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Peer-to-Peer Control Architecture for Multiuser Haptic Collaboration over Undirected Delayed Packet-Switching Network

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***Abstract*— We propose a novel peer-to-peer distributed control architecture for shared haptic collaboration among remotely-located users over undirected packet-switching network (e.g. Internet) with inter-user communication delay. The proposed architecture is distributed, in that each user simulates and interacts with its own local copy of the shared virtual environment. Spring connection among the local copies and local damping are used, which, together, under a certain condition, achieve configuration synchronization among the local copies while enforcing discrete-time passivity of the total peer-to-peer architecture, thereby, rendering the architecture portable/scalable for any (passive) users/devices and ensuring its interaction stability be user/device-invariant. The issue of optimizing communication network is also addressed with some relevant experimental results.**

I. Introduction

Multiuser shared haptic collaboration among remotely-located users over some packet-switching communication networks (e.g. Internet) would enable us to achieve many powerful applications: virtual collaborative surgical training, haptic evaluation of virtual mechanical components, virtual sculpting among remote artists, and haptically-enabled networked computer games, to name a few. Perhaps, even more importantly, this idea of multiuser haptic collaboration may revolutionize our way of interacting with each other in the cyberspace, by complementing with the current vision and audio interaction modalities.

In this paper, relying on our recent results of [1], we propose a novel peer-to-peer control architecture for this problem, which is depicted in Fig. 1 and can be summarized as follows. First, to achieve (near) **real-time** responsiveness of haptic feedback for each remote user, we make our architecture peer-to-peer and distributed, that is, each user simulates and interacts with its own local copy of the shared virtual environment (VE), possibly with 3-D (dimensional) deformable virtual objects (e.g. human organ in surgical simulation) inside it. We then connect these local (discrete-time) VE copies via a (discrete-time) spring connection with local damper so that the local copies' configuration can be synchronized with each other, thereby, providing **consistent** haptic experience among the distributed users. For this, we assume that the communication network among the users is undirected (i.e. if the user i receives data from the user j, so does the user j from the user i). We also assume that

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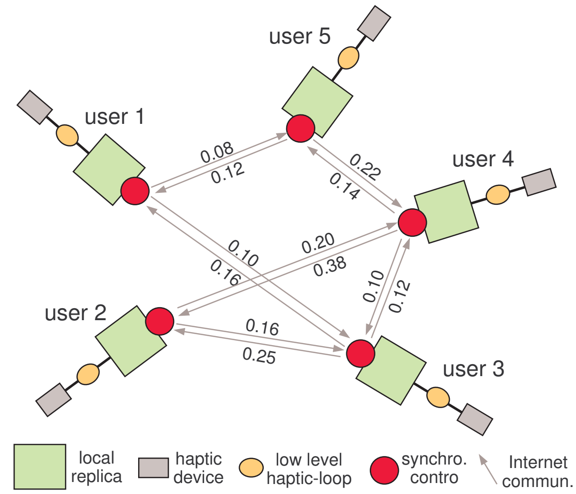

Fig. 1. Distributed peer-to-peer (p2p) multiuser haptic collaboration architecture.

the inter-user communication undergoes constant (indexing) delay (e.g. with some data buffering), although this delay may be asymmetric (i.e. delay from user i to j is not the same as that from user j to user i). See Fig. 1, where numbers on each link represent the communication time delay (= [data indexing delay] $\times$ [data update interval]).

Yet, it is well-known that those spring synchronization connections, once established with the delays, can easily become unstable. The issue of stability is even more challenging here, since the total architecture in Fig. 1 needs to be mechanically coupled with a wide-range of (often unmodeled, uncertain, complicated) human users and haptic devices. To address this interaction stability issue with the delays, we enforce discrete-time passivity of the total peer-to-peer architecture of Fig. 1. More specifically, we utilize: 1) our recent result of [1], which extends the (PD) proportional-derivative scheme of [2] into the discrete-domain, to achieve passivity-enforcing synchronization among the VEs with the delays; and 2) the non-iterative variable-rate passive mechanical integrators (NPMIs) of [3] to passively simulate the local VE copy (e.g. 3D deformable virtual objects). With this passivity, our proposed architecture can: 1) enforce interaction stability for any (passive) humans users and haptic devices (i.e. user/device-invariant stability and scalability); and 2) allow all users to simulate the same VE even if their devices are heterogeneous (i.e. portability - from separation of VE simulation and device servo-loop [4]).

Some results and frameworks have been reported for this multiuser haptic shared collaboration over the packet-

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switching network. Yet, most of them are rather experimental and qualitative (e.g. [5], [6], [7], [8], [9], [10]), without (often useful/important) theoretical stability or performance measures. The works of [11], [12] may be considered as exceptions. Yet, some of the important features of the multiuser shared haptic interaction, which are fully incorporated in this paper, were not considered there (e.g. stability with unknown heterogeneous users/devices; communication delay and network topology; portability and scalability, etc).

The rest of the paper is organized as follows. We review some graph theory and NPMIs of [3] in Sec. II. The peer-to-peer architecture with the synchronization loop is proposed and analyzed in Sec. III. Network topology optimization is performed and related experimental results are discussed in Sec. IV. Concluding remarks and some comments on future research are given in Sec. V.

II. Preliminary

A. Graph Theory

We use an undirected graph to describe the inter-user communication network topology in Fig. 1, with each user as the node and their communication link as the edge of the graph. In particular, we consider an un-weighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} := \{v_1, \ldots, v_N\}$ is the set of nodes (i.e. users); and $\mathcal{E}$ refers to the set of edges (i.e. their communication links). By the edge $e_{ij} := (v_i, v_j) \in \mathcal{E}$, we mean that the information flows from v_i to v_j. Since $\mathcal{G}$ is undirected, $e_{ij} \in \mathcal{E}$ implies $e_{ji} \in \mathcal{E}$. The neighborhood of a node v_i is also defined as $\mathcal{N}_i := \{j \mid e_{ij} \in \mathcal{E}\}$.

The graph Laplacian $\mathcal{L} = \{l_{ij}\} \in \Re^{N \times N}$ of $\mathcal{G}$ is defined by

$$l_{ij} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -1 & \text{if } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$

where $\deg(v_i)$ refers to the degree (number of edges) of v_i. It is well-known that, if $\mathcal{G}$ is undirected and connected, the zero eigenvalue of $\mathcal{L}$ is simple with the eigenvector $\mathbf{1} := [1 \ldots 1]^T \in \Re^N$, and all the other eigenvalues have strictly positive real parts [13].

To address the inter-user communication delay, we assign a constant indexing (integer) delay $N_{ji} \geq 0$ to each edge $e_{ij} \in \mathcal{E}$. Note that this constant indexing delay N_{ji} does not necessarily imply constant time delay, if the data update interval is varying (e.g. variable $T_i(k)$ in Sec. II-B). With a suitably-defined variable $T_i(k)$, we may convert variable time delay into constant indexing delay, although details for this we left for future research.

B. Non-Iterative Passive Mechanical Integrator (NPMI)

In this paper, we utilize the non-iterative variable-rate passive mechanical integrator (NPMI) proposed in [3] to simulate the local copy of the **shared** VE. In particular, we consider a linear $3n$-dimensional mass-spring-damper type deformable virtual object as the shared VE, for which the NPMI description can be written as following: during the integration step $T(k) > 0$, with $x(k), v(k), f(k)$ given,

$$\begin{aligned} &Ma(k) + B\hat{v}(k) + K(\hat{x}(k) - x_d) = f(k) \\ &a(k) := \frac{v(k+1) - v(k)}{T(k)} \\ &\hat{v}(k) := \frac{v(k+1) + v(k)}{2} = \frac{x(k+1) - x(k)}{T(k)} \\ &\hat{x}(k) := \frac{x(k+1) + x(k)}{2} \end{aligned} \tag{1}$$

where $k > 0$ is the integration index; $x_\star, v_\star, a_\star, f_\star \in \Re^{3n}$ are respectively the configuration, velocity, and acceleration of the virtual object's nodes $x^j(k) \in \Re^3$[1] with $x(k) = [x^1(k); x^2(k)....; x^n(k)] \in \Re^{3n}$ (here, we use MatLab appending-like notation), and external force (e.g. virtual coupling [14]; PSPM coupling [15]; contact within the VE); $M \in \Re^{3n \times 3n}$ is the symmetric and positive-definite nodes' mass matrix; and $B, K \in \Re^{3n \times 3n}$ are symmetric damping and spring matrices often decomposable s.t.

$$B := B_{\text{int}} + B_{\text{ext}}, \quad K := K_{\text{int}} + K_{\text{ext}} \tag{2}$$

where the matrix $\star_{\text{int}}$ defines the inter-nodes connection among x^j with the property like that of the graph Laplacian $\mathcal{L}$ in Sec. II-A; while $\star_{\text{ext}}$ is a positive diagonal matrix, binding some nodes x^j to the mechanical ground x_d via its non-zero diagonal terms (with the other terms being all zero).

This NPMI virtual object simulation (1) is implicit, yet, still non-iterative, thus, can be simulated haptically fast. Also, unlike other explicit integrators frequently used in haptics (e.g. [16]), it possesses (open-loop) discrete-time passive [3]:

$$\sum_{k=0}^{\bar{M}} \hat{v}^T(k) f(k) T(k) \geq E(\bar{M}+1) - E(0) \geq -E(0)$$

for all $\bar{M} \geq 0$, where, using the notation that, for $y \in \Re^m$ and $A \in \Re^{m \times m}$ with $A = A^T$, $||y||_A := \sqrt{y^T A y}$, $E(k) := ||v(k)||_M^2/2 + ||x(k) - x_d||_K^2/2$, i.e. the total energy of the virtual object. This open-loop passivity of the NPMI was crucial for us to extend the (continuous-time) PD-scheme of [2] to the discrete-time domain in [1], the result of which will be used in the next Sec. III to achieve passivity-enforcing synchronization among the VEs' configuration with the inter-user communication delay.

III. Peer-to-Peer Control Architecture on Undirected Delayed Network

In our peer-to-peer architecture in Fig. 1, each user will simulate its own local copy of the shared VE (1), while their configurations (i.e. $x_i(k)$ for the i^{th} user) are synchronized with each other. Now, suppose that the synchronization is perfect so that the behaviors of the (distributed) N local copies of the virtual object (1) are exactly coordinated with each other. Then, if a single user tries to deform its own local

[1]This x^j is the node of the virtual object's mesh and should not be confused with v_i denoting users in Sec. II-A.

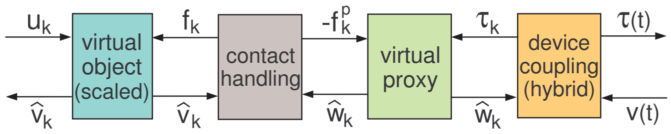

Fig. 2. Network representation of haptic rendering sub-modules.

copy of the virtual object with all the other users not touching their copies, s/he needs to make the same deformation for all the N copies of the virtual object. This implies that, the larger the number of user N gets, the more difficult for each user to move/deform the shared virtual object.

To avoid this problem, here, we utilize the N-scaled virtual object (1), that is, instead of simulating (1), the i^{th} user will simulate its N-scaled version given by:

$$
\begin{aligned}
&\frac{1}{N}\left[Ma_i(k) + B\hat{v}_i(k) + K(\hat{x}_i(k) - x_d)\right] = u_i(k) + f_i(k) \\
&a_i(k) := \frac{v_i(k+1) - v_i(k)}{T_i(k)} \\
&\hat{v}_i(k) := \frac{v_i(k+1) + v_i(k)}{2} = \frac{x_i(k+1) - x_i(k)}{T_i(k)} \\
&\hat{x}_i(k) := \frac{x_i(k+1) + x_i(k)}{2}
\end{aligned}
\tag{3}
$$

where $\star_i$ is the local variable of the user i. Here, note that: 1) the integration step $T_i(k) > 0$ can be variable and also non-uniform across the users; and 2) the presence of M, B, K, x_d to simulate the shared VE among the users. Also, $u_i(k) \in \Re^{3n}$ is the synchronization control defined s.t.: following [1],

$$
u_i(k) := -B_i\hat{v}_i(k) - \sum_{j\in\mathcal{N}_i} K_{ji}\left(\hat{x}_i(k) - \hat{x}_j(k - N_{ij})\right)
$$

where $B_i, K_{ij} \in \Re^{3n\times 3n}$ are the symmetric and positive definite local damping and synchronization spring gains, $\mathcal{N}_i$ is the communication neighbors of the user i, and $N_{ij} \in \mathbb{Z}$ is the constant indexing delay on e_{ji} (i.e. from user j to user i). For K_{ij}, we assume that

$$
K_{ij} = K_{ji}
$$

i.e. symmetric spring connection, even if their delays may not be so (i.e. $N_{ij} \neq N_{ji}$). This is to endow the peer-to-peer architecture in Fig. 1 with a level of flexibility, although asymmetric K_{ij} will provide a higher level of flexibility.

Then, for each user, we interface this local copy of the N-scaled virtual object (3) under the synchronization control $u_i(k)$ with the local haptic device as shown in Fig. 2. First, we use the virtual proxy, simulated with the following NPMI integration equation: for the i^{th} user, similar to (3),

$$
\begin{aligned}
&m_i\frac{w_i(k+1) - w_i(k)}{T_i(k)} = \tau_i(k) - \sum_{p\in\mathcal{C}_i} f_i^p(k) \\
&\hat{w}_i(k) := \frac{w_i(k+1) + w_i(k)}{2} = \frac{y_i(k+1) - y_i(k)}{T_i(k)} \\
&\hat{y}_i(k) := \frac{y_i(k+1) + y_i(k)}{2}
\end{aligned}
$$

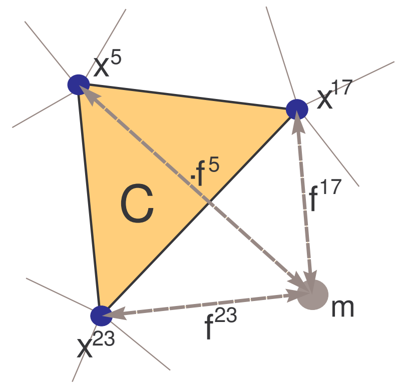

Fig. 3. Contact mesh $\mathcal{C}$, contact node $x^\star$, contact force $f^\star$ and virtual proxy m.

where $m_i > 0$, and $y_i(k), w_i(k) \in \Re^3$ are respectively the mass, position, velocity of the virtual proxy; $\tau_i(k) \in \Re^3$ is the coupling force connected to the (continuous) haptic device via the hybrid device coupling block (e.g. virtual coupling or PSPM coupling); $T_i(k)$ is the integration step same as that in (3); and $f_i^p(k) \in \Re^3$ is the contact force between the virtual object (3) and the virtual proxy, defined s.t.

$$
f_i^p(k) := \begin{cases} -b_i[\hat{v}_i^p(k) - \hat{w}_i(k)] & \text{if } m_i \in \text{int}(\mathcal{VO}) \\ \quad -k_i[\hat{x}_i^p(k) - \hat{y}_i(k)] & \\ 0 & \text{otherwise} \end{cases}
$$

where $m_i \in \text{int}(\mathcal{VO})$ means the virtual proxy m_i is contained within the interior of the virtual object and $\mathcal{C}_i$ is the "contact mesh" of the virtual object (3) with $x_i^p(k)$, $p \in \mathcal{C}_i$, being the "contact" nodes - see Fig. 3. Here, note that only those "contact" nodes in $\mathcal{C}_i$ interact with the virtual proxy. This implies that, for $f_i(k) = [f_i^1(k); f_i^2(k)...; f_i^n(k)] \in \Re^{3n}$, $f_i^j(k) = 0$ if $j \notin \mathcal{C}_i$ (i.e. non-contact nodes).

We assume that all the tasks related to this contact (e.g. contact detection, energy switching due to contact-on and contact-off) are embedded in the contact handling block in Fig. 2. This issue of contact handling is an important and challenging problem on its own in haptics and general mechanical simulation [17]. Since our paper mainly focuses on the peer-to-peer architecture for multiuser haptics, we do not consider this (low-level) contact handling issue any further here and just assume that a (discrete-time) passivity-enforcing contact handling algorithm is in place (e.g. [3] for 1-dim. virtual-wall VE). This assumption, however, is readily granted if the contact is always on (with $\mathcal{C}_i$ invariant) or off. For instance, for the user i, with the contact on, using the property of NPMI, we can easily show that: $\forall \bar{M} \geq 1$,

$$
\begin{aligned}
&\sum_{k=0}^{\bar{M}-1}[\hat{v}_i^T(k)f_i(k) - \sum_{p\in\mathcal{C}_i}\hat{w}_i^T(k)f_i^p(k)]T_i(k) \\
&\leq -\sum_{p\in\mathcal{C}_i}\frac{k_i}{2}||x_i^p(\bar{M}) - y_i(\bar{M})||^2 + \sum_{p\in\mathcal{C}_i}\frac{k_i}{2}||x_i^p(0) - y_i(0)||^2
\end{aligned}
$$

which is upper-bounded by the most right term (i.e. initial energy); or zero if the contact is off (with $f_i(k) = 0$).

We also assume that the device coupling block in Fig. 2 is two-port hybrid passive [15], [18]: $\forall \bar{t} \geq 0$, $\exists\ \bar{M} \geq 0$ and a bounded $d \in \Re$ s.t.

$$\int_0^{\bar{t}} \tau^T(t) v(t) dt + \sum_{k=0}^{\bar{M}} \tau_i^T(k) \hat{v}_i(k) \leq d^2$$

where $\bar{M}$ is defined s.t. $T_i(\bar{M})$ is contained within $[0, \bar{t})$, yet, $T_i(\bar{M}+1)$ is not. This is indeed achievable, e.g. by using the PSPM coupling [15] or by extending the result of [18] for any multi-DOF nonlinear haptic device connected to $(\tau(t), v(t))$ in Fig. 2.

Then, the passivity of the total peer-to-peer architecture in Fig. 1 now hinges on whether the virtual object block (with the synchronization control $u_i(k)$) in Fig. 2 possesses (one-port) discrete-time passivity or not. For this, we utilize our recent result of [1] and obtain the following Th. 1, which summarizes some key properties of our peer-to-peer multiuser haptic collaboration architecture.

Theorem 1 *Consider N distributed users on $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with constant indexing delays N_{ij}, local copy of the shared VE* (3)*, synchronization control $u_i(k)$, and other haptic rendering sub-modules of Fig. 2. Suppose that the synchronization gains B_i, K_{ij} are set to be*

$$B_i \geq \sum_{j \in \mathcal{N}_i} \frac{N_{ij} + N_{ji}}{2} \max_k [T_j(k)] K_{ij}$$

for all users following [1]. Also, assume that $v_i(k) = \hat{v}_i(k) = 0\ \forall k < 0$. Then,

1) *collection of all the (N-scaled) virtual objects with $u_i(k)$* (3) *is N-port discrete-time passive: $\forall \bar{M} > 0$*

$$\sum_{i=1}^{N} \sum_{k=0}^{\bar{M}} \hat{v}_i(k)^T f_i(k) T_i(k) \geq -c^2 \tag{4}$$

where $c \in \Re$ is a bounded constant;

2) *if each user's virtual object* (3) *contains extra positive-definite damping $B_i^e \in \Re^{3n \times 3n}$ and $f_i(k) = 0$,*

$$\left[\mathcal{P} + I_{N \times N} \otimes \frac{K}{N}\right] (x(k) - 1_N \otimes x_d) \rightarrow 0 \tag{5}$$

where $x(k) = [x_1(k); x_2(k), ..., x_N(k)] \in \Re^{3nN}$, $1_N = [1; 1...; 1] \in \Re^N$, $\otimes$ is the Kronecker product, and $\mathcal{P} \in \Re^{3nN \times 3nN}$ is the stiffness matrix [1] with its ij^{th} block $P_{ij} \in \Re^{3n \times 3n}$ given by

$$P_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i} K_{ij} & i = j \\ -K_{ij} & i \neq j \text{ and } e_{ij} \in \mathcal{E} \\ 0_{3n \times 3n} & \text{otherwise} \end{cases}$$

3) *if $v_i(k) \rightarrow 0$ for all users,*

$$\sum_{i=1}^{N} f_i(k) \rightarrow K(\bar{x} - x_d) \tag{6}$$

where $x_i \in \Re^{3n}$ is defined s.t. $x_i(k) \rightarrow x_i$ (with $v_i(k) \rightarrow 0$) and $\bar{x} := (x_1 + x_2 + ...x_N)/N \in \Re^{3n}$.

Proof: For the first item, then, using a similar procedure for the proof in [1], we can show that: for all $\bar{M} \geq 0$,

$$\sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} \hat{v}_i^T(k) f_i(k) T_i(k) \tag{7}$$

$$\geq V(\bar{M}+1) - V(0) + \sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} ||\hat{v}_i(k)||_B^2 T_i(k) - b^2$$

where $b \in \Re$ is a bounded constant (see [1] for the expression of this b) and

$$V(k) := \sum_{i=1}^{N} [\kappa_i(k) + \varphi_i(k) + \sum_{j \in \mathcal{N}_i} \varphi_{ij}(k)]$$

with $\kappa_i(k) := ||v_i(k)||_{M/N}^2/2$ (i.e. kinetic energy of (3)), $\varphi_i(k) := ||x_i(k) - x_d||_{K/N}^2/2$ (i.e. potential energy of (3)) and $\varphi_{ij} := ||x_i(k) - x_j(k)||_{K_{ij}}^2/4$ (i.e. half of synchronization spring energy in e_{ij}). It is then immediate to achieve the N-port passivity (4) from (7) with $c^2 := b^2 + V(0)$.

For the second item, with extra B_i^e and $f_i(k) = 0$, the inequality (7) becomes

$$0 \geq \sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} ||\hat{v}_i(k)||_{B+B_i^e}^2 T_i(k) - V(0) - b^2 \tag{8}$$

$\forall \bar{M} \geq 0$. Since $V(0) + b^2$ is bounded, $\sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} ||\hat{v}_i(k+1)||_{B+B_e^i}^2$ is upper bounded, implying that, with $B + B_e^i$ being positive-definite, $||\hat{v}_i(k)|| \rightarrow 0$. Then, from (3) with $\hat{v}_i(k) \rightarrow 0$, $x_i(k+1) \rightarrow x_i(k)$ and $\hat{x}_i(k+1) \rightarrow \hat{x}_i(k)$. This implies that the term with K and $u_i(k)$ in (3) also converge, i.e. $K(\hat{x}_i(k+1) - x_d) \rightarrow K(\hat{x}_i(k) - x_d)$ and $u_i(k+1) \rightarrow u_i(k)$. Applying these to (3) for $T_i(k)$ and $T_i(k+1)$ integration steps, we then achieve:

$$\frac{v_i(k+1) - v_i(k)}{T_i(k)} \rightarrow \frac{v_i(k+2) - v_i(k+1)}{T_i(k+1)}$$

where $v_i(k+2) \rightarrow v_i(k)$ since $\hat{v}_i(k) \rightarrow 0$. Thus, we have $v_i(k+1) \rightarrow v_i(k)$, which, with $\hat{v}_i(k) \rightarrow 0$, implies $v_i(k) \rightarrow 0$. By inserting this back into (3) with $f_i(k) = 0$, we can further achieve

$$\frac{K}{N}(x_i(k) - x_d) + \sum_{j \in \mathcal{N}_i} K_{ij} (x_i(k) - x_j(k)) \rightarrow 0 \tag{9}$$

$\forall i \in \{1, 2, ..., N\}$, which can be rewritten as (5).

For the third item, similar to (9), by using $v_i(k) \rightarrow 0$ and x_i defined above for the dynamics (3), we have

$$\frac{K}{N}(x_i - x_d) + \sum_{j \in \mathcal{N}_i} K_{ij} (x_i - x_j) \rightarrow f_i(k)$$

Summing this up, we then have:

$$\sum_{i=1}^{N} f_i(k) \rightarrow K(\bar{x} - x_d) + \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} K_{ij}(x_i - x_j)$$

where the right most term is zero, since, due to $\mathcal{G}(\mathcal{V}, \mathcal{E})$ being undirected and $K_{ij} = K_{ji}$, for each K_{ab}, $a, b, = 1, ..., N$, we only have two terms, $K_{ab}(x_a(k) - x_b(k))$ and

$K_{ba}(x_b(k) - x_a(k))$, with their sum being zero. ■

The N-port passivity (4), along with the passivity of other blocks in Fig. 2, guarantees passivity of the total peer-to-peer architecture in Fig. 1, thereby, allowing us to achieve interaction stability with *any* passive haptic device and human users, regardless how unknown, uncertain, complicated, or heterogeneous they are (i.e. user/device-invariant stability and scalability). This also enables us to separate the discrete VE simulation design from the haptic device servo-loop as originally envisioned in [4] (i.e. portability).

The property (6) shows that our peer-to-peer architecture captures the peculiarity of the multiuser shared haptic collaboration, that is: 1) multiple users together induce average deformation $\bar{x}$ in the shared virtual object; 2) if they somehow balance with each other with $\bar{x} = x_d$ (e.g. pushing in opposite directions), their force sum will be zero (e.g. holding together); and 3) if only the i^{th} user pushes the object with perfect synchronization (i.e. $x_i(k) = x_j(k)$), $f_i \to K(x_i - x_d)$ just as in the case of single user haptic interaction.

On the other hand, the condition (5) implies that, when all the virtual objects are released with B_e^i, $x(k) = [x_i(k); x_2(k); ...; x_N(k)]$ will converge to the set null$(\mathcal{P}) \cap$ null$(I \otimes K)$, where the former shows the synchronization effect (due to $u_i(k)$) while the latter that of the local spring K of (3). For instance: 1) if $K = 0$ for (3) and $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is connected, the synchronization will still push $x_i(k) \to x_j(k) \to d$ with an unspecified $d \in \Re^{3n}$ [1, Cor. 1]; 2) if $K_{\text{ext}} = 0$ and K_{int} defines a connected (undirected) graph in (2), $x_i(k) \to x_j(k) \to cI_{3n}$ with an unspecified $c \in \Re$ from the property of the graph Laplacian (e.g. Sec. II-A); and 3) if $K_{\text{ext}} \neq 0$ with connected K_{int}, K becomes positive definite, thus, $x_i(k) \to x_d$, with the synchronization possibly speeding up the convergence speed.

IV. Network Optimization and Experiments

We perform experiments for the case of five users with 1-dim. VE. We use the following network parameters:

$$\text{spring} = \begin{bmatrix} 0 & 60 & 50 & 40 & 100 \\ 60 & 0 & 25 & 75 & 100 \\ 50 & 25 & 0 & 50 & 40 \\ 40 & 75 & 50 & 0 & 75 \\ 100 & 100 & 40 & 75 & 0 \end{bmatrix}$$

$$\text{delay} = \begin{bmatrix} 0 & 0.1 & 0.8 & 0.8 & 1.0 \\ 0.05 & 0 & 0.05 & 0.05 & 0.8 \\ 0.8 & 0.04 & 0 & 0.05 & 0.8 \\ 0.8 & 0.04 & 0.05 & 0 & 0.05 \\ 1.0 & 0.8 & 0.8 & 0.04 & 0 \end{bmatrix}$$

where ij-th component of the spring and delay matrices are respectively associated to the synchronization K_{ij} and N_{ij}.

Given this underlying network characteristics, a question of immediate importance is which network topology $\mathcal{G}(\mathcal{V}, \mathcal{E})$ should be chosen. For this, here, we perform network optimization to achieve the maximum speed of information propagation among the users (i.e. fastest mixing graph [19]).

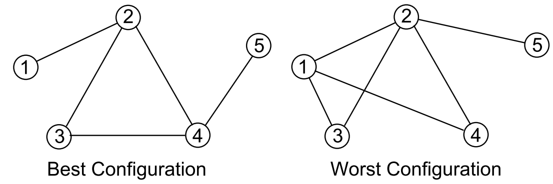

Fig. 4. Best and worst network topology designs

We also assume that, although our virtual object (3) defines second-order systems, this information propagation can be adequately captured by a first-order information propagation model with the spring coefficient K_{ij} defining the strength of the information mixing of e_{ij} with the delays N_{ij}, N_{ji}. In fact, as shown by the results below, this assumption appears to be a reasonable one.

More precisely, we use the following widely-used first-order consensus equation:

$$p_i(k+1) = \left(\sum_{j \in \mathcal{N}_i} K_{ij} \right)^{-1} \sum_{j \in \mathcal{N}_i} K_{ij} p_j(k + 1 - N_{ij})$$

where $p_i(k) \in \Re$ defines an abstract state of information of the user i at the time-index k. From some initial conditions, if this protocol brings all $p_i(k)$ to the same value, we may then say the information is fully propagated within the N users. Moreover, the optimal network topology would be the one, that achieves this consensus with the fastest speed.

This first-order information mixing equation can then be "lifted" into the following matrix form:

$$\begin{pmatrix} p(k+1) \\ p(k) \\ \vdots \\ p(k+1-\bar{N}) \end{pmatrix} = J(\mathcal{G}, K_{ij}, N_{ij}) \begin{pmatrix} p(k) \\ p(k-1) \\ \vdots \\ p(k-\bar{N}) \end{pmatrix}$$

where $p(k) := [p_1(k); p_2(k) \ldots ; p_N(k)] \in \Re^N$; $\bar{N} = \max_{ij}(N_{ij})$,

$$J(\mathcal{G}, K_{ij}, N_{ij}) := \begin{bmatrix} A_0 & A_1 & \ldots & A_{\bar{N}-1} & A_{\bar{N}} \\ I & 0 & \ldots & 0 & 0 \\ 0 & I & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & I & 0 \end{bmatrix}$$

and $A_k \in \Re^{N \times N}$ has its ij-th component as given by

$$A_{kij} = \begin{cases} \left(\sum_{j \in N_i} k_{ij} \right)^{-1} k_{ij} & \text{if } k = N_{ij} \text{ and } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$

where $k_{ij} > 0$ is the spring coefficient of e_{ij}.

Here, the matrix $J(\mathcal{G}, K_{ij}, N_{ij})$ defines the information propagation among the users. We also found that, with connected $\mathcal{G}(\mathcal{V}, \mathcal{E})$, $J(\mathcal{G}, K_{ij}, N_{ij})$ has a simple eigenvalue at 1 with eigenvector $1_{N(\bar{N}+1)}$ and all other eigenvalues are strictly within the unit circle. Thus, the eigenvalue at 1 correspond to the information consensus, while the others related to the residual, which we want to vanish as quickly

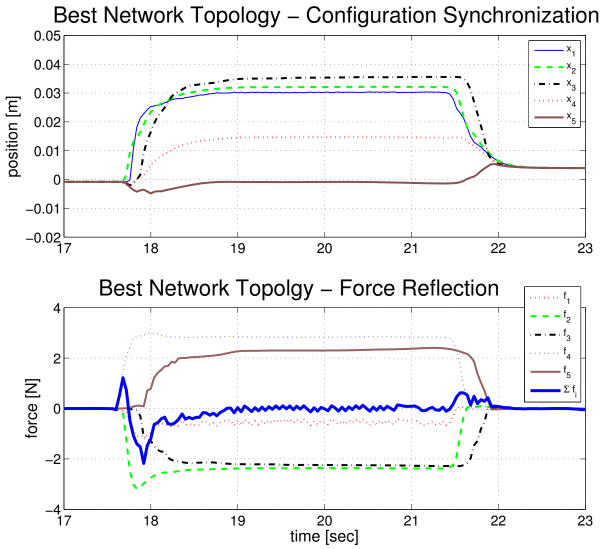

Fig. 5. Experimental results with the best network topology design

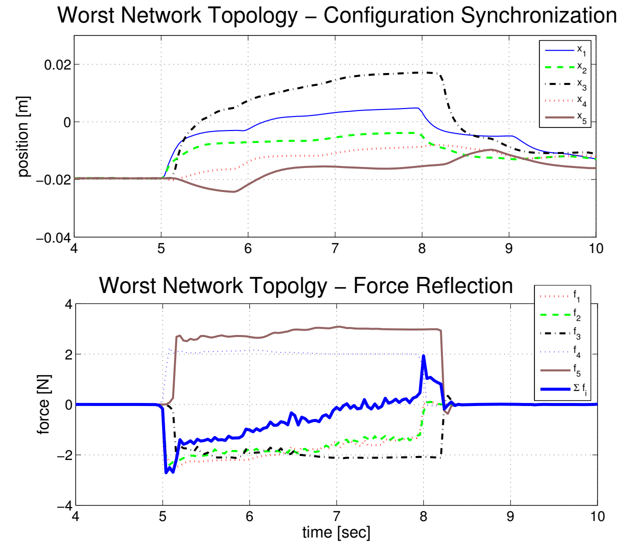

Fig. 6. Experimental results with the worst network topology design

as possible. This suggests the optimal network topology to be the one with the minimum second largest spectral radius (i.e. *algebraic connectivity* [20]).

Using this algebraic connectivity, we perform the network optimization. We also impose the constraint that the number of total edges is six among the users to take the communication cost into the consideration. The best and worst network topologies are shown in Fig. 4, and the experimental results are given in Fig. 5 and Fig. 6 respectively (with $B = K = 0$ for (3)). There, it is clear that the optimal topology provides a faster configuration and force reflection synchronization. Notice also that all the users' forces are balanced with $\sum_{i=1}^{5} f_i \rightarrow 0$ when the VE's motion stops (i.e. they all feel others' forces).

V. Conclusion and Future Works

We present a novel passivity-enforcing peer-to-peer distributed control architecture for multiuser shared haptic collaboration over the Internet with communication delays. The proposed architecture provides real-time responsiveness of haptic feedback and consistent haptic experience among the users, by using local copies of the shared VE for each user and position synchronization among them.

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