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An Improved Semi-Empirical Model for a Body and/or Caudal Fin (BCF) Fish Robot

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Abstract— **To develop useful applications for the underwater robots, the first step is to study its performance, such as thrust force or swimming velocity. In order to avoid numerous parameter studies in evaluating its performance, the prediction model plays an important role. As fish's swimming includes the kinematic of its own body and the hydrodynamic interaction with the surrounding fluid, it is difficult to formulate a precise mathematical model by purely analytical approaches. This paper offers a semi-empirical method to model the performance of a BCF (body and/or caudal fin) biomimetic fish robot. By using a dimensional statistical method, a semi-empirical model for predicting the thrust force generated by a BCF oscillation swimming mode is derived. This model shows good prediction capability. The predicted results are in good agreement with the experiment data. Therefore, the proposed modeling method can be used to solve the engineering problem concerned without a complex theory derivation.**

I. Introduction

In recent years, people become more focusing on the amazing swimming capabilities of fishes. As much as inspiring to build flying machines learnt from the birds, man is also curious in building machines that are learnt from the marine life which navigate through the water [1, 2]. The Nanyang Awana (NAF-I) is an example of a biomimetic robot, which combines fish-like propulsive mechanism and robotic technology [3]. The swimming mode of fish is generally divided into two main categories, namely the Body and/or Caudal Fin (BCF) locomotion and the Median or Pectoral Fin (MPF) locomotion [4], based on their physiological mechanics and propulsion structure. Fishes that generate thrust by bending their bodies into backward-moving propulsive wave that extends to its caudal are classified under BCF locomotion [5] .

To fully develop the applications for these biomimetic robots, the first step will be to study its performance, such as thrust force or swimming velocity. In order to avoid numerous parameter studies in evaluating its performance, the prediction model plays an important role. As fish's swimming involves the kinematic of its own body and the hydrodynamic interaction with the surrounding fluid, it is difficult to formulate a precise mathematical model by purely analytical approaches. Those models derived from theories are either too complex or inaccurate to be used for real life applications. This paper discusses the possibility of modeling the thrust force and swimming speed by a fish robot through semi-empirical model [6]. A dimensional analysis method is used to avoid the complicated derivation process, while still giving relatively good prediction results. The model formulated is believed to be useful in future navigation control of the fish robot. This model can also be used as a guideline for others to develop their fish robot's mechanism based on the predictive thrust and swimming speed required [7].

In this paper, an improved predictive model is developed for the output thrust generated and the swimming velocity of the fish robot. The modeling starts with the construction of predictive output thrust model before progressing to construction of the model for swimming velocity prediction. A dimensional analysis approach is used to find the mathematical relationship between the output thrust generated and the swimming velocity of the fish robot with respect to the various influencing parameters. Finally, the plausibility and predictive capability of the model is assessed both qualitatively and quantitatively by analyzing the trends of the predictive models and by comparing with the experimental results.

II. Thrust Generation of Fish

Thrust forces are generated as fish moves its body and fins relative to water by displacing the water. As fish swims, it generated thrust by transferring the momentum to the surrounding fluid [8]. Since water is an incompressible fluid, any action force will produce an equal and opposite force [9]. Most of the fish's motion comes from the vortices shedding at its caudal fin and it was believed that the fish utilize these vortices to achieve very efficient swimming [10].

The wake left behind the tail of an undulatory swimmer is an array of trailing discrete vortices of alternating sign, generated as the caudal fin moves up and down. Vortices in the wake have a reversed rotational direction as compared to the well-documented von Kármán vortex street, which is observed in the wake of stationary objects such as cylinders or aero foils [11]. Although the generation mechanism of this wake structure is still unclear, the observed phenomenon, named reverse von Kármán vortex street, appears to be closely associated with thrust generation.

For a fish swimming at a constant speed, its thrust must be sufficient to overcome its drag. Hence, our prediction model for the swimming velocity of the fish robot will be based on the convention drag force equation [11-13] as shown in (1):

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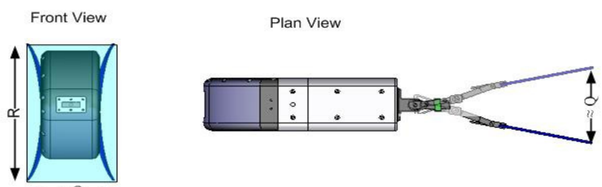
$$F_d = \frac{1}{2}\rho C_d A_c v^2 \quad (1)$$

where F_d is the drag force, which is defined to be the force component in the direction of the flow velocity; ρ is the density of the fluid; C_d is the drag coefficient; v is speed of the fish robot relative to the fluid; A_c is the reference area, which is defined to be the maximum frontal area of the fish robot during motion (For example, the maximum cross-section area when viewed from the ahead). The drag coefficient value used in this study will be based on steady-state estimation from the experimental data, due to the absence of information on the value of unsteady drag coefficient. Through the experiments conducted, the reference area, A_c was found to be a function of the oscillation frequency (f) and amplitude (θ_1). The reference frontal areas (A_c) of the fish robot for various cases are approximated using (2) with an 'Adjusted R-Square' value of 0.991:

$$A_c = R\left(-0.126f + 0.016\theta_1 - 0.004f\theta_1 + 0.042f^2 + 0.00015\theta_1^2 + 0.078\right) \quad (2)$$

where R is the approximate maximum height of the fish robot as shown in Fig. 1.



$A_c = Q \times R$: Reference Area (Approx. maximum frontal area of robotic fish during motion)

Fig. 1. Reference frontal area approximation

III. Design of Fish Robot and Model Derivation

A. Overview of Fish Robot

As shown in Fig. 2 is the fish robot, Nanyang Awana I (NAF-I), which will be use in this study [3, 14]. The thrust force is generated by the oscillation of the tail. The caudal fin mechanism is driven by a DC motor with a set of miter gears to convert the horizontal axis (θ_1) of rotation onto vertical axis (α_1) as shown in Fig. 3.

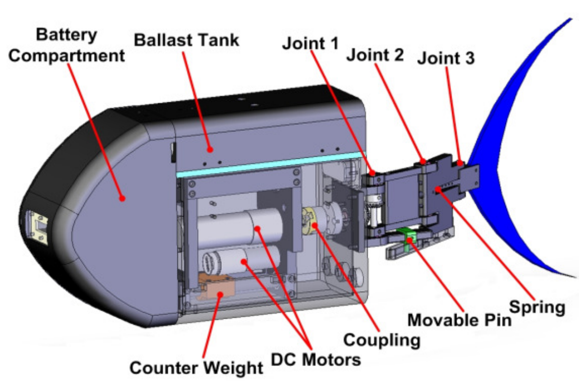


Fig. 2. CAD Model of Nanyang Awana I (NAF-I) [3]

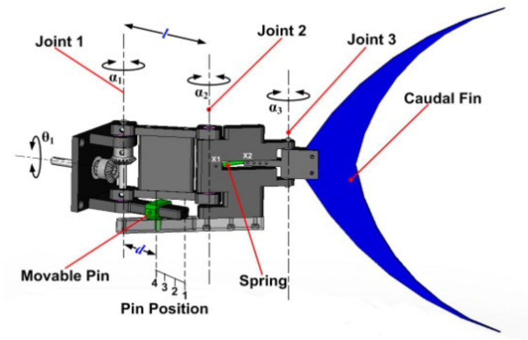


Fig. 3. Cad Model of Caudal Tail Fin Mechanism

B. Model Derivation

A dimensional analysis technique is used to establish a mathematical relationship to describe output thrust in terms of various parameters, while a regression analysis of the experimental data is undertaken to determine the constants in the models. To simplify the model without losing the generality of the proposed work, the following assumption are made: 1) The thrust is being measured under steady swimming speed; 2) Energy loss through mechanical linkages is negligible; 3) The thrust is generated through the pressure difference on the sides of the tail fin; 4) Mechanical losses is negligible (e.g. Input amplitude, θ_1 is equal to output amplitude, α_1) and 5) Amplitude of oscillation (radians) is small, hence $\sin\theta_1 \approx \theta_1$;

It is believed that the oscillation of the tail causes a pressure difference, P between the sides of the tail fin [11, 15]. Thus, the thrust force can be calculated by:

$$F_{Thrust} = PA_1 \sin\alpha_1 + PA_2 \sin\alpha_2 + PA_3 \sin\alpha_3 \quad (3)$$

where A_1, A_2, and A_3 are the area (m^2) of the three respective links, while α_1, α_2, and α_3 are the angles between three links and the horizontal axis. The pressure difference, P acting on the three links of the caudal tail fin can be estimated in terms of oscillation frequency, oscillation amplitude, total fin length, caudal fin area, spring constant, movable pin position and flow properties represented by velocity and density [11]. Therefore, only the major and typical variables are considered. The pressure difference, P can be expressed in the following form:

$$P = \phi(f,\ \theta,\ l,\ \rho_w,\ \bar{u},\ A_3,\ k,\ d) \quad (4)$$

where f is the oscillation frequency, θ_1 is the oscillation amplitude, l is the total fin length, ρ_w is the density of the water, $\bar{u}$ is the average relative velocity, A_3 is area of caudal fin, k is the spring constant at joint 3, and d is the distance between the movable pin position and joint 1.

A dimensional analysis technique is employed to establish the relationship between the various test parameters and the pressure difference generated on the caudal tail fin during the swimming process. With this technique, a relationship between n variables (the physical properties such as velocity, density, etc.) can be expressed as an n-m non-dimensional groups of variables (called π groups) using the constraint that all products formed must have the same dimension, where m is the number of fundamental dimensions (such as mass, length, and time) required to express the variables.

The dimensional analysis is based on Buckingham's theorem [16]. According to Buckingham's theorem, fundamental dimensions m should be first found through the analysis of dimensions of all the variables involved in a problem. Thereafter, we will proceed to select the m repeating variables, which have influences in the problem, to form the n-m dimensionless π groups. Table I shows the nine variables (n = 9), involved in (4), with their respective dimension, required to carry out the dimensional analysis for the pressure difference on the caudal tail fin during oscillation.

TABLE I
PARAMETERS ASSOCIATED WITH PRESSURE (WHERE L, T AND M DENOTE LENGTH, TIME AND MASS, RESPECTIVELY).

Parameter	Symbol	Dimension
Pressure difference	P	M/LT^2
Oscillation frequency	f	1/T
Oscillation amplitude (radians)	θ_1	-
Total fin length	l	L
Water density	ρ_w	M/L^3
Average relative velocity	$\bar{u}$	L/T
Caudal fin area	A_3	L^2
Spring	k	M/T^2
Movable pin distance	d	L

In Table I, Length (L), mass (M) and time (T) are the fundamental dimensions, and θ_1 is a dimensionless variable. Three repeating variables, l, ρ_w, and f are selected, so that six π groups are formed as follows:

$$\pi_1 = \theta_1 \tag{5}$$

$$\pi_2 = \frac{P}{\rho_w f^2 l^2} \tag{6}$$

$$\pi_3 = \frac{\bar{u}}{lf} \tag{7}$$

$$\pi_4 = \frac{A_3}{l^2} \tag{8}$$

$$\pi_5 = \frac{k}{\rho_w f^2 l^3} \tag{9}$$

$$\pi_6 = \frac{d}{l} \tag{10}$$

Based on Buckingham's π theorem, the original relationship defined in (4) can be re-formulated as a function of the dimensionless π groups obtained above. By applying the power law, the pressure difference is given by:

$$P = \rho_w l^2 f^2 \theta^{a1} \left(\frac{\bar{u}}{lf}\right)^{a2} \left(\frac{A_3}{l^2}\right)^{a3} \left(\frac{k}{\rho_w l^3 f^2}\right)^{a4} \left(\frac{d}{l}\right)^{a5} \tag{11}$$

The relative velocity u is the difference between the flow velocity and the tail velocity (factor v and factor v_t respectively) given by:

$$\vec{u} = \vec{v} - \vec{v_t} \tag{12}$$

Hence, the relative velocity in x-direction can be obtained by:

$$u = v + 2\pi f s_1 \theta_1 \cos 2\pi ft \sin \alpha_1 \tag{13}$$

The average velocity can be approximated as:

$$\bar{u} = v + 4 f s_1 \theta_1 \tag{14}$$

By referring to Fig. 3, the geometry relationship between α_1 and α_2 can be expressed as:

$$\sin\alpha_2 = \frac{s_1 \sin\alpha_1}{\sqrt{s_1^2 + d^2 - 2 s_1 d \cos\alpha_1}} \tag{15}$$

where s_1 is the length of first link and d is the distance between the movable pin and joint 1. Since the range of α_1 used in the experiments is relatively small, so based on our assumption, (15) can be estimated by:

$$\sin \alpha_2 = c_1 \frac{s_1}{s_1 - d} \sin \alpha_1 \tag{16}$$

Also from the analysis of the kinematics layout in Fig. 3, we can state that:

$$\sin\alpha_3 = \varphi(k, f, \theta_1, d, \bar{u}, \rho_w, A_3)\sin\alpha_1 \tag{17}$$

where k is the spring constant and the definition of other variables has been defined previously. All the parameters involved in (17) as well as their dimensions are listed in Table II.

TABLE II
PARAMETERS INVOLVED IN EQUATION (17) (WHERE L, T AND M DENOTE LENGTH, TIME AND MASS, RESPECTIVELY).

Parameter	Symbol	Dimension
Spring constant	k	M/T^2
Oscillation frequency	f	1/T
Oscillation amplitude	θ_1	-
Position of movable pin	d	L
Water density	ρ_w	M/L^3
Average relative velocity	$\bar{u}$	L/T
Area of link 3	A_3	L^2

By using the dimensional analysis method, four π groups can now be formed as follows:

$$\pi_1 = \frac{k}{\rho_w d^3 f^2} \tag{18}$$

$$\pi_2 = \theta_1 \tag{19}$$

$$\pi_3 = \frac{\bar{u}}{df} \tag{20}$$

$$\pi_4 = \frac{A_3}{d^2} \tag{21}$$

By applying the power law formulation, we obtain:

$$\sin\alpha_3 = \left(\frac{k}{\rho_w d^3 f^2}\right)^{b1} \theta_1^{b2} \left(\frac{\bar{u}}{df}\right)^{b3} \left(\frac{A_3}{d^2}\right)^{b4} \sin\alpha_1 \tag{22}$$

By substituting (11), (16) and (22) into (3), we obtain:

$$F_{Thrust} = P\left(A_1 + A_2 + A_3\right) \sin\alpha_1 C \tag{23}$$

where

$$C = \left[1 + \frac{c_1 s_1}{s_1 - d} + \left(\frac{k}{\rho_w d^3 f^2}\right)^{b1} \theta_1^{b2} \left(\frac{\bar{u}}{df}\right)^{b3} \left(\frac{A_3}{d^2}\right)^{b4}\right] \tag{24}$$

With the given input oscillation given as:

$$\alpha_1 = \theta_1 \sin 2\pi ft \tag{25}$$

The output thrust force can then be expressed as:

$$F_{Thrust} = P\left(A_1 + A_2 + A_3\right) \sin(\theta_1 \sin 2\pi ft) C \tag{26}$$

It is noted from (27) that the thrust force is a time dependent function, the average thrust force can thus be calculated by:

$$\overline{F}_{Thrust} = 2f\int_0^{1/2f} F_{Thrust}\,dt \tag{27}$$

By using the Simpson rule, we can then estimate the integrations of F_{Thrust} with high accuracy:

$$\overline{F}_{Thrust} = PC(A_1 + A_2 + A_3)\kappa \tag{28}$$

where

$$\kappa = \frac{2}{3} f\left(8\sin\frac{\theta_1}{2} + 4\sin\frac{\sqrt{3}\theta_1}{2} + 4\sin\theta_1\right) \tag{29}$$

The swimming velocity prediction model was built on the predictive thrust model found, the convention drag force equation in (1), the correction factor β and a regression analysis on the experimental data. To simplify the swimming velocity model without losing the generality of the proposed work, the following assumption were made: 1) The fish robot is in steady swimming state; 2) Energy loss due to external disturbances is negligible; 3) The fish robot is under constant swimming velocity when the total drag force is equal to the total thrust force, $F_d = \overline{F}_{Thrust}$; 4) Drag coefficient, C_d is determined experimentally and treated as a constant with a mean value of 0.67; and 5) The 'Free Body Motion' swimming velocity, v_{free} is proportional to the 'Fixed Body Motion' swimming velocity (fish robot's body held rigid for thrust force measurement [3]) by a correction factor of β which was determined through curve fitting using the experimental data collected.

For a fish swimming at a constant speed, its thrust must be sufficient to overcome its drag. Based on our assumption that under constant swimming velocity, (1) will become:

$$F_d = \frac{1}{2}\rho C_d A_c v_{fix}^2 = \overline{F}_{Thrust}$$
$$\overline{F}_{Thrust} = \frac{1}{2}\rho C_d A_c v_{fix}^2 \tag{30}$$

With some manipulation, the 'Fixed Body' swimming velocity will be:

$$v_{fix} = \sqrt{\frac{2\overline{F}_{Thrust}}{\rho C_d A_c}} \tag{31}$$

From the assumption above, the 'Free Body Motion' swimming velocity can be expressed as:

$$v_{free} = \beta\sqrt{\frac{2\overline{F}_{Thrust}}{\rho C_d A_c}} \tag{32}$$

where

$$\beta = 0.153f - 0.132\theta_1 + 0.002f\theta_1 + 0.015f^2 + 0.004\theta_1^2 + 1.603 \tag{33}$$

Substituting (28) into Equation (32), the 'Free Body Motion' swimming velocity can be expressed as

$$v_{free} = \beta\sqrt{\frac{\frac{4}{3}PCf(A_1 + A_2 + A_3)(8\sin\frac{\theta_1}{2} + 4\sin\frac{\sqrt{3}\theta_1}{2} + 4\sin\theta_1)}{\rho C_d A_c}} \tag{34}$$

IV. Model Assessments

With the predictive thrust and swimming model formulated, the constants in (11) and (24) can be determined using regression analysis based on the experimental data found in earlier studied [3]. The experimental data from the orthogonal design found earlier are used here to determine the constants (the coefficient and exponents) in (11) and (24). Since there are only 49 runs data from the orthogonal design, a regression analysis of the as-measured data has been carried out to obtain the thrust generated by the caudal tail fin for all the possible combinations (all the levels of the experimental parameters used in the study). The 5184 data (output thrust) from the regression analysis were statistically used to determine the constants. By substituting the constants found into (11) and (24), we have

$$P = \rho_w l^2 f^2 \theta^{11.011}\left(\frac{\overline{u}}{lf}\right)^{8.354}\left(\frac{A_3}{l^2}\right)^{-21.732}\left(\frac{k}{\rho_w l^3 f^2}\right)^{3.955}\left(\frac{d}{l}\right)^{27.661} \tag{11a}$$

$$C = 1 - \frac{0.229 s_1}{s_1 - d} + \left(\frac{k}{\rho_w d^3 f^2}\right)^{-3.169}\theta_1^{-9.919}\left(\frac{\overline{u}}{df}\right)^{-8.880}\left(\frac{A_3}{d^2}\right)^{22.333} \tag{24a}$$

Through further simplification, (11a) and (24a) become:

$$P = \frac{\theta_1^{11.011}(v + 0.28f\theta_1)^{8.354}k^{3.955}d^{27.661}}{\rho_w^{2.955}l^{2.416}f^{14.264}A_3^{21.732}} \tag{11b}$$

$$C = 1 - 0.229\frac{s_1}{s_1 - d} + \frac{\rho_w^{3.169}f^{15.218}A_3^{22.333}}{k^{3.169}\theta_1^{9.919}d^{26.279}(v + 0.28f\theta_1)^{8.880}} \tag{24b}$$

Substituting the constants such as the water density, ρ_w = 1000 kg/m^3, area of link 1, A_1 = 0.00504 m^2, area of link 2, A_2 = 0.00468 m^2, total fin length, l = 0.339 m, length of link1, s_1 = 0.07 m into (11b) and (24b), we have

$$P = 18.622\times10^{-9}\left[\frac{\theta_1^{11.011}(v + 0.28f\theta_1)^{8.354}k^{3.955}d^{27.661}}{f^{14.264}A_3^{21.732}}\right] \tag{11c}$$

$$C = 1 - \frac{0.01603}{0.07 - d} + \frac{3.214\times10^9\times f^{15.218}A_3^{22.333}}{k^{3.169}\theta_1^{9.919}d^{26.279}(v + 0.28f\theta_1)^{8.880}} \tag{24c}$$

By substituting (11c) and (24c) into (28) and (32) will allows us to predict the output thrust and swimming velocity of the fish robot respectively. An analysis of the output thrust predicted by the model with respect to the input parameters has been carried out to study the model's generality and plausibility. Meanwhile, by comparing the predicted output thrust with the experimental results, the adequacy of the model's prediction is further assessed. Figs. 4 (a-f) show the predicted trends by the developed model and the trends from the experimental results. From the charts, the solid lines represent the predicted data and the experimental results were represented by symbol.

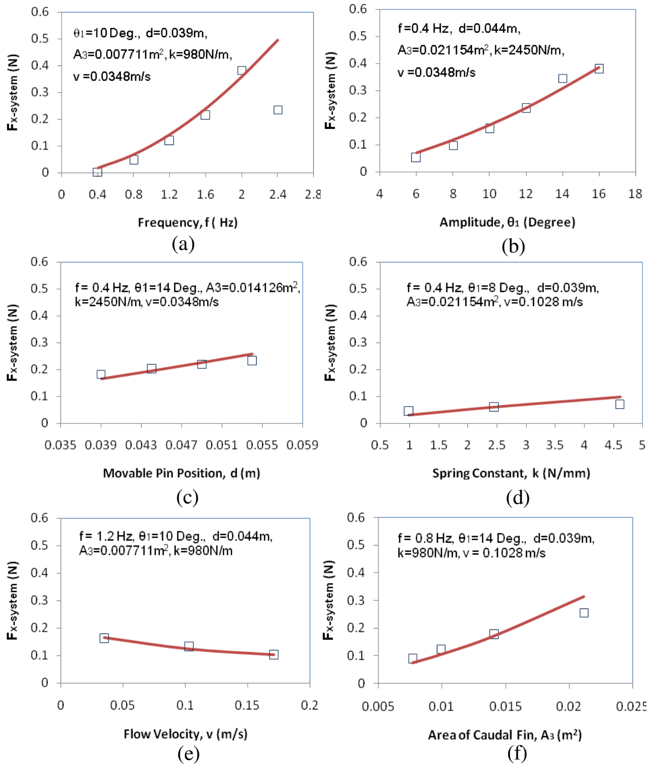


Fig. 4. Predicted trends of thrust (Fx-system) generated with respect to various parameters.

Fig. 4 shows the average predict thrust force with respect to different input parameters. It can be found that the predicted model has a good agreement with the experimental results for most of the parameters. The predicted trend of the output thrust generated with respect to the oscillating frequency is shown in Fig. 4(a). It is seen that the output thrust increases as the frequency increases. These trends are qualitatively in good agreement with the experimental result (symbols) for frequency range of 0.4 Hz to 2.0 Hz as shown in the figures. However, the predicted values do not tally well with the experimental data at a frequency of 2.4 Hz. The drop in thrust force at frequency of 2.4 Hz may be due to the limitation of the tail fin mechanism, which has not been taken care of by the modeling. However, this model is still adequate to predict the thrust force for most of the cases, especially for the other parameters.

The above qualitative analysis and comparison of the output thrust (predicted values versus experimental values) have shown that the model has accurately predicted the relationship between the output thrust and its input parameters. It can be concluded from this qualitative analysis that the model has been formulated with sufficient accuracy to represent the effect of these variables. Thus, the model can be considered being formulated correctly.

Figs. 5 (a-d) show the predicted trends of the 'Free Body Motion' swimming velocity and the trends from the experimental results. From the charts, the solid lines represent the predicted data and the experimental results were represented by symbol.

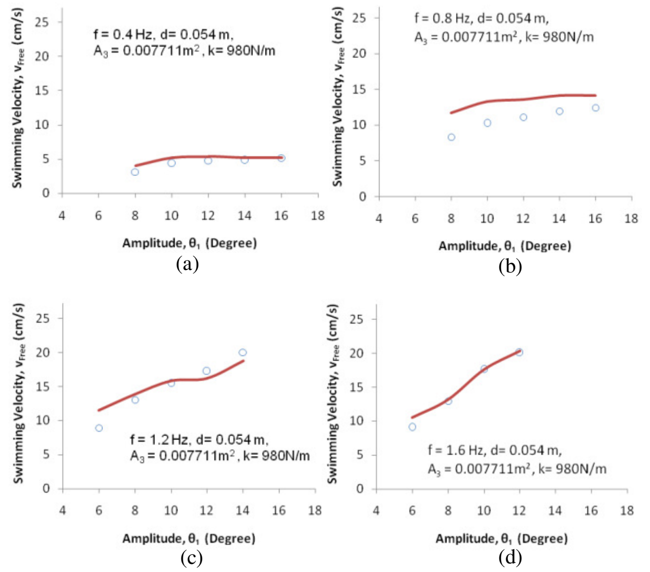


Fig. 5. Predicted trends of the 'Fixed Body Motion' swimming velocity as compared to the experimental results.

It can be noted from Figs. 5 (a-d) that the predicted trends of the 'Free Body Motion' are consistent with the corresponding experimental trends. This result has shows that the model has adequately predicted the swimming velocity of the fish robot. Hence, it can be concluded that the curve-fitting for the approximation of the corrector factor (β) is ample to form the relationship between the 'Fixed Body Motion' swimming velocity and the 'Free Body Motion' swimming velocity. Thus, we believe that this model can be utilized in future navigation control to improve its positioning capability.

V. Concluding Remarks

In this paper, predictive models have been developed for the output thrust generated and the swimming velocity of the fish robot. The modeling will start with the construction of predictive output thrust model before progressing to construct the model for swimming velocity prediction.

The application of the dimensional analysis proposed in this study gives a relatively simplified equation to predict the thrust force generated by the tail oscillation. The predictive model for the thrust force has been assessed qualitatively and quantitatively by comparing with experimental data under the respective conditions. The predicted values are in good agreement with the experimental testing results. The swimming velocity prediction model is built on the predictive thrust model found earlier, with the use of the fundamental drag force equation and regression analysis on the experimental data. This swimming velocity model is being assessed by comparing with experimental data under corresponding conditions. It can be concluded from the qualitative analysis that the model has been formulated with sufficient accuracy to predict the swimming velocity of the fish robot.

With the models being able to adequately predict the output thrust and swimming velocity, we believes that it will be a very useful input to future navigation control of the fish robot.

Especially in the inertial navigation control, where the sensors input can be compared with this prediction data to reduce its positioning error. In future, this semi-empirical method can also be applied for the performance prediction model for median and/or paired fin (MPF) biomimetic fish robots.

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