The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Robust Adaptive Composite Control of Space-based Robot System with Uncertain Parameters and External Disturbances

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Abstract—**In this paper, the control problem of space robot system with uncertain parameters and external disturbances is discussed. With the momentum conservation of the system, the kinematics and dynamics of the system are analyzed, and it is found that the generalized Jacobi matrix and the dynamic equations of the system are nonlinearly dependent on inertial parameters. In order to overcome the problems mentioned above, the idea of augmentation approach is introduced. It is shown that the augmented generalized Jacobi matrix and the dynamic equations of the system can be linearly dependent on a group of inertial parameters with augmented inputs and outputs. Based on the results, a robust adaptive composite control scheme for space-based robot to track the desired trajectories in inertial space is developed. The stability of the overall system is analyzed through Lyapunov direct method. For the proposed approach, the global uniform asymptotic stability of the system is established. In addition, the controller presented possesses the advantage that it needs no measurement of the position, linear velocity and acceleration of the base with respect to the orbit, because of the effective exploitation of the particular property of system dynamics. To show the feasibility of control scheme, a planar space robot system is simulated.**

I. Introduction

SPACE-BASED robot system has been suggested for lots of important tasks in space, such as capturing, handling and assembling space structures in earth orbit and considerable research efforts [1-5] have been focused on the dynamics and control problems of space-based robot system. Because of the high dynamic coupling between the arms and its floating base, the dynamics and control of the space-based robot system become extremely complicated. It is found that a major problem in controlling space-based robot system is that the dynamic control equations of the system cannot be linearly parameterized. This results in infeasibility of most robust control and adaptive control schemes which are currently applied to the fix-based robot system control [6-7], since the linear parameterization is a prerequisite of these schemes. Besides, the kinematic relation between inertial space, in which the tasks are usually specified, and joint space, where the control is executed, is not only dependent on the kinematics parameters but also the dynamic parameters. Therefore, the controller for space robot system to track desired trajectory in inertial space is more complicated than that in joint space, especially when the system is subject to the uncertainties. Y.-L. Gu [8] investigated the control for free-floating space robot systems, and proposed an adaptive control scheme based on the augmentation approach. Yangsheng Xu [9] studied the linear parameterization problem of robot system dynamics, and verified the effectiveness of the proposed adaptive control scheme both in joint space and inertial space. Chen Li [10] proposed the adaptive and robust composite control scheme of coordinated motion of space robot system with prismatic joint. Guo Yishen [11] presented a robust adaptive composite control of dual-arm space robot system in inertial space.

Although these controllers mentioned above are effective in compensating the influence of structured uncertainties, such as uncertain or unknown payload, it is not clear that they can obtain the desired control performance when a space robot system faces unstructured uncertainties, such as sensor noise and external disturbances.

In this paper, the difficulties of nonlinear parameterization of the generalized Jacobi matrix and the dynamic equations of the system are overcome by the augmentation approach. It is demonstrated that the augmented generalized Jacobi matrix and the dynamic equations of the system can be linearly dependent on a group of inertial parameters. Based on the results, a robust adaptive composite control scheme for a space-based robot system with uncertain parameters and external disturbances in inertial space is proposed. The advantage of the presented control scheme is that it needs no measurement of the position, linear velocity and acceleration of the base with respect to the orbit. A planar space robot system is simulated to verify the control scheme.

The remainder of paper is organized as follow: In section II, the dynamics of a space-based robot system is formulated. Section III presents the Jacobi relation of the system. The augmentation approach is introduced in section IV. A robust adaptive composite control of space robot system is proposed in section V. To show the feasibility of control scheme, The simulation studies are presented in section VI, which are followed by conclusions given in section VII.

II. Dynamics of the System

Without a loss of generality, a planar two-link space-based robot system with payload is considered here, Fig. 1. The

Manuscript received February 24, 2009. This work was supported in part by National Natural Science Foundation of PRC under Grant No. 10672040 and Grant No. 10372022 and by Natural Science Foundation of Fujian Province, PRC under Grant No. E0410008..

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system consists of the base B_0, B_1 (link 1) and B_2 (link 2) and the payload P. We assume the end-effector hold the payload rigidly. O_0 coincides with the mass center O_{C0} of B_0, O_i $(i=1,2)$ is the rotational center of the revolute joint between B_{i-1} and B_i, O_{C1} is the mass center of B_1, O_{C2} is the mass center of combination B_2 and P, x_i is the symmetrical axis of each link. The other symbols are defined as follows

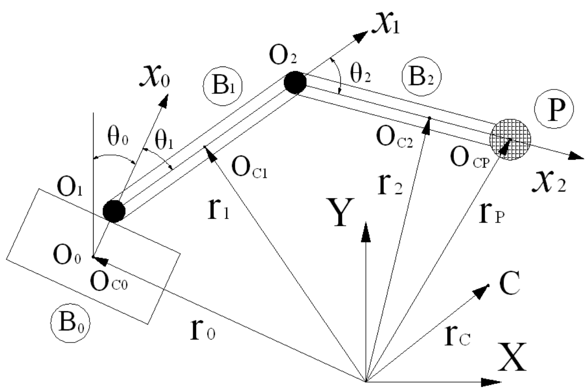

Fig. 1. A planar two-link space-based robot system.

l_0	Distance from joint O_0 to O_1;
l_i	Length of link i $(i=1,2)$;
a_i	Distance from joint O_i to the mass center O_{Ci} $(i=1,2)$;
m_i	Mass of B_i $(i=0,1,2)$;
m_P	Mass of P;
M	Total mass of the entire system;
J_i	Inertial moment of B_i $(i=0,1,2)$ with respect to its mass center;
J_P	Inertial moment of P with respect to its mass center O_{CP};
$(O-XY)$	Inertial coordinate frame of the system;
$(O_i-x_iy_i)$	Local coordinate frame of B_i $(i=0,1,2)$;
C	Mass center of the entire system;
$\vec{e}_i$	Unit vector pointing along with x_i $(i=0,1,2)$;
$\vec{r}_i$	Position vector of mass center of B_i $(i=0,1)$;
$\vec{r}_2$	Position vector of mass center of combination B_2 and P;
$\vec{r}_C$	Position vector of the entire mass center C;
$\vec{r}_P$	Position vector of the end-effector P;
θ_0	Attitude angle of the base, which is the angle between the Y axis and the x_0 axis;
θ_i	Rotational angle of joint O_i $(i=1,2)$, i.e. the angle between the x_{i-1} axis and the x_i axis.

From the geometrical relation of the system and the definition of the mass center of the system, the position vector of O_{Ci} $(i=0,1,2)$ and end-effector P in inertial coordinate system can be written as

$$\vec{r}_0=\vec{r}_C+L_{00}\vec{e}_0+L_{01}\vec{e}_1+L_{02}\vec{e}_2\,,\ \vec{r}_1=\vec{r}_C+\sum_{i=0}^{2}L_{1i}\vec{e}_i\,,$$

$$\vec{r}_2=\vec{r}_C+\sum_{i=0}^{2}L_{2i}\vec{e}_i\,,\ \vec{r}_P=\vec{r}_C+\sum_{i=0}^{2}L_{Pi}\vec{e}_i\,. \tag{1}$$

Where, $L_{00}=-(m_1+m_{2P})l_0/M$, $L_{01}=-(m_1a_1+m_{2P}l_1)/M$, $L_{02}=-(m_{2P}a_2)/M$, $m_{2P}=m_2+m_P$, and the parameters L_{1i},L_{2i},L_{Pi} $(i=0,1,2)$ same as L_{0i} are the functions of the inertial parameters.

Assuming that there are no external forces, the space-based robot system can be regarded as a free-floating mechanical chain, and the momentum are conserved during the operation. Without any loss of generality, the initial momentum of the system is assumed to be zero here, i.e., $\dot{\vec{r}}_C=0$. Obviously, From (1), the velocities of the mass center of B_i $(i=0,1,2)$ are linearly dependent on the inertial parameters L_{0i},L_{1i},L_{2i} $(i=0,1,2)$.

With Lagrange Equation, the dynamic equations of the space-based robot system can be represented by the following form

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{\tau}_d=(0\quad \boldsymbol{\tau}^{\mathrm{T}})^{\mathrm{T}}\,. \tag{2}$$

Where, $\boldsymbol{D}(\boldsymbol{q})\in\Re^{3\times3}$ is symmetric positive-definite inertial matrix; $\boldsymbol{q}=(\theta_0\quad \boldsymbol{\theta}^{\mathrm{T}})^{\mathrm{T}}\in\Re^3$, $\boldsymbol{\theta}=(\theta_1\quad\theta_2)^{\mathrm{T}}$ is generalized coordinate vector of the system; $\boldsymbol{\tau}=(\tau_1\quad\tau_2)^{\mathrm{T}}$ represents the joints input torque vector of O_1 and O_2; $\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\in\Re^3$ is the vector of centripetal and Coriolis torque; $\boldsymbol{\tau}_d\in\Re^3$ represents the effects of external disturbances. If the elements of $\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})\in\Re^{3\times3}$ can be properly given as

$$h_{ij}=\sum_{k=1}^{3}\frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k}+\frac{\partial D_{ik}}{\partial q_j}-\frac{\partial D_{kj}}{\partial q_i}\right)\dot{q}_k\,. \tag{3}$$

For any arbitrary $\boldsymbol{z}\in\Re^3$, the following property exists [12]

$$\boldsymbol{z}^{\mathrm{T}}\boldsymbol{h}\boldsymbol{z}=\frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\dot{\boldsymbol{D}}\boldsymbol{z}\,. \tag{4}$$

Thus, the dynamic equations (2) of the system can be linearly parameterized. The result is good for the designing of robust adaptive composite control scheme.

It is assumed that the above system satisfies the following assumptions.

Assumption 1: The uncertain function $\boldsymbol{\tau}_d$ is bounded in norm by a known constant upper bound. In other word,

$$\|\boldsymbol{\tau}_d\|<d\,. \tag{5}$$

Where, d is a real positive constant.

Assumption 2: The parameter deviation vector between the real plant and the model plant $\overline{\boldsymbol{\Phi}}^*=\boldsymbol{\Phi}^*-\hat{\boldsymbol{\Phi}}^*$ is bounded in norm by a known constant upper bound, which means

$$\left\|\overline{\boldsymbol{\Phi}}^*\right\|\le\Psi\,. \tag{6}$$

Where, $\Psi\in\Re^+$.

III. JACOBI

From the last equation of (1), the position coordinate of end-effector in inertial space can be written as

$$\begin{aligned}x_P&=x_C+L_{P0}\sin(\theta_0)+L_{P1}\sin(\theta_0+\theta_1)\\&\quad+L_{P2}\sin(\theta_0+\theta_1+\theta_2)\,,\\y_P&=y_C+L_{P0}\cos(\theta_0)+L_{P1}\cos(\theta_0+\theta_1)\\&\quad+L_{P2}\cos(\theta_0+\theta_1+\theta_2)\,.\end{aligned} \tag{7}$$

Differentiating (7), and using the assumption which the initial momentum of the system is zero, i.e., $\dot{\vec{r}}_C = 0$, we obtain

$$\begin{pmatrix} \dot{x}_P \\ \dot{y}_P \end{pmatrix} = (\boldsymbol{J}_b \quad \boldsymbol{J}_r) \begin{pmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}. \tag{8}$$

Where, $\boldsymbol{J}_b = \begin{pmatrix} J_{11} \\ J_{21} \end{pmatrix}, \boldsymbol{J}_r = \begin{pmatrix} J_{12} & J_{13} \\ J_{22} & J_{23} \end{pmatrix}$.

Obviously, J_{ij} $(i = 1,2; j = 1,2,3)$ is the function of $\boldsymbol{q}$ and also linearly dependent on a group of inertial parameters L_{P0}, L_{P1}, L_{P2}. However, to obtain the generalized Jacobi matrix of the space robot system, $\dot{\theta}_0$ needs to be eliminated from (8). This will make the generalized Jacobi matrix dependent nonlinearly on inertial parameters and results in the infeasibility of most robust and adaptive control schemes which are currently used in fix-based robot control.

IV. Augmentation Approach

In order to overcome the above problems and guarantee that the dynamic equations of the space-based robot system can be linearly parameterized, we now extend the output vector $\boldsymbol{X}_P = (x_P \quad y_P)^{\mathrm{T}}$ to be the augmented output vector $\boldsymbol{Y} = (\theta_0 \quad \boldsymbol{X}_P{}^{\mathrm{T}})^{\mathrm{T}}$, and assume the variables $\theta_0, \dot{\theta}_0, \ddot{\theta}_0$ can be measured. Then the relationship between the augmented output velocity vector $\dot{\boldsymbol{Y}}$ and the generalized joint motions vector $\dot{\boldsymbol{q}} = (\dot{\theta}_0 \quad \dot{\boldsymbol{\theta}}^{\mathrm{T}})^{\mathrm{T}}$ can be written as

$$\dot{\boldsymbol{Y}} = \begin{pmatrix} \dot{\theta}_0 \\ \dot{\boldsymbol{X}}_P \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{J}_b & \boldsymbol{J}_r \end{pmatrix} \begin{pmatrix} \dot{\theta}_0 \\ \dot{\boldsymbol{\theta}} \end{pmatrix} = \boldsymbol{J}_a \dot{\boldsymbol{q}}. \tag{9}$$

Where, $\boldsymbol{I} \in \Re^{1\times1}$ is identity matrix, $\boldsymbol{O} \in \Re^{1\times2}$ is zero matrix and $\boldsymbol{J}_a \in \Re^{3\times3}$ is the augmented generalized Jacobi matrix.

If $\boldsymbol{J}_r$ is assumed to be nonsingular, the square matrix $\boldsymbol{J}_a$ can be inverted. We obtain the inverse relation of (9)

$$\dot{\boldsymbol{q}} = \boldsymbol{J}_a{}^{-1} \dot{\boldsymbol{Y}} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ -\boldsymbol{J}_r^{-1}\boldsymbol{J}_b & \boldsymbol{J}_r^{-1} \end{pmatrix} \begin{pmatrix} \dot{\theta}_0 \\ \dot{\boldsymbol{X}}_P \end{pmatrix}. \tag{10}$$

We denote $\boldsymbol{Y}_d = (\theta_0 \quad \boldsymbol{X}_{Pd}{}^{\mathrm{T}})^{\mathrm{T}}$, where, $\boldsymbol{X}_{Pd}$ is the desired trajectory of end-effector of the robot system in inertial space, and $\tilde{\boldsymbol{q}}_1 = (\boldsymbol{X}_{Pd} - \boldsymbol{X}_P)$ as the output error function between the actual and the desired trajectory. The augmented output error function $\tilde{\boldsymbol{q}}$ between $\boldsymbol{Y}$ and $\boldsymbol{Y}_d$ can be written as

$$\tilde{\boldsymbol{q}} = (\boldsymbol{Y}_d - \boldsymbol{Y}) = (0 \quad \tilde{\boldsymbol{q}}_1{}^{\mathrm{T}})^{\mathrm{T}}. \tag{11}$$

we define a reference output velocity $\dot{\boldsymbol{\eta}}$ as follows

$$\dot{\boldsymbol{\eta}} = \hat{\boldsymbol{J}}_a{}^{-1}(\dot{\boldsymbol{Y}}_d + \boldsymbol{k}_p \tilde{\boldsymbol{q}}) = (\dot{\theta}_0 \quad \dot{\boldsymbol{\eta}}_r^{\mathrm{T}})^{\mathrm{T}}. \tag{12}$$

Where, $\boldsymbol{k}_p \in \Re^{3\times3}$ is a positive-definite symmetric constant matrix, $\dot{\boldsymbol{\eta}}_r = (-\hat{\boldsymbol{J}}_r^{-1}\hat{\boldsymbol{J}}_b \quad \hat{\boldsymbol{J}}_r^{-1})(\dot{\boldsymbol{Y}}_d + \boldsymbol{k}_p \tilde{\boldsymbol{q}})$.

Let an extended augmented error be defined by

$$\dot{\boldsymbol{s}} = \dot{\boldsymbol{\eta}} - \dot{\boldsymbol{q}} = (0 \quad \dot{\boldsymbol{s}}_1^{\mathrm{T}})^{\mathrm{T}}. \tag{13}$$

Where, $\dot{\boldsymbol{s}}_1 = (\dot{\boldsymbol{\eta}}_r - \dot{\boldsymbol{\theta}})$.

Then, substituting (11), (12) and (13) into (9), we have

$$\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{k}_p \tilde{\boldsymbol{q}} = \boldsymbol{J}_a \dot{\boldsymbol{s}} - \boldsymbol{W}_1 \overline{\boldsymbol{\Phi}}_1. \tag{14}$$

Where, $\boldsymbol{W}_1 \overline{\boldsymbol{\Phi}}_1 = (\boldsymbol{J}_a - \hat{\boldsymbol{J}}_a)\dot{\boldsymbol{\eta}}$, $\boldsymbol{W}_1 \in \Re^{3\times3}$ is a matrix function of $\boldsymbol{q}$, $\boldsymbol{Y}_d$ and $\dot{\boldsymbol{Y}}_d$, and independent of physical parameters; $\overline{\boldsymbol{\Phi}}_1 = \boldsymbol{\Phi}_1 - \hat{\boldsymbol{\Phi}}_1 \in \Re^3$ is the parameter deviation vector between the real plant and the model plant.

Differentiating (12), we obtain

$$\ddot{\boldsymbol{\eta}} = \hat{\boldsymbol{J}}_a^{-1}(\ddot{\boldsymbol{Y}}_d + \boldsymbol{k}_p \dot{\tilde{\boldsymbol{q}}} - \dot{\hat{\boldsymbol{J}}}_a \dot{\boldsymbol{\eta}}) = (\ddot{\theta}_0 \quad \ddot{\boldsymbol{\eta}}_r^{\mathrm{T}})^{\mathrm{T}}. \tag{15}$$

Where, $\ddot{\boldsymbol{\eta}}_r = (-\hat{\boldsymbol{J}}_r^{-1}\hat{\boldsymbol{J}}_b \quad \hat{\boldsymbol{J}}_r^{-1})(\ddot{\boldsymbol{Y}}_d + \boldsymbol{k}_p \dot{\tilde{\boldsymbol{q}}} - \dot{\hat{\boldsymbol{J}}}_a \dot{\boldsymbol{\eta}})$.

From (13), the dynamic equations (2) can be rewritten as

$$\boldsymbol{D}\ddot{\boldsymbol{s}} + \boldsymbol{h}\dot{\boldsymbol{s}} = \boldsymbol{D}\ddot{\boldsymbol{\eta}} + \boldsymbol{h}\dot{\boldsymbol{\eta}} - (0 \quad \boldsymbol{\tau}^{\mathrm{T}})^{\mathrm{T}} + \boldsymbol{\tau}_d. \tag{16}$$

Finally, in order to keep the linear parameterization of the system dynamics equations, we must expand the input vector $\boldsymbol{\tau}$ of the system to the augmented input vector $(0 \quad \boldsymbol{\tau}^{\mathrm{T}})^{\mathrm{T}}$.

V. Robust Adaptive Composite Control Scheme

In this section, a robust adaptive composite control for space-based robot system with uncertain parameters and external disturbances is considered. Our objective is to find a controller which uses control laws to make $\boldsymbol{Y}$ tend to $\boldsymbol{Y}_d$ in the presence of uncertainties.

Now let's define the following control law

$$(0 \quad \boldsymbol{\tau}^{\mathrm{T}})^{\mathrm{T}} = \hat{\boldsymbol{D}}\ddot{\boldsymbol{\eta}} + \hat{\boldsymbol{h}}\dot{\boldsymbol{\eta}} + (\delta \quad (\boldsymbol{K}_1 \dot{\boldsymbol{s}}_1)^{\mathrm{T}})^{\mathrm{T}} + \boldsymbol{u}_\Psi + \boldsymbol{u}_d. \tag{17}$$

Where, $\hat{\boldsymbol{D}}$, $\hat{\boldsymbol{h}}$, respectively, represent the inertial matrix $\boldsymbol{D}$ and the matrix $\boldsymbol{h}$ in the model plant. $\boldsymbol{K}_1 \in \Re^{2\times2}$ is a positive definite symmetric constant matrix. The notations $\boldsymbol{u}_\Psi$ and $\boldsymbol{u}_d$ are considered to compensate for $\overline{\boldsymbol{\Phi}}^*$ and $\boldsymbol{\tau}_d$, respectively. The parameter δ is proposed to guarantee that the input torque of the base's attitude is always zero.

Substituting (17) into (16), we have

$$\boldsymbol{D}\ddot{\boldsymbol{s}} + \boldsymbol{h}\dot{\boldsymbol{s}} + (\delta \quad (\boldsymbol{K}_1 \dot{\boldsymbol{s}}_1)^{\mathrm{T}})^{\mathrm{T}} + \boldsymbol{u}_\Psi + \boldsymbol{u}_d - \boldsymbol{\tau}_d = \boldsymbol{W}^* \overline{\boldsymbol{\Phi}}^*. \tag{18}$$

Where, $\boldsymbol{W}^* \overline{\boldsymbol{\Phi}}^* = (\boldsymbol{D} - \hat{\boldsymbol{D}})\ddot{\boldsymbol{\eta}} + (\boldsymbol{h} - \hat{\boldsymbol{h}})\dot{\boldsymbol{\eta}}$, $\boldsymbol{W}^* \in \Re^{3\times6}$ is a matrix function of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\dot{\boldsymbol{\eta}}$, $\ddot{\boldsymbol{\eta}}$, and also independent of the physical parameters, and $\overline{\boldsymbol{\Phi}}^* = (\boldsymbol{\Phi}^* - \hat{\boldsymbol{\Phi}}^*) \in \Re^{6\times1}$ is the parameter deviation vector between the real plant and the model plant.

Assuming that the actual values $\theta_0, \dot{\theta}_0, \ddot{\theta}_0$ are measurable and the notations $\boldsymbol{u}_\Psi$ and $\boldsymbol{u}_d$ in the control law (17) which need to be specified are properly chosen, we can obtain the theorem as follows

Theorem: The control input law (17) and the following adaptation law

$$\dot{\hat{\boldsymbol{\Phi}}}_1 = -\boldsymbol{W}_1^{\mathrm{T}} \boldsymbol{H} \tilde{\boldsymbol{q}}. \tag{19}$$

Guarantee, $\lim_{t\to\infty} \tilde{\boldsymbol{q}} = 0$, $\lim_{t\to\infty} \dot{\boldsymbol{s}} = 0$. Where, $\boldsymbol{H} \in \Re^{3\times3}$ is a positive-definite symmetric constant matrix.

Proof: Define the Lyapunov function candidate $\boldsymbol{V}$ as

$$\boldsymbol{V} = \frac{1}{2}(\gamma \dot{\boldsymbol{s}}^{\mathrm{T}} \boldsymbol{D} \dot{\boldsymbol{s}} + \tilde{\boldsymbol{q}}^{\mathrm{T}} \boldsymbol{H} \tilde{\boldsymbol{q}} + \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}} \overline{\boldsymbol{\Phi}}_1). \tag{20}$$

Where, $\gamma > 0$ is a scalar constant.

Differentiating $\boldsymbol{V}$, and using (4), (14), (18) and (19), we obtain

$$
\begin{aligned}
\dot{V} &= \frac{1}{2}\gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\dot{\boldsymbol{D}}\dot{\hat{\boldsymbol{s}}} + \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{D}\ddot{\hat{\boldsymbol{s}}} + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\tilde{\boldsymbol{q}}} + \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}}\dot{\overline{\boldsymbol{\Phi}}}_1 \\
&= \frac{1}{2}\gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\dot{\boldsymbol{D}}\dot{\hat{\boldsymbol{s}}} + \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}(\boldsymbol{W}^*\overline{\boldsymbol{\Phi}}^* + \boldsymbol{\tau}_d - (\delta \quad (\boldsymbol{K}_1\dot{\hat{s}}_1)^{\mathrm{T}})^{\mathrm{T}} \\
&\quad - \boldsymbol{u}_\Psi - \boldsymbol{u}_d - \boldsymbol{h}\dot{\hat{\boldsymbol{s}}}) + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\tilde{\boldsymbol{q}}} + \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}}\dot{\overline{\boldsymbol{\Phi}}}^{\mathrm{T}} \\
&= \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}(\boldsymbol{W}^*\overline{\boldsymbol{\Phi}}^* + \boldsymbol{\tau}_d - \boldsymbol{u}_\Psi - \boldsymbol{u}_d - (\delta \quad (\boldsymbol{K}_1\dot{\hat{s}}_1)^{\mathrm{T}})^{\mathrm{T}}) \\
&\quad + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\tilde{\boldsymbol{q}}} + \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}}\dot{\overline{\boldsymbol{\Phi}}}_1 \\
&= \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}(\boldsymbol{W}^*\overline{\boldsymbol{\Phi}}^* + \boldsymbol{\tau}_d - \boldsymbol{u}_\Psi - \boldsymbol{u}_d) + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}(-\boldsymbol{k}_p\tilde{\boldsymbol{q}} \\
&\quad + \boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \boldsymbol{W}_1\overline{\boldsymbol{\Phi}}_1) - \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}}\dot{\hat{\boldsymbol{\Phi}}}_1 - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}} \\
&= \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}(\boldsymbol{W}^*\overline{\boldsymbol{\Phi}}^* + \boldsymbol{\tau}_d - \boldsymbol{u}_\Psi - \boldsymbol{u}_d) - \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{k}_p\tilde{\boldsymbol{q}} \\
&\quad + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \overline{\boldsymbol{\Phi}}_1^{\mathrm{T}}(\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{H}\tilde{\boldsymbol{q}} + \dot{\hat{\boldsymbol{\Phi}}}_1) - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}} \\
&= \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}(\boldsymbol{W}^*\overline{\boldsymbol{\Phi}}^* + \boldsymbol{\tau}_d - \boldsymbol{u}_\Psi - \boldsymbol{u}_d) - \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{k}_p\tilde{\boldsymbol{q}} \\
&\quad + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}}\,. \qquad (21)
\end{aligned}
$$

Where, $\boldsymbol{K} = \mathrm{diag}(1 \quad \boldsymbol{K}_1)$ and (21) can be rewritten as the inequality

$$
\begin{aligned}
\dot{V} &\le \gamma(\|\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{W}^*\|\cdot\|\overline{\boldsymbol{\Phi}}^*\| + \|\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\|\cdot\|\boldsymbol{\tau}_d\| - \dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{u}_\Psi - \dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{u}_d) \\
&\quad - \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{k}_p\tilde{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}}\,. \qquad (22)
\end{aligned}
$$

that regarded to (5) and (6) leads to

$$
\begin{aligned}
\dot{V} &\le \gamma(\|\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}\|\cdot\Psi + \|\dot{\hat{\boldsymbol{s}}}\|\cdot d - \dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{u}_\Psi - \dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{u}_d) \\
&\quad - \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{k}_p\tilde{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}}\,. \qquad (23)
\end{aligned}
$$

Now, we define $\boldsymbol{u}_\Psi$ and $\boldsymbol{u}_d$ as

$$
\boldsymbol{u}_\Psi = \frac{\Psi}{\|\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}\|}\boldsymbol{W}^*\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}p_1\,, \quad \boldsymbol{u}_d = \frac{d}{\|\dot{\hat{\boldsymbol{s}}}\|}\dot{\hat{\boldsymbol{s}}}p_2\,. \qquad (24)
$$

Where, $p_1 = \begin{cases} 1 & \|\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}\| \ne 0 \\ 0 & \|\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}\| = 0 \end{cases}$, $p_2 = \begin{cases} 1 & \|\dot{\hat{\boldsymbol{s}}}\| \ne 0 \\ 0 & \|\dot{\hat{\boldsymbol{s}}}\| = 0 \end{cases}$.

Then, substituting (24) into (23) gives

$$
\begin{aligned}
\dot{V} &\le \gamma(\|\boldsymbol{W}^{*\mathrm{T}}\dot{\hat{\boldsymbol{s}}}\|\cdot\Psi\cdot(1-p_1) + \|\dot{\hat{\boldsymbol{s}}}\|\cdot d\cdot(1-p_2)) \\
&\quad - \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{k}_p\boldsymbol{K}\boldsymbol{k}_p\tilde{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{k}_p\boldsymbol{K}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}}\,. \qquad (25)
\end{aligned}
$$

According to the definitions of p_1, p_2 in (24), the first term in the right-hand side of (25) is always zero, and then (25) is arranged as

$$
\dot{V} \le -\tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{k}_p\boldsymbol{K}\boldsymbol{k}_p\tilde{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{k}_p\boldsymbol{K}\boldsymbol{J}_a\dot{\hat{\boldsymbol{s}}} - \gamma\dot{\hat{\boldsymbol{s}}}^{\mathrm{T}}\boldsymbol{K}\dot{\hat{\boldsymbol{s}}}\,. \qquad (26)
$$

Clearly, $\dot{V}$ is the quadratic form of $\tilde{\boldsymbol{q}}$ and $\dot{\hat{s}}$. Since $\boldsymbol{J}_a$ is bounded and the parameter γ is chosen to be big enough, $\dot{V}$ is non-positive which means $\boldsymbol{V}$ never increases. Since the matrix $\boldsymbol{D}$ is symmetric positive-definite, from (20), we know that $\tilde{\boldsymbol{q}}$, $\dot{\hat{s}}$ and $\overline{\boldsymbol{\Phi}}_1$ are bounded. Considering (14) and (18), $\dot{\tilde{\boldsymbol{q}}}$, $\ddot{\hat{s}}$ and $\overline{\boldsymbol{\Phi}}^*$ are bounded too.

Based on the results, we have $\lim_{t\to\infty}\dot{V} = 0$, which means

$$
\lim_{t\to\infty}\tilde{\boldsymbol{q}} = 0\,, \quad \lim_{t\to\infty}\dot{\hat{s}} = 0\,.
$$

Therefore, the control law (17), the adaptation law (19) and the definition of $\boldsymbol{u}_\Psi$ and $\boldsymbol{u}_d$ given in (24) can asymptotically stabilize the space robot system to track the desired trajectory described in terms of $\boldsymbol{X}_{Pd}$.

VI. Numerical Simulation

For illustrative purposes, a planar two-link space-based robot system shown in Fig. 1 is considered in our simulation study. The actual plant parameters of the system are as follows

$$
m_0 = 40\,\mathrm{kg}\,,\ m_1 = 2m_2 = 2\,\mathrm{kg}\,,\ m_P = 2.5\,\mathrm{kg}\,, l_0 = 1.5\,\mathrm{m}\,,
$$
$$
l_1 = 2a_1 = 3\,\mathrm{m}\,,\ l_2 = 2a_2 = 3\,\mathrm{m}\,, J_0 = 34.17\,\mathrm{kg\cdot m^2}
$$
$$
J_1 = 2J_2 = 1.5\,\mathrm{kg\cdot m^2}\,,\ J_P = 1.5\,\mathrm{kg\cdot m^2}\,.
$$

In this simulation, the parameter vector $\boldsymbol{\Phi}^*$ and $\boldsymbol{\Phi}_1$ are

$$
\boldsymbol{\Phi}^* = (\phi_1^* \quad \phi_2^* \quad \phi_3^* \quad \phi_4^* \quad \phi_5^* \quad \phi_6^*)^{\mathrm{T}}\,,
$$
$$
\boldsymbol{\Phi}_1 = (L_{P0} \quad L_{P1} \quad L_{P2})^{\mathrm{T}}\,.
$$

Where, $\phi_1^* = J_0 + m_0L_{00}^{\ 2} + m_1L_{10}^{\ 2} + m_{2P}L_{20}^{\ 2}$,

$$
\begin{aligned}
\phi_2^* &= J_1 + m_0L_{01}^{\ 2} + m_1L_{11}^{\ 2} + m_{2P}L_{21}^{\ 2}\,, \\
\phi_3^* &= J_{2P} + m_0L_{02}^{\ 2} + m_1L_{12}^{\ 2} + m_{2P}L_{22}^{\ 2}\,, \\
\phi_4^* &= m_0L_{00}L_{01} + m_1L_{10}L_{11} + m_{2P}L_{20}L_{21}\,, \\
\phi_5^* &= m_0L_{00}L_{02} + m_1L_{10}L_{12} + m_{2P}L_{20}L_{22}\,, \\
\phi_6^* &= m_0L_{01}L_{02} + m_1L_{11}L_{12} + m_{2P}L_{21}L_{22}\,, \\
L_{P0} &= L_{00} + l_0\,,\ L_{P1} = L_{01} + l_1\,, L_{P2} = L_{02} + l_2\,.
\end{aligned}
$$

Two different desired trajectories of the end-effector in inertial space are chosen as follows

The first desired trajectory

$$
x_{Pd} = \frac{1}{2} - \frac{1}{2}\cos(\frac{\pi}{5}t)(\mathrm{m})\,,\ y_{Pd} = 1 + \frac{1}{2}\sin(\frac{\pi}{5}t)(\mathrm{m})\,.
$$

The second desired trajectory

$$
x_{Pd} = \frac{7}{10}\sin(\frac{\pi}{5}t)(\mathrm{m})\,,\ y_{Pd} = \frac{6}{10} + \frac{7}{10}\sin(\frac{\pi}{5}t)(\mathrm{m})\,.
$$

and external disturbances are

$$
\boldsymbol{\tau}_d = [2\sin(t) \quad 2\sin(t) \quad 2\cos(t)]^{\mathrm{T}}(\mathrm{N\cdot m})\,.
$$

Here, the inertial parameters of the payload, i.e., m_P and J_P are uncertain, whose estimated values are $\hat{m}_P = 2\,\mathrm{kg}$, $\hat{J}_P = 1.25\,\mathrm{kg\cdot m^2}$. The initial states of space-based robot system are given as

$$
\boldsymbol{X}_P(0) = (0.1782 \quad 0.9137)^{\mathrm{T}}(\mathrm{m})\,,
$$
$$
\boldsymbol{q}(0) = (0.00 \quad 0.14 \quad 1.46)^{\mathrm{T}}(\mathrm{rad})\,.
$$

The gains of the controller are chosen as

$$
\boldsymbol{k}_p = \mathrm{diag}(2.0)\,,\ \boldsymbol{K}_1 = \mathrm{diag}(4.0)\,,
$$
$$
\boldsymbol{H} = \mathrm{diag}(1.0)\,,\ d = 3.4641\,,\ \Psi = 7.3235\,.
$$

The time taken for simulation is 10.0 seconds. Fig. 2-4 are the simulation results when the space-based robot system is specified to track the first desired trajectory. Fig. 2 plots the desired and actual trajectory of end-effector in inertial space. Fig. 3 shows the angle change curves of the base and joints during the operation. Fig. 4 is the tracking error of the whole system. And then, the same controller is utilized to control the end-effector to track the second desired trajectory in inertial space and the corresponding simulation results are shown in Fig. 5-7. The simulation results verify that the proposed control laws for the space-based robot system are feasible and effective. The closed-loop system can deal with not only the parametric uncertainties, but also the external disturbances.

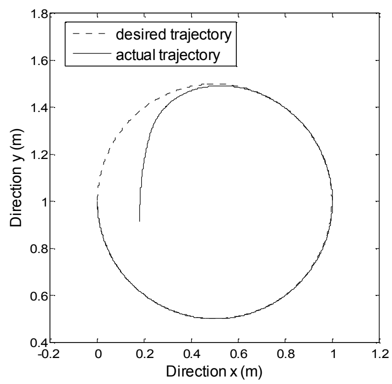

Fig. 2. The desired (dot line) and actual trajectory (solid line) of end-effector.

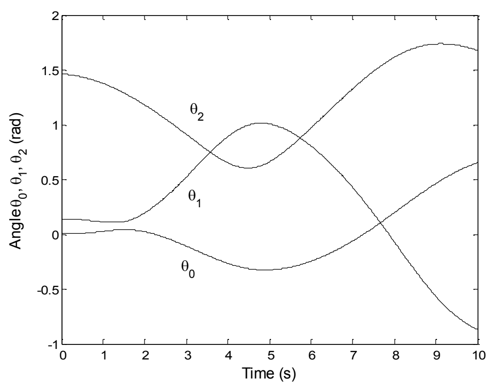

Fig. 3. The angle change curves of the base and joints.

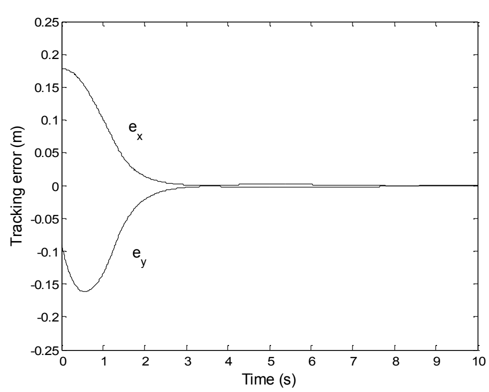

Fig. 4. The tracking error of the system.

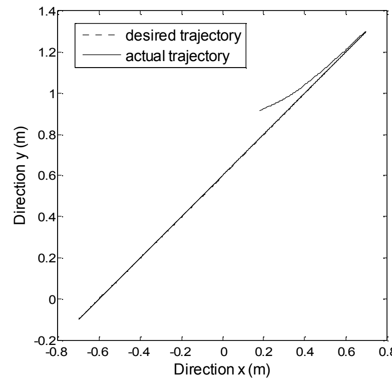

Fig. 5. The desired (dot line) and actual trajectory (solid line) of end-effector.

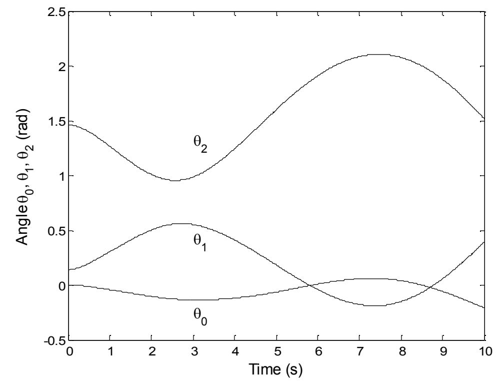

Fig. 6. The angle change curves of the base and joints.

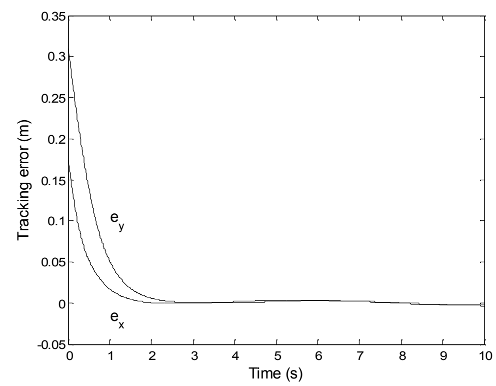

Fig. 7. The tracking error of the system.

VII. CONCLUSION

In this paper, the augmentation approach is adopted to overcome the difficulty of nonlinear parameterization of space robot system and then a robust adaptive control scheme is developed to control the end-effector of the system to track the desired trajectories in inertial space. The controller proposed is designed based on a priori knowledge about uncertain bound and possess the advantages that there are no needs to measure the position, linear velocity and acceleration of the free-floating base. With Lyapunov direct method, the asymptotic stability of the overall system is analyzed. As simulation results illustrate, the proposed control scheme can deal with not only the parametric uncertainties, but also the external disturbances.

ACKNOWLEDGMENT

The work forms part of the research programs sponsored by National Natural Science Foundation of PRC under Grant No. 10672040 and Grant No. 10372022 and by Natural Science Foundation of Fujian province, PRC under Grant No. E0410008.

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