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Maintaining Connectivity in Environments with Obstacles

Onur Tekdas* Patrick A. Plonski Nikhil Karnad Volkan Isler

***Abstract*— Robotic routers (mobile robots with wireless communication capabilities) can create an adaptive wireless network and provide communication services for mobile users on-demand. Robotic routers are especially appealing for applications in which there is a single mobile user whose connectivity to a base station must be maintained in an environment that is large compared to the wireless range.**

In this paper, we study the problem of computing motion strategies for robotic routers in such scenarios, as well as the minimum number of robotic routers necessary to enact our motion strategies. Assuming that the routers are as fast as the user, we present an optimal solution for cases where the environment is a simply-connected polygon, a constant factor approximation for cases where the environment has a single obstacle, and an $O(h)$ approximation for cases where the environment has h circular obstacles. The $O(h)$ approximation also holds for cases where the environment has h arbitrary polygonal obstacles, provided they satisfy certain geometric constraints – e.g. when the set of their minimum bounding circles is disjoint.

I. INTRODUCTION

Suppose a mobile user operating in a large environment needs network connectivity to a base station. The user may be a tele-operated robot and the base station may be a gateway to the Internet. Without a communication infrastructure, the mobility of the user would be restricted by the communication radius of the base-station. The traditional solution for providing long-range network connectivity is to deploy a network of static wireless routers which cover the entire area of interest. However when the environment is large, covering it can be costly. Moreover, in some scenarios (such as natural disasters or hazardous conditions) it might be impossible to manually deploy this network in advance.

On the other hand, we can deploy a small number of robots to act as mobile routers. These robots can autonomously relocate themselves according to the movement of the user and maintain their connectivity with the base station. In order to demonstrate the potential gain attained by using robots, let us consider the scenario shown in Figure 1. In this scenario, a user u navigates inside a circular arena, and wishes to remain connected to the base station at c at all times. Suppose the communication range of all devices is σ. If we deploy a stationary network to cover the arena, the number of necessary routers is $\Theta(R^2/\sigma^2)$. Instead, $\Theta(R/\sigma)$ robotic routers can maintain the user's connectivity by staying on the line segment $[cu]$ (the details of this strategy is given in Section III). Hence, the number of routers used can be drastically reduced by using the mobility of robots.

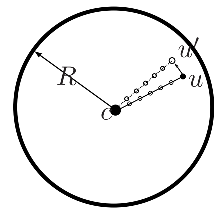

Fig. 1. An example which demonstrates the potential gain of using robotic routers. We need $\Theta(R^2/\sigma^2)$ static routers whereas $\Theta(R/\sigma)$ robots are sufficient to keep the user connected. When the user moves from u to u', robotic routers move with the same angular velocity to keep the user connected.

In this paper, we study the problem of computing the minimum number of robotic routers (and their motion strategies) in order to maintain the connectivity of a single user to a base station. After an overview of related work, we formalize the robotic routers problem in Section II. In Section III, we present an optimal solution for simply-connected polygons. In Section IV, we present a constant-factor approximation for a polygonal environment with a single obstacle. In Section V, we present an $O(h)$-approximation algorithm for polygons with multiple obstacles.

A. Related work

Exploiting the controlled mobility of robots to improve connectivity has received significant attention. Related problems include establishing connectivity [8], [15], building a bridge between two points [20], network coverage [6], [16], repairing a network's connectivity [5], [1], and improving it [4], [10]. In sensor networks literature, robots are used as mobile sensors to extend sensing regions [7], [11] or to collect data from stationary sensors [21], [17]. Moreover, a network of robots can be used for improving the communication in search and rescue tasks [12], [13].

In our recent work, we studied the problem of maintaining connectivity between a mobile user and a base station for two different user motion models [19]. We presented optimal algorithms to compute the minimum number of routers to maintain connectivity for both models. While the algorithms can incorporate arbitrary communication link models, their running times are exponential in the number of robots. In this work, we focus on geometric domains and geodesic distance based connectivity models, and present polynomial time approximation algorithms.

In other related work, Stump et al propose two metrics for characterizing connectivity and present a framework which

*Corresponding author is Onur Tekdas. The authors are with the Department of Computer Science and Engineering at the University of Minnesota, 200 Union St SE, Minneapolis, MN 55455, USA. Emails: {tekdas,plonski,karnad,isler}@cs.umn.edu.

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chooses the best local decision to maintain the connectivity of an independently moving target [18]. Here, we are able to give global guarantees by controlling the number of robots. In a related paper [9], Dixon et al study the problem of forming a chain of robotic relays and present an algorithm to control robots along the chain to improve the signal-to-noise ratio. A similar problem is considered by Kutylowski et. al [14]. They presented a global strategy for using a chain of robots to create a communication bridge between a stationary camp and a mobile explorer. In [2], they extend this strategy to local (distributed) strategies. However, in complex environments with obstacles, topologies more general than chains must be used. This is the main focus of the present work.

II. Problem formulation and notation

In this section, we present the terminology and notation used throughout the paper, and formalize the robotic router problem.

A *robotic router* is a mobile robot which can communicate wirelessly. Robotic routers are subject to communication and motion constraints such as limited communication range and a bounded maximum speed. The *base station* is a static entity to which the *user* wishes to establish connection. All entities are contained in a shared environment denoted by $\mathcal{P}$. The user is connected to the base station through a communicating bridge of robotic routers.

Throughout the paper, we assume that the time domain is continuous. We denote the position of the user at time t as $u(t)$, and that of the i^{th} robotic router as $r_i(t)$. We assume that both the robotic routers and the user have the same maximum speed. We call this requirement as *the motion constraint*. We will prove the correctness of our strategies by showing that the speed of each robotic router at time t is less than or equal to the speed of the user at time t. In other words, let $|\dot{u}(t)|$ and $|\dot{r_i}(t)|$ be their respective speeds; we show that $|\dot{r_i}(t)| \leq |\dot{u}(t)|$ always holds.

We measure the distance between any two points $x, y \in \mathcal{P}$ by the length of the geodesic path from x to y, i.e. the shortest path from x to y that lies inside $\mathcal{P}$ and does not cut through any obstacles. For any time t, we denote the geodesic shortest path from the base station b to the user $u(t)$ as $SP(t)$. The shortest geodesic distance between x and y is denoted by $d(x, y)$

Various models for radio propagation are studied in the literature. Due to various environment dependent effects (such as multi-path, fading, occlusion, etc.), it is difficult to provide a generic model which incorporates all these effects. In this work, we assume that two points $x \in \mathcal{P}$ and $y \in \mathcal{P}$ are connected if $d(x, y) \leq \sigma$ holds. This is the *communication constraint*. With this threshold, we can address the fading effects with our model in which σ is a communication distance threshold. Moreover, this model implicitly addresses the occlusion (absorption) effects. If there exist a line-of-sight between x and y the geodesic distance is same as the Euclidean distance. However, when the polygon or an obstacle occludes between x and y, the geodesic distance increases.

To simplify the notation, we scale all distances by the communication distance threshold σ. Throughout the paper, without loss of generality, we assume that the *communication distance* is the unit distance. Let D be the longest geodesic shortest path from b to any point $\in \mathcal{P}$; $m^* = \lceil D - 1 \rceil + 1 = \lceil D \rceil$ is a lower bound on the minimum number of robotic routers necessary to connect any point in $\mathcal{P}$ to b, including the base station as a robotic router.

We define the number of robots used as follows. For a given user trajectory $\mu = u(t)$, let $n(\mu)$ be the number of robots required to connect the user to the base station. For a given environment, the number of robots required is the maximum number over all possible user trajectories, i.e. $n = \max_{\mu} n(\mu)$. When computing n, we do not require that the routers know the user's trajectory in advance. However we assume that the robotic routers in the network are all continuously made aware of the current position of the user[1], and they can instantaneously choose their movements based on this information.

Problem Statement: Given an environment $\mathcal{P}$ (possibly with obstacles) and a base station $b \in \mathcal{P}$, find the minimum number of robotic routers and their motion strategies such that wherever the user u moves, it is connected to the base station at all times, and the motion and communication constraints are satisfied.

III. Environments with no obstacles

In this section, we present a strategy to maintain connectivity using an optimal number of $m^* = \lceil D \rceil$ routers. The strategy, which we call *EQ-DIST*, involves maintaining an equidistant separation along $SP(t)$. We show that this can be achieved without violating communication and motion constraints.

We say that the *Evenly Spaced Property (ESP)* holds at time t if all the routers are positioned uniformly along $SP(t)$. We will refer to this chain of routers as an *arm*. We assume that ESP holds at time 0 (i.e. the user is willing to wait until the initial connection is established).

It is well known that $SP(t)$ is a polygonal path $\{p_0 = b, p_1, p_2, \ldots, p_j, u\}$ from b to u, where any p_i for $i > 0$ is a vertex of $\mathcal{P}$. Observe that $j \leq n$ where $n = m^*$ is the number of robotic routers used by our strategy. In this sequence, the *parent node* of any point z on SP is defined as the node between b and z that is the closest to z (Figure 2).

We will need the following technical lemma. Due to space limitations, we present its proof in an associated technical report [22].

Lemma 1. *For any t there exists a sufficiently small $dt > 0$ and a shared point $s \in SP(t) \cap SP(t + dt)$ such that $SP(t)$ and $SP(t + dt)$ only differ along a single line segment, from their respective endpoints to s.*

We now show that the robots can maintain connectivity using *EQ-DIST*:

[1] e.g. this information can be provided by the user

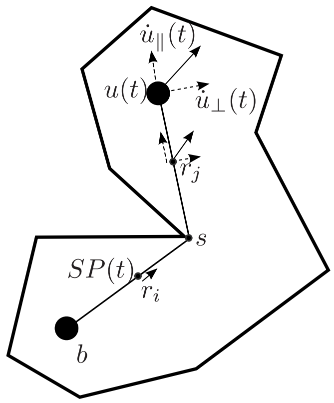

Fig. 2. Robots r_i only translate along SP while robots r_j rotate about s and translate.

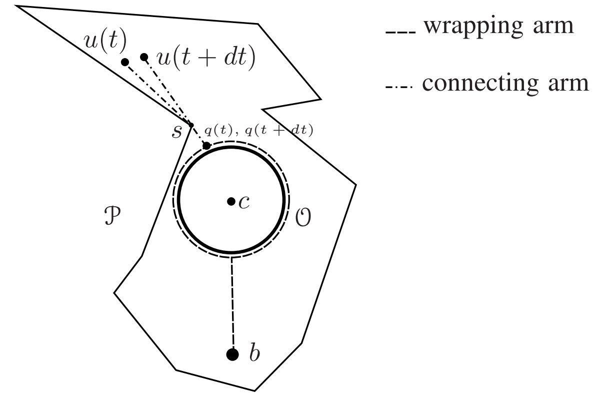

Fig. 3. Illustration of the first case of our strategy.

For any point $z(t) \in SP(t)$, let $z^{\dagger}(t)$ denote its parent node. Parameterize the velocity $\dot{z}(t)$ into a radial component $\dot{z}_{\|}(t)$, along $[z^{\dagger}(t)\ \ z(t)]$, and a tangential component $\dot{z}_{\perp}$ orthogonal to $\dot{z}_{\|}(t)$. We have $|\dot{z}(t)| = \sqrt{|\dot{z}_{\perp}(t)|^2 + |\dot{z}_{\|}(t)|^2}$. For any robot $r_i(t)$, we denote its velocity components as $\dot{r}_{i\|}(t)$ and $\dot{r}_{i\perp}(t)$ (see Figure 2).

Only the radial component of the user's velocity affects the length of SP. Let λ be a differential change in the length of SP. We have $\lambda = \dot{u}_{\|}(t)$. To satisfy ESP, the robotic routers should move proportional to λ along the radial component.

$$\dot{r}_{i\|}(t) = \frac{i}{n+1}\dot{u}_{\|}(t) \tag{1}$$

When the user moves, some line segments along SP rotate, while others remain the same. The tangential velocity of any robot is thus a function of which side of the shared point s the robot lies on.

If r_i lies between s and u, we can show using similar triangles that

$$\frac{|\dot{r}_{i\perp}(t)|}{|\dot{u}_{\perp}(t)|} = \frac{||s\ r_i(t)||}{||s\ u(t)||}$$

Where $||s\ r_i(t)||$ is the length of line segment $[s\ r_i(t)]$ and $||s\ u(t)||$ is the length of line segment $[s\ u(t)]$. Since $r_i(t)$ is closer to b than $u(t)$, we have $||s\ r_i(t)|| < ||s\ u(t)||$, i.e.

$$|\dot{r}_{i\perp}(t)| < |\dot{u}_{\perp}(t)| \tag{2}$$

The robots between s and b have a tangential component of zero.

Therefore, for any robot, (1) and (2) show that the robot only needs to move at most as fast as the user to stay on the geodesic from b to u while maintaining ESP.

Theorem 1. *In a simply-connected polygon, the number of mobile robots that the EQ-DIST strategy requires is optimal.*

Proof: Recall that the cost of the optimal solution is the required number of robots to connect any user trajectory. When the user goes to a location where SP is maximized, the optimum solution has to use at most $m = \lceil D \rceil$ robots. We have seen that *EQ-DIST* can maintain connectivity using the same number of robots. ∎

IV. Convex environments containing a single obstacle

In this section, we present robotic router strategies where a user is connected to b in a convex environment with a single obstacle. First we present a solution for circular obstacles.

Let $\mathcal{O}$ be a circular obstacle and let c and r be the center and radius of $\mathcal{O}$. Our strategy is as follows. First, we connect every point on $\mathcal{O}$ to b by extending an arm starting from b and wrapping it around $\mathcal{O}$. We call this our *wrapping arm*, and the robotic routers in it are stationary (see Figure 3). After connecting $\mathcal{O}$ to b, we use a *connecting arm* which rotates around $\mathcal{O}$ and connects the user to $\mathcal{O}$ which is then connected to b through the wrapping arm.

We place robots so that ESP property is satisfied; these locations can be easily found by using geometric properties of lines and circles. The bounds on the length of the arm and the number of robots used will be obtained in Theorem 2.

The connecting arm's responsibility is to connect user to $\mathcal{O}$ and consequently to b. We achieve this by moving robotic routers on the SP between u and $\mathcal{O}$. To guarantee that the connecting arm is always connected to $\mathcal{O}$, we use an additional robotic router q which moves along the boundary of $\mathcal{O}$. Robot q acts as a base station for the connecting arm. Let $SP_{\mathcal{O}}(t)$ be the shortest geodesic path between $\mathcal{O}$ and $u(t)$ (this path is the subset of SP between c and $u(t)$). Robot q always remains at the beginning of this path on $\mathcal{O}$.

We analyze the connecting arm strategy in two cases: (i) $u(t)$ has a parent node different than $q(t)$ (ii) $u(t)$ has $q(t)$ as the parent node.

Case (i): If there exists a parent node s of $u(t)$ such that $s \neq q$, then we can find a dt and a shared point s such that the shortest paths $SP_{\mathcal{O}}(t)$ and $SP_{\mathcal{O}}(t+dt)$ differ only along their last line segment (Lemma 1). Since both shortest paths pass through s and the shortest path from $\mathcal{O}$ to s is same, q does not move, i.e. $\dot{q}(t) = 0$ (See Figure 3). In this case, the connecting arm can execute EQ-DIST and maintain connectivity.

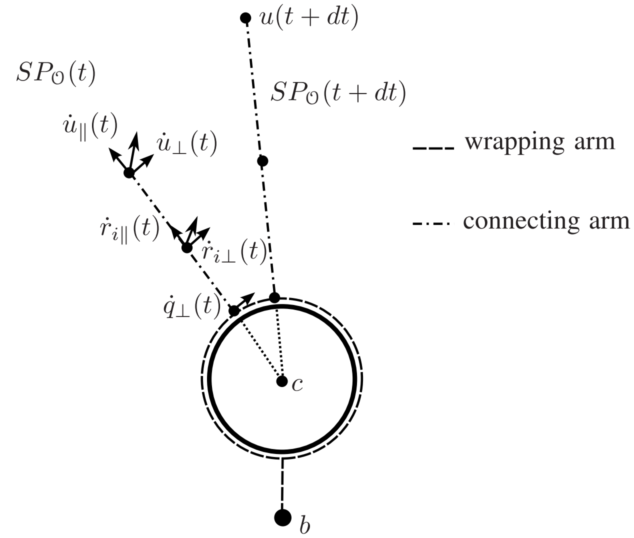

Fig. 4. Illustration of the second case of our strategy.

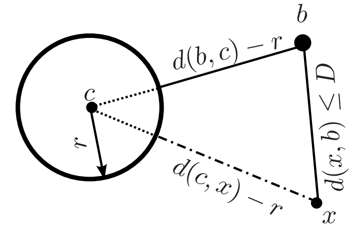

Fig. 5. Bounding the length of the connecting arm using triangle inequality.

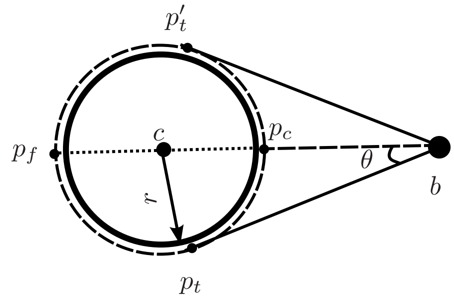

Fig. 6. The shortest geodesic distance from b to p_f and the length of wrapping arm in a convex polygon.

Case (ii): If the parent node of $u(t)$ is q, we can move q and the robots on the connecting arm in such a way that they maintain the ESP property without violating motion and communication constraints. As we did in Section III, we divide the velocity $\dot{u}(t)$ into two components: radial velocity $\dot{u}_{\parallel}$ and tangential velocity $\dot{u}_{\perp}$. Since q is moving on the boundary of $\mathcal{O}$ its radial velocity is 0. If q is the common parent node for $u(t)$ and $u(t+dt)$, these shortest paths rotate around c and rotation is due to the tangential component of u (see Figure 4). As angular velocity is the same for the user and each robot on $SP_{\mathcal{O}}$, we can conclude that $|\dot{q}_{\perp}(t)| \leq |\dot{r}_{i\perp}(t)| \leq |\dot{u}_{\perp}(t)|$. Since $\dot{q}_{\parallel}(t) = 0$ and $|\dot{q}_{\perp}(t)| \leq |\dot{u}|$, we have $|\dot{q}(t)| \leq |\dot{u}(t)|$. Otherwise, if $u(t)$ and $u(t+dt)$ do not have q as their common parent node, we can show that a time interval $dt' < dt$ can be found such that the above condition holds. The proof is the same as in Lemma 1.

Suppose $\dot{u}_{\parallel}(t)$ is positive; in this case $SP_{\mathcal{O}}$ increases in length. To satisfy ESP, the robotic routers have to move towards u. The distance from q to r_i must increase by $\frac{i}{n_c}|\dot{u}_{\parallel}(t)|$ where n_c is the number of robots in the connecting arm, including q, and i is the robot index in the connecting arm (the 0^{th} robot refers to q). The even spacing causes the distance from r_i to u to also increase. This is only possible when $|\dot{r}_{i\parallel}| \leq |\dot{u}_{\parallel}|$. Now suppose $\dot{u}_{\parallel}$ is negative; it can be shown in this case that $|\dot{r}_{i\parallel}| \leq |\dot{u}_{\parallel}|$. Therefore, for all times, the robotic routers have a smaller radial speed than the user. Moreover, from constant angular velocity observation we know that $|\dot{r}_{i\perp}(t)| \leq |\dot{u}_{i\perp}(t)|$ holds, hence we conclude that $|\dot{r}_i(t)| \leq |\dot{u}(t)|$, and the motion constraint is satisfied.

Theorem 2. *In a polygon with single circular obstacle, let m^* be the minimum number of robots required to maintain connectivity. The strategy presented in this section uses at most $5m^*$ robots.*

Proof: Let p_c and p_f be the closest and furthest points on $\mathcal{O}$ from b, respectively. By definition, we know that $d(b, p_f) \leq D$ where D is the maximum SP from b. The length of the wrapping arm equals to the sum of the SP distance from b to $\mathcal{O}$ and the circumference of $\mathcal{O}$, i.e. $2\pi r + d(b, p_c)$.

We find an upper bound on the length of the connecting arm using triangle inequality. For any point x in the polygon, due to triangle inequality, we have $d(c,x) \leq d(c,b) + d(b,x)$. We subtract r from both sides: $(d(c,x) - r) \leq d(x,b) + (d(b,c) - r)$. The connecting arm has length $d(q,x)$ where q is the closest point on $\mathcal{O}$ from x. This distance equals to $d(c,x) - r$ and it is upper bounded by $d(b,p_c) + D \leq 2D$ (Figure 5).

Next, we find a bound on the length of the wrapping arm (Figure 6). We start by showing that $\pi r \leq D$ holds. By definition, we have $d(b, p_f) \leq D$. We prove that $\frac{\pi r}{D} \leq \frac{\pi r}{d(b,p_f)} \leq 1$ holds. First, we calculate the maximum value of this ratio for a special case where $\mathcal{P}$ does not intersect with the tangents $[b\ p_t]$ and $[b\ p'_t]$. In this case, the ratio is $\frac{\pi r}{r(\theta + \cot(\theta) + \pi/2)}$. Using basic calculus, we can show that the maximum value of the ratio is 1.

We now show that if $\mathcal{P}$ intersects with one or both of the tangents, the ratio is reduced. Hence, the upper bound found in the special case is valid for any case.

Two types of vertices of $\mathcal{P}$ exist which can intersect with one or both of the tangents. Figure 7 shows these cases. The furthest point p_f on obstacle has the property that it has two shortest paths from the opposite sides of the obstacle. Any other point on the obstacle has a unique shortest path. For example, in Figure 6, these shortest paths are $SP_1 = \{b, p_t, p_f\}$ and $SP_2 = \{b, p'_t, p_f\}$. Since both shortest paths are equal, we can find $d(b, p_f) = SP_1 = SP_2$. Moreover, since both shortest paths start from b and end at p_f, their union constructs a convex hull around $\mathcal{O}$ and b. Hence, we

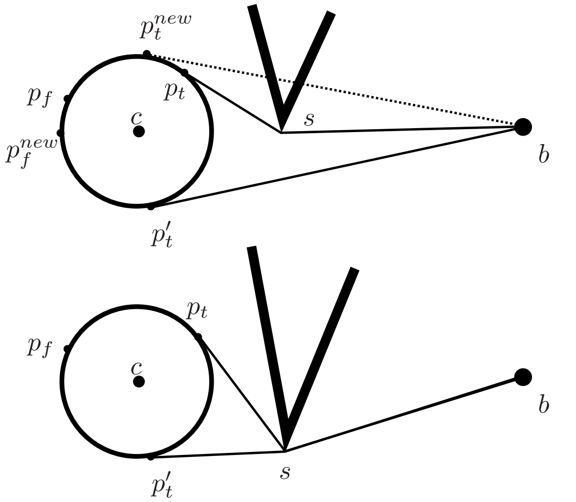

Fig. 7. Two types of vertices of $\mathcal{P}$ which can obscure with $[b\ p_t]$ and $[b\ p'_t]$

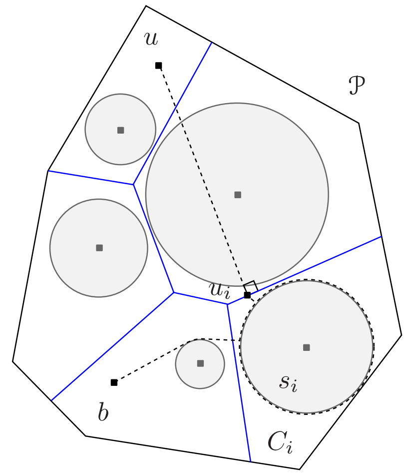

Fig. 8. Power diagram edges are shown in blue (light color). Circular obstacles are shaded. The connection from b to cell C_i is shown, along with the construction of the point u_i.

can say that $d(b, p_f)$ is half of $perim(\mathcal{H})$ where $perim(\mathcal{H})$ is the perimeter of the convex hull $\mathcal{H} = \{b, p_t, p_f, p'_t\}$.

First, we consider the first type of vertex (top Figure 7). Let s be a vertex which interferes with $[b\ p_t]$. The distance of the furthest point on $\mathcal{O}$ from b is half of $perim(\mathcal{H})$ where $\mathcal{H} = \{b, p_t, p_f, p'_t\}$. Now assume that we remove s; the distance of furthest point on $\mathcal{O}$ from b becomes the half of $perim(\mathcal{H}^{new})$ where $\mathcal{H}^{new} = \{b, p_t^{new}, p_f^{new}, p'_t, b\}$. By triangle inequality ($d(b, p_t^{new}) \leq d(b, s) + d(s, p_t) + d(p_t, p_t^{new})$), we can show that $perim(\mathcal{H})$ is longer than $perim(\mathcal{H}^{new})$. Hence, by introducing s, we reduce the ratio: $\frac{\pi r}{d(b,p_f)}$.

Let s be a vertex of the second type (bottom Figure 7) which interferes with both $[b\ p_t]$ and $[b\ p'_t]$. Observe that $\frac{\pi r}{d(s,p_f)} \leq 1$ holds due to the special condition that we discussed before. Because $d(b, p_f) = d(b, s) + d(s, p_f)$, the following inequalities hold: $\frac{\pi r}{d(b,p_f)} \leq \frac{\pi r}{d(s,p_f)} \leq 1$.

Finally, the ratio between the number of robots used by our strategy and the optimal solution including the base station is:

$$\frac{\lceil d(b, p_c) + 2\pi r\rceil + \lceil d(b, p_c) + D\rceil}{\lceil D\rceil} \leq \frac{\lceil 3D\rceil + \lceil 2D\rceil}{\lceil D\rceil} \leq 5$$

■

Remark. We can also extend the cicular obstacle strategy into convex and non-convex polygonal obstacles. For both cases, the same approximation ratio holds as in the circular obstacle case. The details can be found in [22].

V. Convex environments with multiple obstacles

Let $\mathcal{P}$ be a convex polygonal environment containing two or more non-intersecting obstacles. If the convex hulls of the obstacles are disjoint, we can extend the strategies for the single obstacle case as follows. First, we partition $\mathcal{P}$ into cells, such that each cell is convex and contains exactly one obstacle.

For each cell we execute a strategy similar to the one we used in domains with a single obstacle: we have a wrapping arm that connects to b and wraps around the obstacle in the cell, and we have a connecting arm that, whenever u is in the cell, connects u to the closest point to u on the obstacle.

We start by presenting the partitioning strategy for the case when all of the obstacles are circular.

A. Power diagrams for circular obstacles

Our partitioning strategy relies on the concept of power diagrams [3]. The power $pow(x, s)$ of a point x with respect to a circle (or in our case a circular obstacle) s in the Euclidean space $\mathbb{R}^2$ is given by $d^2(x, z) - r^2$, where d is the Euclidean distance function, and z and r are the center and the radius of s. For a finite set of circles S in $\mathbb{R}^2$, the *power diagram* of S, denoted $PD(S)$, is a cell complex that associates each $s \in S$ with the convex domain $\{x \in \mathbb{R}^2 | pow(x, s) < pow(x, t), \forall t \in S - s\}$. An example is shown in Figure 8. When $r = 0$, i.e. the circles degenerate to points, $PD(S)$ becomes the Voronoi diagram. The following properties about $PD(S)$ are relevant to our partitioning strategy (see [3] - §2.2: Observations 1 and 2, and Lemma 1).

- When the circles are non-intersecting, the edges of $PD(S)$ do not intersect any of the circles.
- If the cardinality of S is k, then $PD(S)$ contains at most k cells.

Let S be the set of finite circular obstacles in our environment. We intersect each cell in $PD(S)$ with $\mathcal{P}$ to get a convex tessellation of $\mathcal{P}$, with each resulting cell C_i containing one obstacle. We include the power diagram edges that bound C_i as part of C_i.

The strategy to maintain connectivity is as follows: At any time, let C_u be the cell that contains the user u. The routers in C_u will move according to the strategy presented in Section IV, and maintain the user's connectivity.

The other routers move to "guard" their regions . Let C_i be a region which does not contain u. We project the user onto the boundary of C_i by finding the closest point in C_i to u using the Euclidean distance (i.e. we ignore the obstacles).

Let u_i be the *closest* point to u in C_i (see Figure 8). The routers in C_i maintain u_i's connectivity to b. This can be done by executing the strategy presented in Section IV by exchanging the role of q with u_i. This guarantees that the user's connectivity is maintained by the connecting arm in C_i as soon as the user enters this cell.

We now bound the number of routers.

Lemma 2. *Let m^* be the number of robotic routers used by any optimal solution, including the base station, to guarantee connectivity between u and b in a convex environment with h circular obstacles. Our robotic router strategy uses at most $5m^* \cdot h$ robots.*

Proof: For each cell, we need $5m^*$ robots which directly comes from Theorem 2. Since we have h cells, the proof follows. ∎

Discussion (extension to non-circular obstacles). When the obstacles are non-circular, the notion of a radius is undefined and power diagrams cannot be applied as such. However, if we can find an enclosing circle for each obstacle such that the circles are disjoint, it is straightforward to extend the previous result. In certain cases, a partition exists even if the disks defined by minimum enclosing circles are intersecting. Further details can be found in [22].

VI. Conclusion

In this work, we studied a novel application of robotic sensor networks in which the robots act as mobile routers and maintain the connectivity of a user to a base station. Given a complex environment where two entities can communicate if the geodesic distance between them is less than a threshold, we presented algorithms to compute the minimum number of necessary robotic routers and the strategies they should use. Specifically, we presented an optimal (in terms of the number of routers) algorithm for simply-connected polygons, a constant factor algorithm for a polygonal environment with a single obstacle, and an $O(h)$-approximation algorithm for environments with h obstacles.

Our future work includes improving the $O(h)$ approximation ratio. It is easy to see that the lower bound of $\lceil D \rceil$ is loose in some instances. For example, when h obstacles are arranged on a $\sqrt{h} \times \sqrt{h}$ grid, the number of necessary routers is clearly more than D. Improving the lower bound will yield better a approximation ratio.

Acknowledgement

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