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Control of Robotic Manipulators under Time-Varying Sensing-Control Delays

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Abstract— **Time-varying input/output delays in a control system can significantly degrade the stability and performance of the closed loop system. Recently, passivity based control has emerged as a promising candidate to guarantee delay independent stability of passive systems with delays in the input-output channel. In this paper we study set point control of rigid robots with time-varying sensing/control delays. We first show that the classical PD controller can be modified to regulate the robotic manipulator, provided scattering transformation along with additional gains are used in the communication path. While this results in a stable system, asymptotic regulation cannot be guaranteed. Hence, a (delay dependent) gain margin for a proportional position feedback controller is provided to guarantee stability and asymptotic convergence of the regulation error to the origin. To improve closed loop performance, scattering transformation based design of a damping injection scheme is also discussed. The proposed algorithms are numerically verified on a two-degree-of-freedom manipulator.**

I. Introduction

In this paper we study the problem of motion control of rigid robots when there are time varying delays in their input-output channel. In the last three decades, several control schemes [23] have been developed for control of robots. Starting with the work of [24], passivity-based control [18] has been a fruitful methodology for control design of robotic systems. Several control design have been presented in the literature [17], [13] where the controller and the mechanical system can be represented as a negative feedback interconnection of passive systems. Invoking the fundamental passivity theorem [7], it is then possible to guarantee passivity of the closed loop system. Under additional assumptions, stability of the closed loop system can also be established.

The problem of bilateral teleoperation [1], [15], a classical problem in robot control, highlighted the deleterious effect of time delays on the stability of the closed loop system. This problem has received widespread attention and several results [9] have been developed to address the network delay and lossy nature of the communication network. However, input/output delays may manifest in a robotic control system from many other sources, for example processing delays in visual systems [6] or from communication between different computers on a single humanoid robot [19]. It is well known [20] that guaranteeing stability of a control system with input delays is a challenging problem.

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The issue of time delay instability in dissipative systems has been studied by several authors [14], [11], [4], [2], [19], [21]. Scattering or the wave-variable representation, which was developed in [1], [15] for guaranteeing stability of bilateral teleoperators, has emerged as a novel tool for studying network control systems [14], [11], [4], [2], [19], [21]. The basic idea in these results is to use the scattering variables to guarantee passivity of the communication block, thereby creating a passive two-port network between a passive plant, communication and the passive controller. Furthermore, time-varying gains [12], dependent on the maximum rate of the delay, can be additionally added in the communication path to guarantee stability independent of the time-varying delays [4], [2].

In the sequel, we study the problem of set-point control in rigid robots (with revolute joints) with time-varying input/output delays and which are constrained to move in the horizontal plane. We first show that the closed loop system constituted by a PI controller, a rigid robot in the horizontal plane can be stabilized using the scattering transformation along with gains dependent on the maximum rate of change of the delay [12]. However, simulations indicate that while stability is preserved, the regulation goal is not always achievable. Consequently, a gain margin for a proportional position feedback controller is provided to guarantee stability and asymptotic convergence of the regulation error to the origin. However, the proportional gain in the scheme cannot be arbitrarily assigned and depends on the maximum round trip delay and the innate dissipation in the robotic manipulator. To improve the convergence rate, a second control loop is appended to the proportional controller so as to guarantee arbitrary dissipation in the system independent of the time-varying delays and hence achieve good tracking performance.

The contribution of the paper can be summarized as follows:

- **Theorem 3.1**: This result demonstrates that a PI controller can be used to stabilize the robotic system independent of the time-varying delays between the controller and the robotic manipulator provided gains dependent on the maximum rate of the change of delay are utilized in the communication path. This formulation was earlier used for stabilizing teleoperators against time-varying delays in [5,12]. The application in the current problem of set-point regulation is novel. However, we show by simulations that this result cannot guarantee set-point regulation, hence motivating the next result.
- **Theorem 3.3:** If the manipulator has intrinsic damping, this result proves that a proportional position feedback

controller can be used to to guarantee set-point regulation independent of the time-varying delays. However in this architecture, the robotic system communicates its position signal to the controller and the controller has knowledge of the damping bound at the robotic manipulator. To enable a modular scheme the next result is proposed.

- **Theorem 3.4:** The previous result relies on internal damping in the robotic system. However, the bound on the damping may be unknown at the controller and hence damping may have to be injected by the controller. In this result we show that damping can be injected by using a scattering transformation based loop in conjunction with the proportional feedback discussed in Theorem 3.3.

The outline of the paper is as follows. A brief background on the general concept of passivity and a description of the robot dynamics is presented in Section II. This is followed by the main results and accompanying simulations in Section III. Finally the results are summarized in Section IV.

II. Preliminaries

The concept of passivity is one of the most physically appealing concepts of system theory [22] and, as it is based on input-output behavior of an system, is equally applicable to both linear and nonlinear systems. Consider a dynamical system represented by the state space model

$$\dot{x} = f(x,u) \quad (1)$$

$$y = h(x) \quad (2)$$

where f: $R^n \times R^p \rightarrow R^n$ is locally Lipschitz, h: R^n R^p is continuous, $f(0,0)=0$, $h(0)=0$ and the system has the same number of inputs and outputs.

The dynamical system (1)-(2) is said to be passive if there exists a continuously differentiable non-negative definite scalar function $S(x)$: $R^n \rightarrow R$ (called the storage function) such that

$$u^T y \geq \dot{S}(x), \qquad \forall (x,u) \in R^n \times R^p$$

Following [23], in the absence of friction and disturbances, the Euler-Lagrange equations of motion for an n-degree-of-freedom robotic system in the horizontal plane are given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = -\tau_s + \tau_e = \tau_t \quad (3)$$

where $q(t) \in R^n$ is the vector of generalized configuration coordinates, $\tau_s \in R^n$ is motor torque acting on the system, $\tau_e \in R^n$ is the external torque acting on the system, $M(q) \in R^{n\times n}$ is the positive definite inertia matrix and $C(q,\dot{q})\dot{q} \in R^n$ is the vector of Coriolis/Centrifugal forces. In this paper we consider manipulators with revolute joints. The above equations exhibit certain fundamental properties due to their Lagrangian dynamic structure [23].

- **Property 1:** The matrix $M(q)$ is symmetric positive definite and there exists a positive constants m_1, m_2 such that $m_1 \leq M(q) \leq m_2$.

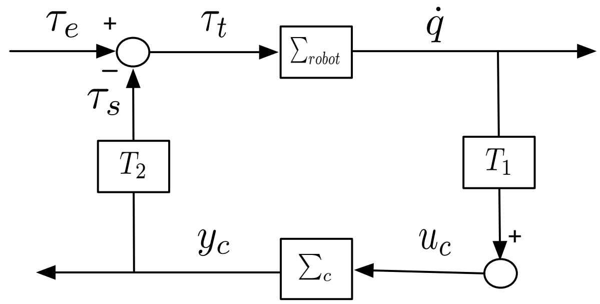

Fig. 1. A negative feedback interconnection of the robot dynamics and the controller

- **Property 2:** Under an appropriate definition of the matrix C, the matrix $\dot{M}$ - $2C$ is skew-symmetric

Moreover, it is well known that the robot dynamics are passive [23] with

$$S(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} \quad (4)$$

as the storage function and $(\tau_t, \dot{q})$ as the input-output pair. The passivity property of the robot dynamics has led to constructive control designs for the robot manipulators. Specifically, several robot control algorithms can be reformulated as a negative feedback interconnection of two passive systems [13]. Observing Figure 1, the controller takes in the robot velocity as the input, and the output of the controller block is fed back to the robot as the desired control input. If the controller is input-output passive, then by the fundamental passivity theorem [7], the closed loop system formed by the robot dynamics and the controller is passive. This interconnection property is exploited in the subsequent control design.

III. Main Results

In this section we study the set point problem for mechanical systems with time-varying input/output delays. The controller dynamics are given as

$$\text{Controller} = \begin{cases} \dot{x}_c = u_c \\ y_c = K_P u_c + K_I(x_c - q_d) \end{cases} \quad (5)$$

where where $K_P, K_I > 0$ are the scalar controller gains, q_d denotes the constant vector for the desired configuration, $u_c(t) = \dot{q}(t - T_1(t))$ and furthermore the control input to the robot is given as $\tau_s(t) = y_c(t - T_2(t))$ where $T_1(t), T_2(t)$ are the heterogeneous time varying delays between the robot and the controller. It is possible to show (see [3]) that the closed loop system easily destabilizes even with small constant delays.

Let $x(t) = [x_c(t) \;\; \dot{q}(t)]^T$ and denote by x_t the state of the system. Denote by $\mathscr{C} = \mathscr{C}([-h,0], R^{2n})$, the Banach space of continuous functions mapping the interval $[-h,0]$ into R^{2n}, with the topology of uniform convergence. Define $x_t = x(t+\phi) \in \mathscr{C}, -h < \phi < 0$ as the state of the system [8]. We assume in this note that $x(\phi) = \eta(\phi), \eta \in \mathscr{C}$ and that all signals belong to $\mathscr{L}_{2e}$, the extended $\mathscr{L}_2$ space. The time delays are

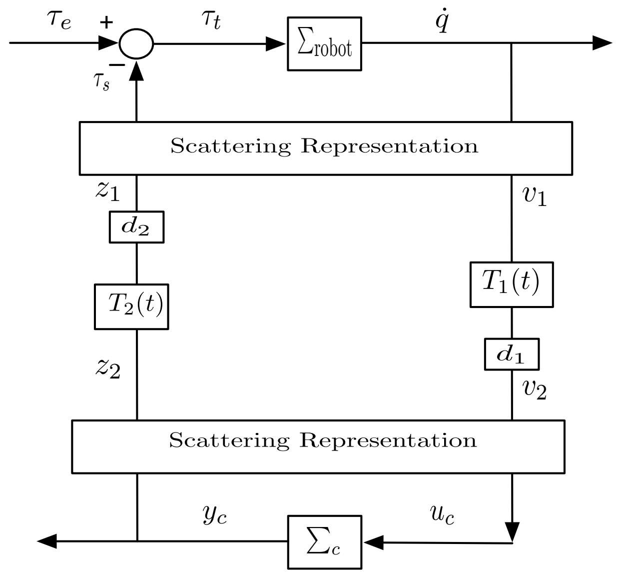

Fig. 2. The scattering transformation, together with the gains (dependent on the rate of change of delay) are used to ensure stability of the closed loop system

assumed to be bounded ($0 < T_i(t) \leq T_m^* < \infty, \quad i = 1,2$) and continuously differentiable with

$$\dot{T}_i(t) \leq T_i^{\max} < 1, \quad i = 1,2 \tag{6}$$

The above condition implies that the time delays cannot grow faster than time itself, and hence is a statement about the causality of the system. To passify the communication block, scattering variables are used between the plant and the controller and are given as

$$\begin{array}{ll} v_1 = \frac{1}{\sqrt{2b}}(\tau_s + b\dot{q}) & ; \quad z_1 = \frac{1}{\sqrt{2b}}(\tau_s - b\dot{q}) \\ v_2 = \frac{1}{\sqrt{2b}}(y_c + bu_c) & ; \quad z_2 = \frac{1}{\sqrt{2b}}(y_c - bu_c) \end{array} \tag{7}$$

where $b > 0$ is a constant. Furthermore, to address time varying delays [12], [5], gains dependent on the maximum rate of change of delay are inserted in the communication between the plant and the controller. The constant gains d_1, d_2 are selected as

$$\begin{array}{l} d_1^2 < (1 - T_1^{\max}) \\ d_2^2 < (1 - T_2^{\max}) \end{array} \tag{8}$$

The proposed architecture is demonstrated in Figure 2. The transmission equations between the robot and the controller can be written as

$$\begin{array}{rl} z_1(t) = & d_2 z_2(t - T_2(t)) \\ v_2(t) = & d_1 v_1(t - T_1(t)) \end{array} \tag{9}$$

The controller dynamics for this system are described by (5), however note that $u_c \neq \dot{q}(t - T_1(t))$ but is derived from the scattering representation (7) and the transmission equations (9).

The first claim in the paper follows

Theorem 3.1: Consider the closed loop system described by (3), (5), (7) and (9). Then the closed loop system is input-output passive with $(\tau_e, \dot{q})$ as the input-output pair. Additionally, if $\tau_e(t) \equiv 0$, then the signals $\dot{q}$ and $q_c - q_d$ are Lyapunov stable.

Proof: Consider a positive definite storage functional for the system as

$$S(x_t) = \frac{1}{2}(\dot{q}^T M(q)\dot{q} + K_I(x_c - q_d)^T(x_c - q_d)) + \frac{1}{2}\left(\int_{t-T_1(t)}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2(t)}^{t} ||z_2(\tau)||^2 d\tau\right)$$

The derivative of the storage function yields

$$\begin{aligned} \dot{S}(x_t) &= \dot{q}^T(-C(q,\dot{q})\dot{q} - \tau_s + \tau_e) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} \\ &+ K_I(x_c - q_d)^T \dot{x}_c + \frac{1}{2}(||v_1||^2 - ||v_1(t - T_1(t))||^2(1 - \dot{T}_1(t)) \\ &+ ||z_2||^2 - ||z_2(t - T_2(t))||^2(1 - \dot{T}_2(t))) \\ &\leq \dot{q}^T(-C(q,\dot{q})\dot{q} - \tau_s + \tau_e) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} \\ &+ K_I(x_c - q_d)^T \dot{x}_c + \frac{1}{2}(||v_1||^2 - ||v_1(t - T_1(t))||^2 d_1^2 + ||z_2||^2 \\ &- ||z_2(t - T_2(t))||^2 d_2^2 \\ &\leq (-\tau_s + \tau_e)^T \dot{q} + y_c^T u_c - K_P u_c^T u_c + \frac{1}{2}(||v_1||^2 - ||z_1||^2 \\ &+ ||z_2||^2 - ||v_2||^2) \\ &\leq (-\tau_s + \tau_e)^T \dot{q} + y_c^T u_c - K_P u_c^T u_c + \tau_s^T \dot{q} - u_c^T y_c \\ &\leq \tau_e^T \dot{q} - K_P u_c^T u_c \end{aligned} \tag{10}$$

Hence the closed loop system is passive with $(\tau_e, \dot{q})$ as the input-output pair. From (10) it is easy to observe that if $\tau_e(t) \equiv 0$, then $\dot{S}(x_t) \leq 0$ and hence the signals $\dot{q}$ and $q_c - q_d$ are Lyapunov stable. ■

The above result demonstrates that the closed loop system constituted by the robotic system, coupled with the PI controller can be made passive independent of the time-varying delays. To observe the regulation capabilities of the the above architecture, the proposed algorithm is simulated using a two-link revolute joint arm [23]. The dynamics of a two link robot, in the absence of gravitational forces, are given as

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_2\dot{q}_1 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 = \tau_1 \tag{11}$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 = \tau_2 \tag{12}$$

where the entries of the inertia matrix are given as

$$\begin{aligned} d_{11} &= m_1 l^2 c1 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2}\cos(q_2)) + I_1 + I_2 \\ d_{12} &= d_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2 \\ d_{22} &= m_2 l_{c2}^2 + I_2 \end{aligned}$$

On the other hand, the $c_{121} = -m_2 l_1 l_{c2} sin(q_2) = p$ and $c_{221} = p, c_{112} = -p$. In the simulations, $m_1 = 7.848, m_2 = 4.49, I_1 = 0.176, I_2 = 0.0411, l_1 = 0.3, l_2 = 1, l_{c1} = 0.1554, l_{c2} = 0.0341$.

The vector $\dot{q} = [\dot{q}_1 \ \ \dot{q}_2]^T$ is the output of the robotic system and is transmitted to the controller described by (5). The time-varying delay in the input-output path was selected to be $T_1(t) = 0.6 + 0.5sin(t); T_2(t) = 0.3 + 0.2sin(t)$ which satisfies

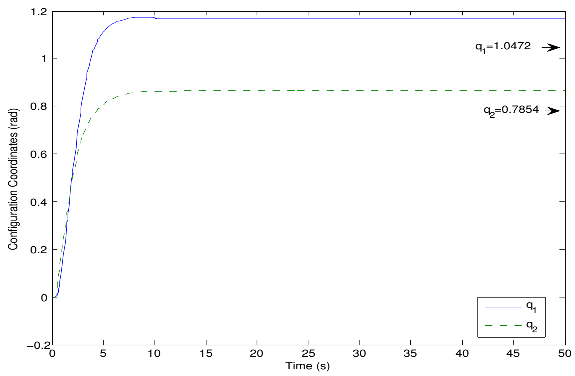

Fig. 3. The joint angles are stable but do not converge to the desired equilibrium as pointed out using the steady state values

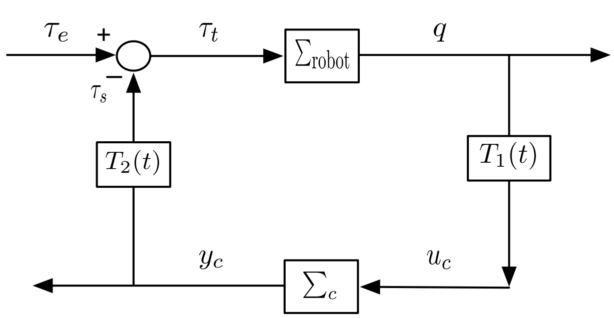

Fig. 4. A proportional position feedback controller architecture for stabilizing the joint angles to the desired equilibrium. Note that in contrast to Figure 2, the joint information is explicitly communicated to the controller

the assumption that $\dot{T}_i \leq 1, i = 1,2$. Following (8), the time-varying gains were calculated as $d_1 = 0.7$ and $d_2 = 0.8$. The desired set point was chosen to be $q_d = [\frac{\pi}{3} \quad \frac{\pi}{4}]^T$. As seen in Figure 3, the system is stable, however the proposed control system is not able to regulate the robotic system to the desired equilibrium. Thus, in contrast with the constant delay studied in [3], the scattering transformation based architecture in the case of time-varying delays is only able to guarantee Lyapunov stability of the closed loop system but not its tracking performance.

To achieve the desired regulation goal in the presence of time-varying delays, an alternative architecture is studied as shown in Figure 4. However, the next result necessitates modification of the system dynamics where it is assumed that there exists innate dissipation in the system. Under this assumption, let the robot dynamics be given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B_n\dot{q} = -\tau_s + \tau_e = \tau_t \tag{13}$$

where $B_n > 0$ is a scalar denoting the natural damping in the system. In the proposed architecture, the signal $q(t)$ is the plant output which is communicated to the controller and the control action is then given as

$$y_c \quad = \quad K(q(t - T_1(t) - q_d)) \tag{14}$$

where $K > 0$ is scalar and as before q_d denotes the constant vector for the desired configuration. The controller output y_c is communicated back to the robot, and assuming $\tau_e \equiv 0$ the closed loop system becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B_n\dot{q} = K(q(t - T_1(t) - T_2(t)) - q_d) \tag{15}$$

We next study the stability of the above closed loop system. For the sake of completeness, we provide a brief overview of a technical result developed in [16].

Lemma 3.2: Given signals $x, y \in R^n$, $\forall T(t)$ such that $0 < T(t) \leq T_m < \infty$, and $\alpha > 0$ the following inequality holds

$$-\int_0^t x^T(\sigma)\int_{-T(\sigma)}^0 y(\sigma+\theta)d\theta d\sigma \leq \frac{\alpha}{2}||x||_2^2 + \frac{T_m^2}{2\alpha}||y||_2^2 \tag{16}$$

where $||\cdot||_2$ denotes the $\mathscr{L}_2$ norm of the enclosed signal. We refer the reader to [16] for a proof the above result. Our next result in the paper follows

Theorem 3.3: Consider the closed loop system described by (15). If the time-varying delays satisfy $0 \leq T_1(t) + T_2(t) \leq T_m < \infty$, $i = 1,2$, then for a range of the gain $0 < K \leq K^*$, the signals $\dot{q}$ and $(q(t - T_1(t) - T_2(t)) - q_d)$ are asymptotically stable.

Proof: Consider a positive definite storage function for the system as

$$V(\dot{q},q) = \frac{1}{2}(\dot{q}^T M(q)\dot{q} + K(q - q_d)^T(q - q_d)) \tag{17}$$

Differentiating along system trajectories we get

$$\begin{aligned}
\dot{V} &= \dot{q}^T(-C(q,\dot{q})\dot{q} - B_n\dot{q} - K(q(t - T_1(t) - T_2(t)) - q_d)) \\
&+ \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} + K\dot{q}^T(q - q_d) \\
&= -B_n\dot{q}^T\dot{q} + \dot{q}^T(q - q(t - T_1(t) - T_2(t))) \\
&= -B_n||\dot{q}||^2 + K\dot{q}^T\int_{-T_1(t)-T_2(t)}^0 \dot{q}(t+\theta)d\theta
\end{aligned} \tag{18}$$

Note that as we try to upper bound the second term, the sign of the second term does not affect the subsequent calculations. Integrating (18) from 0 to t and using Lemma (3.2), we get

$$\begin{aligned}
&V(\dot{q}(t),q(t)) - V(\dot{q}(0),q(0)) \leq -B_n||\dot{q}||_2^2 + K(\frac{\alpha}{2}||\dot{q}||_2^2 \\
&+ \frac{T_m^2}{2\alpha}||q||_2^2) \\
&\leq -||\dot{q}||_2^2(B_n - \frac{K\alpha}{2} - \frac{KT_m^2}{2\alpha})
\end{aligned} \tag{19}$$

If the following inequality given by

$$B_n - \frac{K\alpha}{2} - \frac{KT_m^2}{2\alpha} > 0 \tag{20}$$

is satisfied for $\alpha > 0$, then $V(\dot{q}(t),q(t)) - V(\dot{q}(0),q(0)) \leq 0$ and hence the signal $\dot{q}(t)$ is square integrable. The above inequality has a solution $\alpha > 0$ if $B_n > KT_m$. Thus, if $K < B_n/T_m = K^*$, $V(\dot{q}(t),q(t)) \leq V(\dot{q}(0),q(0))$, $\forall t > 0$. Consequently, for any appropriately selected K as discussed above, the signals $\dot{q}, q - q_d \in \mathscr{L}_\infty$. Noting the system dynamics (15), this additionally implies the robot acceleration $\ddot{q} \in \mathscr{L}_\infty$. Hence as $\dot{q} \in \mathscr{L}_2$ and its derivative is bounded, the robot velocity asymptotically approaches the origin.

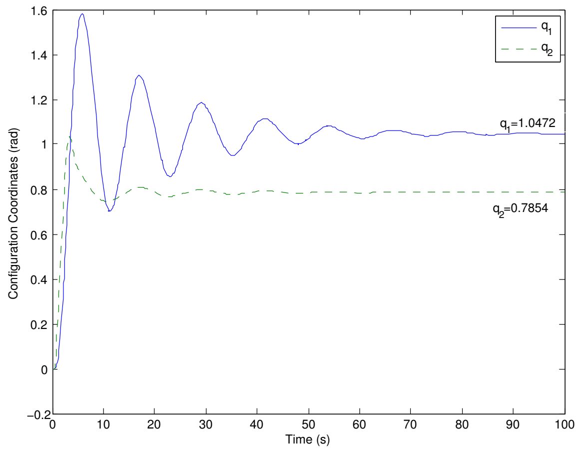

Fig. 5. The proportional position feedback controller, when employed in the permissible gain margin, guarantees asymptotic convergence to the desired configuration but may lead to a large settling time

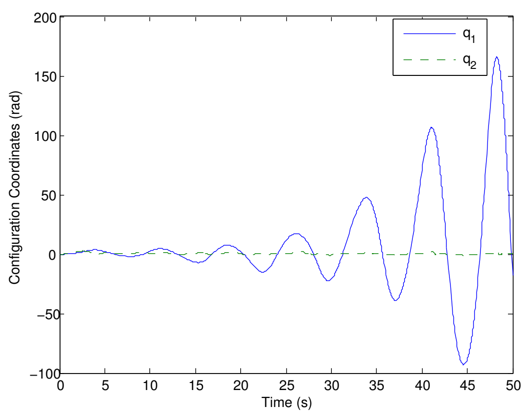

Fig. 6. Increasing the gain K destroys stability of the closed loop system

To demonstrate asymptotic convergence of the tracking error, differentiating (15) yield that the signal $\dddot{q} \in \mathscr{L}_\infty$ (note that the derivative of the Coriolis term is also bounded for revolute joints [16]). Hence, the robot acceleration is uniformly continuous and $\lim_{t\to\infty}\int_0^t \ddot{q}(s)ds$ exists and is finite. Invoking Barbalat's Lemma [10], $\lim_{t\to\infty}\ddot{q}(t)=0$. Observing the closed loop dynamics (15), $\lim_{t\to\infty} q(t-T_1(t)-T_2(t)) - q_d = 0$ and consequently the regulation objective is achieved asymptotically. ■

The above scheme was simulated on the two degree of freedom manipulator described earlier with the same time delays and the desired configuration. The maximum combined delay that be encountered in the sensing and control path is 1.6s. Assume that $B_n = 0.5$ and consequently the allowable limit for $K^* = \frac{B_n}{T_m} = 0.31$. With this value of the gain, as shown in Figure 5 asymptotic tracking was achieved. However the settling time in the proposed architecture in relatively large. With the aim of improving performance, the value of the gain was increased to $K=1$. However, as seen in Figure 6, the closed loop system is rendered unstable. A higher gain control gain K, while maintaining stability of the

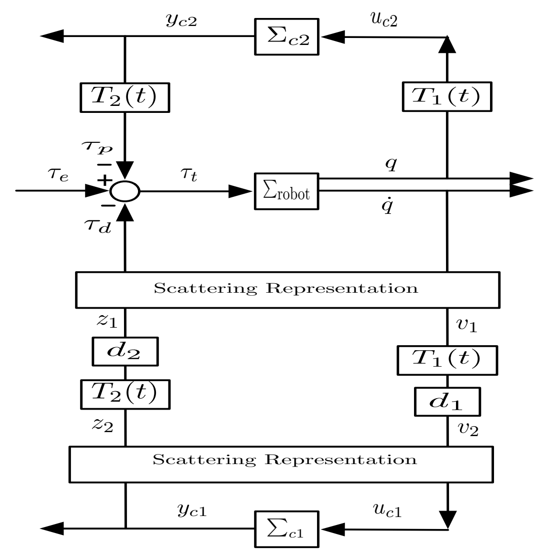

Fig. 7. A two loop architecture where the proportional position feedback controller is used in conjunction with a damping injection loop. The damping injection loop uses scattering variables to ensure stability of the closed loop system

closed loop system, requires more damping in the system. However, this parameter B_n cannot be controlled and hence control action is required to additionally damp the system. To this end, we propose our next control strategy that allows injection of arbitrary damping in the closed loop dynamics independent of the time-varying delays.

The proposed architecture (see Figure 7) incorporates an additional control loop to increase the damping in the system. In this design the robot velocity $\dot{q}$ is communicated to a static controller via the scattering transformation. Thus, the control torque is the sum of two terms and is given as

$$\tau_s = \tau_p + \tau_d \tag{21}$$

where τ_p is the proportional term designed earlier (14) and τ_d is the additional damping injected in the closed loop system. The damping control is derived out of the scattering transformation as shown in Figure 7. The scattering variables are given as

$$\begin{aligned} v_1 &= \frac{1}{\sqrt{2b}}(\tau_d + b\dot{q}) \quad ; \quad z_1 = \frac{1}{\sqrt{2b}}(\tau_d - b\dot{q}) \\ v_2 &= \frac{1}{\sqrt{2b}}(y_{c1} + bu_{c1}) \quad ; \quad z_2 = \frac{1}{\sqrt{2b}}(y_{c1} - bu_{c1}) \end{aligned} \tag{22}$$

where y_{c1} is the output of the static controller and is given as $y_{c1} = K_d u_{c1}$, $K_d > 0$ is a constant. Using this controller, the scattering variables can be rewritten as

$$\begin{aligned} v_1 &= \frac{1}{\sqrt{2b}}(\tau_d + b\dot{q}) \quad ; \quad z_1 = \frac{1}{\sqrt{2b}}(\tau_d - b\dot{q}) \\ v_2 &= \frac{K_d+b}{\sqrt{2b}}u_{c1} \quad ; \quad z_2 = \frac{K_d-b}{\sqrt{2b}}u_c \end{aligned} \tag{23}$$

The transmission equations are given by (9) and substituting (23) in (9) we get

$$\begin{aligned} (K_d+b)u_{c1} &= d_1(\tau_d(t-T_1(t)) + b\dot{q}(t-T_1(t))) \\ d_2(K_d-b)u_{c1}(t-T_2(t)) &= \tau_d - b\dot{q} \end{aligned} \tag{24}$$

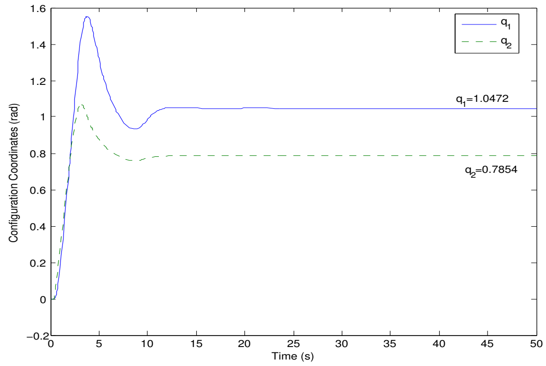

Fig. 8. The proportional controller along with damping injection results in better performance

In the second equation above, choosing $K_d = b$ yields

$$\tau_d - b\dot{q} = 0 \Rightarrow \tau_d = b\dot{q} \qquad (25)$$

Thus by using the scattering transformation and selecting the controller gain appropriately, arbitrary damping can be injected in the closed loop system.

In the previous result, i.e Theorem 3.3, existence of natural damping was required to guarantee stability of the closed loop system for arbitrary input/output delay. However, in the new architecture where arbitrary damping injection is possible, this assumption is not required. The final result in the paper follows

Theorem 3.4: Consider the robot dynamics (3) together with the control input (21), (14) and (23). Then for a range of the gain $0 < K \leq K^*$, the signals $\dot{q}$ and $(q(t - T_1(t) - T_2(t)) - q_d)$ are asymptotically stable.
This result follows from the above discussion and the proof of Theorem 3.3. An important difference between this algorithm and the scheme underlying Theorem 3.3 is that the limit on the proportional gains in the current scheme depends on the injected damping. Following the proof of Theorem 3.3, it is easy to observe that $K^* \leq \frac{K_d}{T_m}$. The gain K_d is equal to the wave impedance b which is a free parameter that can be chosen arbitrarily. The scheme was simulated on the two DOF system with $K_d = b = 2$ and the coupling gain was chosen to satisfy the gain constraint outlined above. As seen in Figure 8, the position error is asymptotically stable.

IV. Conclusions

In this paper we studied the problem of set-point control in rigid robots (with revolute joints) with time-varying input/output delays in the horizontal plane. We demonstrated that the closed loop system constituted by a PI controller, a rigid robot in the horizontal plane can be stabilized using a modified scattering transformation scheme [12]. However, while stability was preserved, simulations indicated that the regulation goal was not always achievable. To improve performance, gain margin for a proportional controller was calculated to guarantee stability and asymptotic convergence of the regulation error to the origin. However, the proportional gain in the scheme was contingent on the maximum round trip delay and the natural dissipation in the robotic manipulator. To improve the convergence rate, a second control loop was appended to the proportional controller so as to guarantee arbitrary dissipation in the system independent of the time-varying delays and hence achieved good closed loop performance. The results in the paper were validated on a two-degree-of-freedom manipulator

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