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Navigation Control for Tracking and Catching a Moving Target

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***Abstract*—This paper presents feedback control laws for pursuing and catching a fly ball by taking Chapman's hypothesis into the closed-loop system connecting perceptions and actions. Through the analysis of the closed-loop system, we make it clear that the hypothetical trajectory Chapman showed is a special dynamic solution of the closed-loop system. Moreover, using a motion-analyzing technique over a finite time, it is shown that the proposed feedback control laws make it possible to generate a pursuing trajectory automatically that a fly ball can be caught in the right place and at the right time. It is also shown that the pursuing trajectory gets closer to the one Chapman showed as a feedback gain increases. In addition, we compare the proposed feedback control laws with Proportional Navigation (PN) which is the most common navigation technique for tracking a moving target, and demonstrate that the proposed control laws perform better than PN.**

***Index Terms*—closed-loop systems, navigation, nonlinear differential equations, theorem proving.**

I. INTRODUCTION

PROPORTIONAL Navigation (PN)[1] is well known in the field of aviation as a guidance control method for enabling a moving body to track and catch a moving target. This method is based on the fact that a moving body tracking a target will always collide with the target as long as the angle representing the line of sight (LOS) of the moving body in the direction of the target (LOS angle) is kept constant. PN accelerates the moving body laterally proportionally to the rate of change of the LOS angle. It continues to provide the basic framework for guidance control of moving bodies even after sensor information other than the LOS angle has become available in pace with the advancement of measurement and control technologies.

There are many hypotheses[2]-[13] to explain the action of a man in tracking and catching a moving target. Some hypotheses explain that tracking and catching are accomplished based on angle information alone as in PN. Chapman[2] examined the task of tracking a moving target over a long distance, such as catching a flying ball in a baseball game. He demonstrated for cases where the fielder is in the vertical plane containing the trajectory of the flying ball that "if the fielder moves at a constant speed so that he reaches the correct catching position at the moment the ball drops to the ground, the changes over time of the tangent of the elevation angle of observation of the ball as viewed by the fielder are constant." Based on this result, Michaels et al. [7] and McLeod et al.[3] proposed the hypothesis of Optical Acceleration Cancellation (OAC) to explain the ball-tracking path of a man as "the path where the acceleration of the tangent of the elevation angle of the ball is constantly zero" and presented the ball-tracking path of a man to support their hypothesis. However, the result described by Chapman merely represents one solution for tracking and catching a moving target but does not mention the method for determining the tracking speed of the fielder based on the elevation angle of the ball. Indeed, for tracking the ball in the manner indicated by Chapman, the correct catching position and catching moment must be known before starting the tracking task in order to determine the tracking speed.

However, if the elevation angle of the ball, on which Chapman focused, is made to correspond to the LOS angle in PN, Chapman's result can be reconsidered as guidance control laws similar to PN. In this paper, we consider common cases where the fielder is both inside and outside the vertical plane that contains the trajectory of the ball and, assuming that the azimuth angle can also be measured constantly in addition to the elevation angle of the ball, propose feedback control laws in which the tracking acceleration of the fielder is determined based on the angle tangents. Although these control laws are similar to Marken's feedback control laws [8], which tried to describe a similar problem in the framework of Perceptual Control Theory (PCT) and Tresilian's feedback control laws[9], which are based on the OAC hypothesis, these laws significantly differ from them in that these laws theoretically take account of the dynamics of the closed-loop system where the proposed control laws are incorporated and clarify the conditions for the fielder to arrive at the correct catching position at the moment when the ball drops to the ground. Furthermore, in this paper, we also discuss the similarity and differences between the proposed control laws and PN.

Section II is presented the feedback control laws for tracking and catching the ball after formulating the subject. Moreover, the trajectory described by Chapman becomes a special solu-

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tion for the dynamics of a closed-loop system is demonstrated. Section III is presented analyzing the action of the fielder in tracking the ball in a two-dimensional plane as the motion in the direction of each axis of the two-dimensional orthogonal coordinate system. The ball remains aloft for a finite period, and thus a motion-analyzing technique within a finite time will be used without applying the stability theory. This analysis will enable us to demonstrate that the ball can be caught at the exact time and in the correct position starting from an arbitrary initial position if the proposed feedback control laws are followed. Furthermore, through comparison with PN, we will clarify the characteristics of the proposed control laws. Section IV is demonstrated that an appropriate ball-tracking and catching path can be generated by means of the proposed feedback control laws for common cases where the fielder may be inside or outside the vertical plane that contains the trajectory of the ball.

II. FEEDBACK CONTROL LAWS FOR FLY BALL CATCHING

To incorporate Chapman's hypothesis into a closed-loop control system that integrates perception and action, we formulate the subject of tracking and catching a fly ball.

A. Problem Formulation

Let o-xyz be a Cartesian coordinate frame fixed to the ground and let the origin o be the position where the fielder is initially located, as seen in Fig. 1. The axis oz is vertically upwards and the fielder runs in the xy-plane. At any instant of time, the fielder's position on the ground is described by $x(t)$ and $y(t)$. The initial condition is $x(0) = y(0) = \dot{x}(0) = \dot{y}(0) = 0$. The equation of motion of the fielder is given by

$$\begin{array}{rcl} \ddot{x} & = & a_x \\ \ddot{y} & = & a_y \end{array} \tag{1}$$

where (a_x, a_y) is the acceleration along each axis produced by the fielder. The problem is how to control the acceleration (a_x, a_y) in order to catch the ball at the right place at the right time, under the condition that the information to be perceived is limited to data within the relative angles with the ball that the fielder can observe with a single eye.

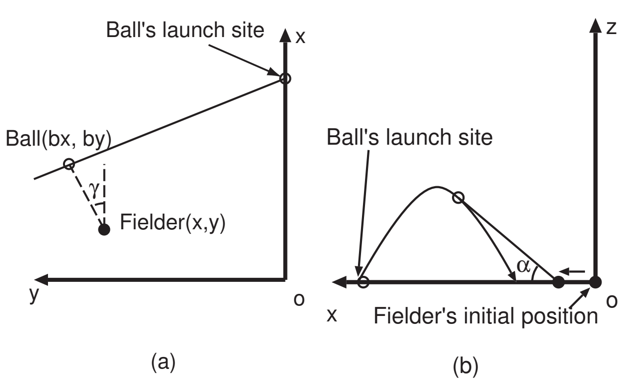

Fig. 1. The angles α, γ defined in a cartesian coordinate frame. (a) Top view. (b) Side view.

B. Feedback Control Laws

Let us consider the feedback control laws for the system of (1) given by

$$\begin{array}{rcl} a_x & = & k_v(r_v - \frac{d}{dt}\tan\alpha) \\ a_y & = & k_l \frac{d}{dt}\tan\gamma \end{array} \tag{2}$$

where α and γ are the elevation and lateral angles of gaze at the ball, k_v and k_l are positive gains, and r_v is the rate of change of the tangent of the elevation angle just after the ball is launched, that is $r_v = (d/dt)\tan\alpha|_{t=0}$, as seen in Fig. 1. The control law in the direction of the x axis incorporates Chapman's hypothesis into the fielder's system model so that the rate of change of the tangent of the elevation angle remains constant. Chapman's hypothesis is presented in detail later. The control law in the direction of the y axis has the same form as Proportional Navigation(PN), except using the tangent of γ. It should be noted that these feedback control laws use only relative angles with the ball that the fielder can observe with a single eye.

C. Chapman's Trajectory as a Special Solution

Chapman demonstrated if the fielder moves at a constant speed so that he reaches the correct catching position at the moment the ball drops to the ground, the changes over time of the tangent of the elevation angle of observation of the ball as viewed by the fielder are constant. We call this ideal trajectory of the fielder "Chapman's trajectory".

Let the ball leave the launcher, located at $(b_0, 0, 0)$, with an initial velocity (u, v, w). b_0 remains positive by defining the x axis as directed toward the launcher. For $0 < t < 2w/g$, the trajectory of the ball $(b_x(t), b_y(t), b_z(t))$ is then given by

$$\begin{array}{rcl} b_x(t) & = & ut + b_0 \\ b_y(t) & = & vt \\ b_z(t) & = & -\frac{1}{2}gt^2 + wt \end{array}$$

where g is the magnitude of the acceleration of gravity. The tangents of α and γ are expressed as

$$\begin{array}{rcl} \tan\alpha & = & b_z(t)/(b_x(t) - x(t)) \\ \tan\gamma & = & (b_y(t) - y(t))/(b_x(t) - x(t)). \end{array} \tag{3}$$

Here, Chapman's trajectory $(x^*(t), y^*(t))$ on the ground is introduced by

$$\begin{array}{rcl} x^*(t) & = & (\frac{gb_0}{2w} + u)t \\ y^*(t) & = & vt \end{array} \tag{4}$$

which means that the fielder runs at the proper speed so as to reach the proper spot to catch the ball just as it arrives. If the fielder follows Chapman's trajectory, then α and γ satisfy $\tan\alpha = wt/b_0$ and $\gamma = 0$ from (3), respectively. This means that Chapman's trajectory is a special solution for the dynamics of the closed-loop control system represented by (1) and (2), because r_v is given by w/b_0.

III. BEHAVIOR OF THE CLOSED-LOOP CONTROL SYSTEM

Chapman's trajectory is not applicable to the case in which the information to be perceived is the relative angles with the ball as detected by a single eye because Chapman's trajectory is obtained using the initial condition of the ball's position and velocity in three-dimensional space. How does the fielder respond to a fly ball if he runs based on the control laws of (2) ? In this section, we demonstrate that the proposed feedback control laws enable the fielder to catch the ball at the right place at the right time. For the application, it is assumed that a measurement r_v of the true $\bar{r}_v = (d/dt)\tan\alpha|_{t=0}$ includes some error Δr_v such that $r_v = \bar{r}_v + \Delta r_v$. We also demonstrate that the fielder's running path approaches Chapman's trajectory, given by (4), as the feedback gains of the control laws increase. For the sake of convenience, combining (1) with (2) and integrating of t, then the equation

$$\begin{array}{rcl} \dot{x}(t) &=& k_v(r_v t - \tan\alpha) \\ \dot{y}(t) &=& k_l \tan\gamma \end{array} \qquad (5)$$

holds.

A. Analysis in the xz-plane

The aim of this part is analyzing the fielder's response to the ball in the xz-plane. Let us consider the deviation $e(t)$ of the fielder's position $x(t)$ from Chapman's trajectory $x^*(t)$ that is $e(t) = x(t) - x^*(t)$. $\tan\alpha$ is rewritten as

$$\tan\alpha = \frac{b_z(t)}{b_x(t) - (e(t) + x^*(t))} = \frac{wt - \frac{1}{2}gt^2}{\frac{gb_0}{2w}\left(\frac{2w}{g} - t\right) - e(t)}. \qquad (6)$$

For $0 < t < 2w/g$, using (4) and (5),

$$\dot{e}(t) = \dot{e}(0) - \frac{\frac{k_v w}{b_0} t\, e(t)}{\frac{gb_0}{2w}\left(\frac{2w}{g} - t\right) - e(t)} + k_v \Delta r_v t \qquad (7)$$

$$e(0) = x(0) - x^*(0) = 0 \qquad (8)$$

$$\dot{e}(0) = \dot{x}(0) - \dot{x}^*(0) = -\dot{x}^*(0) = -\left(\frac{gb_0}{2w} + u\right).$$

are obtained. This means that the deviation $e(t)$ satisfies the first-order nonlinear differential equation. Hence, the following theorem can be established.

Theorem 1: If k_v and Δr_v satisfy the following conditions

[I] $\quad k_v > \left(\frac{w}{b_0} + \Delta r_v\right)^{-1} \frac{g}{2w}\dot{x}^*(0)$

[II] $\quad -\frac{w}{b_0} < \Delta r_v < \frac{w}{b_0}$,

then $\lim_{t \to t_{end}} e(t) = 0$ where $t_{end} = 2w/g$ and $e(t)$ satisfies (7) and (8).

Proof: See the Appendix. ■

B. Examining conditions [I] and [II] of Theorem 1

Let us explain the meaning of conditions [I] and [II] through numerical simulations performed by setting the initial velocity of the ball $(u, w) = (-20, 20)$ [m/s] and the feedback gain k_v=200 (except in Fig. 2, which illustrates the influence of k_v on the fielder's trajectory). In each simulation, we choose two location points for the ball launcher, b_0=75 [m] and 85 [m], placing the ball's landing point 5 [m] behind the fielder's initial location and 5 [m] in front of the fielder's initial location.

Condition [I] is always satisfied for an arbitrary $k_v(> 0)$ if the ball lands on the ground behind the fielder's initial location ($\dot{x}^* < 0$), as long as condition [II] is satisfied. In contrast, if the ball lands on the ground in front of the fielder's initial location ($\dot{x}^* > 0$), k_v is restricted by condition [I]. This condition is satisfied in a normal situation. For example, if we give b_0=85 [m] and $|\Delta r_v|$=0, this condition becomes $k_v > 1.25$ and is easily satisfied. Fig.2 plots the time history of $e(t)$, obtained by setting the feedback gain to k_v=200 and 2000 and assuming $|\Delta r_v| = 0$. We see from the figure that $e(t) \to 0$ as $t \to t_{end}(= 2w/g)$ while maintaining $e > 0$ when the ball lands behind the fielder's initial location and $e < 0$ when the ball lands in front of the fielder's initial location. This figure also demonstrates that the fielder's running path approaches Chapman's trajectory (4) as the feedback gain k_v increases.

Condition [II] is satisfied if a measurement r_v includes less than 10 % error with respect to the true value $\bar{r}_v$ in a normal situation. However, we should note that the proposed control law tends to generate inefficient trajectories as the measurement error increases, as seen in Fig. 3. Fig. 3 plots the time history of $x(t)$ when the ball lands on the ground 5 [m] behind the fielder's initial location with measurement errors of $\Delta r_v = 0, +0.1\bar{r}_v$ and $-0.1\bar{r}_v$. Fig. 4 plots the time history of $(d/dt)\tan\alpha$, corresponding to Fig.3. For example, when a measurement r_v includes a 10 % error with respect to the true value $\bar{r}_v$, first the fielder moves toward the ball to bring the rate of change of $\tan\alpha$ to a desired r_v (with 10 % error), and then moves backward and approaches to the ball's landing point.

If the elevation angle of the ball is made to correspond to the LOS angle in PN, PN can be expressed as

$$a_x = -k_v(d\alpha/dt) \qquad (9)$$

instead of using the control law in the direction of the x axis as given in (2). Although we performed the numerical simulations using the control law of (9) under the same conditions as in Fig. 2, this control law did not work well; that is, the fielder failed to pursue and catch the fly ball. Another control law given by

$$a_x = -k_v(r_v - d\alpha/dt) \qquad (10)$$

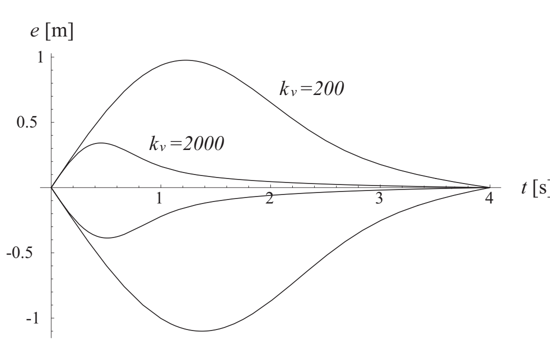

Fig. 2. The time history of e with k_v=200 and 2000. The trajectory satisfies e>0 all the time when the ball lands behind the fielder's initial location and e<0 when the ball lands in front of the fielder's initial location.

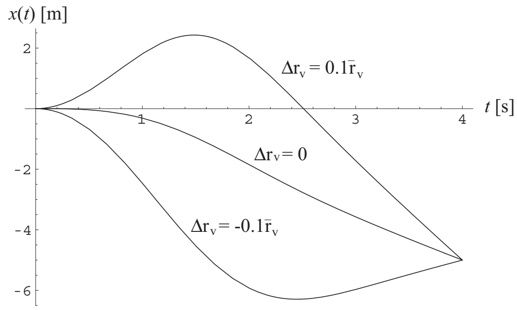

Fig. 3. The time history of x when the ball lands 5[m] behind the fielder's initial location. (Δr_v=0, +0.1$\bar{r}_v$, -0.1$\bar{r}_v$)

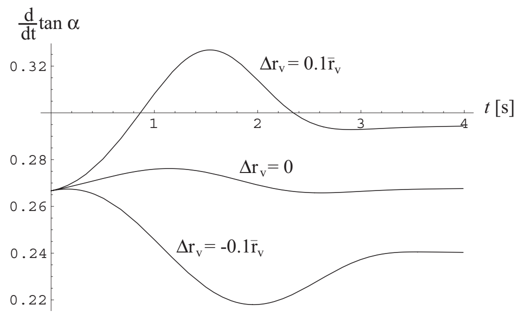

Fig. 4. The time history of $\frac{d}{dt}\tan\alpha$ corresponding to Fig.3.

can generate a ball-tracking and catching trajectory as seen in Fig. 5, where r_v is the rate of change of α just after the ball is launched and r_v includes the measurement error $\Delta r_v = 0, +0.1\bar{r}_v, -0.1\bar{r}_v$. Comparing Fig. 5 with Fig. 3, we can observe rapid movement of the fielder near the ball's landing point in Fig. 5. This is undesirable for the ball-tracking and catching task.

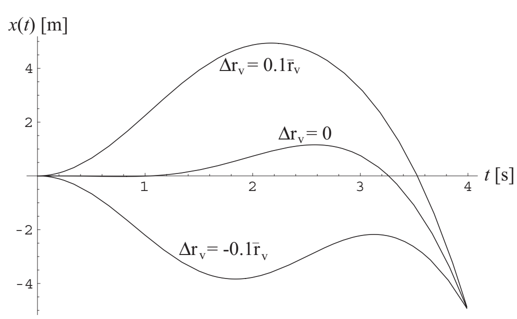

Fig. 5. The time history of x with $a_x = k_v(r_v - \frac{d\alpha}{dt})$. ($\Delta r_v$=0, +0.1$\bar{r}_v$, -0.1$\bar{r}_v$)

Although the proof of Theorem 1 given in the Appendix assumes parabolic motion of the ball, Theorem 1 holds even if the ball moves along a straight line at constant speed. Fig.6 represents the result obtained by assuming the ball travels with a constant velocity $(u, w) = (-20, -20)$[m/s] from the initial position $(x(0), z(0)) = (75, 80)$[m] (the ball's landing point is $(x(4), z(4)) = (-5, 0)$[m]). The result obtained by applying a PN of (10) is also given in Fig. 6. We can see from the figure that the proposed control law generates a more efficient trajectory, that is, the variations in moving speed of the fielder are small in comparison with PN.

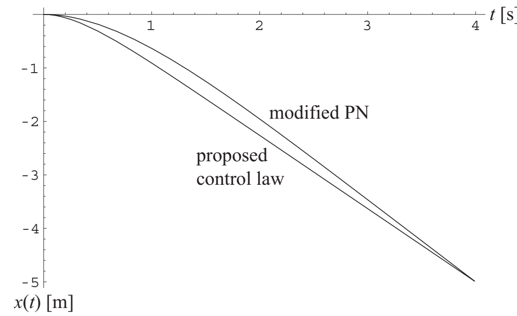

Fig. 6. The time history of x obtained by the proposed control law and the modified PN of (11).

C. Analysis in the yz-plane

We now analyze the fielder's behavior with respect to the ball in the yz-plane. Let us assume that the fielder's motion along the x axis is approximated by Chapman's trajectory $x^*(t)$ in order to solve for the motion along the y axis analytically. In addition, we introduce an inertial coordinate frame parallel to the coordinate frame in Fig. 1 whose origin moves along the x axis following Chapman's trajectory $x^*(t)$. The distance in the direction of the x axis between the fielder and the ball is then represented by

$$p^*(t) = b_x(t) - x^*(t) = -(b_0 g/2w)t + b_0.$$

From the control laws of (2), we have

$$\dot{y} = k_l(vt - y)/p^*. \qquad (11)$$

This nonlinear differential equation can be solved analytically, and its solution is given by

$$y = \frac{vt}{1+\frac{u^*}{k_l}} \left\{ 1 + \frac{b_0}{tk_l}(-1 + b_0^{\frac{k_l}{u^*}}(u^*t + b_0)^{\frac{-k_l}{u^*}}) \right\} \qquad (12)$$

$$\dot{y} = \frac{v}{1+\frac{u^*}{k_l}} \left\{ 1 - b_0^{1+\frac{k_l}{u^*}}(u^*t + b_0)^{-1-\frac{k_l}{u^*}} \right\} \qquad (13)$$

where $u^* = -b_0 g/2w$ and $u^* + k_l \neq 0$. Thus, we can derive the following theorem.

Theorem 2: In the direction of the y axis, the fielder's position coincides with the ball's landing point at the moment $t = t_{end}(= 2w/g)$ if the feedback gain k_l satisfies a condition of the form

$$k_l/u^* < -2. \qquad (14)$$

In addition, the fielder's velocity along y axis at that moment is $v/(1 + u^*/k_l)$.

Proof: Using (13), we can easily verify that $\dot{y}$ has the value $v/(1+u^*/k_l)$ at the moment the ball lands on the ground if the condition of (14) is satisfied. It is also confirmed from (12) that the fielder's position coincides with the ball's landing point. ■

Note that the value $v/(1 + u^*/k_l)$ is slightly different from Chapman's trajectory of (4), ($\dot{y}^*(t) = v$).

D. Examining Theorem 2

Let us examine Theorem 2 through numerical simulations. Fig.7 presents the time history of $y(t)$ and $y^*(t)$, obtained by assuming that the initial velocity of the ball is $(v, w) = (-4, 20)$ [m/s] and that the feedback gain is k_l = 200. We see from the figure that the fielder's velocity approaches a constant value as time passes and finally has the value given in Theorem 2 at the moment the ball lands on the ground $t = t_{end}$ $(= 4$ [s]).

Note that the control law in the direction of the y axis generates a more efficient trajectory than PN, that is, the variations in the speed of the fielder are small in comparison with PN, even though it takes the same form as PN except for using the tangent of γ.

IV. BEHAVIOR OF THE PROPOSED CONTROL LAW IN COMMON CASES

In this section, we demonstrate through numerical simulations that an appropriate ball-tracking and catching path can be generated by means of the proposed feedback control laws in common cases where the fielder may be inside or outside of the vertical plane that contains the trajectory of the ball. It is assumed that the ball is launched from $(x, y, z) = (80, 0, 0)$ [m] and lands on the ground at $t = t_{end} = 4$ [s] at points $(x, y) = (5, 5), (-5, 5), (-5, -5), (5, -5)$, and that the feedback gains are given as $k_v = k_l = 200$. Fig.8 plots the paths followed by simulated fielders starting the pursuit from $t = 0$ [s] and using the proposed control laws. We can see that each fielder starts the pursuit from $(x, y) = (0, 0)$ [m] and arrives at the landing point of the fly ball. Moreover, we notice that the paths are concave towards the ball launch site when the fielders run forward and convex towards the projection point when they run backward as indicated by McLeod et al. [10].

Fig. 9 plots the paths followed by simulated fielders starting the pursuit from $t = 1$ [s] and using the proposed control laws after standing still for one second at the beginning. In this case, the rate of change of the tangent of the elevation angle r_v is measured at $t = 1$ [s]. We can see that the paths in Fig.9 are straighter than those in Fig. 8. This result means that it is possible to take enough time to measure r_v with accuracy, thus allowing practical use of the proposed control laws.

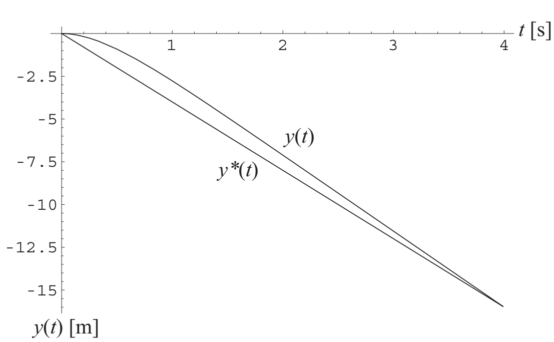

Fig. 7. The time history of y and y^* obtained by the proposed control law.

V. CONCLUSION

This paper presented feedback control laws for tracking and catching a fly ball and analyzed the actions of the fielder in tracking the ball using a motion-analyzing technique over a finite time, without applying stability theory. As a result, we demonstrated that the running path described by Chapman becomes a special solution for the dynamics of a closed-loop control system. We also revealed that the ball can be caught at the right time and in the right place starting from an arbitrary initial position and that appropriate running paths are generated automatically if the proposed feedback control laws are followed. In addition, we compared the proposed feedback control laws with PN and demonstrated that the proposed control laws perform better than PN.

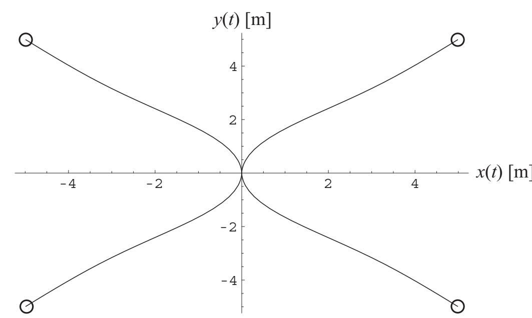

Fig. 8. The paths followed by simulated fielders starting the pursuit from the beginning, viewed from above.

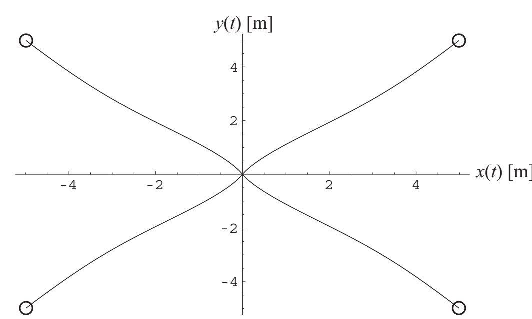

Fig. 9. The paths followed by simulated fielders starting the pursuit 1 second after the beginning, viewed from above.

APPENDIX

PROOF OF THEOREM 1

Putting $a = \dot{e}(0)$, $b = \frac{k_v w}{b_0}$, $c = \frac{g b_0}{2w}$, $d = \frac{2w}{g}$, $k = k_v \Delta r_v$ in (7), $e = e(t)$ satisfies the following differential equation

$$\dot{e} = a - \frac{bte}{c(d-t) - e} + kt \qquad (0 < t < d), \tag{15}$$

where $b, c, d > 0$. Moreover, for the disturbance term kt, k satisfies $-b < k < b$. The aim of this appendix is to show the following conclusion.

Theorem 1.1: For $-(b+k)d < a < \infty, 0 < b, c, d, -b < k < b$, we have $\lim_{t \to d} e(t) = 0$ where $e(t)$ satisfies (15).

Putting $\tilde{e} = e/c$, (15) is rewritten as

$$c\dot{\tilde{e}} = a - \frac{bt\tilde{e}}{(d-t)-\tilde{e}} + kt \qquad (0 < t < d). \tag{16}$$

Hereafter, $\tilde{}$ is abbreviated. (16) can be expressed as

$$c\Big[\ (d-t) - e\ \Big]\,\dot{e} = \Big[\ a + (b+k)t\ \Big]\Big[f(t) - e(t)\Big] \tag{17}$$

where

$$f(t) = \frac{(a+kt)(d-t)}{a+(b+k)t}.$$

It should be noted that $f(0) = d > 0,\ f(d) = 0$ and

$$d - t - f(t) = \frac{b\,t(d-t)}{a+(b+k)\,t}.$$

Now let us prove Theorem 1.1 for two cases where (I) $0 < a < \infty$ and (II) $-(b+k)d < a < 0$ (the proof of the case of $a = 0$ is omitted because it is easy to prove).

(I) The following relation holds:

$$a + (b+k)t > 0, \quad (d-t) - f(t) > 0 \quad (0 < t < d).$$

If k satisfies $0 \le k < b$, then we have $f(t) > 0$ $(0 < t < d)$. If k satisfies $-b < k < 0$, then we have

$$f(t)\ \begin{cases} > 0 & (0 < t < (\frac{a}{|k|} \wedge d)) \\ = 0 & (t = (\frac{a}{|k|} \wedge d)) \\ < 0 & ((\frac{a}{|k|} \wedge d) < t < d) \\ = 0 & (t = d) \end{cases}$$

where $A \wedge B$ means $\min\{A, B\}$. For $e(0) = 0,\ \dot{e}(0) = a/c > 0$, there exist $t = t_0$ $(0 < t_0 < d)$ such that $e(t_0) = f(t_0)$. Hence $\dot{e}(t) > 0$ $(0 < t < t_0)$ and $\dot{e}(t_0) = 0$ are obtained. If there exist $t_1 (t_0 < t_1 \le d)$ which satisfies $d - t_1 = e(t_1)$,

$$0 = c\Big[\ (d-t_1) - e(t_1)\ \Big]\,\dot{e}(t_1) = \Big[\ a + (b+k)t_1\ \Big]\Big[\ f(t_1) - e(t_1)\ \Big]$$

is holds. From this, $e(t_1) = f(t_1)$ is obtained. Because of $d - t_1 = e(t_1) = f(t_1)$, $t_1 = d$ and $e(d) = 0$ are hold.

(II) Putting $a = -|a|$, the following relation holds:

$$g(t) = -|a| + (b \pm |k|)t\ \begin{cases} < 0 & (0 < t < \frac{|a|}{b \pm |k|}) \\ = 0 & (t = \frac{|a|}{b \pm |k|}) \\ > 0 & (\frac{|a|}{b \pm |k|} < t < d). \end{cases}$$

If k satisfies $0 < k < b$, then we have

$$f(t) = \frac{(-|a| + |k|t)(d-t)}{-|a| + (b+|k|)t}$$

$$\begin{cases} = d & (t = 0) \\ > 0 & (0 < t < \frac{|a|}{b+|k|}) \\ = \infty & (t = \frac{|a|}{b+|k|} - 0) \\ = -\infty & (t = \frac{|a|}{b+|k|} + 0) \\ < 0 & (\frac{|a|}{b+|k|} < t < (\frac{|a|}{|k|} \wedge d)) \\ = 0 & (t = (\frac{|a|}{|k|} \wedge d)) \\ > 0 & ((\frac{|a|}{|k|} \wedge d) < t < d) \\ = 0 & (t = d). \end{cases}$$

If k satisfies $-b < k \le 0$, then we have

$$f(t) = -\frac{(|a| + |k|t)(d-t)}{-|a| + (b-|k|)t}$$

$$\begin{cases} = d & (t = 0) \\ > 0 & (0 < t < \frac{|a|}{b-|k|}) \\ = \infty & (t = \frac{|a|}{b-|k|} - 0) \\ = -\infty & (t = \frac{|a|}{b-|k|} + 0) \\ < 0 & (\frac{|a|}{b-|k|} < t < d) \\ = 0 & (t = d). \end{cases}$$

For $e(0) = 0,\ \dot{e}(0) = -|a|/c < 0$, there exist $t = t_0$ $(0 < t_0 < d)$ such that $e(t_0) = f(t_0)$. Hence $g(t_0) > 0$, $\dot{e}(t) < 0$ $(0 < t < t_0)$ and $\dot{e}(t_0) = 0$ are obtained. If there exist $t_1 (t_0 < t_1 \le d)$ which satisfies $d - t_1 = e(t_1)$, then we have

$$0 = c\Big[\ (d-t_1) - e(t_1)\ \Big]\,\dot{e}(t_1) = g(t_1)\Big[\ f(t_1) - e(t_1)\ \Big].$$

From this, $e(t_1) = f(t_1)$ is obtained. Because of $d - t_1 = e(t_1) = f(t_1)$, $t_1 = d$ and $e(d) = 0$ are obtained.

From the above (I) and (II), Theorem 1.1. is proved. ■

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