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Flocking Control of Multiple Agents in Noisy Environments

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Abstract— **Birds, bees, and fish often flock together in groups to find the source of food (target) based on local information. Inspired by this natural phenomenon, a flocking control algorithm is designed to coordinate the activities of multiple agents in noisy environments. Based on this algorithm, all agents can form a network and maintain connectivity. This is of great advantage for agents to exchange information. In addition, collision avoidance among agents is guaranteed in the whole process of target tracking. We show that even with noisy measurements the flocks can achieve cohesion and follow the moving target. We also investigate the stability and scalability of our algorithm. The numerical simulations are performed to demonstrate the effectiveness of the proposed algorithm.**

Keywords: Flocking control, multi-agent systems.

I. Introduction

Flocking is a phenomenon in which a number of agents move together and interact with each other. In nature, schools of fish, birds, ants, and bees, etc. demonstrate the phenomena of flocking. Flocking control for multiple mobile agents has been studied in recent years [1], [2], [3], and it is designed based on three basic flocking rules proposed by Reynolds in [4]: flock centering (agents try to stay close to nearby flock-mates), collision avoidance (agents try to avoid collision with nearby flock-mates), and velocity matching (agents try to match their velocity with nearby flock-mates). The problems of flocking have also attracted many researchers in physics [5] and biology [6].

Early work on flocking control stability includes [1], [2], [3]. Tanner *et al.* [1] and [2] studied the stability properties of a system of multiple mobile agents with double integrator dynamics in case of fixed and dynamic topologies. In [3], the theoretical framework for design and analysis of distributed flocking algorithms was proposed. This established a background for flocking control design for a group of agents. As an extension of the flocking algorithm in [3], flocking of agents with a virtual leader in case of a minority of informed agents and varying velocity of virtual leader was presented in [7].

In this paper we study the stability properties and connectivity preservation during the flocking of a multi-agent system. The main difference with the above related work is that we consider the effect of position and velocity measurement errors (noises) in sensing the agent's neighbors and the agent itself, and the noises in sensing the position and velocity of the target. We propose a new flocking control algorithm that allows the flocks to preserve connectivity, avoid collision, and follow the target even in such noisy environments. We demonstrate that by applying our algorithm the agents can flock together and maintain connectivity better compared with those in existing flocking control algorithms.

The rest of this paper is organized as follows. In the next section we present the background of flocking control and problem formulation. Section III describes flocking control to track a moving target in noisy environments. Section IV presents the main results on stability analysis of flocking control in noisy environments. Section V shows the simulation results. Finally, Section VI concludes this paper.

II. Flocking Backgrounds and Problem Formulation

In this section we will present flocking control background and the problems in existing flocking control algorithm.

We consider n agents moving in an m (e.g., $m = 2,3$) dimensional Euclidean space. The dynamic equations of each agent are described as:

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i, \quad i = 1,2,...,n. \end{cases} \tag{1}$$

here $q_i, p_i \in R^m$ are the position and velocity of node i, respectively, and u_i is the control input of agent i.

To describe the topology of flocks we consider a dynamic graph G consisting of a vertex set $\vartheta = \{1,2...,n\}$ and an edge set $E \subseteq \{(i,j) : i,j \in \vartheta, j \neq i\}$. In this topology each vertex denotes one member of the flock, and each edge denotes the communication link between two members.

We know that during the movement of agents, the relative distance between them may change, hence the neighbors of each agent also change. Therefore, we can define a neighborhood set of agent i as follows:

$$N_i = \left\{ j \in \vartheta : \|q_j - q_i\| \leq r,\ \vartheta = \{1,2,...,n\}, j \neq i \right\},$$

here r is an active range (radius of neighborhood circle in the case of two dimensions, $m = 2$, or radius of neighborhood sphere in the case of three dimensions, $m = 3$), and $\|.\|$ is the Euclidean distance.

The geometry of flocks is modeled by an α-lattice [3] that meets the following condition:

$$\|q_j - q_i\| = d, j \in N_i, \tag{2}$$

here d is a positive constant indicating the distance between agent i and its neighbor j. However, at singular configuration ($q_i = q_j$) the collective potential used to construct the

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geometry of flocks is not differentiable. Therefore, the set of algebraic constrains in (2) is rewritten in term of σ - norm [3] as follows:

$$\|q_j - q_i\|_\sigma = d^\alpha, j \in N_i, \quad (3)$$

here the constraint $d^\alpha = \|d\|_\sigma$ with $d = r/k_c$, where k_c is the scaling factor. The σ - norm, $\|.\|_\sigma$, of a vector is a map $R^m \Longrightarrow R_+$ defined as $\|z\|_\sigma = \frac{1}{\varepsilon}[\sqrt{1+\varepsilon\|z\|^2}-1]$ with $\varepsilon > 0$. Unlike the Euclidean norm $\|z\|$, which is not differentiable at $z = 0$, the σ - norm $\|z\|_\sigma$, is differentiable everywhere. This property allows to construct a smooth collective potential function for agents.

The flocking control law in [3] controls all agents to form an α-lattice configuration. This algorithm consists of two components as follows:

$$u_i = f_i^\alpha + f_i^t. \quad (4)$$

The first component of (4) f_i^α, which consists of a gradient-based component and a consensus component, is used to regulate the potentials (repulsive or attractive forces) and the velocity among agents,

$$f_i^\alpha = \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)n_{ij} + \sum_{j\in N_i} a_{ij}(q)(p_j - p_i), \quad (5)$$

where each term in (5) is computed as follows [3]:

1. The action function $\phi_\alpha(z)$ that vanishes for all $z \geq r^\alpha$ with $r^\alpha = \|r\|_\sigma$ is defined as follows:

$$\phi_\alpha(z) = \rho_h(z/r_\alpha)\phi(z - d^\alpha)$$

with the uneven sigmoidal function $\phi(z)$ defined as $\phi(z) = 0.5[(a+b)\sigma_1(z+c)+(a-b)]$, here $\sigma_1(z) = z/\sqrt{1+z^2}$, and parameters $0 < a \leq b$, $c = |a-b|/\sqrt{4ab}$ to guarantee $\phi(0) = 0$. The bump function $\rho_h(z)$ with $h \in (0,1)$ is

$$\rho_h(z) = \begin{cases} 1, & z \in [0,h) \\ 0.5[1+cos(\pi(\frac{z-h}{1-h}))], & z \in [h,1) \\ 0, & otherwise. \end{cases}$$

2. The vector along the line connecting q_i to q_j is

$$n_{ij} = (q_j - q_i)/\sqrt{1+\varepsilon\|q_j - q_i\|^2}.$$

3. The elements $a_{ij}(q)$ of the adjacency matrix $[a_{ij}(q)]$ are defined as

$$a_{ij}(q) = \begin{cases} \rho_h(\|q_j - q_i\|_\sigma/r_\alpha), & if \quad j \neq i \\ 0, & if \quad j = i. \end{cases}$$

The second component of (4) f_i^t is designed for distributed target tracking,

$$f_i^t = -c_1^t(q_i - q_t) - c_2^t(p_i - p_t) \quad (6)$$

where c_1^t and c_2^t are positive constants, and (q_t, p_t) are the position and velocity of the moving target defined as follows

$$\begin{cases} \dot{q}_t = p_t \\ \dot{p}_t = f_t(q_t, p_t) \end{cases}$$

Finally, the Olfati-Saber flocking control law [3] in free space is:

$$\begin{aligned} u_i \quad = \quad & \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)n_{ij} + \sum_{j\in N_i} a_{ij}(q)(p_j - p_i) \\ & -c_1^t(q_i - q_t) - c_2^t(p_i - p_t). \end{aligned} \quad (7)$$

The control law (7) is designed under the following assumptions:

1. Each agent can sense its own position and velocity precisely (without noises).

2. Each agent can obtain its neighbors position and velocity via sensing or message broadcasting precisely.

3. Each agent can sense the target position and velocity precisely.

However, in reality these assumptions are not valid because sensing noise always exists. Motivated by this observation we will study how to design a distributed flocking control law to handle noises.

III. Flocking control of multiple agents in noisy environments

In this section, first we design a distributed flocking control law in noisy environments. Then we will derive the dynamic error model. We assume that each agent senses its own position and velocity with noises, and each agent can obtain its neighbors position and velocity via sensing or message broadcasting with noises. We also assume that each agent senses the target position and velocity with noises.

A. Algorithm Description

We have the following definitions:

The local average of position and velocity $(\overline{q}_i, \overline{p}_i)$ of agent i and its neighbors is defined:

$$\begin{cases} \overline{q}_i = \frac{1}{|N_i|+1}\sum_{i=1}^{|N_i|+1} q_i \\ \overline{p}_i = \frac{1}{|N_i|+1}\sum_{i=1}^{|N_i|+1} p_i, \end{cases} \quad (8)$$

and the global average of position and velocity $(\overline{q}, \overline{p})$ is defined as:

$$\begin{cases} \overline{q} = \frac{1}{n}\sum_{i=1}^{n} q_i \\ \overline{p} = \frac{1}{n}\sum_{i=1}^{n} p_i. \end{cases} \quad (9)$$

$d_{il} = q_i - \overline{q}_i$ is relative distance between node i and its local average of position;

$v_{il} = p_i - \overline{p}_i$ is relative velocity between node i and its local average of velocity;

$d_{ig} = q_i - \overline{q}$ is relative distance between node i and its global average of position;

$v_{ig} = p_i - \overline{p}$ is relative velocity between node i and its global average of velocity;

Assume that the estimates of position and velocity of agent i are: $\hat{q}_i = q_i + \varepsilon_q^i$ and $\hat{p}_i = p_i + \varepsilon_p^i$, where ε_q^i and ε_p^i are position and velocity errors (noises), respectively. Then we have:

$\hat{q}_i - \hat{q}_j = q_i - q_j + \varepsilon_q^{ij}$,

$\hat{p}_i - \hat{p}_j = p_i - p_j + \varepsilon_p^{ij}$, here $\varepsilon_q^{ij} = \varepsilon_q^j - \varepsilon_q^i$ and $\varepsilon_p^{ij} = \varepsilon_p^j - \varepsilon_p^i$.

Similarly, the estimates of position and velocity of the target are: $\hat{q}_t = q_t + \varepsilon_q^t$ and $\hat{p}_t = p_t + \varepsilon_p^t$, where ε_q^t and ε_p^t are position and velocity noises, respectively. Then we have:

$\hat{q}_i - \hat{q}_t = q_i - q_t + \varepsilon_q^{it}$,
$\hat{p}_i - \hat{p}_t = q_i - p_t + \varepsilon_p^{it}$, here $\varepsilon_q^{it} = \varepsilon_q^t - \varepsilon_q^i$ and $\varepsilon_p^{it} = \varepsilon_p^t - \varepsilon_p^i$.

Now, we propose a distributed flocking control law in noisy environments as:

$$\begin{aligned} u_i &= c_1^\alpha \sum_{j\in N_i} \phi_\alpha(\|\hat{q}_j - \hat{q}_i\|_\sigma)\hat{n}_{ij} + c_2^\alpha \sum_{j\in N_i} \hat{a}_{ij}(q)(\hat{p}_j - \hat{p}_i) \\ &\quad - c_{pos}\hat{d}_{il} - c_{ve}\hat{v}_{il} - c_1^t(\hat{q}_i - \hat{q}_t) - c_2^t(\hat{p}_i - \hat{p}_t) \\ &\quad - c_1^g(\hat{\overline{q}}_i - \hat{q}_t) - c_2^g(\hat{\overline{p}}_i - \hat{p}_t), \end{aligned} \tag{10}$$

here $\hat{\overline{q}}_i$ and $\hat{\overline{p}}_i$ are computed as

$$\begin{cases} \hat{\overline{q}}_i = \frac{1}{|N_i|+1}\sum_{i=1}^{|N_i|+1}\hat{q}_i \\ \hat{\overline{p}}_i = \frac{1}{|N_i|+1}\sum_{i=1}^{|N_i|+1}\hat{p}_i, \end{cases} \tag{11}$$

and $\hat{n}_{ij}$, $\hat{a}_{ij}(q)$ are computed as
$\hat{n}_{ij} = (\hat{q}_j - \hat{q}_i)/\sqrt{1+\varepsilon\|\hat{q}_j - \hat{q}_i\|^2}$,

$$\hat{a}_{ij}(q) = \begin{cases} \rho_h(\|\hat{q}_j - \hat{q}_i\|_\sigma / r_\alpha), & if \quad j \neq i \\ 0, & if \quad j = i \end{cases},$$

and $\hat{d}_{il}$, $\hat{v}_{il}$ are the estimates of d_{il} and v_{il}, respectively; and $c_1^\alpha, c_2^\alpha, c_{pos}, c_{ve}, c_1^t$, c_2^t, c_1^g and c_2^g are positive constants. In this control protocol, the first two terms are used to control the formation (α-lattice configuration) and to allow agents to avoid collision. $-c_{pos}\hat{d}_{il}$ and $-c_{ve}\hat{v}_{il}$ are called position and velocity cohesion feedbacks, respectively. The role of these negative feedbacks is to maintain position and velocity cohesions. This means that each agent tries to stay close to the local average of position (8) and minimize the velocity mismatch between its velocity and the local average of velocity (8) in noisy environments. That allows the network to shrink in order to maintain network connectivity, and also allows the error dynamics of the system to be bounded (proved in Theorem 1). In addition, the terms $-c_1^t(\hat{q}_i - \hat{q}_t) - c_2^t(\hat{p}_i - \hat{p}_t)$ and $-c_1^g(\hat{\overline{q}}_i - \hat{q}_t) - c_2^g(\hat{\overline{p}}_i - \hat{p}_t)$ allow each agent and its neighbors to closely follow the target.

B. Dynamic Error Model

To study the stability properties, we have the error dynamics of the system given as follows:

$$\begin{cases} \dot{d}_{ig} = v_{ig} \\ \dot{v}_{ig} = u_i - \frac{1}{n}\sum_{j=1}^n u_j = u_i - \overline{u}, \quad i = 1,2,...,n, \end{cases} \tag{12}$$

here $\overline{u} = \frac{1}{n}\sum_{j=1}^n u_j$.

Firstly, we have the following relations:

$$\begin{aligned} d_{il} &= q_i - \overline{q}_i = d_{ig} + \overline{q} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} q_j \\ &= d_{ig} + \overline{q} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1}(d_{jg} + \overline{q}) \\ &= d_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} d_{jg}. \end{aligned} \tag{13}$$

Then similar to d_{il}, v_{il} is obtained as follows:

$$v_{il} = v_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} v_{jg}. \tag{14}$$

However, because agent i senses its own position and velocity with noises, hence the estimates $\hat{d}_{il}$ and $\hat{v}_{il}$ are also corrupted by noises $(\varepsilon_d^i, \varepsilon_v^i)$ as:

$$\begin{cases} \hat{d}_{il} = d_{il} - \varepsilon_d^i \\ \hat{v}_{il} = v_{il} - \varepsilon_v^i. \end{cases} \tag{15}$$

Also, the estimates of the local average of position and velocity, respectively in (11) corrupted by noises $(\overline{\varepsilon}_q^i, \overline{\varepsilon}_p^i)$ can be rewritten as

$$\hat{\overline{q}}_i = q_i - d_{ig} + \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} d_{jg} + \overline{\varepsilon}_q^i. \tag{16}$$

$$\hat{\overline{p}}_i = p_i - v_{ig} + \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} v_{jg} + \overline{\varepsilon}_p^i. \tag{17}$$

Now, we can rewrite the control law (10) with considering (15), (16) and (17):

$$\begin{aligned} u_i &= c_1^\alpha \sum_{j\in N_i} \phi_\alpha(\|\hat{q}_j - \hat{q}_i\|_\sigma)\hat{n}_{ij} + c_2^\alpha \sum_{j\in N_i} \hat{a}_{ij}(q)(\hat{p}_j - \hat{p}_i) \\ &\quad + (c_1^g - c_{pos})(d_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} d_{jg}) \\ &\quad + (c_2^g - c_{ve})(v_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} v_{jg}) \\ &\quad - (c_1^t + c_1^g)(q_i - q_t) - (c_2^t + c_2^g)(p_i - p_t) \\ &\quad + c_{pos}\varepsilon_d^i + c_{ve}\varepsilon_v^i - c_1^g\overline{\varepsilon}_q^i - c_2^g\overline{\varepsilon}_p^i \\ &\quad - (c_1^t + c_1^g)\varepsilon_q^{it} - (c_2^t + c_2^g)\varepsilon_p^{it} \end{aligned} \tag{18}$$

Compute the average of control law (18), then substitute obtained average $\overline{u}$, and u_i in (18) into (12) we obtain:

$$\begin{aligned} \dot{v}_{ig} &= -(c_1^t + c_{pos})d_{ig} - (c_2^t + c_{ve})v_{ig} \\ &\quad + \Phi_i(V) + \Omega_i(V) + \zeta_i(V), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \Phi_i(V) &= c_1^\alpha \sum_{j\in N_i} \phi_\alpha(\|\hat{q}_j - \hat{q}_i\|_\sigma)\hat{n}_{ij} \\ &\quad - \frac{c_1^\alpha}{n}\sum_{i=1}^n[\sum_{j\in N_i} \phi_\alpha(\|\hat{q}_j - \hat{q}_i\|_\sigma)\hat{n}_{ij}] \\ &\quad + c_2^\alpha \sum_{j\in N_i} \hat{a}_{ij}(q)(\hat{p}_j - p_i) \\ &\quad - \frac{c_2^\alpha}{n}\sum_{i=1}^n[\sum_{j\in N_i} \hat{a}_{ij}(q)(\hat{p}_j - p_i)]; \end{aligned}$$

$$\begin{aligned} \Omega_i(V) &= -(\frac{c_1^g - c_{pos}}{|N_j|+1})\sum_{j=1}^{|N_j|+1} d_{jg} - (\frac{c_2^g - c_{ve}}{|N_j|+1})\sum_{j=1}^{|N_j|+1} v_{jg} \\ &\quad - (\frac{c_1^g - c_{pos}}{n})\sum_{i=1}^n(d_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} d_{jg}) \\ &\quad - (\frac{c_2^g - c_{ve}}{n})\sum_{i=1}^n(v_{ig} - \frac{1}{|N_j|+1}\sum_{j=1}^{|N_j|+1} v_{jg}); \end{aligned}$$

$$\begin{aligned}\zeta_i(V) &= c_{pos}\varepsilon_d^i + c_{ve}\varepsilon_v^i - c_1^g\bar{\varepsilon}_q^i - c_2^g\bar{\varepsilon}_p^i \\ &-(c_1^t + c_1^g)\varepsilon_q^{it} - (c_2^t + c_2^g)\varepsilon_p^{it} \\ &-\frac{1}{n}\sum_{i=1}^{n}[c_{pos}\varepsilon_d^i + c_{ve}\varepsilon_v^i - c_1^g\bar{\varepsilon}_q^i - c_2^g\bar{\varepsilon}_p^i \\ &-(c_1^t + c_1^g)\varepsilon_q^{it} - (c_2^t + c_2^g)\varepsilon_p^{it}].\end{aligned}$$

Rewrite (19) in state space representation

$$\begin{aligned}\begin{bmatrix} \dot{d}_{ig} \\ \dot{v}_{ig} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -k_1 I & -k_2 I \end{bmatrix}\begin{bmatrix} d_{ig} \\ v_{ig} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ I \end{bmatrix}(\Phi_i(V) + \Omega_i(V) + \zeta_i(V)), \quad (20)\end{aligned}$$

here $k_1 = (c_1^t + c_{pos})$, $k_2 = (c_2^t + c_{ve})$, and I is an m x m identity matrix.

Let $V_i = [d_{ig}\ \ v_{ig}]^T$, then we can rewrite (20) as

$$\begin{aligned}\dot{V}_i &= \begin{bmatrix} 0 & I \\ -k_1 I & -k_2 I \end{bmatrix} V_i \\ &+ \begin{bmatrix} 0 \\ I \end{bmatrix}(\Phi_i(V) + \Omega_i(V) + \zeta_i(V)) \qquad (21)\end{aligned}$$

Note that the matrix $A_i = \begin{bmatrix} 0 & I \\ -k_1 I & -k_2 I \end{bmatrix}$ is Hurwitz because with selected constants such that $k_1 > 0$ and $k_2 > 0$ this matrix has eigenvalues given by the roots of $(s^2 + k_2 s + k_1)^m$, which are in the strict left half plane.

IV. Stability Analysis

In this section we will analyze the stability of our proposed flocking control algorithm in noisy environments based on the Lyapunov approach.

We assume that the errors of sensing position and velocity have linear relationship with the magnitude of the state of the error system. That is because two agents are far away from each other, the sensing errors will usually increase. Hence, we have

$$\begin{cases} \|\varepsilon_d^i(t)\| \le c_{ed_1}^i\|V_i(t)\| + c_{ed_2}^i \\ \|\varepsilon_v^i(t)\| \le c_{ev_1}^i\|V_i(t)\| + c_{ev_2}^i, \quad i = 1,2,...,n. \end{cases} \qquad (22)$$

We also assume that the noises ε_q^{it} and ε_p^{it} on the target tracking terms (negative feedbacks) are bounded as

$$\begin{cases} \|\varepsilon_q^{it}(t)\| \le c_{eq}^i \\ \|\varepsilon_p^{it}(t)\| \le c_{ep}^i, \quad i = 1,2,...,n, \end{cases} \qquad (23)$$

and the noises $\bar{\varepsilon}_q^i$ and $\bar{\varepsilon}_p^i$ on the estimates of local average of position and velocity of agent i and its neighbors, respectively are bounded as

$$\begin{cases} \|\bar{\varepsilon}_q^i(t)\| \le \bar{c}_{eq}^i \\ \|\bar{\varepsilon}_p^i(t)\| \le \bar{c}_{ep}^i, \quad i = 1,2,...,n. \end{cases} \qquad (24)$$

Theorem 1. Consider a system of n mobile agents, that have dynamics (1) and are controlled by the control law (10), and all noises are bounded by (22), (23) and (24) . Let

$$c_{pv}^1 = \frac{(c_{pos}+1)^2 + c_{ve}^2}{2c_{pos}c_{ve}} + \sqrt{(\frac{c_{pos} + c_{ve}^2 - 1}{2c_{pos}c_{ve}})^2 + \frac{1}{c_{pos}^2}}, \quad (25)$$

and if

$$c_{pos}c_{ed_1}^i + c_{ve}c_{ev_1}^i \le \frac{1}{c_{pv}^1}, \qquad (26)$$

and the parameters are such that

$$\sum_{j=1}^{m}\frac{2c_{pv}^1[\sqrt{(c_1^g - c_{pos})^2 + (c_2^g - c_{ve})^2} - \frac{1}{n}(c_{pos}c_{ed_1}^i + c_{ve}c_{ev_1}^i)]}{(1-\varepsilon_i)[1 - c_{pv}^1(c_{pos}c_{ed_1}^i + c_{ve}c_{ev_1}^i)]} \qquad (27)$$
$$< 1$$

here $0 < \varepsilon_i < 1$ for $\forall i$, then the trajectories of (21) are bounded.

Proof: To study the stability of the error dynamics (21), one possible choice is to choose the Lyapunov function for each agent as

$$L_i(V_i) = V_i^T P V_i, \qquad (28)$$

here $P = P^T$ is a $2m$ x $2m$ positive-definite matrix ($P > 0$). Then, the Lyapunov function for the composite system is

$$L(V) = \sum_{i=1}^{n} V_i^T P V_i. \qquad (29)$$

From (28) we have

$$\dot{L}_i(V_i) = V_i^T P \dot{V}_i + \dot{V}_i^T P V_i. \qquad (30)$$

Then, substitute $\dot{V}_i$ in (21) into (30) we obtain: $\dot{L}_i(V_i) = -V_i^T C V_i + 2V_i^T PB(\Phi_i(V) + \Omega_i(V) + f_i^t(V) + \zeta_i(V))$, here $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$, and $C = -(PA_i + A_i^T P)$.

Since for any matrix $D = D^T > 0$ and vector X we have $\lambda_{min}(D)X^T X \le X^T D X \le \lambda_{max}(D)X^T X$, where $\lambda_{min}(D)$ and $\lambda_{max}(D)$ are the minimum and maximum eigenvalue of the matrix D. Then, from (29) we have $\sum_{i=1}^{n}(\lambda_{min}(P)\|V_i\|^2) \le L(V) \le \sum_{i=1}^{n}(\lambda_{max}(P)\|V_i\|^2)$. From this property and the fact $\|B\| = 1$ we obtain

$$\begin{aligned}\dot{L}(V) &= \sum_{i=1}^{n}\dot{L}_i(V_i) = \sum_{i=1}^{n}[-V_i^T C V_i \\ &+2V_i^T PB(\Phi_i(V) + \Omega_i(V) + \zeta_i(V))] \\ &\le \sum_{i=1}^{n}(-c_1^i\|V_i\|^2 + c_2^i\|V_i\| \\ &+\|V_i\|\sum_{j=1}^{n} b_{ij}\|V_j\|), \qquad (31)\end{aligned}$$

here

$$c_1^i = \lambda_{min}(C)[1 - \frac{2\lambda_{max}(P)}{\lambda_{min}(C)}(c_{pos}c_{ed_1}^i + c_{ve}c_{ev_1}^i)],$$

$$\begin{aligned}c_2^i &= 2\lambda_{max}(P)[\delta_\Phi^{max} + \delta_a^{max} + c_{pos}c_{ed_2}^i + c_{ve}c_{ev_2}^i \\ &-(c_1^t + c_1^g)c_{eq}^i - (c_2^t + c_2^g)c_{ep}^i - c_1^g\bar{c}_{eq}^i - c_2^g\bar{c}_{ep}^i \\ &-\frac{1}{n}\sum_{j=1}^{n}(c_{pos}c_{ed_2}^i + c_{ve}c_{ev_2}^i - (c_1^t + c_1^g)c_{eq}^i \\ &-(c_2^t + c_2^g)c_{ep}^i - c_1^g\bar{c}_{eq}^i - c_2^g\bar{c}_{ep}^i)],\end{aligned}$$

where

$$\delta_{\Phi}^{max} = max[|c_1^{\alpha}\sum_{j\in N_i}\phi_{\alpha}(\|\hat{q}_j-\hat{q}_i\|_{\sigma})\hat{n}_{ij} - \frac{c_1^{\alpha}}{n}\sum_{i=1}^{n}[\sum_{j\in N_i}\phi_{\alpha}(\|\hat{q}_j-\hat{q}_i\|_{\sigma})\hat{n}_{ij}]|],$$

$$\delta_{a}^{max} = max[|c_2^{\alpha}\sum_{j\in N_i}\hat{a}_{ij}(q)(\hat{p}_j-\hat{p}_i) - \frac{c_2^{\alpha}}{n}\sum_{i=1}^{n}[\sum_{j\in N_i}\hat{a}_{ij}(q)(\hat{p}_j-\hat{p}_i)]|],$$

$$b_i = 2\lambda_{max}(P)[\sqrt{(c_1^g-c_{pos})^2+(c_2^g-c_{ve})^2}(\frac{1}{|N_i|+1}+\frac{1}{n} + \frac{1}{n(|N_i|+1)})-\frac{1}{n}(c_{pos}c_{ed_1}^i+c_{ve}c_{ev_1}^i)].$$

Look at the first term in (31) we see that if $[1-\frac{2\lambda_{max}(P)}{\lambda_{min}(C)}(c_{pos}c_{ed_1}^i+c_{ve}c_{ev_1}^i)]\geq 0$ or $c_{pos}c_{ed_1}^i+c_{ve}c_{ev_1}^i\leq\frac{1}{c_{pv}^0}$ with $c_{pv}^0=\frac{2\lambda_{max}(P)}{\lambda_{min}(C)}$ then $c_1^i>0$. Hence, the first term in (31) gives a negative contribution to $\dot{L}(V)$. It is noted that if c_{pv}^0 is as small as possible the system will tolerate noise with largest bounds ($c_{ed_1}^i$ and $c_{ev_1}^i$) while keeping stability. Here, c_{pv}^0 is minimized by taking $C=I$, or $min(c_{pv}^0)=\frac{2\lambda_{max}(P)}{\lambda_{min}(C)}|_{C=I}=c_{pv}^1$.

Now, let us analyze the first two terms in (31) as:

$$\begin{aligned}-c_1^i\|V_i\|^2+c_2^i\|V_i\| &= -(1-\varepsilon_i)c_1^i\|V_i\|^2-\varepsilon_i c_1^i\|V_i\|^2 + c_2^i\|V_i\| \\ &\leq -(1-\varepsilon_i)c_1^i\|V_i\|^2=\delta_i\|V_i\|^2 \\ &(\forall -\varepsilon_i c_1^i\|V_i\|^2+c_2^i\|V_i\|\geq 0 \\ &\Leftrightarrow \|V_i\|\geq\frac{c_2^i}{\varepsilon_i c_1^i}=\theta_i)\end{aligned} \quad (32)$$

here $0<\varepsilon_i<1$, and $\delta_i=-(1-\varepsilon_i)c_1^i$. Clearly we can see that if $\|V_i\|\geq\theta_i$ then the first two terms in (31) give a negative contribution to $\dot{L}(V)$.

Next, consider the last term in (31) being over-bounded by replacing b_i with b_i^*, and $b_i^*=max(b_i)|_{1\leq i\leq n}$. For the flocking control of multiple agents, $n\geq 2$, we know that at the initial state the agents are randomly distributed. Hence, some agents may not have any neighbor or $|N_i|=0$. Based on this fact we have

$$b_i^* = max(b_i)=2c_{pv}^1[\sqrt{(c_1^g-c_{pos})^2+(c_2^g-c_{ve})^2} - \frac{1}{n}(c_{pos}c_{ed_1}^i+c_{ve}c_{ev_1}^i)]. \quad (33)$$

Then, we consider the general situation ($\|V_i\|\geq\theta_i$ and $\|V_i\|<\theta_i$). Accordingly, we define the set

$M_O=\{i:\|V_i\|\geq\theta_i, 1\leq i\leq n\}=\{i_O^1,i_O^2,...,i_O^{n_O}\}$

$M_I=\{i:\|V_i\|<\theta_i, 1\leq i\leq n\}=\{i_I^1,i_I^2,...,i_I^{n_I}\}$

here n_O and n_I are the size of M_O and M_I, respectively. $n_O+n_I=n$; $M_O\cup M_I=\{1,2,..,n\}$; and $M_O\cap M_I=\emptyset$

Here we only need to prove $\dot{L}(V)\leq 0$ for the case $\|V_i\|\geq\theta_i$ since for the case $\|V_i\|<\theta_i$ the proof is trivial.

First we assume that there exist positive constants such that

$K_1\geq\sum_{j=1}^{n_I}b_i^*\|V_j\|$; $K_2\geq\sum_{j=1}^{n_I}\|V_i\|$; $K_3\geq\sum_{i=1}^{n_I}(-c_1^i\|V_i\|^2+c_2^i\|V_i\|)$; and $K_4\geq\sum_{i=1}^{n_I}(\|V_i\|\sum_{j=1}^{n_I}b_i^*\|V_j\|)$;

Then from (31), (32) and (33) we have:

$$\dot{L}(V) \leq \sum_{i=1}^{n_O}\delta_i\|V_i\|^2+\sum_{i=1}^{n_O}(\|V_i\|\sum_{j=1}^{n_O}b_i^*\|V_j\|) + \sum_{i=1}^{n_O}(K_1+K_2b_i^*)\|V_i\|+K_3+K_4$$

Let $Z^T=[\|V_{i_O^1}\|,\|V_{i_O^2}\|,...,\|V_{i_O^{n_O}}\|]$ and the n_O x n_O matrix $S=[s_{ij}]$ be specified by

$$s_{ij}=\begin{cases}-b_{i_O^j}^*, & if \quad j\neq i\\ -(\delta_{i_O^j}+b_{i_O^j}^*), & if \quad j=i.\end{cases}$$

Then we have

$\dot{L}(V)\leq -Z^TSZ+\sum_{i=1}^{n_O}(K_1+K_2b_i^*)\|V_i\|+K_3+K_4$.

Suppose that $S\geq 0$, thus $\lambda_{min}(S)\geq 0$, hence we obtain

$$\dot{L}(V) \leq -\lambda_{min}(S)\sum_{i=1}^{n_O}\|V_i\|^2 + \sum_{i=1}^{n_O}(K_1+K_2b_i^*)\|V_i\|+K_3+K_4 \quad (34)$$

From (34) we can see that if $\|V_i\|$ for $i\in M_O$ are sufficiently large, then the sign of $\dot{L}(V)$ is determined by $-\lambda_{min}(S)\sum_{i=1}^{n_O}\|V_i\|^2$ or $\dot{L}(V)\leq 0$.

A necessary and sufficient condition for $S>0$ is that its successive principal minors are all positive. Let us define $|s_m|$ as determinants of the principal minors of matrix S, then we have

$$|s_m|=(1+\sum_{j=1}^{m}\frac{b_{i_O^j}^*}{\delta_{i_O^j}})\prod_{k=1}^{m}(-\delta_{i_O^k}), m=1,2,...,n_O.$$

Due to $-\delta_{i_O^k}>0$ with $k=1,2,...,m$, to have all above determinants positive, we need:

$\sum_{j=1}^{m}\frac{b_{i_O^j}^*}{\delta_{i_O^j}}>-1$ or

$$\sum_{j=1}^{m}\frac{2c_{pv}^1[\sqrt{(c_1^g-c_{pos})^2+(c_2^g-c_{ve})^2}-\frac{1}{n}(c_{pos}c_{ed_1}^{i_O^j}+c_{ve}c_{ev_1}^{i_O^j})]}{(1-\varepsilon_i)[1-c_{pv}^1(c_{pos}c_{ed_1}^{i_O^j}+c_{ve}c_{ev_1}^{i_O^j})]} \quad (35)$$

< 1

for $m=1,2,...,n_O$. Since $1\leq m\leq n_O\leq n$, the inequality (35) is satisfied when (27) is satisfied. ∎

V. Experimental Results

In this section we discuss a metric to evaluate the network connectivity. Then we test our proposed flocking control algorithm (10) and compare it with the existing flocking control algorithm (7) in noisy environments. The parameters used in this simulation are specified as follows:

- Parameters of flocking: number of agents = 50 (randomly distributed in the square area of 120 x 120 size); $a=b=5$; $d=16$; the scaling factor $k_c=1.2$; the active range $r=$

$k_c * d = 19.2$; $\varepsilon = 0.1$ for the σ-norm; h = 0.2 for the bump function ($\phi_\alpha(z)$); h = 0.9 for the bump function ($\phi_\beta(z)$).

- Parameters of target movement: The target moves in a sine wave trajectory: $q_t = [50+50t,\ 295-50sin(t)]^T$ with $0 \leq t \leq 6$.

- The noises used in the simulation are Gaussian with zero mean, variance of 1 and standard deviation of 1.

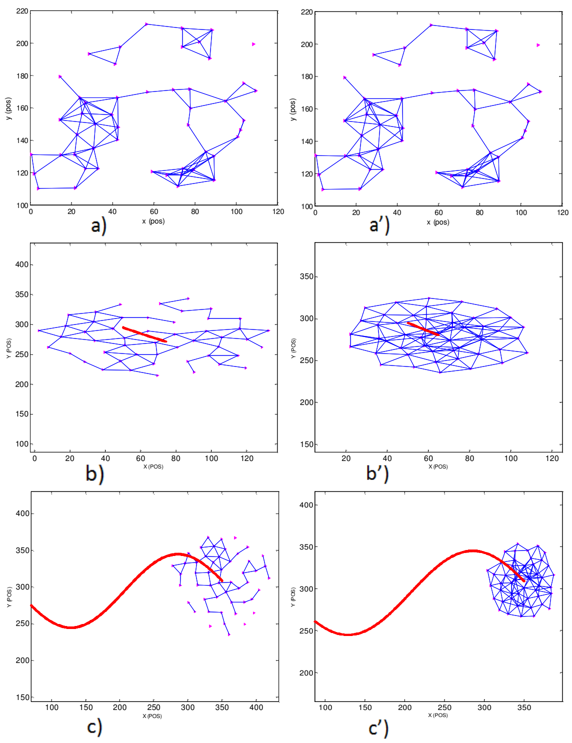

Fig. 1. Snapshots of agents when they are randomly distributed (a, a'), when they form a network (b, b'), and when they track a target (red/dark line) moving in sine wave trajectory (c, c'), here (a, b, c) are for algorithm (7) and (a', b', c') are for algorithm (10).

To evaluate the the network connectivity maintenance, first we know that the link (connectivity) between node i and node j is maintained if the distance $0 < \|q_i - q_j\| \leq r$. Otherwise this link is considered broken. Then for graph connectivity: a dynamic graph $G(\vartheta, E)$ is connected at time t if there exists a path between any two vertices. Based on the above analysis, to analyze the connectivity of the network we define a connectivity matrix $[c_{ij}(t)]$ as follows:

$$[c_{ij}(t)] = \begin{cases} 1, & if \quad j \in N_i(t), i \neq j \\ 0, & if \quad j \notin N_i(t), i \neq j \end{cases}$$

and $c_{ii} = 0$. Since the rank of Laplacian of a connected graph [3] $[c_{ij}(t)]$ of order n is at most $(n-1)$ or $rank([c_{ij}(t)]) \leq (n-1)$, the relative connectivity of a network at time t is defined as: $C(t) = \frac{1}{n-1} rank([c_{ij}(t)])$. If $0 \leq C(t) < 1$ the network is broken, and if $C(t) = 1$ the network is connected. Based on this metric we can evaluate the network connectivity in our proposed flocking control algorithm (10) and the existing flocking algorithm (7).

Figure 1 represents the results of the moving target (red/dark line) tracking in the sine wave trajectory in noisy environments where (a, b, c) represent the results of the existing flocking control algorithm (7), and (a', b', c') represent the results of the proposed flocking control algorithm (10). Figure 2 shows the results of connectivity: (a) for the algorithm (7), and (b) for the algorithm (10). To compare these algorithms we use the same initial state (position and velocity) of the mobile agents. Comparing these figures we see that by applying the proposed flocking control algorithm (10) the connectivity is maintained while the existing flocking control algorithm (7) fails to do this. Also, by applying the algorithm (10) we can see that after only five iterations the agents form a network and then maintain connectivity in presence of noises. In both algorithms, collision avoidance among agents is guaranteed. For more details please see some video files which are available at our ASCC Lab's website.

http://ascc.okstate.edu/projectshung.html

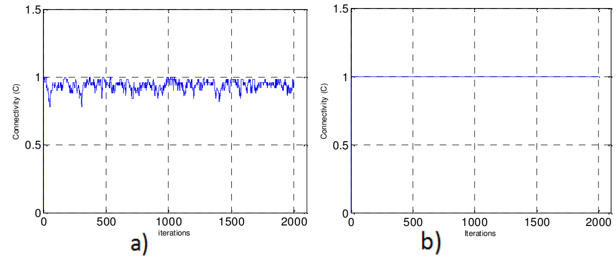

Fig. 2. The connectivity is evaluated by the $C(t)$ value: (a) algorithm (7), and (b) algorithm (10).

VI. Conclusion and Future Work

In this paper, we considered the problem of controlling a group of agents to form a network and track a target in noisy environments. Our approach is based on flocking control that integrates position and velocity cohesion feedbacks in order to deal with the noises. The stability of the proposed flocking control law is investigated based on the Lyapunov approach. Also, the network connectivity preservation is improved, and collision avoidance among agents is guaranteed. In the future work we intend to study how the communication time delays affect to the performance of the proposed approach.

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