

# Strength Analysis of Miniature Folded Right Angle Tetrahedron Chain Programmable Matter

Paul J. White, Michael L. Posner, and Mark Yim

**Abstract**—Miniaturization of Programmable Matter is a major challenge. Much of the difficulty stems from size and power requirements of internal actuators. This paper demonstrates that external energy can be used to both move modules and actuate their bonding mechanism. It presents a lattice style Programmable Matter system whose neighbor to neighbor lattice distance is 14mm. Previous work has shown a chain of edge connected right angle tetrahedrons can fold to arbitrary shapes. To form useful shapes such as tools, the chain should be folded to meet the functional requirements of the task such as mechanical strength. This paper also introduces the analysis of the strength of Programmable Matter systems. Module connections are defined by 6DOF stiffness matrices. The paper analyzes the strength of a heterogeneous system with some rigid and some soft connections.

## I. INTRODUCTION

The term *Programmable Matter* has been used in a variety of ways. In [9], Goldstein et al. describe small robot ensembles that can rearrange themselves to form different 3D shapes. DARPA has a program called Programmable Matter [34] in which systems are based on mesoscale particles, that can reversibly assemble into complex 3D objects upon external command. In addition, these 3D objects exhibit all the functionality of their conventional counterparts.

These systems resemble lattice based modular self-reconfigurable robots in that the robots rearrange their modules to form different shapes. The modules combine in almost any fashion so long as the modules lie on a lattice and remain one connected component. There are several methods used to reconfigure a module to an adjacent lattice site including module deformation, cooperation, climbing, and unconventional methods. [4] explores the kinematics of reconfiguration and presents a hexagonal metamorphic module which deforms to reconfigure along the surface of the configuration. Deforming modules have also been demonstrated in 2D [20] and in 3D [23], [25]. The ATRON [15] and Fracta 3D [17] modules bond to neighbors with mechanical latches and cooperate to reconfigure by moving another module to a new location. In stochastic systems, 2D modules move randomly on an air table and bond magnetically [1], [28] and in [27], cube modules float in a fluid and bond using fluid flow. The Miche [8] system forms shapes by disassembly: modules not in the desired shape release magnetic latches and fall off the structure.

One differentiator between the system presented in this paper and lattice based reconfigurable robots is that the modules are permanently attached together in a chain that

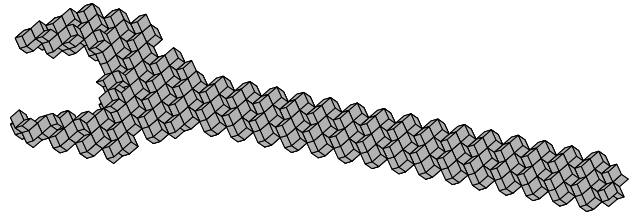


Fig. 1: A wrench voxelated by over 400 hexahedrons. Six right angle tetrahedrons make up each hexahedron.

can be folded into shapes rather than individual blocks. Intuitively, permanent joints can be made stronger than temporary bonds. Structures assembled from these chain systems can exploit these stronger bonds by folding to a configuration that remains most rigid under a given load.

The main question is then whether adding a chain constraint to lattice systems limits the types of shapes that can be made. In his thesis [11], Griffith shows that in fact, any arbitrary 3D shape can be approximated with a chain of right angle tetrahedrons in a space-filling sense.

In the work of Griffith et al. [12], the information for folding a desired shape is encoded in the order the two types of right angle tetrahedrons are placed in the chain. This ordering of types in a given chain hard codes the form of the resulting unique self-assembled shape. We extend this work by having the ability to form arbitrary shapes and further presenting a strength analysis of the conglomerated shape.

### A. Size

In Programmable Matter systems, reducing module size is a goal as this sets the resolution of the system, much like a computer display is limited in resolution by the pixel size. Currently, the smallest modules of a reprogrammable self-reconfiguring system is  $40 \times 40 \times 50$  mm module called *miniature* [33]. The Claytronics project has also shown planar modules that are 44mm in diameter [9].

On the other hand, arbitrary 3D structures have been attempted at the sub-millimeter scale. Results in the field of self-assembly have demonstrated crystalline structures that are prone to defects and are limited in complexity [32], [13]. [7] presents several methods for forming cellular automaton patterns by encoding assembly information in DNA tiles. These are one-time mechanisms that can self-assemble into one shape that is programmed at manufacture time, whereas Programmable Matter is aimed at developing systems that can be reprogrammed and reconfigured to form arbitrary shapes.

One effort to reduce the size of modular reconfigurable robots has been to simplify the modules. One way to do this

Paul J. White, Michael L. Posner, and Mark Yim are with the Mechanical Engineering and Applied Mechanics Department at the University of Pennsylvania (e-mail: {whitepj, posnerml, yim}@seas.upenn.edu).

is to remove the main actuator and use external energy to cause the modules to move. This can be done with shaker tables, or with modules suspended in a moving fluid [1], [28], [27]. [30], [31] demonstrates that using deterministic external actuation for moving a module facilitates miniaturization.

In scale, the proposed work sits between the onetime programmable self-assembly work and the lattice-based modular self-reconfigurable robot work. The actuation system requires and external energy source much like stochastic self-assembly systems. However, the external actuation method purposely attempts to minimize entropy by executing deterministic motions that assemble the module chain in a known time. In addition, modules connect with reversible latches allowing the system to reconfigure in the sense of a modular robot.

In [29], we showed that right angle tetrahedrons can be combined with external actuation to form arbitrary shapes. The work in this paper utilizes this external actuation principle to reduce modules to less than half of the characteristic length of the system reported in [29].

### B. Strength

In Programmable Matter work, Goldstein and Mowry present an application called *telepario* [10] in which many (maybe millions) of millimeter or smaller modules form 3D shapes for visualization. Much like television or telephone, telepario transmits shape that moves in real time. On the other hand, Programmable Matter as proposed by Zakin at DARPA focuses on *functionality*, typically mechanical functionality for example a wrench, rather than real-time motion.

Mechanical strength in reconfiguring systems is a difficult issue. Classically, strength of materials is referred to in terms of yield strength, or Young's modulus or limit strengths depending on the application. In Programmable Matter which is made up of modules that can be rearranged, Young's modulus doesn't make sense since that property assumes a homogeneous material, not a system made up of units. A better determination of strength can be seen from a functional task. For example, what will be the deflection of the wrench handle of Figure 1 in a typical load case? The model presented here is a step in the direction of simulating such behavior.

For most systems assembled from reconfigurable modules, the compressive strength can be quite large as the material properties of the modules are the limiting factor. Imagine a LEGO<sup>TM</sup> structure which when compressed is limited by the geometry and material properties. However, tensile strength is determined by the bonds which hold the system together. Since Programmable Matter systems are made to actively reconfigure, these bonds are typically much weaker than the compressive strength.

Combining the strength with the desire to make modules as small as possible makes this problem very difficult. Utilizing Griffith's right angle tetrahedron chain idea, we can make chain hinges that are stronger than the reconfigurable bonds. Since the system is now composed of heterogeneous strength characteristics, we need to find the path the chain makes through the lattice structure that maximizes the needed

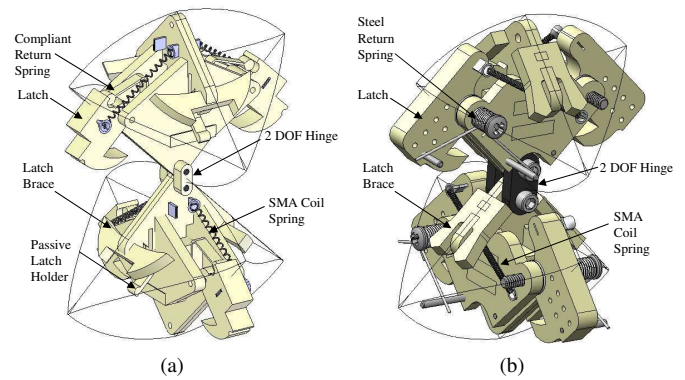


Fig. 2: Chain of two RATCHET14mm modules (first (a) and second (b) generation.) Each module is shown inside a full radiused right angle tetrahedron.

strength in the needed directions. One step towards this goal is to determine the strength of a given configuration.

## II. RATCHET14MM DESIGN

The major goal of the design process is miniaturization. When assembled into a shape, the centroids of the modules presented here are 14mm apart. This section presents the design of two generations of Right Angle Tetrahedron Chain Externally-actuation Testbed (RATCHET14mm) modules. The design focuses on a critical aspect of all modular robots: the latching mechanism that bonds modules together. The latch must be reversible so that a single chain can be folded to a desired shape and then reconfigured to another shape. The key principle is the use of external energy to both move and actuate modules.

Figure 2 shows two RATCHET14mm modules of first (Figure 2a) and second (Figure 2b) generation systems. Modules are assembled from 1.5mm thick laser cut acrylic and joined by two axis hinges.

### A. Latch

Modules bond to each other using a mechanical latch. Each module has two active faces with latches and two passive faces with notches to receive the latch. As the top module in Figure 2 rotates beyond 90° about the hinge axis the latch gets pushed up and engages with the passive face of module below. The curvature of the latch brace is used to push up a latch when a module rotates into a tightly packed portion of the configuration. The first generation module uses a compliant passive latch holder to maintain a latch connection after it has engaged.

A module unlatches using a Shape Memory Alloy (SMA) coil spring (Toki Corp.) The 0.15mm wire diameter SMA coil spring provides approximately 0.3N which is sufficient to overcome friction and the force of the return spring [26]. The SMA coil spring requires a return spring to stretch it back to its relaxed length. A first generation module (Figures 2a and 3a, top) utilizes a 250 μm thick compliant return spring. While providing sufficient return force, they are susceptible to breaking when handled and deforming when the SMA springs are heated. A second generation module (Figures 2b and 3a, bottom) utilizes stainless steel torsion springs (Century Spring Corp.)

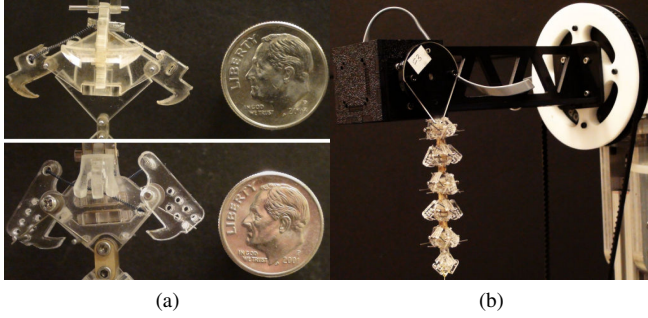


Fig. 3: (a) First (top) and second (bottom) generation RATChET14mm modules. (b) External actuation manipulator with six modules.

### B. External Actuator

The external actuation system consists of two parts: (1) a 2 DOF manipulator to reposition modules and (2) a heating source (such as a heat gun or oven) used to actuate the SMA coil springs.

As in [29], the external actuator uses gravity to latch modules during the folding process. The 2 DOF manipulator (Figure 3b) comprises two CKBot [19], [21] motor modules. One motor module drives a pulley (shown in white in Figure 3b). A truss extends along the pulley axis to the second motor module with axis parallel to the floor and normal to the pulley axis. The root RATChET14mm module is mounted to the second motor module with an acrylic bracket.

The SMA coil spring contracts when heated to  $70^\circ$  to  $90^\circ\text{C}$ . To disassemble a configuration, we manually use a heat gun across the configuration to actuate the SMAs and retract the latches. In the case of acrylic modules, disassembling with an oven is not robust as temperatures approach the glass temperature of the material. Future work will explore more localized (e.g. laser) and global (e.g. oven) disassembly methods with next generation modules.

## III. STRENGTH ANALYSIS

In general, the goal of Programmable Matter is to form arbitrary shapes from a single collection of modules. Many applications require that the configuration is rigid under certain loading conditions. In order to find the optimal folded arrangement that minimizes displacement, it is necessary to model the configuration's behavior under load. This section presents the analysis of a given folded configuration of right angle tetrahedrons subjected to static loads.

A major assumption of the model is that the modules in a Programmable Matter configuration are much stiffer than the connections between modules. We present a lumped parameter model that considers the stiffness of inter-module connections. This seems reasonable as the connections between modules must be able to automatically attach and detach. It would be difficult to design a system in which these bonds would be stronger than any other part.

### A. Physical Model

There are several requirements for the physical model. It must have enough degrees of freedom to accurately study the effects of different methods of joining modules. It must be

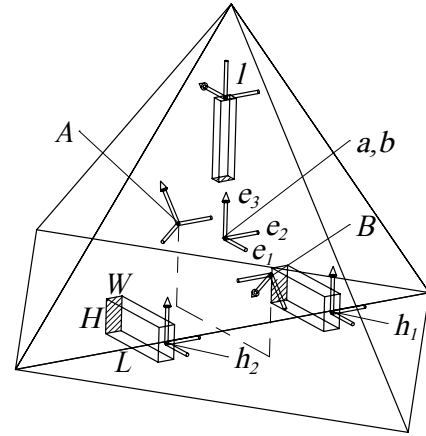


Fig. 4: Model of two tetrahedron modules with centroid frames  $A$  and  $B$  and respective center of stiffness frame  $a$  and  $b$ . Modules connect with three elastic beams: two hinges (frames  $h_1$  and  $h_2$ ) and a latch catch  $l$ . Dashed lines depict projections of centroid frame position. Hatch denotes fixed end of beam. Right handed basis annotated for frames  $a$  and  $b$ ; others not shown for clarity.

computationally efficient so that it can be used in many iterations of an optimization routine. The finite element method allows for arbitrary modeling of joined modules handling collision detection between bodies. However, the time cost for each solution is too great. A lower fidelity method models the joints as simple zero-length linear springs, but cannot model arbitrary stiffness matrices [14].

A 6 DOF spring has sufficient degrees of freedom to model arbitrary connection methods. [6], [35] furthered the methods of [3], [2] to model elastically coupled rigid bodies. The model presented in this section is based on the quaternion-based potential function in [35] and uses notation from [5].

Two elastically coupled modules with centroid frames  $A$  and  $B$  each have centers of stiffness located at frames  $a$  and  $b$  respectively. Under no load, frames  $a$  and  $b$  are coincident. The center of stiffness is analogous to the center of mass. A small translational displacement of  $b$  relative to  $a$  results in a pure force along an axis through the center of stiffness. A relative rotation between  $b$  and  $a$  results in a pure moment about an axis through the center of stiffness. [16] shows that there may not exist a point that perfectly decouples translational and rotational stiffness but the center of stiffness maximally decouples them.

A  $6 \times 6$  stiffness matrix  $K$  maps the change in configuration  $b$  relative to  $a$  to equal and opposite wrenches acting on both bodies. For a relative twist displacement  $\Delta T_b^a$  of frame  $b$  relative to  $a$  with respect to  $a$ , the wrench  $w_b^a$  module  $B$  applies to the elastic coupling is given by:

$$w_b^a = K \Delta T_b^a \rightarrow \begin{bmatrix} f_b^a \\ \tau_b^a \end{bmatrix} = \begin{bmatrix} K_t & K_c \\ K_c^t & K_o \end{bmatrix} \begin{bmatrix} \Delta p_b^a \\ \Delta \theta_b^a \end{bmatrix} \quad (1)$$

where  $K_t$  is the translational stiffness matrix,  $K_o$  is the rotational stiffness matrix and  $K_c$  is the coupling stiffness matrix.

For complex connection methods, finite element analysis

can determine the stiffness matrix [36]. The connections between RATCHET14mm are approximated as beams as shown in Figure 4. The latch connection beam (with frame  $l$ ) represents the catch the paw latches to. We model the latch catch because it is significantly more compliant than the paw. The distance between hinge beams (with frames  $h_1$  and  $h_2$ ) is exaggerated for clarity. The stiffness matrix at the free end of a fixed free beam is given by:

$$K = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & 0 & 0 & 0 & -\frac{6EI_3}{L^2} \\ 0 & 0 & \frac{12EI_2}{L^3} & 0 & \frac{6EI_2}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_2}{L^2} & 0 & \frac{4EI_2}{L} & 0 \\ 0 & -\frac{6EI_3}{L^2} & 0 & 0 & 0 & \frac{4EI_3}{L} \end{bmatrix} \quad (2)$$

where  $J = I_2 + I_3$  and dimensions and coordinate systems for hinge and latch beams are as defined in Figure 4.

A pair of modules connect to each other in one of two ways. If the modules are chain neighbors, they connect with a pair of hinges and a latch. Otherwise, they simply connect with a latch.

Nominally, the latch stiffness matrix  $K^l$  and hinge stiffness matrices  $K^{h1}$  and  $K^{h2}$  take the form of Equation 2. However, in the case of the hinge, an angular displacement about the hinge axis  $\mathbf{e}_2$  of e.g.  $h_1$  causes no moment and thus  $K_{55}^{h1}$  is zero. Due to slip, when the paw of the latch undergoes a linear displacement along  $\mathbf{e}_2$  of  $l$  or an angular displacement about  $\mathbf{e}_1$  or  $\mathbf{e}_3$  it does not cause the latch catch to bend. Hence, latch stiffness terms  $K_{22}^l$ ,  $K_{44}^l$ ,  $K_{66}^l$ ,  $K_{25}^l$  and  $K_{52}^l$  are zero.

The effective stiffness between hinged modules is given by the sum of the stiffnesses of each beam. In order to sum the stiffnesses, they must be transformed to the center of stiffness frame using [6]:

$$K = Ad_{H_a}^t K^l Ad_{H_a} + Ad_{H_{h1}}^t K^{h1} Ad_{H_{h1}} + Ad_{H_{h2}}^t K^{h2} Ad_{H_{h2}} \quad (3)$$

where frame  $a$  is the location of the center of stiffness and  $h1$  and  $h2$  denote each hinge.

The center of stiffness can be computed similar to the center of mass. The center of stiffness for modules which are not hinged together is simply the centroid of the latch beam. In the case of connections including hinges, solving Equation 3 such that  $K_c$  is symmetric [36] determines the center of stiffness position.

Under the assumption that the bodies are coupled perfectly elastically, [35] defines a potential function based on the relative configuration of the two bodies. The relative orientation of frame  $\beta$  with respect to  $\alpha$  is expressed by a quaternion  $q_\beta^\alpha = [\eta_\beta^\alpha (e_\beta^\alpha)^t]^t$ . [5] proves several useful properties of their potential function. It is sufficiently diverse and therefore can model an arbitrary local stiffness. The potential functions are frame indifferent (i.e. equal and opposite wrenches are applied to the elastically coupled bodies.) They are also port indifferent meaning the choice of body  $A$  and  $B$  does not matter. The method is valid for small displacements between the frames.

	$K_{t11}$	$K_{t22}$	$K_{t33}$	$K_{o11}$	$K_{o22}$	$K_{o33}$
w/ hinge	1.6E6	8.3E4	4.6E5	0.14	0.27	3.9
w/o hinge	6.2E3	0	4.2E5	0	0.14	0

TABLE I: Diagonal stiffness terms for modules with and without a hinge connection.  $K_t$  has units of  $N/m$ ;  $K_o$  has units of  $Nm$ .

Using the principal of virtual work, [35] computes the wrench body  $B$  applies to the elastic body with respect to the global frame as:

$$f_b = \frac{1}{2} R_a K_t p_b^a - \frac{1}{2} R_b K_t p_a^b + \eta_a^b (R_a + R_b) K_c e_b^a \quad (4a)$$

$$\begin{aligned} \tau_b = & \frac{1}{2} \tilde{p}_b R_a K_t p_b^a - \frac{1}{2} \tilde{p}_a R_b K_t p_a^b + 2R_b (E_a^b)^t K_o e_b^a \\ & \eta_a^b (\tilde{p}_b R_a + \tilde{p}_a R_b) K_c e_b^a + \\ & \frac{1}{2} R_b (\eta_a^b (E_a^b)^t - e_b^a (e_b^a)^t) K_c (I + R_a^b) p_b^a \end{aligned} \quad (4b)$$

where  $E_a^b = \eta_a^b I - \tilde{e}_a^b$  and  $\tilde{p}$  is the cross product matrix. Though the equation is somewhat complex, it requires only algebraic operations.

#### IV. SIMULATION

The simulation written in MATLAB has two main functions: (1) `shape2hamiltonian` that computes the tetrahedron chain path and (2) `tet_sim` that finds the equilibrium position of a statically loaded configuration of tetrahedron modules.

`shape2hamiltonian` voxelates a shape defined by an STL file (a common 3D file format) superimposed on a dodecahedron lattice. Each dodecahedron comprises four hexahedrons. It finds all hexahedrons within the shape and creates a graph of the hexahedrons. Figure 1 shows an example of a wrench shape voxelated by over 400 hexahedrons; each hexahedron comprises six right angle tetrahedrons. The path the tetrahedron chain makes through the voxelated shape is a Hamiltonian path for the tetrahedron graph. [11], [12] shows the tetrahedron Hamiltonian can be found by taking the path around the ‘‘perimeter’’ of a spanning tree of the hexahedron graph.

`tet_sim` determines the stiffness matrix between each pair of adjacent modules based on whether or not they are chain neighbors. It uses MATLAB’s nonlinear equation solver `fsolve` to determine the equilibrium position and orientation of each tetrahedron given the load and fixed boundary conditions using Equation 4.

The model uses the material properties of acrylic ( $E = 2.4\text{GPa}$ ,  $G = 0.89\text{GPa}$ .) The latch and hinge beam dimensions are respectively  $L = 8.0\text{mm}$ ,  $W = 1.5\text{mm}$ ,  $H = 1.0\text{mm}$  and  $L = 8.0\text{mm}$ ,  $W = 1.5\text{mm}$ , and  $H = 2\text{mm}$ .

Table I reports the diagonal stiffness terms in the center of stiffness frame for modules connection with and without a hinge. Comparing the terms indicates hinge connected modules have significantly higher stiffness compared to modules that are only latched together. Note that the hinge provides resistance to linear displacements along  $\mathbf{e}_2$  of frame  $a$  and angular displacements about  $\mathbf{e}_1$  and  $\mathbf{e}_3$ . Resistance to angular displacements is further increased due to the center of stiffness location between the latch and hinges as shown as frame  $a$  in Figure 4.

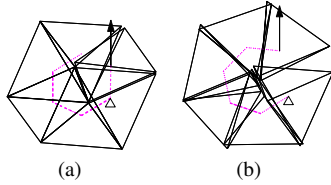


Fig. 5: Comparison of chain path under given load conditions. The white triangle indicates the fixed module. Hamiltonian paths are shown by the dashed lines.

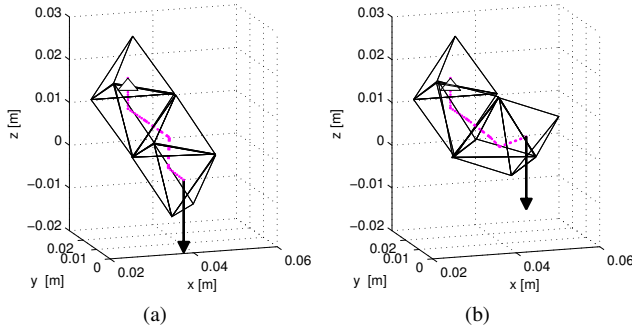


Fig. 6: Predicted displacement behavior of rigid bar (a) and arc (b). The white triangle indicates the fixed module and dark arrow indicates 0.4 N applied load.

The strength of a folded configuration depends on the path the chain takes through the shape. Figure 5 shows the displacement for two Hamiltonian cases of a six module configuration under a given loading. In Figure 5a, the fixed module (marked with a white triangle) is a chain neighbor to the module where the load is applied. Because it shares a relatively stiff connection with the fixed neighbor, the configuration displaces an order of magnitude less than the same loading case with a different Hamiltonian (Figure 5b.)

To compare the model with the physical system we loaded two configurations of six modules as shown in Figure 6. The simulation predicts loaded module in the rigid bar case (Figure 6a) and the arc case (Figure 6b) displaces 0.8mm and 1.8mm respectively due to a 0.4 N load. This corresponds to an apparent stiffness at the load point of 500 N/m and 222 N/m respectively in the load direction.

The stiffness model so far assumes modules are in contact and apply forces that elastically deform configurations; however, nonzero clearance between components is needed to allow modules to move during assembly. Future work will incorporate such non-linearities into the model. To experimentally determine the stiffness of the physical configurations corresponding to Figures 6a and 6b, we first apply a 0.5 N preload. We then apply an additional 0.4 N load and observe the displacement. The rigid bar displaces 0.9 mm corresponding to a stiffness of 444 N/m; the arc displaces 1.9 mm corresponding to a stiffness of 211 N/m.

Comparing experiment and simulation results indicates the model accurately predicts the rigid bar configuration is approximately twice as stiff as the arc. In addition, absolute errors for the line and arc cases are 11% and 5% respectively validating the analytical approximations.

## V. DEMONSTRATION

To demonstrate the capabilities of the RATChET14mm system, a module chain is folded into a rigid bar configuration (Figure 7), heated to reconfigure back to the chain, and folded into an arc configuration.

The external actuator motion planning is similar to [29] however the current workspace is less constrained. Given a reconfiguration motion plan, an active research topic [18], [20], [22], [24], the goal is to find an external actuator motion path to fold the chain to the desired shape. During the folding process, one module moves at a time. To fold module  $i + 1$  above it, the manipulator positions the root module such that the hinge axis between  $i$  and  $i + 1$  is parallel to one of the motor modules axes which then rotates latching  $i$  to  $i + 1$ .

A 24g weight attached to the end of the chain (Figure 7a) provides sufficient force to engage a latch. The weight and relative slow motor speed mitigates oscillations that could cause unintended latching. Assembly of a six module chain takes approximately 100 seconds.

Figure 7 depicts the assembly sequence of first generation modules for a rigid bar configuration. The first motor module (driving pulley) rotates the root RATChET14mm module approximately  $120^\circ$  to engage the root module's latch with the module below (Figure 7b) and then back to  $90^\circ$ . This process continues (Figures 7c through 7g) until the line configuration is completely folded (Figure 7h.) After assembly, a heat gun is manually waved across the configuration activating the SMA coil springs and retracting the latches. The disassembling process takes approximately 40 seconds. The same chain is subsequently folded to an arc configuration.

The supplemental video shows rigid bar assembly, external heat to disassemble and arc assembly with second generation modules. To test reliability, we successfully ran 20 trials between assembling into rigid bar and arc shapes.

## VI. CONCLUSION

This paper addresses two important design issues for developing functional Programmable Matter: size and strength.

The external actuation method presented demonstrates that module size can be significantly reduced using external energy. The RATChET14mm system improves on the work of [29]: the module size is reduced by a factor of two and the latching mechanism is reversible. The design focuses on a fundamental aspect of modular self-reconfigurable robots and Programmable Matter: a reversible bonding mechanism. Repeated trials demonstrate the mechanism's reliability in latching and unlatching.

Programmable Matter structures to be used in the physical world must meet some mechanical functional requirements. Comparing the strength of configurations requires a simulator that accurately models how modules are mechanically connected. This paper presents a simulator that models an inter-module connection with a 6 DOF generalized spring. The framework is sufficiently expressive to model arbitrary connection methods.

This paper shows that folded chain structures exhibit heterogeneous stiffness properties. Such properties can be exploited to form structures the best suit the task.

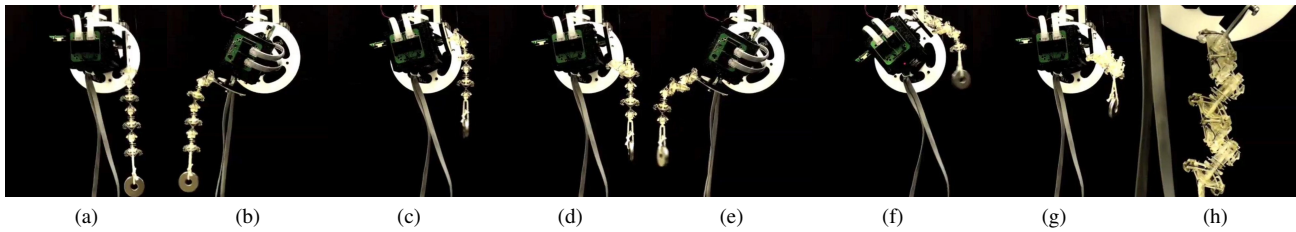


Fig. 7: Demonstration reconfiguration sequence for chain of seven modules forming a rigid bar.

Future work will use the simulator to find the optimum folded structure given functional requirements. Future work also includes finding new methods for externally actuating module bonding mechanisms and further module miniaturization.

#### REFERENCES

- [1] J. Bishop, S. Burden, E. Klavins, R. Kreisberg, W. Malone, N. Napp, and T. Nguyen. Programmable parts: a demonstration of the grammatical approach to self-organization. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3684–3691, Edmonton, 2005.
- [2] F. Caccavale, C. Natale, B. Siciliano, and L. Villani. Six-dof impedance control based on angle/axis representations. *IEEE Transactions on Robotics and Automation*, 15(2):289–300, 1999.
- [3] F. Caccavale, B. Siciliano, and L. Villani. Quaternion-based impedance with nondiagonal stiffness for robotmanipulators. In *American Control Conference, 1998. Proceedings of the 1998*, volume 1, 1998.
- [4] G.S. Chirikjian. Kinematics of a metamorphic robotic system. In *Proceedings of IEEE/RSJ International Conference on Robotics and Automation*, pages 449–455 vol.1, May 1994.
- [5] E.D. Fasse and P.C. Breedveld. Modeling of Elastically Coupled Bodies: Part I - General Theory and Geometric Potential Function Method. *Journal of Dynamic Systems, Measurement, and Control*, 120:496–500, 1998.
- [6] E.D. Fasse and P.C. Breedveld. Modeling of Elastically Coupled Bodies: Part II Exponential and Generalized Coordinate Methods. *Journal of Dynamic Systems, Measurement, and Control*, 120:501–506, 1998.
- [7] K. Fujibayashi, R. Hariadi, S.H. Park, E. Winfree, and S. Murata. Toward reliable algorithmic self-assembly of DNA tiles: a fixed-width cellular automaton pattern. *Nano Lett.*, 8(7):1791–1797, 2008.
- [8] K. Gilpin, K. Kotay, D. Rus, and I. Vasilescu. Mische: Modular shape formation by self-disassembly. *The International Journal of Robotics Research*, 27(3-4):345–372, 2008.
- [9] S. C. Goldstein, J. D. Campbell, and T. C. Mowry. Programmable matter. *Computer*, 38(6):99, 2005.
- [10] S.C. Goldstein and T. Mowry. Claytronics: A scalable basis for future robots. *Robosphere*, Nov, 2004.
- [11] S. Griffith. *Growing Machines*. PhD thesis, Massachusetts Institute of Technology, 2004.
- [12] S. Griffith, J. McBride, B. Su, B. Ren, and J. M. Jacobson. Folding any 3D shape. pre published.
- [13] K. Hosokawa, I. Shimoyama, and H. Miura. Two-dimensional micro-self-assembly using the surface tension of water. *Micro Electro Mechanical Systems, 1996, MEMS '96, Proceedings. 'An Investigation of Micro Structures, Sensors, Actuators, Machines and Systems'. IEEE, The Ninth Annual International Workshop on*, pages 67–72, Feb 1996.
- [14] S. Huang and J.M. Schimmels. The bounds and realization of spatial stiffnesses achieved with simple springs connected in parallel. *Robotics and Automation, IEEE Transactions on*, 14(3):466–475, Jun 1998.
- [15] M. W. Jorgensen, E. Hallundbaek Østergaard, and H. H. Lund. Modular ATRON: Modules for a self-reconfigurable robot. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, volume 2, pages 2068–2073, Sendai, Japan, 2004.
- [16] J. Lončarić. Normal forms of stiffness and compliance matrices. *Robotics and Automation, IEEE Journal of*, 3(6):567–572, December 1987.
- [17] S. Murata, H. Kurokawa, E. Yoshida, K. Tomita, and S. Kokaji. A 3-d self-reconfigurable structure. In *Proceedings of IEEE/RSJ IEEE International Conference on Robotics and Automation*, volume 1, pages 432–439 vol.1, Leuven, Belgium, May 1998.
- [18] A. Pamecha and G. Chirikjian. Useful metric for modular robot motion planning. In *Proceedings of IEEE/RSJ IEEE International Conference on Robotics and Automation*, volume 1, page 442, Minneapolis, MN, USA, 1996.
- [19] M. Park, S. Chitta, and M. Yim. Isomorphic gait execution in homogeneous modular robots. In *Robotics: Science and Systems Workshop on Self-reconfigurable Modular Robots*, Philadelphia, 2006.
- [20] D. Rus and M. Vona. Self-reconfiguration planning with compressible unit modules. In *Proceedings of IEEE/RSJ IEEE International Conference on Robotics and Automation*, volume 4, pages 2513–2520, Detroit, 1999.
- [21] J. Sastra, S. Chitta, and M. Yim. Dynamic rolling for a modular loop robot. *Intl. J. of Robotics Research*, 2007.
- [22] K. Stoy. Using cellular automata and gradients to control self-reconfiguration. *Robotics and Autonomous Systems*, 54(2):135, 2006.
- [23] J. W. Suh, S. B. Homans, and M. Yim. Telecubes: mechanical design of a module for self-reconfigurable robotics. In *Proceedings of IEEE/RSJ IEEE International Conference on Robotics and Automation*, volume 4, pages 4095–4101, Washington, DC, 2002.
- [24] C. Unsal and P. K. Khosla. A multi-layered planner for self-reconfiguration of a uniform group of I-Cube modules. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, volume 1, pages 598–605, Maui, 2001.
- [25] S. Vassilvitskii, J. Kubica, E. Rieffel, J. Suh, and M. Yim. On the general reconfiguration problem for expanding cube style modular robots. In *IEEE/RSJ International Conference on Robotics and Automation*, volume 1, pages 801–808 vol.1, 2002.
- [26] T. Waram. *Actuator Design Using Shape Memory Alloys*. T. C. Waram, 2nd edition, 1993.
- [27] P. White, V. Zykov, J. Bongard, and H. Lipson. Three dimensional stochastic reconfiguration of modular robots. In *Robotics: Science and Systems*, pages 161–168, Cambridge, 2005.
- [28] P. J. White, K. Kopanski, and H. Lipson. Stochastic self-reconfigurable cellular robotics. In *Proceedings of IEEE/RSJ International Conference on Robotics and Automation*, volume 3, pages 2888–2893, New Orleans, LA, USA, 2004.
- [29] P. J. White, C. E. Thorne, and M. Yim. Right Angle Tetrahedron Chain Externally-actuated Testbed (RATCHET): A Shape Changing System. In *Proceedings of IDETC/CIE*, San Diego, CA, USA, 2009.
- [30] P. J. White and M. Yim. Scalable modular self-reconfigurable robots using external actuation. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 2773–2778, San Diego, 2007.
- [31] P. J. White and M. Yim. Reliable external actuation for extending reachable robotic modular self-reconfiguration. In *International Symposium on Experimental Robotics*. IFRR, July 2008.
- [32] G.M. Whitesides and M. Boncheva. Supramolecular Chemistry And Self-assembly Special Feature: Beyond molecules: Self-assembly of mesoscopic and macroscopic components. *Proceedings of the National Academy of Sciences of the United States of America*, 99(8):4769, 2002.
- [33] E. Yoshida, S. Kokaji, S. Murata, H. Kurokawa, and K. Tomita. Miniaturized self-reconfigurable system using shape memory alloy. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, volume 3, 1999.
- [34] M. Zakin. Programmable Matter-The Next Revolution in Materials. *MILITARY TECHNOLOGY*, 32(5):98, 2008.
- [35] S. Zhang and E.D. Fasse. Spatial compliance modeling using a quaternion-based potential function method. *Multibody System Dynamics*, 4(1):75–101, 2000.
- [36] S. Zhang and E.D. Fasse. A finite-element-based method to determine the spatial stiffness properties of a notch hinge. *Journal of Mechanical Design*, 123:141, 2001.