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Flight Formation of Multiple Mini Rotorcraft via Coordination Control

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Abstract— **In this paper, the coordination and trajectory tracking control design of multiple mini rotorcraft systems are discussed. The dynamic model of a mini rotorcraft is presented using the Newton-Euler formalism. Our approach is based on a leader/follower structure of multiple robot systems. The centroid of the coordinated control subsystem is used for trajectory tracking purposes. A nonlinear coordinated control design for multiple autonomous vehicle synchronization is developed. The analytic results are supported by simulation tests.**

I. Introduction

An important challenge in automatic control field is the problem of multiple spacecraft flying in formation which has been intensively investigated during the last decades.

Different approaches for multiple spacecraft flying in formation have been proposed in the literature for coordination of multiple robot systems. There are mainly three approaches: Leader/Follower, Virtual Structure and Behavioral Control.

In the leader/follower architecture, one agent is designated as leader while the others are designated as followers which should track the leader. Leader/follower approaches are described in [1], [2]. The virtual structure approach considers every agent as an element of a larger structure [3] and [4]. Finally, the behavioral control in [5] and [6] is based on the decomposition of the main control goal into tasks or behaviors. This approach also deals with collision avoidance, flock centering, obstacle avoidance and barycenter.

Generally, to analyze the communication between agents, directed or undirected graphs are used. Every node in a graph is considered as an agent which can have information exchange with all or several agents. In [4], [7], [8], and [9], the authors use algebraic graph theory in order to model the information exchange between vehicles. By using this technique, several control strategies have been developed, e.g. [9], [10], [11], [12] and [13]. In [9], the authors present several algorithms for consensus and obstacle avoidance for multiple-agent systems. [10] presents an algorithm for trajectory tracking of a time varying reference for a single integrator multi-agent system. [11] and [12] present a passive decomposition approach for consensus and formation control. In [13], the authors present a bilateral teleoperation control approach for the multi-agent trajectory tracking problem.

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We are interested in the problem of multiple mini rotorcraft flying in formation using a nonlinear control with a coordination control strategy. A coordination algorithm assumes that there are n-agents which have some kind of information exchange between them. In this approach, roll, pitch and yaw angles are considered as agents with an information exchange topology. Thus, the lateral and longitudinal dynamical systems of each mini rotorcraft are considered as agents to be coordinated and to follow a desired trajectory. To do this, combined with a nonlinear control, we use an algebraic graph theoretical approach to model information exchange where every node in a graph is considered as an agent which can have information exchange with all or several agents. Several approaches of nonlinear control of a mini rotorcraft can be found in the literature; for instance, in [14] a nonlinear control based on nested saturations is presented. In this approach, the dynamics is decoupled into lateral and longitudinal dynamical subsystems. Thus, nested saturations control was used to stabilize each subsystem. In [15], the authors proposed a robust linear PD controller considering parametric interval uncertainty. The authors also presented a robust stability analysis and computed the robustness margin of the system with respect to the parameters uncertainty.

This work addresses the nonlinear control for multiple mini rotorcraft flying in formation, shown in Figure 1, based on coordination control strategy. The novelty of this approach is to consider that the lateral and longitudinal dynamical systems of each mini rotorcraft as agents to be coordinated which follow a virtual reference. In this way, the multiple mini rotorcraft platoon can hover and thus keeping the desired formation by following a constant zero-reference. Another contribution of this work is that the centroid of a virtual center of mass can be used to follow a given smooth trajectory and a four integrator coordination problem is developed.

This paper is organized as follows: Section II presents some preliminary results on algebraic graph theory and passivity systems. Section III presents the dynamical model of the proposed architecture. In section IV the nonlinear control design is presented. Simulation results are presented in section V. Section VI presents the conclusions and future work.

II. Preliminaries

A. Graph Theory

A multi-agent dynamic system can be modelled as a group of dynamical systems which has a information exchange topology represented by information graphs. A graph $\mathcal{G}$ is a pair $\mathcal{G}(\mathcal{N},\mathcal{E})$ consisting of a set of nodes $N =$

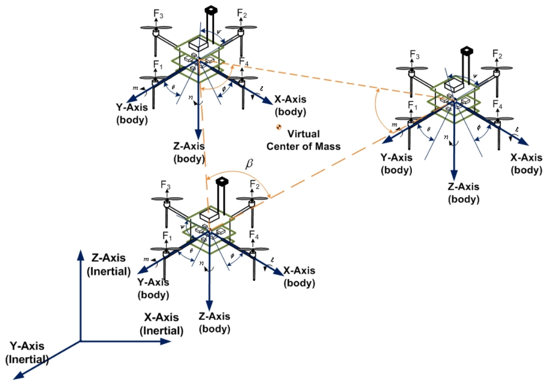

Fig. 1. Multiple mini rotorcraft flying in formation

$\{n_i\colon n_i \in N, \forall i = 1, ..., n\}$ together with their interconnections $\mathcal{E}$ on $\mathcal{N}$ [16]. Each pair (n_1, n_2) is called an edge $e \in \mathcal{E}$. An undirected graph is one where nodes i and j can get information from each other. In a digraph, the i^{th} node can get information from the j^{th} node but not necessarily viceversa. We can think of the information exchange between agents as an undirected graph but also as a digraph which implies a more complicated problem. One important characterization of graphs is their connectivity. A graph is said to be connected if for every pair $\{x, y\}$ of distinct vertices there is a path from x to y. A connected graph allows the communication between all agents through the network. A directed graph is said to be strongly connected if any two vertices can be joined by a path. A graph is said to be balanced if its in-degree (number of communication links arriving at the node) is equal to its out-degree (number of communication links leaving the node).

A typical consensus algorithm considers a first integrator system of the form

$$\begin{aligned} \dot{x} &= -Lx + bu \\ y &= c^T x \end{aligned} \tag{1}$$

where L is the Laplacian matrix having the following properties:

1) L has a single eigenvalue at 0, $\lambda_1(L) = 0$ with right eigenvector $w_1^T = \left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array}\right], i.e.\ Lw_1 = 0$.
2) The remaining eigenvalues are all positive, i.e. $\lambda_i(L) > 0$ and $Lw_i = \lambda_i w_i$ for $i = 2, ..n$, and $w_i \in R^n$.

We assume that the information exchange graph is balanced. Let us assume also that in the coordinating controller the gains multiplying the signals in between agents are all equal to 1. For the $i-th$ row of L, the entries $l_{ij} = -1$ for $i \neq j$ correspond to the gains multiplying the signals from other agents coming to agent i. For the $i-th$ column of L, the entries $l_{ji} = -1$ for $i \neq j$ correspond to the gains multiplying the signals going out of agent i towards the other agents. We then have the following property.

3) w_1 defined above is also the left eigenvalue of L corresponding to the eigenvalue 0, i.e. $w_1^T L = 0$.

It is worth to mention that dynamics (1) can be rewritten as

$$\dot{x}_i = u_i \tag{2}$$

with multiple agent consensus achieved using a consensus algorithm proposed in [17].

$$u_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j) \tag{3}$$

III. Dynamic Model

To obtain the vehicle dynamical model, it will be assumed that it flies over a local area in the Earth. Then, the Flat-Earth model equations will be used [18]. The equations representing the kinematic and the moments are written as

$$C_{b/n} = fn(\Phi) \tag{4}$$

$$\dot{\Phi} = H(\Phi)\,\omega^b_{b/e} \tag{5}$$

$$\dot{\omega}^b_{b/e} = \left(J^b\right)^{-1}\left[\mathbf{M}^b_{A/T} - \Omega^b_{b/e} J^b \omega^b_{b/e}\right] \tag{6}$$

The vehicle center of mass, CM, is coincident with the body frame origin, F_b. The angular velocity in terms of the body system is given by $\omega^b_{b/e} = \left[\begin{array}{ccc} P & Q & R \end{array}\right]^T$ and its cross product matrix is denoted by $\Omega^b_{b/e}$. The angular velocity in the local inertial system has components $\dot{\Phi} = \left[\begin{array}{ccc} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{array}\right]^T$. The matrix of rotation from F_e to F_b is denoted by $C_{b/n}$. The set of attitude equations can be

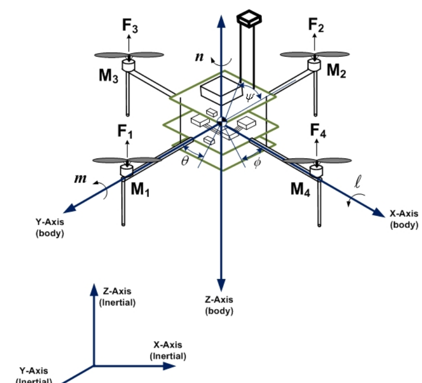

Fig. 2. Quadrotor vehicle schematic for vertical flight mode

obtained using the equations (5) and (6). The transformation of the components of the angular velocity generated by a sequence of Euler rotations from the body to the local reference system is written as follows:

$$H(\Phi) = \left[\begin{array}{ccc} 1 & t\theta s\phi & t\theta c\phi \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{array}\right] \tag{7}$$

where s, c and t are used to denote the $\sin$, $\cos$ and the $\tan$ respectively.

The term J^b in (6) represents the inertia matrix, and is defined by

$$J^b = \left[\begin{array}{ccc} J_x & J_{xy} & J_{xz} \\ J_{yx} & J_y & J_{yz} \\ J_{zx} & J_{zy} & J_z \end{array}\right]$$

Since the quadrotor prototype is symmetrical in the xz-plane and the xy-plane, the products of inertia J_{xy}, J_{yz} and J_{xz} vanish. Then J^b and its inverse can be written by

$$J^b = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{8}$$

The aerodynamics and thrust moments can be denoted by $M^b_{A,T} = \begin{bmatrix} \ell & m & n \end{bmatrix}^T$, and are shown in Figure 2. Then differentiating (5) we get

$$\ddot{\Phi} = \dot{H}(\Phi)\,\omega^b_{b/e} + H(\Phi)\,\dot{\omega}^b_{b/e} \tag{9}$$

Introducing the RHS of (6) into (9),

$$\ddot{\Phi} = \dot{H}(\Phi)\,\omega^b_{b/e} + H(\Phi)\left(J^b\right)^{-1}\left[\mathbf{M}^b_{A/T} - \Omega^b_{b/e}J^b\omega^b_{b/e}\right] \tag{10}$$

It is proposed that

$$\mathbf{M}^b_{A/T} \triangleq \Omega^b_{b/e}J^b\omega^b_{b/e} + H(\Phi)^{-1}J^b\left[\tilde{\tau} - \dot{H}(\Phi)\,\omega^b_{b/e}\right] \tag{11}$$

where $\tilde{\tau} = \begin{bmatrix} \tilde{\tau_\phi} & \tilde{\tau_\theta} & \tilde{\tau_\psi} \end{bmatrix}^T$. Then (9) can be rewritten as

$$\ddot{\phi} = \tilde{\tau}_\phi \tag{12}$$
$$\ddot{\theta} = \tilde{\tau}_\theta \tag{13}$$
$$\ddot{\psi} = \tilde{\tau}_\psi \tag{14}$$

Newton's second law is used to obtain the equations of translational motion in the inertial frame of reference as

$$\ddot{p}^n_{CM/T} = C_{n/b}\frac{\mathbf{F}^b_{A,T}}{m_b} + \mathbf{g}^n \tag{15}$$

where m_b represents the mass of the vehicle. The position of CM in the NED (North-East-Down) coordinate system with respect to the inertial frame origin, F_e, is given by $\mathbf{p}^n_{CM/T} = \begin{bmatrix} x & y & z \end{bmatrix}^T$. The aerodynamic and thrust force vector in the body system is represented by $F_{A,T} = \begin{bmatrix} X_{A,T} & Y_{A,T} & Z_{A,T} \end{bmatrix}^T$. The aerodynamic and thrust forces in the body frame of reference is given by

$$F_{A,T} = \sum_{i=1}^{4} F_i Z_{A,T} \tag{16}$$

Then (15) can be rewritten as

$$\ddot{x} = -F_{A,T}sin\theta \tag{17}$$
$$\ddot{y} = F_{A,T}cos\theta sin\phi \tag{18}$$
$$\ddot{z} = F_{A,T}cos\theta cos\phi - mg \tag{19}$$

where the constant m is the mass of the mini rotorcraft and g is the gravitational acceleration.

IV. Nonlinear Control Design

A. *Vehicle Stabilization and Consensus Agreement*

In this section, a nonlinear controller with a coordination control strategy is developed. It will be proved that the proposed control scheme stabilizes the rotorcraft in hover flight. The altitude and the vehicle's heading can be stabilized by using

$$F_{A,T} \triangleq \frac{-a_1\dot{z} - a_2(z - z^d) + mg}{cos\phi cos\theta} \tag{20}$$

where a_1 and a_2 are positive constants; z^d is the desired altitude. Around the origin, using (20) in (17)-(19), the lateral dynamic model of the mini rotorcraft is given by the following set of equations:

$$\ddot{y} = \tan\phi \tag{21}$$
$$\ddot{\phi} = \tilde{\tau}_\phi \tag{22}$$

and the longitudinal dynamic model can be represented by

$$\ddot{x} = \frac{-\tan\theta}{\cos\phi} \tag{23}$$
$$\ddot{\theta} = \tilde{\tau}_\theta \tag{24}$$

Then, we aim at synchronizing the lateral and longitudinal dynamical systems of two mini rotorcraft. Therefore, we propose a consensus algorithm which allows to stabilize the platform in attitude and position. The cases of having three agent dynamical systems with cyclic and chain information exchange topology are considered. It will be shown that the considered approach reaches consensus to a desired position while maintaining a stable attitude. It is assumed that pitch angle and roll angle are operated in a neighborhood of the origin, i.e., $|\varphi| < \pi/10\ \forall\varphi = \theta, \phi$. Then, the longitudinal dynamical systems, equations (23)-(24) are reduced to

$$\ddot{x}_i = -\theta_i \tag{25}$$
$$\ddot{\theta}_i = \tilde{\tau}_{\theta_i} \tag{26}$$

for all $i \in \mathcal{G}$. The lateral dynamical systems, equations (21)-(22) are reduced to

$$\ddot{y}_i = \phi_i \tag{27}$$
$$\ddot{\phi}_i = \tilde{\tau}_{\phi_i} \tag{28}$$

for all $i \in \mathcal{G}$. It is clear that systems (25)-(26) and (27)-(28) are four integrators in cascade and we propose to use a consensus algorithm to stabilize the rotorcraft. Let us consider the system given by

$$x_i^{(iv)} = \tilde{\tau}_{\theta_i} = u_i \tag{29}$$

A first change of variable is proposed

$$\xi_i \triangleq \dot{x}_i + \lambda x_i \tag{30}$$

Then, the third derivative of ξ_i is

$$\xi_i^{(iii)} = x_i^{(iv)} + \lambda x_i^{(iii)}$$
$$\xi_i^{(iii)} = u_i + \lambda x_i^{(iii)} \tag{31}$$

and the control u_i is defined as

$$u_i \triangleq u_i^{'} - \lambda x_i^{(iii)} \tag{32}$$

Equation (31) can be rewritten as

$$\xi_i^{(iii)} = u_i^{'}$$

By following the iterative algorithm, we define a new variable

$$\zeta_i \triangleq \dot{\xi}_i + \lambda \xi_i \quad (33)$$

The second derivative ζ_i is given by

$$\begin{aligned} \ddot{\zeta}_i &= \xi_i^{(iii)} + \lambda \ddot{\xi}_i \\ \ddot{\zeta}_i &= u_i^{'} + \lambda \ddot{\xi}_i \end{aligned} \quad (34)$$

and the control $u_i^{'}$ is defined as

$$u_i^{'} \triangleq \tilde{u}_i - \lambda \ddot{\xi}_i \quad (35)$$

Therefore, equation (34) can be rewritten as

$$\ddot{\zeta}_i = \tilde{u}_i$$

By following the iterative algorithm, we define the last change of variable

$$w_i \triangleq \dot{\zeta}_i + \lambda \zeta_i \quad (36)$$

The first derivative of w_i is given by

$$\begin{aligned} \dot{w}_i &= \ddot{\zeta}_i + \lambda \dot{\zeta}_i \\ \dot{w}_i &= \tilde{u}_i + \lambda \dot{\zeta}_i \end{aligned} \quad (37)$$

The control $\tilde{u}_i$ is defined as

$$\tilde{u}_i \triangleq \bar{u}_i - \lambda \dot{\zeta}_i \quad (38)$$

Equation (37) can then be rewritten as

$$\dot{w}_i = \bar{u}_i \quad (39)$$

We define control $\bar{u}_i$ as

$$\bar{u}_i \triangleq - \sum_{j \in \mathcal{N}_i} (w_i - w_j) \quad (40)$$

Introducing (40) into (38),

$$\bar{u}_i = - \sum_{j \in \mathcal{N}_i} (w_i - w_j) - \lambda \dot{\zeta}_i \quad (41)$$

Introducing (41) into (35),

$$u_i^{'} = - \sum_{j \in \mathcal{N}_i} (w_i - w_j) - \lambda \dot{\zeta}_i - \lambda \ddot{\xi}_i \quad (42)$$

Introducing (42) into (32),

$$u_i = - \sum_{j \in \mathcal{N}_i} (w_i - w_j) - \lambda \dot{\zeta}_i - \lambda \ddot{\xi}_i - \lambda x_i^{(iii)} \quad (43)$$

Introducing (43) into (29),

$$x_i^{(iv)} = - \sum_{j \in \mathcal{N}_i} (w_i - w_j) - \lambda \dot{\zeta}_i - \lambda \ddot{\xi}_i - \lambda x_i^{(iii)} \quad (44)$$

where

$$\begin{aligned} \ddot{\xi}_i &= x_i^{(iii)} + \lambda \ddot{x}_i \quad (45) \\ \dot{\zeta}_i &= x_i^{(iii)} + 2\lambda \ddot{x}_i + \lambda^2 \dot{x}_i \quad (46) \end{aligned}$$

Then, (44) can be rewritten as

$$x_i^{(iv)} = - \sum_{j \in \mathcal{N}_i} (w_i - w_j) - 3\lambda x_i^{(iii)} - 3\lambda^2 \ddot{x}_i - \lambda^3 \dot{x}_i \quad (47)$$

where $w_i = \left[\begin{array}{cccc} x_i^{(iii)} & 3\lambda \ddot{x}_i & 3\lambda^2 \dot{x}_i & \lambda^3 x_i \end{array} \right]$.

A positive definite Lyapunov function is proposed as in [17]

$$V = 2(V_1 + ... + V_N) \quad (48)$$

where V_i is the storage function for each quadrotor vehicle. Then $\dot{V}$ is

$$\dot{V} = -S_i(w_i) - K \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} (w_i - w_j)^T (w_i - w_j) \quad (49)$$

Nothing that $S_i(w_i) = 0 \ \forall i$ and integrating the above equation we can see that $(w_i - w_j) \in \mathcal{L}_2$. Using (36),

$$(w_i - w_j) = (\dot{\zeta}_i - \dot{\zeta}_j) + \lambda(\zeta_i - \zeta_j) \quad (50)$$

Defining $e_{ij} = \zeta_i - \zeta_j$ and differentiating w.r.t. we get that $\dot{e}_{ij} = \dot{\zeta}_i - \dot{\zeta}_j$, then (50) is rewritten as

$$(w_i - w_j) = \dot{e}_{ij} + \lambda e_{ij} \quad (51)$$

Then $\dot{e}_{ij} + \lambda e_{ij} \in \mathcal{L}_2$ which guaranties the exponential convergence of e_{ij} to the origin. Assuming that the information exchange graph is strongly connected then

$$\lim_{t \to \infty} |\zeta_i - \zeta_j| = 0$$

Since $(\zeta_i - \zeta_j) \in \mathcal{L}_2$. Using (33),

$$(\zeta_i - \zeta_j) = (\dot{\xi}_i - \dot{\xi}_j) + \lambda(\xi_i - \xi_j) \quad (52)$$

Defining $e_{ij}^{'} = \xi_i - \xi_j$ and differentiating w.r.t. we get that $\dot{e}_{ij}^{'} = \dot{\xi}_i - \dot{\xi}_j$, then (52) is rewritten as

$$(\zeta_i - \zeta_j) = \dot{e}_{ij}^{'} + \lambda e_{ij}^{'} \quad (53)$$

Then $\dot{e}_{ij}^{'} + \lambda e_{ij}^{'} \in \mathcal{L}_2$ which guaranties the exponential convergence of $e_{ij}^{'}$ to the origin. Assuming that the information exchange graph is strongly connected then

$$\lim_{t \to \infty} |\xi_i - \xi_j| = 0$$

We can see that $(\xi_i - \xi_j) \in \mathcal{L}_2$. Using (30),

$$(\xi_i - \xi_j) = (\dot{x}_i - \dot{x}_j) + \lambda(x_i - x_j) \quad (54)$$

Defining $\tilde{e}_{ij} = x_i - x_j$ and differentiating w.r.t. we get that $\dot{\tilde{e}}_{ij} = \dot{x}_i - \dot{x}_j$, then (54) is rewritten as

$$(\xi_i - \xi_j) = \dot{\tilde{e}}_{ij} + \lambda \tilde{e}_{ij} \quad (55)$$

Then $\dot{\tilde{e}}_{ij} + \lambda \tilde{e}_{ij} \in \mathcal{L}_2$ which guaranties the exponential convergence of $\tilde{e}_{ij}$ to the origin. Assuming that the information exchange graph is strongly connected then

$$\lim_{t \to \infty} |x_i - x_j| = 0$$

After a time $T > 0$, $\sum_{j \in \mathcal{N}_i} (w_i - w_j) \to 0$; therefore, from (47) it can be seen that $x_i^{(iii)}, \ddot{x}_i, \dot{x}_i \to 0$.

To achieve yaw angle synchronization, let us recall the yaw dynamics given by

$$\ddot{\psi}_i = \tilde{\tau}_{\psi_i} \quad (56)$$

By using the same approach described above, we propose a control law such that

$$\tilde{\tau}_{\psi_i} \triangleq -\sum_{j\in\mathcal{N}_i}\left[(\psi_i-\psi_j)+(\dot{\psi}_i-\dot{\psi}_j)\right]-\lambda\dot{\psi}_i \quad (57)$$

which ensures the consensus agreement in the sense that

$$\lim_{t\to\infty}|\psi_i-\psi_j|=0 \quad (58)$$

Control law (47) guaranties the synchronization of all quadrotor vehicles to the origin. However, this makes no sense in practical situations. Instead, we propose a formation control described in next section.

B. X4 Formation Control

In this section, we propose a leader-relative position consensus (UAV formation) for the multi quadrotor system, i.e. the quadrotor vehicles will converge to a desired position with respect to the leader of the group. We define the control $\bar{u}_i$ as

$$\bar{u}_i=-\sum_{j\in\mathcal{N}_i}(w_i-w_j)-\sum_{j\in\mathcal{N}_i}\lambda^3(x_i^{(d)}) \quad (59)$$

where the x_i^d are constants. Then the formation control law is rewritten as:

$$\begin{aligned}x_i^{(iv)} &= -\sum_{j\in\mathcal{N}_i}(w_i-w_j)-3\lambda x_i^{(iii)}-3\lambda^2\ddot{x}_i\\ &\quad -\lambda^3\dot{x}_i-\sum_{j\in\mathcal{N}_i}\lambda^3(x_i^{(d)}) \quad (60)\end{aligned}$$

It is important to note that

$$\begin{aligned}(w_i-w_j) &= x_i^{(iii)}-x_j^{(iii)}+3\lambda\ddot{x}_i-3\lambda\ddot{x}_j\\ &\quad +3\lambda^2\dot{x}_i-3\lambda^2\dot{x}_j+\lambda^3x_i-\lambda^3x_j \quad (61)\end{aligned}$$

Then,(59) can be rewritten as

$$\begin{aligned}\bar{u}_i &= -\sum_{j\in\mathcal{N}_i}(x_i^{(iii)}-x_j^{(iii)})-\sum_{j\in\mathcal{N}_i}3\lambda(\ddot{x}_i-\ddot{x}_j)\\ &\quad -\sum_{j\in\mathcal{N}_i}3\lambda^2(\dot{x}_i-\dot{x}_j)-\sum_{j\in\mathcal{N}_i}\lambda^3(x_i-x_j)\\ &\quad -\sum_{j\in N_i}\lambda^3(x_i^{(d)}) \quad (62)\\ \bar{u}_i &= -\sum_{j\in\mathcal{N}_i}(x_i^{(iii)}-x_j^{(iii)})-\sum_{j\in\mathcal{N}_i}3\lambda(\ddot{x}_i-\ddot{x}_j)\\ &\quad -\sum_{j\in\mathcal{N}_i}3\lambda^2(\dot{x}_i-\dot{x}_j)\\ &\quad -\sum_{j\in\mathcal{N}_i}\lambda^3(x_i-x_i^{(d)}-x_j) \quad (63)\end{aligned}$$

The control law $\bar{u}_i$ can be written as

$$\bar{u}_i = -\sum_{j\in\mathcal{N}_i}(w_i-w_j) \quad (64)$$

Then, (29) can be rewritten as

$$x_i^{(iv)}=-\sum_{j\in\mathcal{N}_i}(w_i-w_j)-3\lambda x_i^{(iii)}-3\lambda^2\ddot{x}_i-\lambda^3\dot{x}_i \quad (65)$$

where $w_i = \left[\begin{array}{cccc} x_i^{(iii)} & 3\lambda\ddot{x}_i & 3\lambda^2\dot{x}_i & \lambda^3(x_i-x_i^{(d)})\end{array}\right]$ and $w_j=\left[\begin{array}{cccc} x_j^{(iii)} & 3\lambda\ddot{x}_j & 3\lambda^2\dot{x}_j & \lambda^3x_j\end{array}\right]\forall j\in\mathcal{N}_i$.

The change of variable (30) can be rewritten as

$$\xi_i=\dot{x}_i+\lambda(x_i-x_i^{(d)})$$

Since $x_i^{(d)}$ is a constant reference position for $i-th$ vehicle, the third derivative is given by (31).

Defining $\tilde{e}_{ij}=x_i-x_i^{(d)}-x_j$ and differentiating w.r.t., we get $\dot{\tilde{e}}_{ij}=\dot{x}_i-\dot{x}_j$, then (54) is rewritten as

$$(\xi_i-\xi_j)=\dot{\tilde{e}}_{ij}+\lambda\tilde{e}_{ij}$$

Then $\dot{\tilde{e}}_{ij}+\lambda\tilde{e}_{ij}\in\mathcal{L}_2$ which guaranties the exponential convergence of $\tilde{e}_{ij}$ to the constant reference $x_i^{(d)}$. Assuming that the information exchange graph is strongly connected then

$$\lim_{t\to\infty}|x_i-x_j|=x_i^{(d)}$$

1) Triangular Formation: A triangular formation around a circle of radius r for the team of three quadrotor vehicles is proposed. Assuming a cyclic information exchange topology, the relative position is given by

$$\begin{aligned}x_1-x_2 &= r\cos(\pi/6) \quad (66)\\ x_3-x_1 &= -r\cos(\pi/6) \quad (67)\\ x_2-x_3 &= r\cos(\pi/2) \quad (68)\\ y_1-y_2 &= r\sin(\pi/6) \quad (69)\\ y_3-y_1 &= -r\sin(\pi/6) \quad (70)\\ y_2-y_3 &= 2r\sin(\pi/6) \quad (71)\end{aligned}$$

Therefore, we can use (66)-(71) as a relative position reference with respect to each other.

Assuming a chain information exchange topology, the relative position is given by

$$\begin{aligned}x_1-x_2 &= \cos(\pi/6) \quad (72)\\ x_2-x_3 &= \cos(\pi/2) \quad (73)\\ y_1-y_2 &= \sin(\pi/6) \quad (74)\\ y_2-y_3 &= 2\sin(\pi/6) \quad (75)\end{aligned}$$

Therefore, we can use (72)-(75) as a relative position reference with respect to each other.

C. X4 Trajectory Tracking Control

Now, we will consider the case of trajectory tracking of a multiple vehicle system. It is assumed that the leader of the group is always vehicle 1. Also, let us assume that the leader is the only vehicle with access to the desired trajectory. Then, $\bar{u}_i$ is rewritten as

$$\bar{u}_i=-\sum_{j\in\mathcal{N}_i}(w_i-w_j)+bu_{CM} \quad (76)$$

where $b^T=\left[\begin{array}{cccc}1 & 0 & \ldots & 0\end{array}\right]$ and u_{CM} is the input given to the leader. Define $w_{CM}=\frac{1}{N}\sum_{i=1}^{N}w_i$ where N is the number of agents in the formation. Let w_{CM}^dbe the desired value for w_{CM}. Assume for simplicity that agent 1 is the leader, i.e. $c^T = b^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$ and that the control law is

$$u_{CM}(w) \triangleq Nksat\left\{w_{CM}^d - w_{CM}\right\} \tag{77}$$

where $sat(\cdot)$ represents the saturation function and k is a positive gain. Note that w_{CM} may not be directly measurable for the leader (agent 1). We assume the system is observable from the input and output of the leader. The state can therefore be observed from the input and output of agent 1.

Trajectory tracking control law is such that $w_{CM} \to w_{CM}^d$ as $t \to \infty$ which implies that $\zeta \to w_{CM}^d/\lambda$, which in turn implies that $\xi \to w_{CM}^d/\lambda^2$ and $x \to w_{CM}^d/\lambda^3$.

The reference for the center of mass is defined as $w_{CM}^d = \lambda^3(x_{CM}^{(d)})$ which implies that $|x_i - x_i^d| \to x_{CM}^d$.

V. Results

A. Simulation

Here, we apply the results obtained in the previous section. Extensive simulations were run on a platoon of three rotorcraft considering the 6-DOF nonlinear dynamical model. Cyclic and chain topologies of information exchange were consider. The initial conditions for inertial position and velocity are [2,-1,0](m) and [-0.1,-0.1,0.2](m/s) for the first vehicle; [-1,2,0](m) and [-0.1,-0.2,0.3](m/s) for the second vehicle and [-1,-1,0](m) and [0.2,0.3,-0.5](m/s) for the third vehicle It is clear that the nonlinear coordinated control strategy can be used to synchronize the lateral and longitudinal dynamical subsystems (21)-(24) and (21)-(24) as well as the yaw angle system (21) of multiple mini rotorcraft. Thus, using control inputs (20), (57) and (47) on the quadrotor dynamical system (17)-(19) in simulation we get the results shown in Figures 3 and 4.

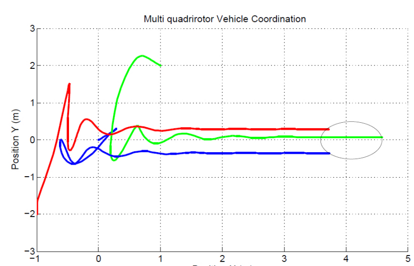

Fig. 3. Formation flying of multiple quadrirotors

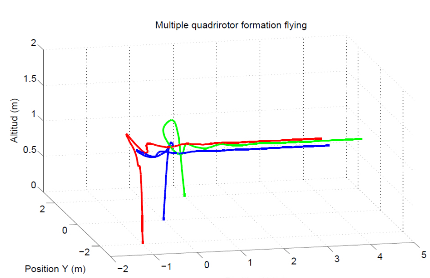

Fig. 4. Formation flying of multiple quadrirotors

VI. Conclusions and Future Work

A nonlinear dynamical model of the mini rotorcraft has been presented using the Newton-Euler formulation. Nonlinear control based on coordination control scheme for flight formation mini rotorcraft was presented. Pitch, roll and yaw angles were considered as dynamical agents with full information access. Tracking of the virtual center of mass of the agents formation has been achieved by using state feedback control. Extensive simulations were run in order to show the performance of the developed control scheme. Future work in this area includes experimental tests on mini rotorcraft with real-time embedded control systems.

References

[1] X. Chen and A. Serrani, "ISS-Based Robust Leader/Follower Trailing Control", *LNCIS 336 Group Coordination and Cooperative Control*, Springer-Verlag, Germany, 2006.

[2] R. Kristiansen, A. Loría, A. Chaillet and P.J. Nicklasson, "Output Feedback Control of Relative Translation in a Leader-Follower Spacecraft Formation", *LNCIS 336 Group Coordination and Cooperative Control*, Springer-Verlag, Germany, 2006.

[3] N.E. Leonard and E. Fiorelli, "Virtual Leaders, Artificial Potentials and Coordinated Control of Groups", *Proceedings of the 40th IEEE Conf. on Decision and Control*, 2001.

[4] R.W. Beard, J. Lawton and F.Y. Hadaegh, "A Coordination Architecture for Spacecraft Formation Control", *IEEE Transactions on Control Systems Technology*, Vol. 9, No. 6, 2001.

[5] F. Arrichiello, S. Chiaverini and T.I. Fossen, "Formation Control of Marine Vessels using the Null-Space-Based Behavioral Control", *LNCIS 336 Group Coordination and Cooperative Control*, Springer-Verlag, Germany, 2006.

[6] T. Balch, R.C. Arkin, "Behavior-based Formation Control for multirobot teams", *IEEE Transactions on Robotics and Automation*, Vol. 14, No. 6, 1998.

[7] H.G. Tanner, A. Jadbabaie and G.J. Pappas, "Stable Flocking of Mobile Agents, Part I: Fixed Topology", *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, 2003.

[8] H.G. Tanner, A. Jadbabaie and G.J. Pappas, "Stable Flocking of Mobile Agents, Part II: Dynamic Topology", *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, 2003.

[9] R. Olfati-Saber, "Flocking for Multi-Agent Dynamic Systems: Algorithms and Theory", *IEEE Transactions on Automatic Control*, Vol. 51, No.3, 2006.

[10] W. Ren, "Consensus Seeking in Multi-vehicule Systems with a Time Varying Reference State", *Proceedings of the 2007 American Control Conference*, N.Y., 2007.

[11] D.J. Lee, P.Y. Li, "Formation and Maneuver Control of Multiple Spacecraft", *IEEE Proceedings of the American Control Conference*, Denver, 2003.

[12] D.J. Lee, M.W. Spong, "Flocking of Multiple Inertial Agents on Balanced Graph", *IEEE Proceedings of the American Control Conference*, 2006.

[13] P. Hokayem, D. Stipanovic and M.W. Spong, "Reliable Control of Multi-agent Formations", *IEEE Proceedings of the American Control Conference*, New York, 2007.

[14] P. Castillo, R. Lozano and A. Dzul, "Stabilization of a mini-rotorcraft having four rotors", *IEEE Control Systems Magazine*, Vol.25, No.6, pp. 45-55, Dec. 2005.

[15] D. Lara, A. Sanchez, R. Lozano and P. Castillo, "Real Time Embedded Control System for VTOL Aircrafts: Application to Stabilize a quad-rotor helicopter", *IEEE/CCA*, 2006.

[16] W. Ren, R.W. Beard and E.M. Atkins, "Information Consensus in Multivehicule Cooperative Control", *IEEE Control Systems Magazine*, Vol. 27, No. 2, April 2007.

[17] N. Chopra, "Output Synchronization of Mechanical Systems", University of Illinois at Urbana Champaign, PhD Thesis, 2006.

[18] B.L. Stevens and F.L. Lewis, *Aircraft Control and Simulation*, John Wiley and Sons, New Jersey USA, 2003.