2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Adaptive Admittance Control of a Robot Manipulator Under Task Space Constraint

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***Abstract*— We present adaptive admittance control of a robotic manipulator, with uncertain dynamic parameters, operating in a constrained task space. To provide compliance to external forces, we generate a differentiable reference trajectory that remains in the constrained task space. Then, adaptive backstepping control, based on a time-varying asymmetric Barrier Lyapunov Function (BLF), is designed to achieve tracking of the reference trajectory while guaranteeing constraint satisfaction. The improved BLF-based control renders the entire constrained task space positively invariant. Despite transient perturbations by external forces and online parameter adaptation, practical tracking of the reference trajectory is achieved without transgression of the constrained task space. In the absence of interaction forces, asymptotic tracking of the desired trajectory is achieved.**

I. Introduction

As robotics applications migrate from controlled factory environments to unstructured human environments, the role of interaction control is becoming increasingly important, driven by the need for safe human-robot physical interactions. Consider a rehabilitation robot that is required to guide the motion of the patient's arm, and simultaneously comply with the forces exerted by the patient (Figure 1). Such interaction tasks cannot be handled by pure motion control that rejects forces exerted by the patient as disturbances.

Since the pioneering works on impedance and compliance control [1], [2], there have been many contributions to this area, including robust impedance control [3], and hybrid force/motion impedance control [4]. In parallel, adaptive control of robotic manipulators has advanced considerably in recent decades to reduce dependency on a precise knowledge of the dynamics of the robot and the environment. This has led to works on adaptive impedance control [5], [6], [7], adaptive admittance control [8], and approximation-based impedance control [9], [10].

While adaptive control and interaction control of robot manipulators have received much attention, position and orientation constraints in the operating environment are usually neglected in the control design. Violation of the constraints may result in hazards or damage. While interacting physically with humans, interactive forces can be unpredictable. Therefore, constraints need to be enforced to prevent endangerment to human safety as well as collision with itself or the surroundings. For example, the motion of a rehabilitation robot needs to be limited to avoid injuring the patient (Figure 1). As such, there is a need for rigorous handling of constraints in adaptive interaction control.

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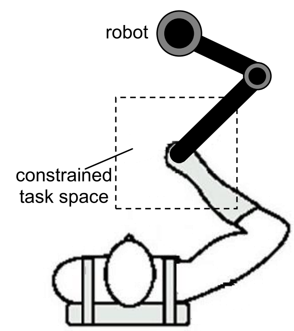

Fig. 1. Constrained task space for a rehabilitation robot.

This paper considers an uncertain robotic manipulator whose end-effector needs to track a desired trajectory while complying with external forces. Furthermore, the end-effector is to remain inside a constrained task space. A promising approach to ensure constraint satisfaction is to employ an asymmetric Barrier Lyapunov Function (BLF) when designing the control [11], [12]. In this paper, we design the asymmetric barrier limits to vary in time with the desired trajectory, unlike [11], [12] which determine a constant barrier limit according to the worst case bound of the desired trajectory over time. A significant improvement obtained with asymmetric time-varying barrier limits is that the set of feasible initial positions is maximized to the entire constrained task space. Furthermore, compliance with external forces cannot be handled with the motion control approach in [11], [12]. To tackle this problem, we shape the reference trajectory based on the external force and the constrained task space, and then track it with a control designed with the time-varying asymmetric BLF.

The remainder of this paper is organized as follows. In Section II, we present a model of the robot, and explain the idea of using a BLF for constraint satisfaction. Following that, in Section III, we present an adaptive admittance control, comprising a reference trajectory generation scheme and an improved BLF-based adaptive backstepping control. The simulation study in Section IV illustrates the performance of the control, and Section V presents concluding remarks.

II. Problem Formulation and Preliminaries

Consider a robot manipulator described by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau + \tau_e(t) \qquad (1)$$

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where $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ the Coriolis and centrifugal forces, $G(q) \in \mathbb{R}^n$ the gravitational forces, $F(\dot{q}) \in \mathbb{R}^n$ the frictional forces, $q \in \mathbb{R}^n$ the robot joint position, $\tau \in \mathbb{R}^n$ the input torque, and $\tau_e \in \mathbb{R}^n$ the interaction torque from the environment (or human). The terms $M(q)$, $C(q,\dot{q})$, $F(\dot{q})$ and $G(q)$ contain uncertain dynamic parameters.

To track a desired trajectory in task space, the joint space dynamics (1) are transformed into task space dynamics [13]:

$$M_x(x)\ddot{x} + C_x(x,\dot{x})\dot{x} + G_x(x) + F_x(\dot{x}) = f + f_e(t) \qquad (2)$$

via the forward kinematics and the Jacobian:

$$x = \Omega(q), \quad \dot{x} = \frac{\partial \Omega}{\partial q}\dot{q} =: J(q)\dot{q} \qquad (3)$$

where $x = [x_1, x_2, ..., x_m]^T$ is a vector of task variables, and the coefficient matrices are defined as

$$\begin{aligned} M_x &= J^{-T}MJ^{-1}, \quad G_x = J^{-T}G, \quad F_x = J^{-T}F \\ C_x &= J^{-T}(C - MJ^{-1}\dot{J})J^{-1}, \\ f &= J^{-T}\tau, \quad f_e = J^{-T}\tau_e \end{aligned} \qquad (4)$$

For simplicity, we consider only non-redundant $(m = n)$ non-singular manipulators with known Jacobian J in this paper. Henceforth, we consider $x \in \mathbb{R}^n$.

The following properties hold [13].

Property 1: The inertia matrix M_x is symmetric positive definite.

Property 2: The matrix $\dot{M}_x - 2C_x$ is skew symmetric.

Property 3: The left-hand-side expression of (2) can be linearly parameterized in terms of the robot system parameters as follows:

$$M_x(x)\phi_1 + C_x(x,\dot{x})\phi_2 + G_x(x) + F_x(\dot{x}) = \psi(\phi_1,\phi_2,x,\dot{x})\theta \qquad (5)$$

for any $\phi_1, \phi_2 \in \mathbb{R}^m$, where $\theta \in \mathbb{R}^l$ are constant parameters and $\psi \in \mathbb{R}^l$ is a known regressor function.

For ease of control design, denote $\eta_1 = x$, $\eta_2 = \dot{x}$, and rewrite (2) into the following form suitable for backstepping:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= M_x^{-1}(-C_x\eta_2 - G_x - F_x + f + f_e) \end{aligned} \qquad (6)$$

The control objective is to ensure that the task variable $x = [x_1, ..., x_n]^T$ tracks a desired trajectory $x_d = [x_{d1}, ..., x_{dn}]^T$ while complying with the external force $f_e(t)$. Additionally, it is required to keep all closed loop signals bounded and prevent the position constraints $|x_i(t)| < k_{c_i}$, $i = 1, ..., n$, from being violated $\forall t > 0$.

Assumption 1: There exists positive constants k_{d_i}, $i = 1, ..., n$, such that $|x_{d_i}(t)| \le k_{d_i} < k_{c_i}$, $i = 1, ..., n$, $\forall\, t \ge 0$.

Assumption 2: There exists a positive constant F_e such that $\|f_e(t)\| \le F_e$ $\forall t \ge 0$.

To prevent the robot end-effector from transgressing the constraints, we employ a Barrier Lyapunov Function [12], whose value approaches infinity as its arguments approaches some finite limits. The following lemma is used in subsequent analysis to show that the constrained task space is not transgressed.

Lemma 1: Let $\mathcal{Z} := \{\xi \in \mathbb{R}^n : |\xi_i| < 1, i = 1, ..., n\} \subset \mathbb{R}^n$ and $\mathcal{N} := \mathbb{R}^l \times \mathcal{Z} \subset \mathbb{R}^{l+n}$ be open sets. Consider the system

$$\dot{\eta} = h(t, \eta) \qquad (7)$$

where $\eta := [w, \xi]^T \in \mathcal{N}$, and $h : \mathbb{R}_+ \times \mathcal{N} \to \mathbb{R}^{l+n}$ is piecewise continuous in t and locally Lipschitz in η, uniformly in t, on $\mathbb{R}_+ \times \mathcal{N}$. Let $\mathcal{Z}_i := \{\xi_i \in \mathbb{R} : |\xi_i| < 1\} \subset \mathbb{R}$. Suppose that there exist functions $U : \mathbb{R}^l \to \mathbb{R}_+$ and $V_i : \mathcal{Z}_i \to \mathbb{R}_+$, $i = 1, ..., n$, continuously differentiable and positive definite in their respective domains, such that

$$V_i(\xi_i) \to \infty \;\text{ as }\; |\xi_i| \to 1, \quad i = 1, ..., n \qquad (8)$$

$$\gamma_1(\|w\|) \le U(w) \le \gamma_2(\|w\|) \qquad (9)$$

where γ_1 and γ_2 are class K_∞ functions. Let $V(\eta) := \sum_{i=1}^n V_i(\xi_i) + U(w)$, and $\xi(0) \in \mathcal{Z}$. If the inequality holds:

$$\dot{V} = \frac{\partial V}{\partial \eta} h \le -\mu V + c \qquad (10)$$

in the set $\xi \in \mathcal{Z}$, where μ and c are positive constants, then $\xi(t) \in \mathcal{Z}$ $\forall t \in [0, \infty)$.

Proof: The conditions on h ensure the existence and uniqueness of a maximal solution $\eta(t)$ on the time interval $[0, \tau_{\max})$, according to [14, p.476 Theorem 54]. From the fact that $\xi(0) \in \mathcal{Z}$, we know that $V_i(\xi_i(0))$, $i = 1, ..., n$, and thus $V(\eta(0))$, exist.

Integrating both sides of inequality (10), it can be shown that $V(\eta(t)) \le V(\eta(0)) + c/\mu$, $\forall t \in [0, \tau_{\max})$. From $V(\eta) = \sum_{i=1}^n V_i(\xi_i) + U(w)$ and the fact that $V_i(\xi_i)$, $i = 1, ..., n$ are positive functions, it is clear that each $V_i(\xi_i(t))$ is bounded $\forall t \in [0, \tau_{\max})$. Since $V_i(\xi_i) \to \infty$ only if $\xi_i \to \pm 1$, we conclude, from the boundedness of $V_i(\xi_i(t))$, that $|\xi_i(t)| < 1$ $\forall t \in [0, \tau_{\max})$.

Therefore, there is a compact subset $K \subseteq \mathcal{N}$ such that the maximal solution of (7) satisfies $\eta(t) \in K$ $\forall t \in [0, \tau_{\max})$. As a direct consequence of [14, p.481 Proposition C.3.6], we have that $\eta(t)$ is defined $\forall t \in [0, \infty)$. It follows that $\xi(t) \in \mathcal{Z}$ $\forall t \in [0, \infty)$. ∎

The following lemma is useful in the stability analysis to obtain the time-derivative of the BLF in the form of (10).

Lemma 2: For any positive constants k_a and k_b, the following inequality holds for all $|\xi| < 1$:

$$\log \frac{1}{1 - \xi^2} < \frac{\xi^2}{1 - \xi^2} \qquad (11)$$

III. Adaptive Admittance Control with Task Space Constraint

Admittance control involves compliant motion generation and control. The first part generates a reference trajectory, deviating from the desired trajectory, so as to achieve a target dynamic behavior of the end-effector with respect to an external interaction force. The motion control part ensures tracking of the online-generated reference trajectory with disturbance rejection. To ensure that the end-effector trajectory remains in the constrained task space for all time, we first shape the behavior of the reference trajectory near the constraint boundaries, and then employ adaptive control based on a time-varying asymmetric BLF.

A. *Reference Trajectory Shaping*

Denote $\zeta_i := \bar{x}_{r_i} - x_{d_i}$, where $\bar{x}_{r_i}$ is an intermediate variable. To compute the reference trajectory x_r, we first integrate the impedance equation

$$k_{m_i}\ddot{\zeta}_i + k_{v_i}\dot{\zeta}_i + k_{k_i}\zeta_i = f_{e_i}, \quad i = 1, ..., n \tag{12}$$

where $\zeta_i(0) = \dot{\zeta}_i(0) = 0$, and k_{m_i}, k_{v_i}, k_{k_i} are positive constants selected to provide the desired admittance at the end-effector. After solving for $\bar{x}_{r_i}$, we obtain x_{r_i} by a soft saturation function (Figure 2), defined by

$$x_{r_i} = \begin{cases} \bar{x}_{r_i} & \text{if } |\bar{x}_{r_i}| \le \beta k_{c_i} \\ -\gamma_i(1 - e^{(\bar{x}_{r_i} + \beta k_{c_i})/\gamma_i}) - \beta k_{c_i} & \text{if } \bar{x}_{r_i} < -\beta k_{c_i} \\ \gamma_i(1 - e^{(\beta k_{c_i} - \bar{x}_{r_i})/\gamma_i}) + \beta k_{c_i} & \text{if } \bar{x}_{r_i} > \beta k_{c_i} \end{cases} \tag{13}$$

for $i = 1, ..., n$, where $\gamma_i := (1 - \beta)k_{c_i}$, $0 \ll \beta < 1$ is a constant that is selected to satisfy

$$\beta k_{c_i} > k_{d_i} \ge |x_{d_i}(t)| \quad \forall t \ge 0 \tag{14}$$

where k_{d_i} is defined in Assumption 1. The soft saturation in (13) ensures that $x_r(t)$ is twice differentiable and belongs to the constrained task space.

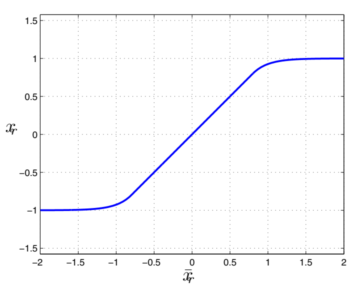

Fig. 2. The soft saturation (13) is differentiable near the corners.

Property 4: The reference trajectory $x_r(t)$ in (13) satisfies $|x_{r_i}(t)| < k_{c_i}(t)\ \forall\ t \ge 0$, and there exist positive constants Y_i, W_i, $i = 1, ..., n$, such that $|\dot{x}_{r_i}(t)| < Y_i, |\ddot{x}_{r_i}(t)| < W_i$ $\forall t \ge 0$.

Lemma 3: In the absence of an external force, i.e. $f_e = 0$, $x_r(t) \to x_d(t)$ as $t \to \infty$.

Proof: When $f_e = 0$, it follows from (12) that $\zeta(t) \to 0$ as $t \to \infty$, i.e. $\bar{x}_r(t) \to x_d(t)$ as $t \to \infty$. Since $|x_{d_i}(t)| < \beta k_{c_i}$ $\forall t \ge 0$, there exists a positive number T such that $|\bar{x}_{r_i}(t)| < \beta k_{c_i}$ $\forall t > T$, $i = 1, ..., n$. As a result, from (13), we have $x_r(t) \to \bar{x}_r(t)$ as $t \to \infty$, and thus, $x_r(t) \to x_d(t)$ as $t \to \infty$. ■

B. *Adaptive Control with Time-Varying Asymmetric BLF*

We design a control to track the reference trajectory $x_r(t)$ while respecting the task space constraint $|x_i(t)| < k_{c_i}$ $\forall t > 0$. Unlike [11], [12], which used time-invariant BLFs, we employ a *time-varying asymmetric* BLF here to maximize the set of feasible initial conditions, i.e. $|x_i(0)| < k_{c_i}$, $i = 1, ..., n$. The additional steps in the design include a change of error coordinates and a term in the stabilizing control that suppresses the effects of the time-varying barrier limits.

Step 1 Denote error $z = [z_1, ..., z_n]^T = \eta_1 - x_r$ and $v = [v_1, ..., v_n]^T = \eta_2 - \alpha$, where $\alpha = [\alpha_1, \alpha_2]^T$ is a stabilizing function to be designed shortly. Consider the time-varying asymmetric barrier function:

$$\begin{aligned} V_1 \;=\; & \frac{1}{2}\sum_{i=1}^{n}\Bigg(p(z_i)\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t) - z_i^2} \\ & + (1 - p(z_i))\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t) - z_i^2}\Bigg) \end{aligned} \tag{15}$$

where

$$k_{a_i}(t) := k_{c_i} + x_{r_i}(t) \tag{16}$$

$$k_{b_i}(t) := k_{c_i} - x_{r_i}(t) \tag{17}$$

$$p(\bullet) := \begin{cases} 1, & \text{if } \bullet > 0 \\ 0, & \text{if } \bullet \le 0 \end{cases} \tag{18}$$

for $i = 1, ..., n$. Due to Assumptions 1-2, there exist positive constants $\underline{k}_{b_1}$, $\overline{k}_{b_1}$, $\underline{k}_{a_1}$ and $\overline{k}_{a_1}$ such that

$$\begin{aligned} 0 < \underline{k}_{b_1} \le \; & k_{b_1}(t) \; \le \overline{k}_{b_1}, \quad \forall\ t \ge 0 \\ 0 < \underline{k}_{a_1} \le \; & k_{a_1}(t) \; \le \overline{k}_{a_1}, \quad \forall\ t \ge 0 \end{aligned} \tag{19}$$

By a change of error coordinates

$$\xi_{a_i} = \frac{z_i}{k_{a_i}}, \quad \xi_{b_i} = \frac{z_i}{k_{b_i}} \tag{20}$$

$$\xi_i = \begin{cases} \xi_{a_i}, & z_i \le 0 \\ \xi_{b_i}, & z_i > 0 \end{cases} \tag{21}$$

for $i = 1, ..., n$, we can rewrite (15) as

$$V_1 = \frac{1}{2}\sum_{i=1}^{n}\log\frac{1}{1 - \xi_i^2} \tag{22}$$

The time derivative of V_1 is given by

$$\begin{aligned} \dot{V}_1 \;=\; & \sum_{i=1}^{n}\Bigg[\frac{p(z_i)\xi_{b_i}}{k_{b_i}(1 - \xi_{b_i}^2)}\left(v_i + \alpha_i - \dot{x}_{d_i} - z_i\frac{\dot{k}_{b_i}}{k_{b_i}}\right) \\ & + \frac{(1 - p(z_i))\xi_{a_i}}{k_{a_i}(1 - \xi_{a_i}^2)}\left(v_i + \alpha_i - \dot{x}_{d_i} - z_i\frac{\dot{k}_{a_i}}{k_{a_i}}\right)\Bigg] \end{aligned} \tag{23}$$

Design the stabilizing functions α_i, $i = 1, ..., n$ as:

$$\alpha_i = -\kappa_{z_i}z_i + \dot{x}_{r_i} - z_i\sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda} \tag{24}$$

where λ and κ_{z_i} are positive constants. The last term of (24) is designed to dominate the term $-z_i\dot{k}_{b_i}/k_{b_i}$ when $z_i > 0$, and the term $-z_i\dot{k}_{a_i}/k_{a_i}$ when $z_i \le 0$ in (23), since

$$\sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda} + p\frac{\dot{k}_{b_i}}{k_{b_i}} + (1 - p)\frac{\dot{k}_{a_i}}{k_{a_i}} \ge 0 \tag{25}$$

Then, the time derivative of V_1 satisfies

$$\begin{aligned} \dot{V}_1 \;\le & -\sum_{i=1}^{n}\left(\frac{p(z_i)\kappa_{z_i}\xi_{b_i}^2}{1 - \xi_{b_i}^2} + \frac{(1 - p(z_i))\kappa_{z_i}\xi_{a_i}^2}{1 - \xi_{a_i}^2}\right) \\ & + \sum_{i=1}^{n}\left(\frac{p(z_i)\xi_{b_i}v_i}{k_{b_i}(1 - \xi_{b_i}^2)} + \frac{(1 - p(z_i))\xi_{a_i}v_i}{k_{a_i}(1 - \xi_{a_i}^2)}\right) \end{aligned} \tag{26}$$

From (20)-(21) and the fact that $k_{a_i}, k_{b_i} > 0$, we note that $\xi_i > 0$ when $z_i > 0$ and that $\xi_i \le 0$ when $z_i \le 0$. Hence, the first term on the right hand side of (26) is nonpositive in the set $\xi = [\xi_1, ..., \xi_n]^T \in \mathcal{Z}$, where

$$\mathcal{Z} \quad := \quad \{(\xi_1, ..., \xi_n) \in \mathbb{R}^n : |\xi_i| < 1,\ i = 1, ..., n\} \quad (27)$$

Step 2 The control τ will be designed in this step. Consider the Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} v^T M_x(\eta_1) v + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (28)$$

where $\tilde{\theta} := \hat{\theta} - \theta$, $\Gamma = \Gamma^T > 0$, and $M_x(\eta_1)$ is positive definite from Property 1. Using Properties 2 and 3, we obtain the time-derivative of V_2 as

$$\dot{V}_2 = v^T[-\psi(\dot{\alpha}, \alpha, \eta_1, \eta_2)\theta + f + f_e] + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + \dot{V}_1 \quad (29)$$

Design the adaptive control law as

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma(\psi^T v - \sigma \hat{\theta}) && (30) \\ f &= -K_v v + \psi \hat{\theta} + g - f_e && (31) \\ \tau &= J^T f && (32) \end{aligned}$$

where σ is a positive constant, $K_v := \text{diag}(\kappa_{v_1}, ..., \kappa_{v_n}) > 0$, and $g = [g_1, ..., g_n]^T$, with

$$g_i := -\left(\frac{p(z_i)\xi_{b_i}}{k_{b_i}(1 - \xi_{b_i}^2)} + \frac{(1 - p(z_i))\xi_{a_i}}{k_{a_i}(1 - \xi_{a_i}^2)} \right) \quad (33)$$

for $i = 1, ..., n$. The term g is used to cancel the coupling term in (26). Note that leakage term in (30) provides robustness against unmodelled disturbances and noisy force measurements.

Substituting (30) and (31) into (29), and using Lemma 2, we can show that

$$\dot{V}_2 \quad \le \quad -\mu V_2 + c \quad (34)$$

in the set $\xi \in \mathcal{Z}$, where $\mathcal{Z}$ is defined in (27), and

$$\begin{aligned} \mu &= \min \left\{ 2\kappa_{z_1}, ..., 2\kappa_{z_n}, 2\frac{\lambda_{\min}(K_v - \frac{1}{2}I)}{\lambda_{\max}(M_x)}, \frac{\sigma}{\lambda_{\max}(\Gamma^{-1})} \right\} \\ c &= \frac{\sigma}{2} \|\tilde{\theta}\|^2 && (35) \end{aligned}$$

where $\lambda_{\min}(\bullet)$ and $\lambda_{\max}(\bullet)$ denote the minimum and maximum eigenvalues of $(\bullet)$, respectively.

Theorem 1: Consider the robot manipulator (1) under the adaptive controller (24), (30)-(32), and Assumptions 1-2. If the initial end-effector position lies in the constrained task space, i.e.

$$|x_i(0)| < k_{c_i}, \quad i = 1, ..., n \quad (36)$$

then the following properties hold.

i) The tracking error $z = [z_1, ..., z_n]^T$ satisfies

$$-\underline{D}_{z_i}(t) \le z_i(t) \le \overline{D}_{z_i}(t), \quad i = 1, ..., n \quad \forall\ t > 0 \quad (37)$$

where

$$\begin{aligned} \overline{D}_{z_i}(t) &:= k_{b_i}(t)\sqrt{1 - e^{-2(V_n(0) + c/\mu)}} && (38) \\ \underline{D}_{z_1}(t) &:= k_{a_i}(t)\sqrt{1 - e^{-2(V_n(0) + c/\mu)}} && (39) \end{aligned}$$

ii) The end-effector position $x(t)$ satisfies $|x_i(t)| < k_{c_i}(t)$, $i = 1, ..., n$, $\forall t > 0$, i.e. the constraint is never violated.

iii) All closed loop signals are bounded.

Proof:

i) The closed loop system can be written as

$$\begin{aligned} \dot{z} &= v - [p(z_i) + (1 - p(z_i))]\kappa_{z_i} z_i \\ &\quad - z_i \sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda} \\ \dot{v} &= M_x^{-1}(\psi\tilde{\theta} - K_v v + g) \\ \dot{\hat{\theta}} &= \Gamma(\psi^T v - \sigma\hat{\theta}) && (40) \end{aligned}$$

where the right hand side is piecewise continuous in t and locally Lipschitz in $(z, v, \tilde{\theta})$, uniformly in t. Based on the definitions of k_{a_i} and k_{b_i} in (16)-(17), we rewrite the initial conditions (36) as $-k_{a_i}(0) < z_i(0) < k_{b_i}(0)$. This is equivalent to

$$|\xi_i(0)| < 1, \quad i = 1, ..., n \quad (41)$$

as follows from (20)-(21). Then, from the fact that $\dot{V}_2 \le -\mu V_2 + c$ in the set $\xi \in \mathcal{Z}$, and that $|\xi_i(0)| < 1$, we invoke Lemma 1 to establish that $|\xi_i(t)| < 1$, $i = 1, ..., n$, $\forall\ t > 0$.

From (21), consider $z_i(t) \le 0$ for some $t > 0$, which yields $-1 < \xi_{a_i}(t) \le 0$. Since $\xi_{a_i} = z_i/k_{a_i}$ for $z_i \le 0$, and $k_{a_i} > 0$, we obtain

$$-k_{a_i}(t) < z_i(t) \le 0 \quad (42)$$

for $i = 1, ..., n$. Similarly, considering $z_i(t) > 0$ for some $t > 0$ yields $0 < \xi_{b_i}(t) \le 1$ and, in turn,

$$0 < z_i(t) < k_{b_i}(t) \quad (43)$$

for $i = 1, ..., n$. Combining both cases, we conclude that

$$-k_{a_i}(t) < z_i(t) < k_{b_i}(t), \quad i = 1, 2, \quad \forall\ t > 0$$

Integrating both sides of inequality (34) yields $V_2(\eta(t)) \le V_2(\eta(0)) + c/\mu$, $\forall t > 0$. For $i = 1, ..., n$, we have

$$V_n(0) + c/\mu \ge \begin{cases} \frac{1}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t) - z_i^2(t)}, & 0 < z_i < k_{b_i} \\ \frac{1}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t) - z_i^2(t)}, & -k_{a_i} < z_i \le 0 \end{cases}$$

Taking exponentials on both sides of the inequality, it can be shown that

$$z_i^2(t) \le \begin{cases} k_{b_i}^2(t)(1 - e^{-2(V_n(0) + c/\mu)}), & 0 < z_i < k_{b_i} \\ k_{a_i}^2(t)(1 - e^{-2(V_n(0) + c/\mu)}), & -k_{a_i} < z_i \le 0 \end{cases}$$

Taking square root of both sides of the inequality, we obtain that $z_i(t) \le \overline{D}_{z_i}(t)$ for positive $z_i(t)$, and that $z_i(t) \ge -\underline{D}_{z_i}(t)$ for negative $z_i(t)$. Combining both cases, it is obvious that $-\underline{D}_{z_i}(t) \le z_i(t) \le \overline{D}_{z_i}(t)$ $\forall t \ge 0$, $i = 1, ..., n$.

ii) Since $x_i(t) = z_i(t) + x_{r_i}(t)$ and $-k_{a_i} < z_i(t) < k_{b_i}$, for $i = 1, ..., n$, we infer that

$$-k_{a_i}(t) + x_{r_i}(t) < x_i(t) < k_{b_i}(t) + x_{r_i}(t) \quad (44)$$

for all $t > 0$. From the definitions of k_{a_i} and k_{b_i} in (16) and (17) respectively, we conclude that $|x_i(t)| < k_{c_i}(t)$, $i = 1, ..., n$, $\forall t > 0$.

iii) Since $V_2(t) \leq V_2(0)+c/\mu$, we know that $\hat{\theta}(t)$ and $z_2(t)$ are bounded $\forall\ t > 0$. From (24), the stabilizing function $\alpha(t)$ is bounded, since $|\dot{x}_{r_i}(t)| < Y_i$ from Property 4, $-\underline{k}_{a_i} < z_i(t) < \overline{k}_{b_i}$, $|\dot{k}_{b_i}| = |\dot{k}_{a_i}| \leq Y_i$, and k_{a_i}, k_{b_i} are bounded away from 0. This leads to the boundedness of $\eta_2(t)$, since $\eta_2 = v+\alpha$. Since f is a continuous function of bounded signals in the set $|\xi_i| < 1$, $i = 1, ..., n$, we know that $f(t)$ is bounded $\forall\ t > 0$. Since the Jacobian J is smooth and nonsingular, $\tau(t)$ is bounded $\forall\ t > 0$. Hence, all closed loop signals are bounded. ■

Remark 1: The set of feasible initial conditions (36) is maximal in the sense that the robot end-effector is able to start from anywhere in the constrained task space, i.e. $|x_i(0)| < k_{c_i}$, and remains in the same set $\forall t > 0$. In other words, the proposed control renders the set $|x_i| < k_{c_i}$ positively invariant – a key improvement over [12].

Corollary 1: Consider the robot manipulator (1) under the special case $f_e \equiv 0$. Then the control law (24), (31)-(32), and adaptation law $\dot{\hat{\theta}} = \Gamma\psi^T v$, with initial conditions (36), ensure that the tracking error $z(t)$ converges to zero asymptotically, i.e., $x(t) \to x_d(t)$ as $t \to \infty$.

Proof: Since $f_e \equiv 0$ and $\sigma = 0$, we obtain, from (34):

$$\dot{V}_2 \quad \leq \quad -\sum_{i=1}^{n} \kappa_{z_i} \left(\frac{p(z_i)\xi_{b_i}^2}{1-\xi_{b_i}^2} + \frac{(1-p(z_i))\xi_{a_i}^2}{1-\xi_{a_i}^2} \right) =: \rho$$

Since $|\xi_i(t)| < 1$, $i = 1, ..., n$, $\forall t > 0$, it can be shown that $\lim_{t\to\infty} \int_0^t \rho(\tau)\ d\tau < \infty$ and that $\dot{\rho}(t)$ is bounded. Then, Barbalat's Lemma [15] is used to show that $\rho(t) \to 0$ as $t \to \infty$. Thus, $\xi_i(t) \to 0$ as $t \to \infty$. Since $k_{a_i}(t), k_{b_i}(t)$ are bounded away from 0 $\forall\ t > 0$, we have $\xi_i = 0 \Rightarrow z_i = 0$, and conclude that $z_i(t) \to 0$ as $t \to \infty$, $i = 1, ..., n$. By virtue of Lemma 3, we have $x_r(t) \to x_d(t)$ as $t \to \infty$. Therefore, $x(t) \to x_d(t)$ as $t \to \infty$. ■

Remark 2: The adaptation law (30) has a leakage term, which provides robustness to unmodelled disturbances and noisy force measurements. The tradeoff for this robustness is that only practical tracking of $x_r(t)$ is ensured. If the leakage term is removed, then asymptotic tracking can be achieved.

IV. Simulation

In the simulation study, we consider, for simplicity, a two-link frictionless robot moving in a horizontal plane subject to a rectangular constraint in the task space. All units are S.I. The robot dynamics are modeled by [16]

$$\begin{aligned} \tau_1 &= m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2(2\ddot{q}_1 + \ddot{q}_2) - \tau_{e_1} \\ &\quad +(m_1 + m_2)l_1^2\ddot{q}_1 - m_2 l_1 l_2 s_2 \dot{q}_2^2 - 2m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 \\ \tau_2 &= m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2 \ddot{q}_1 + m_2 l_1 l_2 s_2 \dot{q}_1^2 - \tau_{e_2} \end{aligned} \quad (45)$$

where $c_i = \cos(q_i)$, $s_i = \sin(q_i)$, $s_{ij} = \sin(q_i + q_j)$, and $c_{ij} = \cos(q_i + q_j)$. The uncertain parameters are m_1 and m_2, whose true values are $1.5kg$ and $1.0kg$ respectively. The lengths of the links are $l_1 = l_2 = 0.3m$.

For the rectangular constraint region, $x_1(t)$ and $x_2(t)$ are to satisfy

$$|x_1(t)| < k_{c_1} = 0.15, \quad |x_2(t)| < k_{c_2} = 0.15, \quad \forall\ t \geq 0$$

where the origin of the task variable $x = [x_1, x_2]^T$ is at $[0, 0.3]^T$m with respect to the position of the base joint. Initially, the end-effector is at rest at the origin, i.e. $x(0) = [0, 0]^T$, $q(0) = [0.5236, 2.0944]^T$. The design parameters are $\lambda = 1.0$, $\Gamma = 1.0I$, $K_v = 1.0I$, $K_z = 4.0I$, $\beta = 0.97$, $\sigma = 0.01$, $k_{m_i} = k_{v_i} = 20$, and $k_{k_i} = 100$, $i = 1, 2$.

The desired trajectory traces a circular path in the task space, and is described by

$$\begin{aligned} x_{d1}(t) &= 0.14\cos(0.5t) \\ x_{d2}(t) &= 0.14\sin(0.5t) \end{aligned} \quad (46)$$

The external force $f_e = [f_{e_1}, f_{e_2}]^T$, as shown in Figure 3, is described by

$$f_{e_i}(t) = \begin{cases} 0 & t < 10 \text{ or } t \geq 21 \\ a_i(1-\cos\pi t) & 10 \leq t < 11 \\ 2a_i & 11 \leq t < 20 \\ a_i(1+\cos\pi t) & 20 \leq t < 21 \end{cases} \quad (47)$$

where $a_1 = 1$ and $a_2 = 2$.

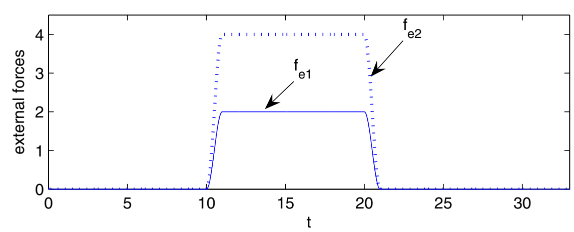

Fig. 3. The external force f_e has an onset at $t = 10s$ and is removed at $t = 21s$.

Figure 5 shows that despite the desired trajectory approaching close to the boundaries of the constrained region, and despite the transients of online parameter estimation as well as the effects of external forces, the robot end-effector never transgresses the position constraints in task space, i.e. $|x_1(t)| < 0.15$ and $|x_2(t)| < 0.15\ \forall\ t > 0$. Initial fluctuations in the trajectories are caused by parametric uncertainty, but these are quickly minimized such that close tracking of $x_r(t) = x_d(t)$ is achieved. Upon the onset of the external force at $t = 10s$, $x(t)$ deviates from $x_d(t)$ to track $x_r(t)$. However, after the external force is removed at $t = 21s$, tracking of $x_d(t)$ resumes.

Figure 6 shows that the tracking errors z_i $i = 1, 2$, converge to zero and never transgresses the asymmetric and time-varying barriers, i.e. $-k_{a_i}(t) < z_i(t) < k_{b_i}(t)\ \forall\ t > 0$.

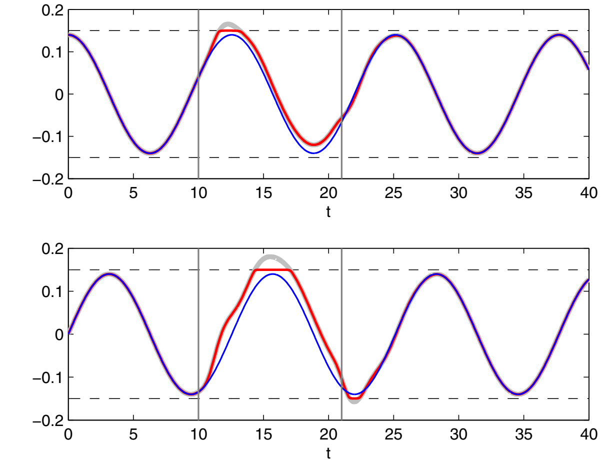

Fig. 4. The reference trajectory $x_r(t)$ (red), which deviates from the desired trajectory $x_d(t)$ (blue), is obtained by saturating $\bar{x}_r(t)$ (thick grey). Vertical lines indicate onset and offset of external force.

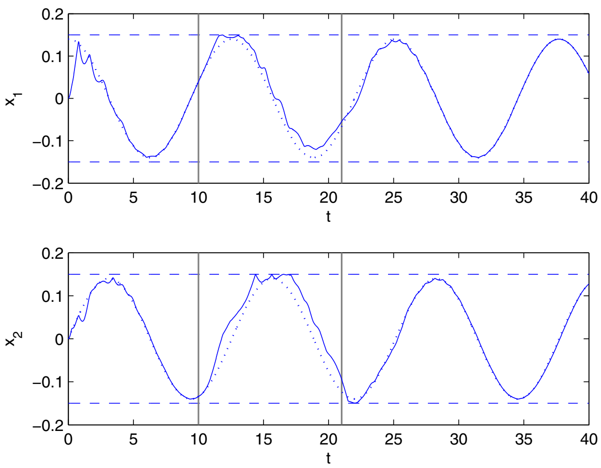

Fig. 5. The end-effector tracks the reference trajectory $x_r(t)$ within the constrained task space. It deviates from the desired trajectory $x_d(t)$ (dotted line) upon the onset of the external force (t=10s), and rejoins $x_d(t)$ after the external force is removed (t=21s).

V. Conclusions

We have presented adaptive admittance control of uncertain robot manipulator under task space constraint. To provide compliance to external forces, a reference trajectory has been shaped by solving an impedance equation online, followed by a soft saturation. Subsequently, adaptive control based on a time-varying asymmetric BLF has been designed to track the reference trajectory while satisfying the constraints. By incorporating both asymmetric and time-varying barrier limits, we have maximized the set of feasible initial positions. We have shown that practical tracking of the reference trajectory is achieved without transgression of the constrained task space. When the external forces vanish, asymptotic tracking of the desired trajectory is guaranteed. The performance of the proposed adaptive control has been illustrated through a simulation.

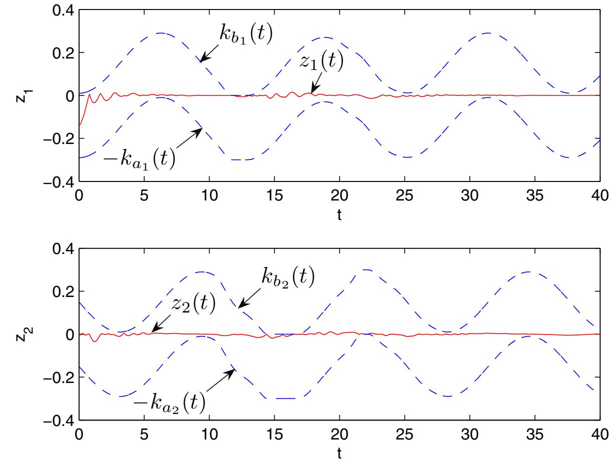

Fig. 6. The tracking errors converge to a neighborhood of zero and never transgresses the time-varying barriers.

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