistencies. In this particular case, experimental results showed that 8 hypotheses are enough to provide a good balance between results and efficiency.

Known the number of hypotheses k, the next step is to estimate the values of w_{ij}, θ_{ij} and σ_{ij} that better approximate the Gaussian Mixture Model of (4) to an Uniform Distribution between 0 and 2π, as in Fig. 2.b. It is easy to see that the value of the hypothesis weights has to be equal to all of them because of the uniform characteristic of the distribution of θ_i, so the weights will be set as $w_{ij} = 1/k$.

Estimating the mean value θ_{ij} of each hypothesis is also simple considering that they have to be uniformly distributed from 0 to 2π. Thus, depending on the number of hypotheses, the values of the mean will be defined as:

$$\theta_{ij} = 2\pi j/k, \qquad j = 0, \ldots, k-1 \quad (5)$$

Finally, given the values of w_{ij} and θ_{ij} for each hypothesis, the value of σ_{ij} has to be determined. Given the uniform distribution of θ_{ij} from 0 to 2π, a good and simple criteria is to make all the standard deviations σ_{ij} equal each other. The value has been empirically calculated by the authors and is based on the following expression:

$$\sigma_{i0} = \sigma_{i1} = \ldots = \sigma_{i(k-1)} = 2\pi/(1.5k) \quad (6)$$

To test the above expressions, several simulations have been made for $k = 2$ to $k = 20$. The simulations showed that the model is a good approximation of the objective uniform distribution for $k > 5$.

C. Incorporating measurements

Once the beacon has been initialized into the filter with the first range information, next measurements will be used to update the estimation of each hypothesis and also to refine the weights w_{ij} associated to them.

The measurements provided by the system are the distances of the robot to the set of beacons that are in communication range. Thus, let r_i be the measured distance from the robot to the beacon i and σ_i^2 the measurement error variance. Considering (2), the following measurement equation is applicable for each of the hypotheses j of beacon i:

$$r_i = \sqrt{(x_i + \rho_i cos(\theta_{ij}) - x_r)^2 + (y_i + \rho_i sin(\theta_{ij}) - y_r)^2} \quad (7)$$

The question now is how to deal with the variance associated with the measurement, σ_i^2. A single measurement is available but it has to be applied to all the existing hypotheses for beacon i. Notice that the measurement cannot be simply applied to all the hypothesis separately because then the same information would be count k times in the filter which finally would lead to filter divergence. In [9], the solution to this problem is shown for the case the information comes from a unique source, as it is our case. It is stated that the correction of the estimate of a random variable by a set of measurement pairs $(\mathbf{z}, \mathbf{R}_{ij})$ is equivalent to the unique correction by $(\mathbf{z}, \mathbf{R}_i)$ if:

$$\mathbf{R}_i^{-1} = \sum_{j=0}^{k-1} \mathbf{R}_{ij}^{-1} \quad (8)$$

This means that the original information can be divided into k new measurements with the same mean and with covariances according to (8). Sharing the information according to the likelihood l_{ij} of each hypothesis is proposed in [9]. Thus, if we are able to compute a weight λ_{ij} proportional

Algorithm 1 Build Measures and Update Weights $\{\{i_i, \sigma_{i0}^2\}, ..., \{r_i, \sigma_{i(k-1)}^2\}\} \leftarrow \{\{x_r, y_r, \mathbf{b}_i\}, \{r_i, \sigma_i^2\}\}$

```
/*Compute likelihood of each hypothesis */
for j = 1 to k do
    l_ij = p(r_i | x_r, y_r, x_i, y_i, ρ_i, θ_ij)
end for
/* Compute measurement variance of each hypothesis */
/* Update weight of each hypothesis */
for j = 1 to k do
    λ_ij = l_ij / Σ_{j=0}^{k-1} l_ij
    σ_ij^2 = σ_i^2 / λ_ij
    w_ij = w_ij l_ij
end for
Normalize weights w_ij such as Σ_{j=0}^{k-1} w_ij = 1
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to the likelihood of each hypothesis of beacon i such as $\sum_{j=0}^{k-1} \lambda_{ij} = 1$, the measurement variance associated to each hypothesis could be computed as $\sigma_{ij}^2 = \sigma_i^2/\lambda_{ij}$. Then, once the likelihood l_{ij} of each of the hypotheses has been computed, it is normalized using the following expression to obtain the values of λ_{ij}:

$$\lambda_{ij} = l_{ij} / \sum_{j=0}^{k-1} l_{ij} \tag{9}$$

Following this procedure, all the range measurements are applied to the corresponding beacon hypotheses.

Finally, it is necessary to properly update the weight associated to each hypothesis, w_{ij}. The key idea is to make evolve the weights according to the closeness of the hypotheses with the real beacon position. For doing this, the likelihood is used again according with the following equation:

$$w_{ij} = w_{ij} l_{ij} \tag{10}$$

Later, the new weights are normalized.

The whole procedure is summarized in Algorithm 1 for the beacon i. Once the weights have been updated, all the measurement pairs $\{r_i, \sigma_{ij}^2\}$ are arranged into the measurement vector and its covariance matrix (which is diagonal), and used to update the hypotheses into the Extended Kalman Filter (EKF) using the standard EKF updating equations. The conditional probability $p(r_i|x_r, y_r, x_i, y_i, \rho_i, \theta_{ij})$ is modeled as a Gaussian distribution, with mean obtained evaluating eq. (7) at the current hypothesis j, and propagating the corresponding state covariances through the Jacobian of the cited equation.

D. Pruning hypotheses

A rule to remove useless hypotheses from the filter have to be defined. Basically, a hypothesis has to be removed if it satisfies at least one of the following constraints:

- The associated weight w_{ij} is below a certain threshold. A general value for this threshold is introduced in [9]:

$$w_{ij} \leq 0.00001/k \tag{11}$$

  where k is the current number of hypotheses for the beacon i. The simulations and experimental results shown in this paper show that this threshold works well for the localization problem.
- The Euclidean distance among hypotheses is smaller than a certain threshold. This constraint allows removing hypotheses very similar each other, saving computation time. The value of the threshold depends on the kind of application. In this case, experiments show that one meter is a good value for this threshold.
  If a set of hypotheses are below this value, the one with the higher weigh stays in the filter and the rest are removed.

IV. Simulation results

A set of simulations has been carried out in order to test the approach for mapping using range-only information. The setup consists of a car-like mobile robot equipped with a wireless sensor node moving in an area in which a wireless sensor network composed by other fifteen static nodes has been deployed.

The robot estimates its own position based on local odometry (using the speed and the direction of the wheels). Each time a message arrives at the sensor node onboard the robot, the distance to the emitter is calculated based on the Received Signal Strength Information (RSSI), as in [10], and this information is used to update the node position into the filter. In the simulation, the signal propagation model described in [10] has been used to generate random samples of the distance between the robot and the sensor node. The maximum transmission range and the rate of messages sent by each node of the network (about one per second) have been also considered into the simulation to be as realistic as possible.

Figure 3 shows some screenshots of the urban simulation environment for the localization of node 5. The different stages of the mapping approach can be easily seen in the figure. At the beginning, all the hypotheses are placed around the vehicle according to the range information, which is very noisy at such distances. Later, the localization of the hypotheses in front the vehicle is improved because the range-RSSI relation is more accurate as smaller is the distance. In the next steps, the hypotheses behind the robot are removed from the filter because their weights are too low. Next, only the hypotheses on the sides of the vehicle remain. This is because the robot is moving in a straight line with respect to the node, so both hypotheses are possible if range-only measurements are considered. Finally, after the robot moved to a different position, the wrong hypothesis is removed and the filter converged to the correct node position.

The evolution of the weights associated to each of the hypotheses related to node 5 (analyzed in Fig. 3) is shown in Fig. 4. Three main stages can be seen in the evolution of the hypotheses' weights: First, most of the hypotheses are removed quickly after the integration of the new measurements around sample 6 (it corresponds with the transition from image (b) to (c) in Fig. 3). Second, another hypothesis

(a) (b) (c) (d) (e)

Fig. 3. Evolution of the multiple hypotheses for the localization of one sensor node. The blue circle denotes the beacon position, the red square represents the robot position and the yellow ellipses are the multiple hypotheses for the localization.

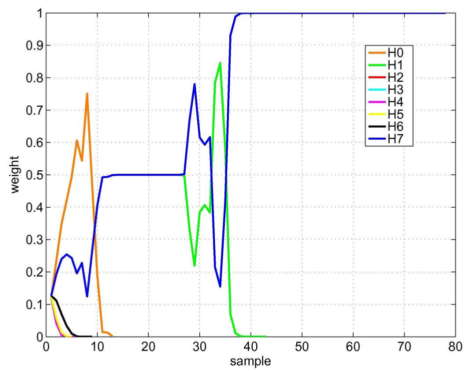
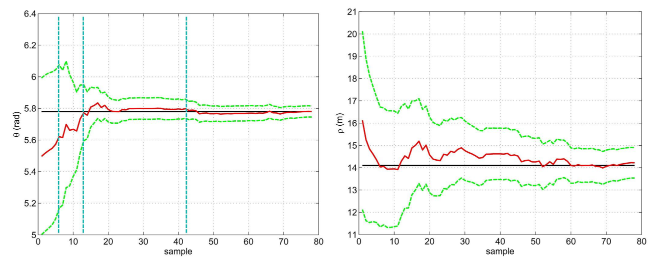


Fig. 4. Values of the weights associated to the hypotheses of node 5. The weight value range from 0 to 1. Notice how hypotheses 3, 4, 5 and 6 are removed from the filter around time step 6. Hypothesis 0 is removed at time 13. Hypothesis 1 is removed at time step 43, becoming hypothesis 7 dominant



(a) (b)

Fig. 5. (a) Estimated value of θ for hypothesis 7 (red solid line) and its 3σ confidence interval (green dashed line) for node 5. The black solid line shows the actual value of θ. Blue dashed lines show the time stamps in which different hypotheses are removed from the filter. (b) Estimated value of ρ (red solid line) and its 3σ confidence interval (green dashed line) for node 5. The black solid line shows the actual value of ρ

is removed at sample 13 and only two possible solutions remain in the filter (transition from (c) to (d) in Fig. 3). Third, after triangulating from different positions, the wrong hypothesis is removed from the filter around sample 43 and the localization converges to a single solution.

For the case of node 5, Fig. 5 shows the estimated mean and standard deviation of θ for the hypothesis 7 (which finally becomes the proper solution of the localization for node 5) and ρ. It can be seen how the estimated θ slowly converges to the correct solution together with the estimated range ρ. It can also be seen how the ground truth is always into the estimated 3σ interval. Due to paper space restrictions, the values of x and y associated to the beacon 5 are not shown because their estimation mainly depends on the robot localization[1].

V. Experimental results

The approach has been also tested in real experiments involving a mobile robot equipped with a sensor node and a wireless sensor network of three nodes. This setup is very similar to the simulation setup but this time with actual equipment.

The distance between wireless sensor nodes is computed based on the RSSI measured by the node on board the robot. The signal propagation model described in [10] is used to estimate such distance based on the RSSI, and its standard deviation. The nodes of the network are Mica2 nodes, with the following characteristics:

- Atmega128L microcontroller at 7.3728 Mhz
- Chipcon CC1000 FSK radio transceiver, 900 Mhz band. Up to 4 dBm transmission power. Up to 78.4 kbps 100 meter outdoor range.
- 512 KB non-volatile memory for programs and retrieved data.

The final mapping result corresponding to this experiments can be seen in Fig. 6. The figure shows the localization of each sensor node and the trajectory of the mobile robot. It can be seen how the localization follows the real position with small errors.

As an example, the localization of node 2 will be analyzed in this paper. Thus, Fig. 7 shows the evolution of the weights associated to the different hypotheses. It can be seen how most of the hypotheses are quickly removed with the first range measurements and finally hypothesis 0 becomes the dominant one.

Figure 8 shows the estimated mean and standard deviation, together with the ground truth, for θ and ρ for node 2. In this case only the value of θ for hypothesis 0 is shown. It can be seen how the estimation converges to the proper solution in both range and angle. It is worth to mention that the algorithm converges even in the presence of a significant error in the range initialization (about fifteen meters) due to

[1]A video with the complete result of this simulation for all the sensor nodes has been created and placed in the following URL: *http://grvc.us.es/staff/caba/share/range_only_sym.mpg*.

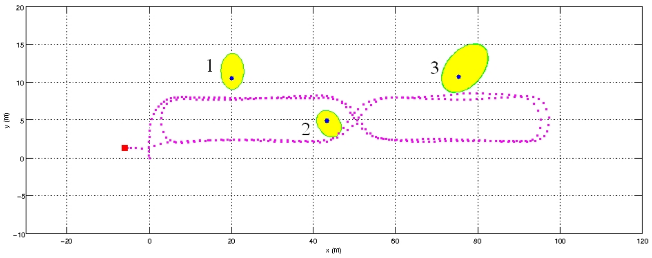


Fig. 6. Experimental results on localizing three sensor nodes (blue dots) using a mobile robot equipped with a sensor node (red square). The yellow ellipses stand for the uncertainty in the localization of the nodes. For simplicity, the uncertainties are a X-Y linear approximation of the uncertainties in polar coordinates. The dotted pink line represents the actual trajectory followed by the robot.

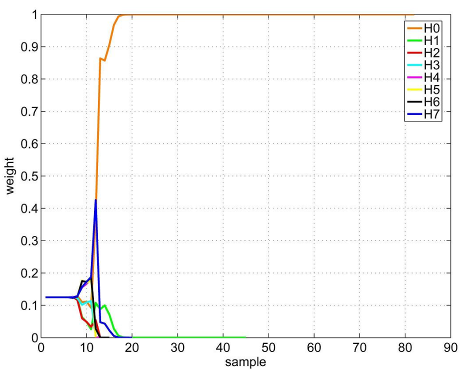
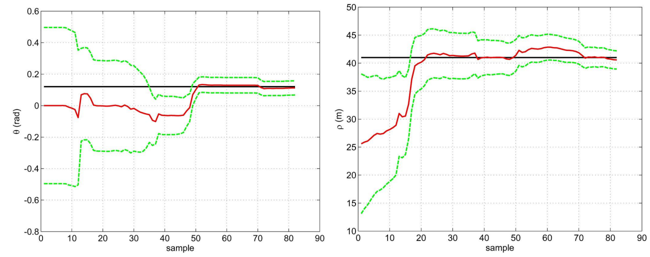


Fig. 7. Values of the weights associated to the hypotheses of node 2. The weight value range from 0 to 1. Most of the hypotheses are removed between samples 11 and 20. Hypothesis 1 is removed at sample 45 and then hypothesis 0 becomes dominant



(a) (b)

Fig. 8. (a) Estimated value of θ for hypothesis 0 (red solid line) and its 3σ confidence interval (green dashed line) for node 2. The black solid line shows the actual value of θ. Blue dashed lines show the time stamps in which different hypotheses are removed from the filter. (b) Estimated value of ρ (red solid line) and its 3σ confidence interval (green dashed line) for node 2. The black solid line shows the actual value of ρ

outliers in the RSSI/distance computation.

VI. Conclusions and Future Work

This paper presented a Gaussian Mixture approach to solve the mapping problem in presence of radio signal-based range-only measurements. The paper described how the Mixture Model can be integrated into a Kalman Filter, leading to a multiple hypotheses filter able to deal with the range-only mapping problem. This approach allows to consider information about the nodes since the first measurement. The initialization and updating steps of such multiple hypotheses approach have been also detailed into the paper.

The paper has presented simulations that show that the approach is feasible for localizing wireless sensor nodes based on the strength of the received signal on a node installed on a mobile robot. Furthermore, experimental results with actual equipment also shows good mapping results.

Future research will consider active sensing strategies to determine the robot commands that optimize the information gain when using the range-only measurements. Thus, the expected variation of the entropy of the Gaussian Mixture can be used to determine the actions that have to be carried out by the robot in order to better acquire the range information. Recent results on bounds on the entropy on a Gaussian Mixture, like [11], can be used for that. At the same time, active sensing strategies can be used to avoid non-observable motions that can arise this problem.

VII. Acknowledgments

The authors thank the help and support of Jose Manuel Sánchez-Matamoros, Pablo Soriano, Fran Real, Alberto de San Bernabé and Jesus Captitán during the URUS Project experiments.

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