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Optimal Passive Dynamics for Torque/Force Control

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Abstract— **For robotic manipulation tasks in uncertain environments, good force control can provide significant benefits. The design of force or torque controlled actuators typically revolves around developing the best possible software control strategy. However, the passive dynamics of the mechanical system, including inertia, stiffness, damping and torque limits, often impose performance limitations that cannot be overcome with software control. Discussions about the passive dynamics are often imprecise, lacking comprehensive details about the physical limitations. In this paper, we develop relationships between an actuator's passive dynamics and the resulting performance, for the purpose of better understanding how to tune the passive dynamics for a force control task. We present two distinct scenarios for the actuator system and calculate the required input to produce a desired output. These exact solutions provide a basis for understanding how the parameters of the mechanical system affect the overall system's bandwidth limit. Our model does not include active control; we computed the optimal input to the system to produce the required torque at the load with zero error. This is important so that our results only reflect the physical system's performance.**

I. INTRODUCTION

Robots excel at precise position control and are useful for tasks that make use of this ability, such as CNC machining. However, physical interaction tasks such as catching a ball, walking, running, grasping unknown objects, constrained contact and even simple force or torque control have historically been difficult for robots. Each of these tasks involve dynamic effects such as unexpected impacts and/or a significant transfer of kinetic energy between the robot and its environment. Animals far outperform robots at many of these tasks, and we contend that this is due to inherent mechanical limitations in traditional robotic mechanisms rather than software control inadequacies. This paper focuses on how an actuator's passive dynamics affect force or torque control.

Consider a traditional industrial robot arm, powered by electric motors with large gear reductions and rigid links. The traditional approach to force control utilizes such an arm, with a force sensor placed at the end-effector. Forces are measured, software controllers calculate the desired motor torques and the motors move accordingly. However, the motors have inertia, which is amplified through the gearbox into a significant reflected inertia, and combines with torque limitations on the motors to limit their acceleration. These passive dynamics cannot be overcome using software control.

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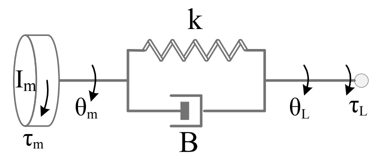


Fig. 1. The system we investigate in this paper is entirely rotational and includes damping, elasticity, motor inertia and torque limits. The actuator is constrained such that only θ_m and θ_L can move.

If an object impacts the arm, such as a baseball, the motors will have no chance to respond, the arm will behave as a rigid inertial object and the software control will have no part in its dynamic response.

Passive dynamics are not always harmful. As an example of passive dynamics improving performance, a mechanical spring in series with a motor can dramatically improve force control bandwidth in response to position disturbances. However, this improvement applies only to the specific case of force control and its robustness to position disturbances; a series spring will reduce the performance of the system for position control. For peak performance in a robotic system, the passive dynamics must be tailored to the specific task. This is roughly analogous to impedance matching in electrical systems.

In this paper, we lay out a mathematical framework for mechanical systems that includes a motor with inertia and torque limits, a series spring and a series damper, as shown in Fig. 1. We investigate two examples; applying constant force to a moving object and applying changing force to a stationary object. We then describe the mathematically optimal passive dynamics required to achieve the best possible bandwidth, based on fundamental physical limits. Based on this work, roboticists will be able to estimate that a mechanical system has the bandwidth necessary for a particular task, especially tasks involving force or torque control, spring-like behavior, impacts and kinetic energy transfer.

II. BACKGROUND

Muscular systems in animals incorporate elastic elements, which are most often examined while investigating locomotion, and are generally discussed in the context of energy storage [1][2][4][7]. Roboticists have built machines designed to mimic this spring-like behavior [5][9]. Although the designers of these running machines acknowledge that elasticity provides robustness, their studies generally focus on energy storage and efficiency, with little attention to force control.

Early investigations into force control found that series compliance in an actuator can increase stability, and in some cases is required for stable operation [15][11]. Researchers at the Massachusetts Institute of Technology (MIT) Leg Laboratory explored these ideas and created an actuator designed specifically to include an elastic element as a force sensor and low impedance coupling between the drive system and the load to improve force control. The system is aptly dubbed a series elastic actuator (MIT-SEA) and it has been shown that this configuration provides filtering to handle shock loads as well as higher resolution/bandwidth force control [8][10]. MIT-SEAs offer great advantages, however, there are only approximate guidelines for choosing an appropriate spring. Further work to improve the MIT-SEA has focused on control architecture [16][13] or transmission design [14][12].

Chew et al. proposed a similar actuator design using a viscous damper in place of the elastic element, dubbed a series damper actuator (SDA) [3]. They hypothesize that using damping, rather than elasticity, allows for greater bandwidth, and can be easily constructed to allow a variable damping coefficient. They admit that the main disadvantage of the SDA is the energy dissipation property, which limits the energy efficiency of the design. The developers of the SDA do not provide concrete relationships between damping and bandwidth, but present a conjecture relating the two.

A hybrid of the SDA and MIT-SEA has been proposed by Hurst et al. [6]. They concluded that the added damping provides higher bandwidth than a purely series-elastic element and reduces unwanted oscillations in specific situations. Initial force spikes observed by the drive system at impact are greater than would be observed by just an elastic element, but are still much less than for a perfectly stiff system.

III. System model

In this paper, we define relationships between series stiffness, series damping, drive system inertia and the drive system torque limits in specific experimental scenarios. To simplify the discussion, we use "motor" to describe the drive system as a whole - transmission and motor characteristics. The following symbols describe our model:

ω	Angular frequency	$\frac{rad}{s}$
k	Spring constant	$\frac{N \cdot m}{rad}$
B	Damping constant	$\frac{kg \cdot m^2}{s \cdot rad}$
I_m	Motor inertia	$kg \cdot m^2$
τ_m	Motor torque	$N \cdot m$
τ_{limit}	Motor torque limit	$N \cdot m$
τ_L	Load torque	$N \cdot m$
θ_m	Motor angle	rad
θ_L	Load angle	rad
θ_A	Load angle amplitude	rad

Our goal in this paper is to calculate the fundamental limitations of the physical system. Our model does not include active control; we compute the optimal input to the system to produce a desired torque at the load. This is an important distinction from previous attempts to develop actuators of this nature. By eliminating controller error, we are able isolate the physical limitations of our model.

To develop the relationships between an actuator's design parameters, we investigate the series elastic/damping actuator (SEDA) in Fig. 1. Our actuator includes damping and elasticity because they are both physically unavoidable and possibly useful. We want to know how to select these elements (k, B and I_m) to design the best possible actuator around a force or torque control task.

Our system model is entirely rotational because our lab, the Dynamic Robotics Laboratory, is interested in developing robots that use electric motors. However, the concepts in this paper relate directly to force control as well as to torque control. Roboticists designing actuators with linear drive systems (such as hydraulic pistons) can use the relationships presented in this paper to develop linear systems.

In addition to the reactive elements k and B, we include motor torque limits as well as motor inertia. The torque limit and motor inertia are important for the calculation of the bandwidth. If infinite torque were possible, there would be no requirements for designing the impedance of the actuator. In other words, it would not matter how soft, or stiff, the elements were, just as long as they existed.

In the case of zero motor inertia with motor torque limits, the elastic and damping elements are no longer important. The elements just need to exist to provide for transmission of torque. In this case the largest torque the actuator could produce at the load would be the torque limit. In either case the system is optimal, has infinite bandwidth for any task and the impedance of the actuator is irrelevant. Unfortunately, this is not the case with real systems because all motors have torque limits and rotor inertia.

IV. Actuation scenarios

Each scenario is designed to show that there is an optimal relationship between k, B and I_m for a distinct task. This paper focuses on simple, fundamental motions that might be expected from a force or torque controlled actuator. The goal is to relate k, B, I_m and τ_{limit} to the performance of a robotic actuator under specific conditions.

To determine the effect of k, B and I_m on the performance of the system in any test scenario, we first solve for the motor torque, τ_m, that produces the desired load torque, τ_L. If τ_m remains below the motor's peak torque limit, the system is able to achieve the desired performance goals.

In most cases, as the frequency of a task increases, the required motor torque increases and eventually meets the motor torque limit. The function for the exact motor torque, evaluated with torque limits, becomes the basis for describing the relationships that parameters have on achieving the maximum frequency of each task.

To find the required motor torque, we start by defining the differential equations that describe the motion of the system:

$$I_m\ddot{\theta}_m = \tau_m - \tau_B - \tau_k \quad (1)$$
$$0 = \tau_B + \tau_k - \tau_L \quad (2)$$

where:

$$\tau_k = k[\theta_m - \theta_L]$$
$$\tau_B = B[\dot{\theta}_m - \dot{\theta}_L].$$

We then take the Laplace transform of (1) and (2), and solve for the s-domain equation of the motor torque ($T_m(s)$). With initial conditions ignored, this is calculated as:

$$T_m(s) = \Theta_L(s)\left(I_m s^2\right) + T_L(s)\left(\frac{I_m s^2 + Bs + k}{Bs + k}\right). \quad (3)$$

Equation (3) describes how the load motion and desired load torque affect the required motor torque, where $\Theta_L(s)$ is the s-domain representation of the load motion and $T_L(s)$ is the s-domain representation of the load torque. With this equation, we can define any motion for the load and a desired load torque and determine the exact requirement for the motor torque. At steady state, this computed motor torque will produce the torque at the load with zero error.

V. Changing torque against a static surface

For the first task, our model applies a sinusoidal torque to a fixed load (Fig. 2). We demonstrate how k, B and I_m affect the maximum frequency at which the actuator can vary the applied torque. The maximum frequency for this case is defined as the frequency that the actuator can oscillate the torque at the load before steady-state error is encountered.

To evaluate the maximum frequency the actuator can achieve under a given set of values for k, B and I_m, we consider the point where the motor's torque becomes greater than the torque limit. At this point the motor is no longer able to produce the required torque to exactly generate the desired τ_L.

To find the motor torque as a function of time, $\tau_m(t)$, we define the motion of the load, $\theta_L(t)$ and the desired load torque, $\tau_L(t)$. For this scenario, we hold the load position constant (Fig. 2). We then define the desired load torque to be a sinusoidal function with some angular frequency, ω, and a fixed amplitude of $1\ N \cdot m$. Note that the amplitude can be greater or smaller without affecting the relationships as long as it is less than the torque limit:

$$\theta_L(t) = 0$$
$$\tau_L(t) = sin(\omega t). \quad (4)$$

Taking the Laplace transform of $\tau_L(t)$ gives:

$$T_L(s) = \frac{\omega}{s^2 + \omega^2}. \quad (5)$$

Plugging equation (5) back into (3) and taking the inverse Laplace transform, we find the $\tau_m(t)$ required to produce the $\tau_L(t)$ defined in (4) at steady state ($t \gg 0$):

$$\tau_m(t) = \left(\frac{I_m\omega^3 B}{\omega^2 B^2 + k^2}\right) cos(\omega t) + \left(\frac{\omega^2 B^2 - I_m\omega^2 k + k^2}{\omega^2 B^2 + k^2}\right) sin(\omega t). \quad (6)$$

If we consider the extremes of equation (6), we can begin to draw conclusions about the motor requirements and relationships between the passive dynamic parameters. One extreme occurs when $B = 0$, and equation (6) simplifies to:

$$\tau_m(t) = \left(1 - \frac{I_m\omega^2}{k}\right) sin(\omega t). \quad (7)$$

Equation (7) implies that if the system has very little or no damping, the only way to reduce the torque requirement is to increase k or decrease I_m.

In contrast, if the system has very little or no elasticity, such that $k \approx 0$, (6) simplifies to:

$$\tau_m(t) = \left(\frac{I_m\omega}{B}\right) cos(\omega t) + sin(\omega t). \quad (8)$$

Equation (8) implies that to reduce the torque requirement, increasing B or decreasing I_m are the only options.

Comparing (7) and (8), we note that as the frequency increases, B has a much greater effect than k on reducing the required motor torque.

The graphs in Fig. 3 show the maximum frequency the system can achieve for a set of parameters k, B and I_m. We arbitrarily set $\tau_{limit} = 10$ for each graph and hold I_m constant for Fig. 3(a) and Fig. 3(b). The graphs demonstrate the effects of modifying the various parameters of equation (6).

It follows from these equations that increasing stiffness provides higher bandwidth for applying varying torques to a fixed load. The equations indicate that there is an inverse relationship between the maximum frequency and the motor inertia (as shown in Fig. 3(c)). An increase in k or B will increase the bandwidth but an increase in I_m will decrease the bandwidth.

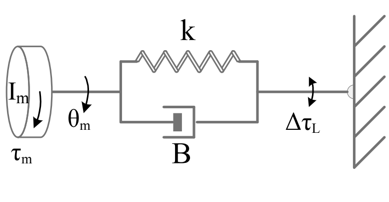
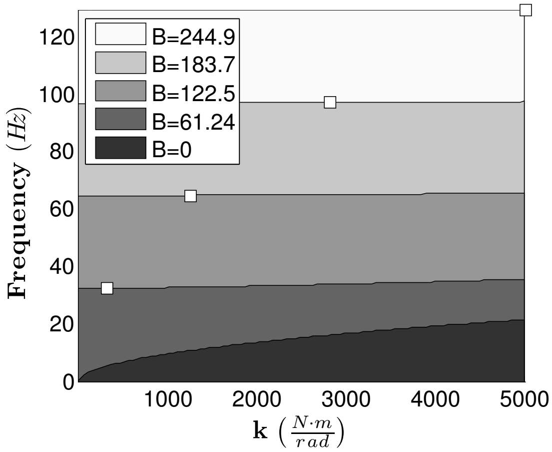
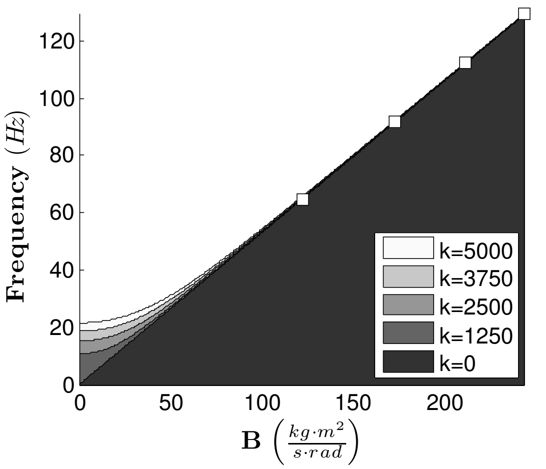


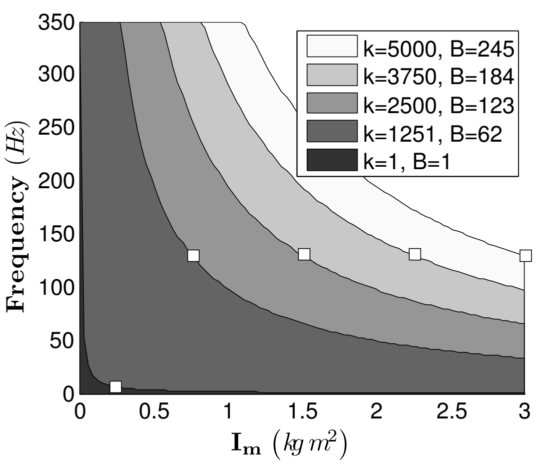
Fig. 2. For the first scenario, the load is fixed to ground ($\theta_L = 0$) while the motor attempts to produce the desired τ_L through the passive dynamic elements k and B.



(a) : Frequency achieved vs. series elasticity, k. Increasing the elasticity slowly increases the maximum frequency.



(b) : Frequency achieved vs. series damping, B. Increasing the damping increases the maximum frequency.



(c) : Frequency achieved vs. motor inertia, I_m. Increasing the inertia greatly decreases the maximum frequency.

Fig. 3. Performance of the series elastic/damped actuator applying a sinusoidal torque against a stationary load (Fig. 2). The maximum frequency occurs at the point where the load torque error exceeds 0. For reference, the squares on each figure indicate where the system is critically damped. For figures 3(a) and 3(b), $I_m = 3$.

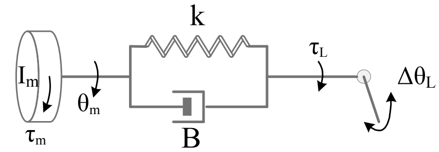


Fig. 4. For the second scenario, the load is forced to move by θ_L while the motor attempts to keep the load torque, τ_L, zero with the passive dynamic elements k and B.

VI. Zero torque against a moving load

The second task requires the actuator to maintain zero torque against a moving load (Fig. 4). We again demonstrate how k, B and I_m affect the maximum frequency, which we define for this task as the frequency at which the load position can oscillate before a prescribed torque error at the load is exceeded. This situation might occur if the goal of the actuator is to keep contact with an object, while maintaining a constant applied torque. An example of this task might be carrying a coffee cup while walking or the iso-elastic system in a Steadicam. Note that there is no inertia at the load, as its motion is predefined and is not affected by the applied torque.

We start by looking at the point where the torque required of the motor becomes greater than the torque limit. For this task we want to find the motor torque as a function of time, $\tau_m(t)$, for a predefined motion of the load, $\theta_L(t)$ and the desired load torque, $\tau_L(t)$. For this scenario, we hold the load torque constant at zero. We then define the desired load position to follow a sinusoidal function at some angular frequency, ω, and an amplitude of θ_A (Fig. 4)

$$\begin{aligned} \theta_L(t) &= \theta_A sin(\omega t) \\ \tau_L(t) &= 0. \end{aligned} \tag{9}$$

Taking the Laplace transform of $\theta_L(t)$ gives:

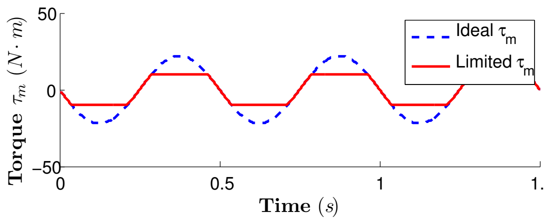
$$T_L(s) = \theta_A \frac{\omega}{s^2 + \omega^2}. \tag{10}$$

Plugging (10) back into (3) and taking the inverse Laplace transform we find the $\tau_m(t)$ required to produce the $\tau_L(t)$ defined in (9) at steady state ($t \gg 0$):

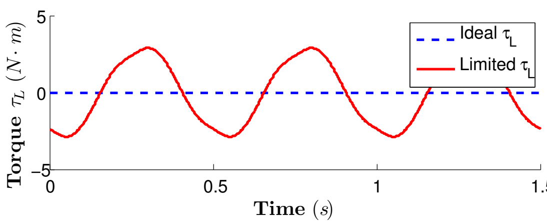
$$\tau_m(t) = \left(-\theta_A I_m \omega^2\right) sin(\omega t). \tag{11}$$

Intuitively this shows that for the motor to exactly produce zero torque at the load, it would have to generate a torque that would cause the motor position (θ_m) to exactly follow the load position (θ_L). We can also conclude that k and B do not matter when trying to follow the load motion. Instead, the only parameter we have for reducing the motor torque requirement is the motor inertia.

However, it may be more useful to measure the load torque within some error tolerance. To actually investigate how k and B affect the system, we now assume that there can be error in the load torque. To produce an error, we take the optimal output defined in (11) and clip it when the torque limits are encountered as shown in Fig. 5(a).



(a) The input torque, τ_m. The ideal input represents what is needed to produce zero error. The limited input results from the torque limit being applied to the ideal input.



(b) The resulting load torques from the inputs in Fig. 5(a). Notice how the limited motor torque, τ_m, no longer generates zero torque at the load, τ_L.

Fig. 5. Example load torque, τ_L, responses to an ideal and limited motor torque, τ_m, generated while attempting to apply zero torque against a moving load. $I_m = 0.4$, $k = 10$, $B = 1$ and $\tau_{limit} = 10$.

With the limited τ_m as the input, we can find the response at τ_L. This new response contains an error for which we can choose a threshold based on system requirements. We can now use the error threshold as a metric for defining the maximum frequency the actuator can provide zero τ_L. The response now also depends on k and B. Fig. 5(b) shows an example of how τ_L responds to a limited τ_m.

To gain an understanding of how the actuator responds with different passive dynamic parameters, we present the graphs in Fig. 6. Notice that in this scenario, the maximum achievable frequencies quickly become relatively low even with modest values of k and B (Fig. 6(a) and Fig. 6(b)).

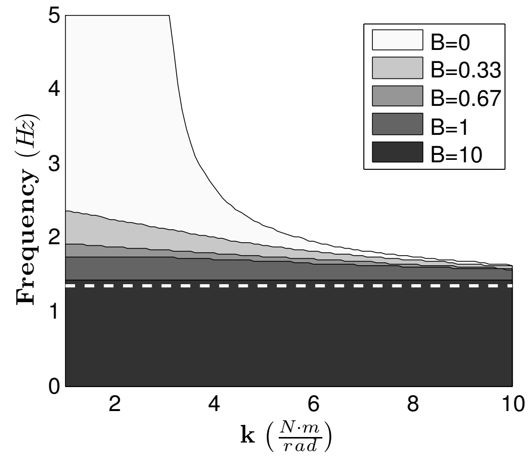
These graphs highlight the result that decreasing stiffness provides higher bandwidth for tracking the load motion, while maintaining acceptable output error. They also indicate that there is an inverse relationship between the maximum frequency, f_{max}, and the parameters, k, B and I_m. In other words, a decrease in k, B or I_m increases the bandwidth.

Even as the stiffness increases to infinity ($k, B \to \infty$), the maximum frequency will never dip below:

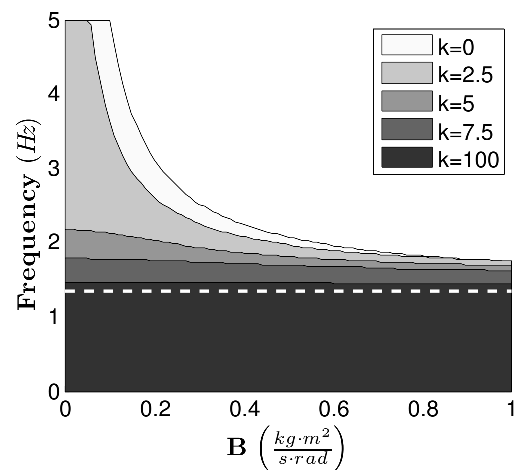
$$f_{worst} = \left(\frac{1}{2\pi}\right)\sqrt{\frac{\tau_{limit}}{\theta_A I_m}}. \tag{12}$$

Equation (12) was found by setting (11) equal to τ_{limit} and solving for frequency.

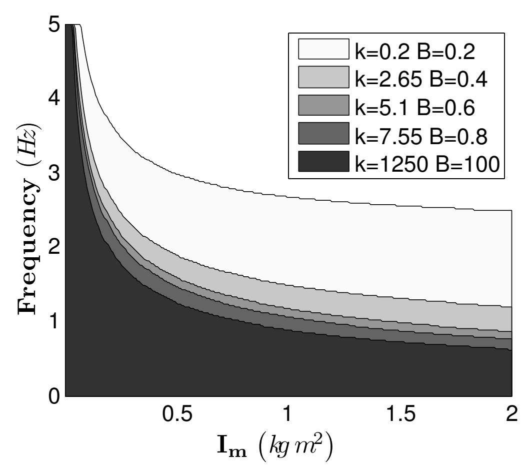
The frequency, f_{worst}, represents the maximum frequency the load motion can move at before the motor torque limit, τ_{limit}, is reached. For any frequency beyond f_{worst} there will be an error, whose magnitude depends on the inertia of the motor, k and B. This frequency is plotted as the dashed white line in Fig. 6(a) and Fig. 6(b).



(a) : Frequency achieved vs. series elasticity, k. Increasing the elasticity decreases the maximum frequency.



(b) : Frequency achieved vs. series damping, B. Increasing the damping decreases the maximum frequency.



(c) : Frequency achieved vs. motor inertia, I_m. Increasing the motor inertia decreases the maximum frequency.

Fig. 6. Performance of a series elastic/damped actuator applying zero torque against a moving load with some allowable error. The maximum frequency is the point where the load torque error exceeds 1 $N \cdot m$. The white dashed lines in figures 6(a) and 6(b) are the worst case maximum frequency, and occur when the system stiffness approaches infinity. For figures 6(a) and 6(b), $I_m = 0.4$.

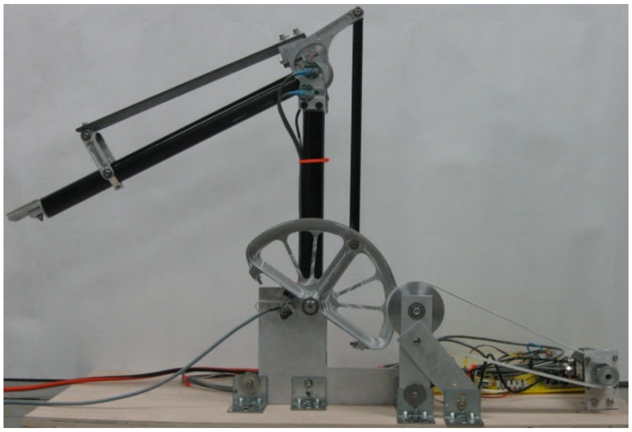


Fig. 7. Test platform for a single degree of freedom force controlled actuator. The system tracks its output force by measuring the deflection in its spring. Springs of varying stiffness can quickly be interchanged.

VII. Conclusions and Future Work

In this paper, we derived the physical limitations of actuators with passive dynamics that can be described by the dynamic model shown in Fig. 1. Our model does not include active control; we computed the optimal input to the system to produce the required torque at the load with zero or acceptable error. This is important so that our results only reflect the physical system's performance. These exact solutions provide the basis for understanding how the parameters affect bandwidth and how to select parameters for a torque control task. Each of these tasks are designed to represent extreme applications of torque and force control.

For our model to generate a varying torque against a fixed load, the system should have higher stiffness and/or lower inertia. Perhaps less obvious is that both damping and inertia play a much larger role in increasing the maximum frequency than stiffness.

For the actuator to produce exactly zero torque against a moving load, the system's stiffness does not matter. Instead, the stiffness only determines how quickly the error increases with increased frequency. We found that reducing stiffness decreases error caused by motor torque limits. But as the stiffness approaches infinity, the performance of the actuator is governed solely by the motor inertia and torque limit.

It is evident that designing an actuator optimized for both varying torque against a stationary load and applying zero torque against a moving load, is very difficult. In fact, they require exact opposite optimizations and share no set of parameters that provide good results for both tasks. The only way to improve the bandwidth of both tasks simultaneously is to reduce the motor inertia or increase the torque limit. This implies that actuators designed to perform a wide set of tasks require variable impedance.

Additional work will include the development of relationships for more complex actuation scenarios such as stopping an inertia or mass with initial velocity, or commanding the actuator to behave like a spring. Real examples of these tasks are space ship docking and legged locomotion. This work will inform engineers and robot designers on the roles of elasticity and damping. They will provide insight into how each parameter contributes for complex motions.

The next step in our work is to validate the calculations presented on a real system. We have begun constructing an actuator that embodies the model presented in this paper (Fig. 7). Our goal is to develop guidelines to allow engineers to understand the compromises and requirements of the mechanical system for all types of robotic physical interaction tasks.

VIII. Acknowledgments

Thanks to Benjamin Goska for discussion and mathematical advice.

References

[1] R. McN. Alexander. Three uses for springs in legged locomotion. *The International Journal of Robotics Research*, 9(2):53–61, April 1990.

[2] Giovanni A. Cavagna. Elastic bounce of the body. *Journal of Applied Physiology*, 29(3):279–282, 1970.

[3] Chee-Meng Chew, Geok-Soon Hong, and Wei Zhou. Series damper actuator: a novel force/torque control actuator. In *2004 4th IEEE/RAS International Conference on Humanoid Robots*, 2004.

[4] Robert J. Full and Claire T. Farley. Musculoskeletal dynamics in rhythmic systems - a comparative approach to legged locomotion. In J. M. Winters and P. E. Crago, editors, *Biomechanics and Neural Control of Posture and Movement*. Springer-Verlag, New York, 2000.

[5] Jonathan W. Hurst and Alfred A. Rizzis. Series compliance for an efficient running gait. *IEEE Robotics & Automation Magazine*, 15:42–51, September 2008.

[6] J.W. Hurst, D. Hobbelen, and A. Rizzi. Series elastic actuation: Potential and pitfalls. In *International Conference on Climbing and Walking Robots*, 2004.

[7] T. A. McMahon. The role of compliance in mammalian running gaits. *Journal of Experimental Biology*, 115:263–282, 1985.

[8] Gill A. Pratt and Matthew M. Williamson. Series elastic actuators. In *IEEE International Conference on Intelligent Robots and Systems*, volume 1, pages 399–406, 1995.

[9] Marc Raibert. *Legged Robots That Balance*. MIT Press, Cambridge, Mass., 1986.

[10] David William Robinson. *Design and Analysis of Series Elasticity in Closed-Loop Actuator Force Control*. PhD thesis, Massachusetts Institute of Technology, June 2000.

[11] J. De Schutter. A study of active compliant motion control methods for rigid manipulators based on a generic control scheme. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 1060–1065, 1987.

[12] Jonathon W. Sensinger and Richard F. Weir. Design and analysis of a non-backdrivable series elastic actuator. In *International Conference on Rehabilitation Robotics*, 2005.

[13] Jonathon W. Sensinger and Richard F. Weir. Improvements to series elastic actuators. In *Proceedings of the 2nd IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications*, 2006.

[14] N.G. Tsagarakis, Matteo Laffranchi, Bram Vanderborght, and D.G. Caldwell. A compact soft actuator unit for small scale human friendly robots. In *IEEE International Conference on Robotics and Automation*, 2009.

[15] Daniel E. Whitney. Force feedback control of manipulator fine motions. In *Joint Automatic Control Confrence*, pages 687–693, 1976.

[16] Gordon Wyeth. Demonstrating the safety and performance of a velocity sourced series elastic actuator. In *IEEE International Conference on Robotics and Automation*.