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Dynamic Model and Adaptive Tracking Controller for 4-Powered Caster Vehicle

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Abstract— **A new approach for adaptive torque distribution of 4-Powered Caster Vehicle (4-PCV) is presented on complex terrain without any additional sensor. The objective is that torques applied to wheels are dynamically redistributed based on the real time conditions of the whole wheel-ground interactions in order to track the desired trajectory. A novel approach based on the redundant actuated wheels is proposed to identify the status of the vehicle and the wheel slip ratio by only observing the velocity feedback from motors encoders. A dynamic model considering the wheel-ground interaction is described. Based on the slip ratio of the wheel joints and the null space of the operational space, control strategies are employed to redistribute the torques applied to the wheel joints so that each wheel can be self-adapted to meet a complex wheel-ground condition to eliminate slippage with high rate. Simulation results show the effectiveness of the proposed estimation approach and the performance of the torque distribution schemes.**

I. INTRODUCTION

Many researchers have been working on holonomic mobile robots, which have 3 degrees of freedom (DOF), due to the unique motion and its high maneuverability in narrow spaces. Compared to either Swedish wheel or most of other omni-directional wheels, the powered castor wheel is not sensitive to road condition and even suitable for uneven terrain [1].

Kinematic modeling and control for holonomic mobile robot have been addressed [2][3], and singularity problems have been analyzed [3][4]. A dynamical modeling and control for holonomic mobile robot is particularly desirable in many fields such as a mobile manipulation system or complex wheel-ground conditions. Holmberg and Khatib [5] propose an approach for a modular, efficient dynamic modeling of powered caster vehicles based on the augmented object model originally developed for the study of cooperative manipulation. The dynamic modeling and control for this type of wheel are strictly limited to non-slip condition. However, the powered caster vehicle with redundant actuators causes internal forces between wheels, which magnify the occurrence of slip phenomenon. Li, et al [6][7] extend the dynamic model taking into account the wheel-ground interaction. The non-slip condition for motion is utilized to plan slip avoidance paths. The approach is a static way to meet non-slip condition, and the parameters on wheel-ground interaction should be known.

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Fig. 1. Updated holonomic mobile robot system

A holonomic mobile robot system with four powered casters, as shown in figure 1, is updated with new control hardware and software by our effort. Due to the redundant actuated wheels of the vehicle and the saturation of admissible forces by the ground, dynamically force distribution controlled mobile robot considering the real time wheel-ground condition is especially important when used in the uneven terrain. The key challenge is to know the wheel-ground interaction in real time. The motivation is to take full use of the redundant actuators to keep the wheels in the same order regarding to the vehicle velocity at any wheel-ground condition. In this paper a novel approach based on the redundant actuated wheels is proposed to identify the status of the wheels and the wheel slip ratio by only observing the velocity feedback from encoders without any additional sensor. In order to redistribute the forces on the wheel-ground contact points, a dynamic model considering the wheel-ground interaction is presented. Based on the status of the wheels and the null space of the operational space, control strategies are employed to adjust the forces applied on the wheels for the purpose that each wheel can self-adaptively coordinate together under complicated wheel-ground conditions. Note that the forces applied on the wheels can be distributed dynamically according to the desired motions and the feedback reflecting the wheel-ground interaction at a very high rate.

This paper is structured as follows. The dynamic modeling considering the wheel-ground interaction is presented in Section II. An approach to identify the slippage status of the wheels by only observing the velocity feedback from motors is described In section III. Dynamic control strategies for force distribution based on the wheel slip rate are proposed in section IV. Experimental results are demonstrated in Section IV. Finally, we give the conclusions of the work.

II. DYNAMICS MODEL OF THE HOLONOMIC 4-PCV

A. Platform dynamics

For the platform without the wheels, let $(\dot{x}, \dot{y})$ denote velocity of the vehicle with respect to the global frame in the directions given by local frame. Let $\dot{\varphi}$ denote the angular velocity of the vehicle along the vehicle center parallel to the x-axis direction given by the global frame. Let $\dot{X}_L = (\dot{x}, \dot{y}, \dot{\varphi})^T$ and suppose no slip happens.

The kinetic energy of the platform in the operational space is

$$K_p = \frac{1}{2}m_p\dot{x}^2 + \frac{1}{2}m_p\dot{y}^2 + \frac{1}{2}I_p\dot{\varphi}^2 \qquad (1)$$

where m_p is the mass of the platform, and I_p is the inertia of the platform in terms of vertical direction.

From the Lagrangian principle and ignoring the potential energy of the vehicle, we have

$$M_p\ddot{X}_L = F_L \qquad (2)$$

where M_p is (3×3) inertia matrix of the platform, and F_L is (3×1) vector of generalized forces/moments on the platform in the direction given by local frame.

B. Powered caster dynamics and contact forces

There are k (≤ 4) powered casters (PCs) holding the rigid platform by steering joint. They can be reconfigured according to the wheel-ground interaction and normally k is equal to 4. For the powered caster vehicle, the coordinate assignments and parameters definition are shown in Figure 2. KW, KL, and KSi denote the global reference frame, local (platform attached) reference frame and frame fixed at i^{th} steering joint respectively.

Suppose the vehicle move in the two dimensional space and only consider the force/moment in the plane. The powered caster i can apply n_i $(= 3)$ wrenches at the i^{th} steering joint, and the total wrenches are $\sum_{i=1}^{k} n_i = n$. For this case, $n \equiv 12$. Note the magnitude of the wrenches can be changed, and even can be zeros when the wrenches cannot be applied.

Let $g_i \in R^3$ denote the vector whose entries are the magnitude of the wrenches applied at i^{th} steering joint in

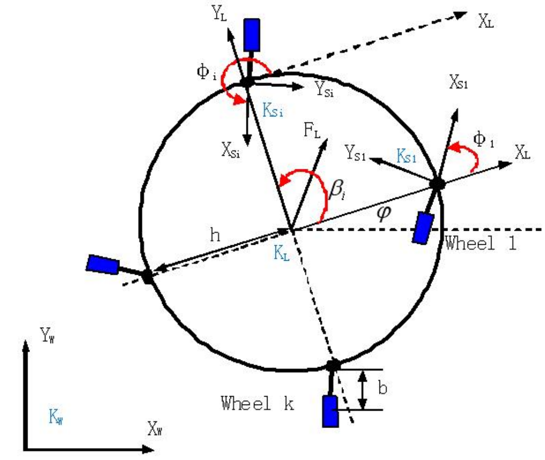

Fig. 2. The coordinate assignments and parameters definition for PCV

the basis directions given by frame k_{Si}. It is easily found that$g_i \in R^3$ include two forces and one torque, and the torque is produced by a steering joint. Thus we can rewrite the force $g_i \in R^3$as: $g_i = \begin{bmatrix} f_i & \tau_{si} \end{bmatrix}^T$. Let $J_{Pi} \in R^{(3\times3)}$ denote matrix to map the motion of the origin in the frame $k_L$with respect to the global frame $k_W$in the basis directions given by the frame $k_L$to the motion of the origin in the fame k_{Si} with respect to the global frame $k_W$in the basis directions given by the frame k_{Si}. $F_i = J_{Pi}^T g_i \in R^3$denote the forces/moments expressed in the local reference frame acting at the i^{th} steering joint due to the $n_i$wrenches. $J_{Pi} \in R^{(3\times3)}$ can be described as

$$J_{Pi} = \begin{bmatrix} c\phi & s\phi & hs(\phi-\beta) \\ -s\phi & c\phi & hc(\phi-\beta) \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

We can move the constant items to S_i and (4) can be partitioned as

$$J_{fi} = \begin{bmatrix} c\phi & s\phi & hs(\phi-\beta) \\ -s\phi & c\phi & hc(\phi-\beta) \end{bmatrix} \qquad (4)$$

and $S_i = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$ denotes the torque in the i^{th} steering joint can be applied to the center of the platform. Let $S = [S_1, S_2, ..., S_k] \in R^{3\times k}$, $f = [f_1, f_2, ..., f_k]^T \in R^{2k\times1}$,$\tau_S = [\tau_{s1}, \tau_s 2, ..., \tau_s k]^T \in R^{k\times1}$ and $J_f = [J_f 1^T, J_f 2^T, ..., J_{fk}^T]^T \in R^{(2k\times3)}$. Therefore

$$F_L = \sum{}_{i=1}^{4} F_i = J_f^T f - S\tau_S \qquad (5)$$

The coordinate assignments and parameters definition of a powered caster i are shown in Figure 3. Let $\dot{\phi}$, $\dot{\theta}$, and $\dot{\eta}$ denote the steering velocity, rolling velocity and twisting velocity at the wheel contact. r and b denote the radius and offset of the caster wheel, respectively. φdenote the cross angle of the x axis directions in the frames between k_L and k_W. $\phi_i$denote the cross angle of the x axis directions in the frames between $k_{Si}$and k_L. $\eta_i$denote the cross angle of the x axis directions in the frames between k_{Si} and k_W. It is

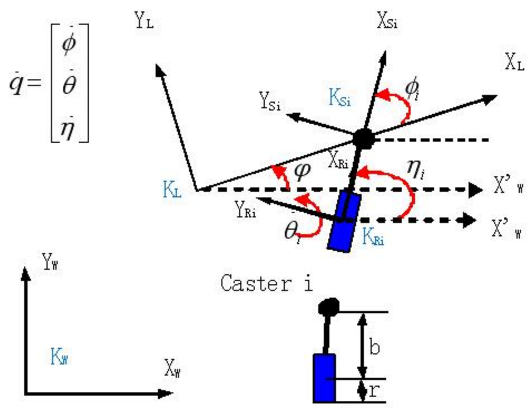

Fig. 3. The coordinate assignments and parameters definition of a powered caster i

obvious that we have the holonomic constraints: $\eta_i = \phi_i + \varphi$, where $i = 1, ..., k$. We use $\dot{q}_i = \begin{bmatrix} \dot{\phi}_i & \dot{\theta}_i & \dot{\eta}_i \end{bmatrix}^T$ and $\dot{\Theta}_i = \begin{bmatrix} \dot{\theta}_i & \dot{\phi}_i \end{bmatrix}^T$ to denote the total joint velocity and actuated joint velocity of the i^{th} powered caster.

The kinetic energy of the i^{th} wheel is

$$K_i = \frac{1}{2} I'_{WLi} \dot{\theta}^2 + \frac{1}{2} J'_{WLi} \dot{\eta}^2 \tag{6}$$

where I'_{WLi} is the computed inertia of the i^{th} wheel and link with respect to the y-axis given by frame k_{Ri} , and $I'_{WLi}$is the computed inertia of the i^{th} wheel and link with respect to the vertical direction given by frame k_{Ri}.

Using the Lagrangian approach and considering the interaction friction with ground, the dynamic equations of the i^{th} PC in contact with external force $g_i \in R^3$ are given by

$$D_i \ddot{w}_i + J_{Ci}^T g_i + M_i = \tau_i \tag{7}$$

where $w_i = \begin{bmatrix} \theta_i & \eta_i \end{bmatrix}^T$, $J_{Ci} = \begin{bmatrix} r & 0 \\ 0 & b \\ 0 & 1 \end{bmatrix}$, $M_i = \begin{bmatrix} M_{ri} & M_{si} \end{bmatrix}^T$denote the rolling and twisting friction torque, $\tau_i = \begin{bmatrix} \tau_{\theta i} & \tau_{\eta i} \end{bmatrix}^T$, and $D_i = \begin{bmatrix} I'_{WL} & 0 \\ 0 & J'_{WL} \end{bmatrix}^T$. Note the twist is passive joint, and then $\tau_{\eta i} = 0$. (7) can be rewritten as

$$D_i \ddot{w}_i + J_{hi}^T f_i + M_i = T_i \tag{8}$$

where $T_i = \begin{bmatrix} \tau_{\theta i} & \tau_{\phi i} \end{bmatrix}^T$,$J_{hi} = \begin{bmatrix} r & 0 \\ 0 & b \end{bmatrix}$ Let $D = Diag(D_1, D_2, ..., D_k)$, $w = [w_1^T, w_2^T, ..., w_k^T]^T$ $J_h \in R^{(8\times8)} = Diag(J_{h1}, J_{h2}, ..., J_{hk})$,$\mathrm{M} = [M_1^T, M_2^T, ..., M_k^T]^T$, and $\tau = [T_1^T, T_2^T, ..., T_k^T]^T$. Considering the friction with ground, the dynamic equation of the PCs in contact with the platform is given by

$$D\ddot{w} + J_h^T f + \mathrm{M} = \tau \tag{9}$$

Substituting (9) into (5) and eliminating the$f$, we can have

$$F_L + J^T D\ddot{w} + J^T \mathrm{M} = J^T \tau \tag{10}$$

where $J_{Ai} = \begin{bmatrix} c\phi & s\phi & hs(\phi-\beta) \\ -s\phi & c\phi & hc(\phi-\beta) - b \end{bmatrix}$, $J_A = [J_A 1^T, J_A 2^T, ..., J_{Ak}^T]^T \in R^{(8\times3)}$, and$J = J_h^{-1} J_A$ According to the kinematic equation of the powered caster vehicle, we have

$$J_h \dot{w} = J_f \dot{X}_L \tag{11}$$

Substituting (2) and (11) into (10), we have

$$A\ddot{X}_L + B(X_L, \dot{X}_L) + J^T \mathrm{M} = J^T \tau \tag{12}$$

where $A = M_p + J_A^T J_h^{-T} D J_h^{-1} J_f$, $B(X, \dot{X}) = J_A^T J_h^{-T} D J_h^{-1} \dot{J}_f$

III. ESTIMATION OF TASK SPACE STATES AND WHEEL SLIPPAGE

When there is no wheel slippage in the mobile robot system, the task space velocity could be obtained by the kinematics model and measured angular velocities of all the motors via speed sensors such as encoders. In case some joints should happen to slip, however, the obtained task space velocity is no longer reliable because the kinematics items corresponding to the slipped joints are no longer correct. In the face of such cases, a reliable estimation of the actual task space velocity should be designed, which could represent most probably the current velocity of the mobile robot.

Most papers we reviewed utilize some sensors amounted on the mobile robot's body to get the task space velocities and positions, such as GPS (Global Positioning System), IMU (Inertial Measurement Unit), etc. This paper, however, proposes a new estimation approach which no longer needs any additional task space sensors. The new estimation approach is based on a probability schemes. The basic idea is to take full use of the redundant actuated motors and identify the motor angular velocities of the joints with the maximal probability of no slip, rather than all of the motor angular velocities, to estimate the task velocity of the mobile robot. From the kinematics of the robot, we may see that the actuators are redundant, which indicates that we could obtain the task space velocity of the robot from any three angular velocities and their corresponding rows in the kinematics matrix. So the key point of the approach is to find a subset of joints containing more than three joints which have the maximal probability of no slippage.

The kinematics model of the mobile robot is written as,

$$\dot{Q} = C(Q)\dot{X} \tag{13}$$

where, $\dot{Q}$ is a 8x1 vector denoting the angular velocity of the 8 joints. $\dot{X}$ is a 3x1 vector denoting the task space velocity of the mobile robot. $C(Q)$ is a 8x3 matrix representing the kinematic relationship, which is related with Q , the current angular values of the 8 joints.

This kinematics model indicates that the task space velocity $\dot{X}$ could be obtained given any three joints' angular velocities if the symmetric matrix composed of the corresponding rows of the given three joints in $C(Q)$ is inversible. Given any three angular velocities $\dot{q}_i$,$\dot{q}_j$, $\dot{q}_k$, we could get their corresponding rows c_i , c_j , c_k in matrix C to form a symmetric matrix $\bar{C}$. And then according to the kinematics model we have

$$\begin{bmatrix} \dot{q}_i \\ \dot{q}_j \\ \dot{q}_k \end{bmatrix} = \begin{bmatrix} c_i \\ c_j \\ c_k \end{bmatrix} \dot{X} = \bar{C}\dot{X} \tag{14}$$

If $\bar{C}$ is inversible, the task space velocity can be obtained by $\dot{X} = \bar{C}^{-1} \begin{bmatrix} \dot{q}_i & \dot{q}_j & \dot{q}_k \end{bmatrix}^T$. There are 8 motors in our system, so there are totally 56 kinds of combination of the three angular velocities. Assume that there are N kinds of combination where the corresponding transform matrix $\bar{C}$ is inversible. Thus we have

$$\dot{X}_m = \bar{C}_m^{-1} \begin{bmatrix} \dot{q}_{m1} & \dot{q}_{m2} & \dot{q}_{m3} \end{bmatrix}^T, \;\; m = 1, \ldots, N \tag{15}$$

where, m denotes a kind of combination of the three angular velocities.

As discussed above, the key point of estimation of real task space parameter under slip is to find a subset of joints containing maximum joints which have the maximal probability of non-slip. Without the knowledge of the task space state, it is impossible to derive the slipped joints with only the knowledge of the state of each joint. However, we can find a subset of joints with the maximal probability of non-slip based on the states of the joints and presented kinematics model. As long as the number of the subset is more than three, and their corresponding matrix is inversible, the task space states of the mobile robot could be derived according to the kinematic model.

When no slippage happens in all of the joints, the computed $\dot{X}_m$, $m = 1, \ldots, N$ in (3) should be all the same. If we choose a subset of M joints where M is more than three, then we have C_N^M subsets. For each of the M subsets, we have C_M^3 kinds of combination of three angular velocities. According to the analysis, if all the joints in the p^{th} subset Π_p do not slip, all the computed task space velocities $\dot{X}_m^p$,$m = 1, \ldots, C_M^3$ should be the same, which indicates that the variance of this subset is zero. In contrast, if there exists some slipped joints in the p^{th} subset, the computed $\dot{X}_m^p$, $m = 1, \ldots, C_M^3$ should not be all the same. Furthermore, the bigger the number of slipped joints is and the more seriously they slipped, the bigger difference exists among the computed task space states, which indicates that the bigger variance in this subset. Based on this analysis, we can find that the variance of a subset reflects the number of slipped joints and the extent they slip in this subset. Thus we can get a subset of joints with the maximal probability of non-slip by finding the subset with the smallest variance. This subset can be found by

$$\Pi_{\min} = \underset{\Pi_p, p=1,2,\ldots,C_N^M}{\arg\min} \left\{ \frac{\mathrm{var}(\Pi_p)}{M} \right\} \tag{16}$$

where, Π_p is a subset which contains M joints, and $\mathrm{var}(\Pi_p)$ denotes the variance of the computed task space states based on subset Π_p . It can be expressed as

$$\mathrm{var}(\Pi_p) = \frac{1}{C_M^3} \sum_{m=1}^{C_M^3} (\dot{X}_m^p - \bar{\dot{X}}^p)^2 \tag{17}$$

where $\dot{X}_m^p$ and $\bar{\dot{X}}^p$ denote the velocity and average velocity of the computed task space velocities in the p^{th} subset. Once we get the subset of joints with the maximal probability of no slip as the q^{th} subset, the estimation of the task space velocity could be obtained as

$$\hat{\dot{X}} = \bar{C}_q^{-1} \begin{bmatrix} \dot{q}_1^q \\ \vdots \\ \dot{q}_M^q \end{bmatrix} \tag{18}$$

where, $\dot{q}_i^q, i = 1, ..., M$ are the angular velocities of the joints in the p^{th} subset. $\bar{C}_q$ is composed of the rows in kinematics matrix C, which are corresponding to the M joints in this subset. When $\bar{C}_q$ is not a symmetric matrix, pseudo inverse $\bar{C}_q^\dagger$ is used to replace $\bar{C}_q^{-1}$. Also, slippage statuses of joints are identified. Thus the computed task space velocity $\hat{\dot{X}}$ can approach the real value by using the joint space state with the maximal probability of non-slippage. As long as the task space velocity is known, the task space position of the mobile robot can be obtained by integrating the velocity. These task space states and joint space states can be used as feedbacks to the controller to implement some advanced tasks, such as trajectory tracking. More importantly, slip ratio and force compensation for slippage, will be deduced in details in the next section.

IV. ADAPTIVE TRACKING CONTROLLER

A. Normal Tracking Controller Design without Slips

The objective of tracking controller is to design a controller to track the given task space trajectory of the mobile robot. According to the dynamic model of the robot, 8 input torques of the 8 motors are to be determined. The task space velocity and position can be obtained based on the angular velocities of all the motors according to the estimation approach referred above. Typically, for such kind of nonlinear system, nonlinear feedback technique can be applied to linearize and decouple the system. This technique needs the inverse of the matrix in the dynamic model. The corresponding matrix J^T in this system model, however, is a 3x8 matrix, which does not have an inverse. We use pseudo inverse to solve this problem. Pseudo inverse is used to get the inverse of an asymmetric matrix in least square sense.

Combining with nonlinear feedback technique and pseudo inverse, we can design the controller in the form as

$$\tau = (J^T)^{\dagger}(Au + B(X_L, \dot{X}_L) + J^T \mathrm{M}) \tag{19}$$

Substitute (19) into the dynamic model (12), we have,

$$\ddot{X} = u \tag{20}$$

From (20), we can see that the nonlinear dynamic system is totally linearized and also decoupled. There are lots of approaches to control this kind of linear system. A typical PD controller can be applied as:

$$u = \ddot{X}^d + k_D(\dot{X}^d - \dot{X}) + k_P(X^d - X) \tag{21}$$

where, $\ddot{X}^d, \dot{X}^d, X^d$ denote desired task space acceleration, velocity and position. $\dot{X}, X$ denote the feedback of task space velocity and position which can be obtained by the proposed estimation approach. K_D and K_P denote the PD controller parameters which can be designed according to the system requirements. Combining the nonlinear feedback law and PD controller together, we get the general controller of the dynamic system as following:

$$\begin{aligned} \tau \quad &= (J^T)^{\dagger}(A(\ddot{X}^d + k_D(\dot{X}^d - \dot{X}) + k_P(X^d - X)) \\ &\quad + B(X_L, \dot{X}_L) + J^T \mathrm{M}) \end{aligned} \tag{22}$$

B. Adaptive Controller for Torque Distribution with Slips

The tracking controller in (21) is used with the assumption that there exists no slip during the robot moving. In uneven terrain environment, the friction of terrain changes dynamically, which will cause the slip in some joints. When slippage occurs in the system, the tracking controller in (7) no longer works, and even can make the mobile robot unstable. So the controller under slip is to be designed to adjust the force/toques adaptively. A torque distribution scheme is proposed to cope with this issue. Based on our estimation approach, the estimated slip ratio of a joint can be defined as:

$$s_i = \frac{\dot{\tilde{q}}_i - \dot{q}_i}{\dot{q}_i} \tag{23}$$

where, $\dot{\tilde{q}}_i$ denotes the computed angular velocity of a joint based on the estimated task space velocity according to the kinematics model, which is obtained by$\dot{\tilde{q}}_i = c_i \dot{\tilde{X}}$, where c_i is the corresponding row in kinematics matrix C.

When a joint slips, it means that the torque can not be translated into the traction force due to the decrease of the terrain friction coefficients. Thus part of the torque is used to drive the wheel rotate in the same task space position. To avoid this slip, the desired torque should be reduced compared with the torque computed under no slip case. Based on this slip ratio s_i , a torque compensator for each motor can be obtained as,

$$Q = (I - (J^T)^{+} J^T) K_1 S \tag{24}$$

where, $S = \left[\begin{array}{ccc} s_1 & \ldots & s_8 \end{array} \right]^T$, and $K_1$is a 8x8 matrix with constant entries representing the compensation coefficients. The term on right side of (24) is the torque component in the null space of matrix J^T that has no effect on operational space force.

Thus, by combining (22), (24), the adaptive controller for torque distribution can be written as:

$$\tau' = \tau + Q \tag{25}$$

V. SIMULATION RESULTS

A simulation is conducted to validate the proposed approaches of slip detection and adaptive torque distribution for trajectory tracking of a holonomic mobile robot with four powered caster joints. The parameters of the holonomic mobile robot are presented as following: robot's weight m is 230 kg; robot's radius R is 0.3 m; caster wheel's radius r is 0.055 m; caster wheel's offset b = 0.02 m; and the distance between the contact of caster wheel with robot body and center of the robot is h = 0.2159 m.

In the simulation, the desired trajectory is to let the mobile robot move along a sine curve in the 2D plane with the speed of 0.2 m/s for 50 seconds firstly, and then move along another sine curve with the speed of 0.15 m/s for another 50 seconds. The orientation is kept the same during all the motion of the robot. Also, the terrain is assumed to be varying. And we assume that the first wheel of the mobile robot always slips both in driving and steering at some random slip ratio during the motion. The control system frame is shown in fig. 4. The details of each block have been introduced in section IV.

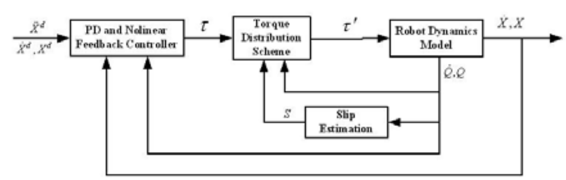

Fig. 4. Control system frame

The simulation results are shown in figures 5 and 6. Fig. 5 (a) depicts the trajectory tracking result of the holonomic mobile robot using common PD controller under random slips. The dashed curve shows the desired trajectory to be tracked. The real curve shows the actual trajectory of the mobile robot. The mobile robot is only controlled by a common PD tracking controller. The estimated slip ratio is shown in figure 5 (b). From the results, we can see that the mobile robot can not track the desired trajectory when the random slips occur. Especially, it moves out of the desired trajectory in the later part of the motion because the slips cause some changes in the motion direction of the robot, which can not be revised by the common PD tracking controller.

Figure 6 (a) and (b) depict the trajectory tracking result of the proposed method and estimated slip ratio under random slips. The mobile robot is controlled by the proposed tracking controller and torque distribution scheme. To better compare the results, we define the tracking error as,

$$E(t) = \sqrt{(x^d(t) - x(t))^2 + (y^d(t) - y(t))^2} \tag{26}$$

where, $x^d(t)$ and $y^d(t)$ denote the desired x axis and y axis positions in the world reference frame; $x(t)$ and $y(t)$ denote the actual x axis and y axis positions in the world reference frame.

According to the simulation results, the average tracking error of the original one is 19.18 cm. The average tracking error of the proposed method decreases to 5.24 cm. This indicates that the proposed method can make the robot track the desired trajectory with distinctly smaller tracking errors under random slips. In addition, from figure 6 (b), we can see that the slip ration are obviously reduced and well controlled by our proposed method compared with the slip ratio in figure 5 (b).

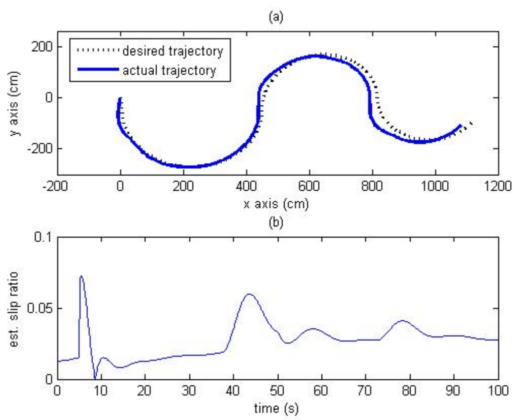

Fig. 5. Trajectory tracking with common PD controller under random slips

It is worth noticing that the slip ratio is a little big at the beginning of the motion. This is because that the terrain condition is changing dramatically and sharply at the beginning, which causes the slip ratio to change dramatically and sharply as well. From the simulation results, we can see that the proposed method can reduce the slip ratio to some extent in this case, but fails to keep the slip ratio stable in some small values when the terrain condition changes dramatically and sharply. So this will be a problem to be investigated in our following research. In general, however, our proposed method has been proved to work effectively in trajectory tracking under random slips.

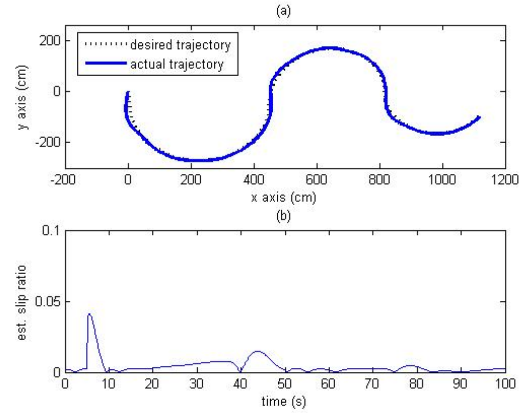

Fig. 6. Trajectory tracking with proposed method under random slips

VI. VI. CONCLUSIONS

This paper proposes a new approach to deal with the problem of trajectory tracking for the holonomic 4-PCV by dynamically distributing the force/torque applied on the wheels under complex terrain. First, the dynamics model of the 4-PCV is derived. A new approach for slip ratio estimation is proposed. Compared with the other methods, this method does not need any additional task space sensors but only some common motor encoders. The adaptive controller for torque distribution is designed based on the dynamics model and the estimated slip ratio. The simulation results show that our proposed approaches and controllers can achieve a good performance. Trajectory tracking under dramatical and sharp changes in slip ratio will be the next consecutive topic in our following research.

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