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# Attitude Determination and Localization of Mobile Robots Using Two RTK GPSs and IMU

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*Abstract*— **This paper focuses on the design and test results of an adaptive variation of Kalman filter (KF) estimator based on fusing data from Inertial Measurement Unit (IMU) and two Real Time Kinematic (RTK) Global Positioning Systems (GPS) for driftless 3-D attitude determination and robust position estimation of mobile robots. GPS devices are notorious for their measurement errors vary from one point to the next. Therefore in order to improve the quality of the attitude estimates, the covariance matrix of measurement noise is estimated in real time upon information obtained from the differential GPS measurements, so that the KF filter continually is "tuned" as well as possible. No *a priori* knowledge on the direction cosines of the gravity vector in the inertial frame is required as these parameters can be also identified by the KF, relieving any need for calibration. Next, taking advantage of the redundant GPS measurements, a weight least-squares estimator is derived to weight the GPS measurement with the "good" data more heavily than the one with "poor" data in the estimation process leading to a robust position estimation. Test results are presented showing the performance of the integrated IMU and two GPS to estimate the attitude and location of a mobile robot moving across uneven terrain.**

## I. Introduction

Both position and attitude determination of a mobile robot are necessary for navigation, guidance and steering control of the robot. A good survey of the state-of-the-art in sensors, systems, and methods that aim at finding the position of a mobile robot may be found in [1]. *Dead-reckoning* using vehicle kinematic model and incremental measurement of wheel encoders are the common techniques to determine the position and orientation of mobile robots for indoors applications. However, the application of these techniques for localization of outdoor robots is limited, particularly when the robot has to traverse an uneven terrain or loose soils. This is because wheel slippage and wheel imperfection cause quick accumulation of the position and attitude errors [2]. Other research utilizes inertial measurement unit and wheel encoders to obtain close estimate of robot position [3]–[5]. The problem with inertial systems, however, is that they require additional information about absolute position and orientation to overcome long-term drift [6]. An attitude estimation system based on utilization of multiple inertial measurements of a mobile robots is proposed in [3]. Only pitch and roll angles may be estimated in this method by using gravity components deduced from measurements of two accelerometers [3], while the yaw angle is not detectable.

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Nowadays, differential GPSs to centimeter-level accuracy are commercially available making them attractive for localization of outdoor mobile robots [7], [8]. Although the GPS satellite infrastructure does not exist on the Moon or Mars, development of the low-power transmitters called "pseudolites" instead of orbiting satellites is a promising technology that can be used for localization of a rover in planetary exploration [9].

In this work, we present a method for estimating robot attitude and position, in three dimensions, by optimally fusing data from two RTK GPS measurements, accelerometric measurements of gravity from an accelerometer and angular rate measurements from a rate gyro in an adaptive Kalman filter. Unlike the case of GPS-based attitude determination of spacecraft, the two differential GPS receivers give their ranges with respect to a stationary GPS base antenna. Therefore, the GPS bias resulting from the multipath is nicely eliminated from measurement of the antenna-to-antenna baseline due to the common mode rejection. However, RTK GPS devices notoriously suffer from signal robustness issue as their signal can be easily disturbed by many factors such as satellite geometry, atmospheric condition and shadow. This means that the covariance matrix of the GPS measurement noise can not be given beforehand as it may change from one point to another. In this work, the covariance matrix associated with GPS measurement noise is simultaneously estimated in an online fashion base upon information obtained in real time from the measurements, so that the filter continually is "tuned" as well as possible. In addition to the attitude, the KF also estimates all required parameters such as the direction cosines of the gravity vector in the inertial frame and the gyro bias. We also show that the redundancy in the GPS measurements data plus the knowledge of the GPS noise characteristics can be utilized to enhance the accuracy and robustness of the GPS-based localization of mobile robots. Tests have been conducted on the Canadian Space Agency (CSA) *red rover* for assessing the performance of our pose estimator.

Section II describes the observation equations and their linearization in terms of two GPS measurements and the accelerometric measurements of gravity by the accelerometer. In Sections III and IV, fusing accelerometer, rate gyro and GPS information in a self-tuning adaptive KF for estimating robot attitude is developed, while another stochastic estimator for reliable position estimation of the robot is described in Section V. Finally, the experimental results are reported in Section VII.

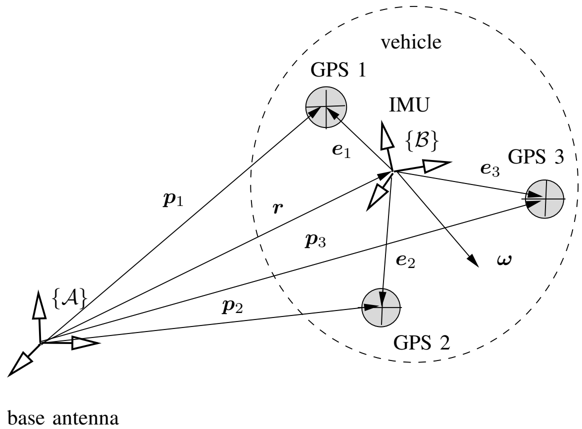


Fig. 1. Multiple GPS antennas and IMU attached on a vehicle body.

## II. Kinematics

Fig. 1 schematically illustrates a vehicle as a rigid body to which multiple differential GPS-antennas and an IMU device are attached. Coordinate frame $\{\mathcal{A}\}$ is an inertial frame while $\{\mathcal{B}\}$ is a vehicle-fixed (body frame) coordinate system. The origin of frame $\{\mathcal{A}\}$ coincides with that of the GPS base antenna, i.e., the vehicle GPS measurements are expressed in $\{\mathcal{A}\}$. On the other hand, the origin of $\{\mathcal{B}\}$ coincides with the IMU center and their axes are parallel, i.e., the IMU measurements are expressed in $\{\mathcal{B}\}$. The orientation of $\{\mathcal{B}\}$ with respect to $\{\mathcal{A}\}$ is represented by the unit quaternion $\boldsymbol{q} = \mathrm{col}(\boldsymbol{q}_v, q_o)$, where subscripts $v$ and $o$ denote the vector and scalar parts of the quaternion, respectively. Below, we review some basic definitions and properties of quaternions used in the rest of the paper. The rotation matrix $\boldsymbol{A}$ representing the rotation of frame $\{\mathcal{B}\}$ with respect to frame $\{\mathcal{A}\}$ is related to the corresponding quaternion $\boldsymbol{q}$ by

$$\boldsymbol{A}(\boldsymbol{q}) = (2q_o^2 - 1)\mathbf{1}_3 + 2q_o[\boldsymbol{q}_v\times] + 2\boldsymbol{q}_v\boldsymbol{q}_v^T, \qquad (1)$$

where $[\cdot\times]$ denotes the matrix form of the cross-product. Consider quaternions $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, and $\boldsymbol{q}_3$. Then, $\boldsymbol{A}(\boldsymbol{q}_3) = \boldsymbol{A}(\boldsymbol{q}_1)\boldsymbol{A}(\boldsymbol{q}_1)$ and $\boldsymbol{q}_3 = \boldsymbol{q}_2 \otimes \boldsymbol{q}_1$ are equivalent, where $\otimes$ denotes the quaternion-product and the operators $[\boldsymbol{q}\otimes]$ is defined, analogous to the cross-product matrix as

$$[\boldsymbol{q}\otimes] = \boldsymbol{\Upsilon} - \mathrm{diag}([\boldsymbol{q}_v\times], 0), \quad \text{where} \quad \boldsymbol{\Upsilon} = \begin{bmatrix} q_0\mathbf{1}_3 & \boldsymbol{q}_v \\ -\boldsymbol{q}_v^T & q_0 \end{bmatrix}.$$

The conjugate $\boldsymbol{q}^*$ of a quaternion is defined such that $\boldsymbol{q}^* \otimes \boldsymbol{q} = \boldsymbol{q} \otimes \boldsymbol{q}^* = [0\ 0\ 0\ 1]^T$.

Now, assume that vector $\boldsymbol{r}$ represents the location of the origin of frame $\{\mathcal{B}\}$ that is expressed in coordinate frame $\{\mathcal{A}\}$, and $\boldsymbol{p}_i$ is the output of the $i$th GPS measurement. Apparently, from Fig. 1, we have

$$\boldsymbol{p}_i = \boldsymbol{r} + \boldsymbol{A}(\boldsymbol{q})\boldsymbol{e}_i + \boldsymbol{v}_{p_i} \qquad \forall i = 1, 2 \qquad (2)$$

where constant vectors $\boldsymbol{e}_i$s are the locations of the GPS antennas in the vehicle-fixed frame and $\boldsymbol{v}_{p_i}$s represent the GPS measurement noises, which are assumed random walk noises with covariance $E[\boldsymbol{v}_{p_i}\boldsymbol{v}_{p_i}^T] = \boldsymbol{R}_{p_i}$.

The IMU is equipped with an accelerometer, which can be used for accelerometric measurements of gravity. In general, accelerometers' outputs contain components of the acceleration of gravity and the inertial acceleration. In mobile robots, the level of inertial acceleration is negligible compared to the acceleration of gravity [1]—typically maximum inertial acceleration is about $0.03g$. Therefore, despite the fact that estimation of the inertial acceleration of the robot can be obtained from the GPS data, it is sufficient to model the inertial acceleration as a measurement noise in the KF. Let assume that $\boldsymbol{a}$ be the accelerometer output. Then, the acceleration equation can be written as

$$\frac{\boldsymbol{a}}{\|\boldsymbol{a}\|} = \boldsymbol{A}^T\boldsymbol{k} + \boldsymbol{v}_a, \qquad (3)$$

where $\|\cdot\|$ denotes the Euclidean norm, and unit vector $\boldsymbol{k}$ is defined to be aligned with the gravity vector in frame $\{\mathcal{A}\}$, while $\boldsymbol{v}_a$ captures the accelerometer noise and inertial acceleration all together. We treat $\boldsymbol{v}_a$ as a random walk noise with covariance $E[\boldsymbol{v}_a\boldsymbol{v}_a^T] = \sigma_a^2\mathbf{1}_3$. The unit vector $\boldsymbol{k}$ can be defined in terms of the corresponding direction cosines $\boldsymbol{\eta} = \mathrm{col}(\alpha, \beta)$ as

$$\boldsymbol{k}(\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{\eta} \\ -(1 - \|\boldsymbol{\eta}\|^2)^{1/2} \end{bmatrix} \qquad (4)$$

### *A. Observation Equations*

The objective of EKF is to determine the vehicle attitude and position by fusing the IMU and GPS measurements. This section presents the measurement equations and measurement sensitivity matrix, while the state transition matrix and the discrete-time process noise needed for covariance propagation are developed later.

In principle, the attitude of a rigid body can be determined from expressions of two non-collinear position vectors in two coordinate systems. The gravity vector is given in both frames $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$ in (3), while the baseline vector

$$\Delta\boldsymbol{p} \triangleq \boldsymbol{p}_1 - \boldsymbol{p}_2$$

is the rotated version of vector

$$\Delta\boldsymbol{e} \triangleq \boldsymbol{e}_1 - \boldsymbol{e}_2,$$

which is expressed in the vehicle frame $\{\mathcal{B}\}$. Therefore, equations (2) and (3) are sufficient to obtain the attitude $\boldsymbol{q}$ and the position $\boldsymbol{r}$. However, the accuracy of this attitude determination method is closely related to the distance between the antennas. Typically, the antenna-to-antenna baseline distance, $\|\Delta\boldsymbol{e}\|$, is approximately 1 m, whereas the GPS error is bout $\pm 5$ cm. This means that the error in measurement of orientation of vector $\Delta\boldsymbol{p}$ is about $10\%$, which is far from desired accuracy. As will be discussed in the followings, the two GPS observations in conjunction with the measurements of the acceleration gravity will be used as external updates in an elaborate Kalman filter integrating a rate gyro data with the observation data.

In our implementation, the IMU signals are given at the rate of 20 Hz, whereas the GPS data can be acquired at the rate of 1 Hz. Therefore, an average of the IMU signal can be obtained between two consecutive GPS data acquisitions whereby decreasing the IMU noise. The discrete-time measurement of the acceleration is obtained through integration of the IMU signals at interval $t_k - t_\Delta < t \leq t_k$, where $t_\Delta$ denotes the GPS sampling rate, i.e.,

$$\bar{\boldsymbol{a}}_k = \frac{1}{t_\Delta}\int_{t_{k-1}}^{t_k} \boldsymbol{a}(\xi)d\xi \tag{5}$$

Let us define the discrete-time measurement vector as

$$\boldsymbol{z}_k = \begin{bmatrix} \boldsymbol{p}_{1_k} - \boldsymbol{p}_{2_k} \\ \bar{\boldsymbol{a}}_k / \|\bar{\boldsymbol{a}}_k\| \end{bmatrix}. \tag{6}$$

Then, in view of (2), (3) and (6), the observation vector as a function of the discrete-time variables $(\boldsymbol{q}_k, \boldsymbol{k}_k)$ can be written as

$$\boldsymbol{h}_k = \begin{bmatrix} \boldsymbol{A}(\boldsymbol{q}_k)(\boldsymbol{e}_1 - \boldsymbol{e}_2) \\ \boldsymbol{A}^T(\boldsymbol{q}_k)\boldsymbol{k}_k \end{bmatrix}. \tag{7}$$

Thus, the observation equation is

$$\boldsymbol{z}_k = \boldsymbol{h}_k + \boldsymbol{v}_k, \quad \text{where} \quad \boldsymbol{v}_k = \begin{bmatrix} \boldsymbol{v}_{\Delta_k} \\ \boldsymbol{v}_{a_k} \end{bmatrix}$$

represents the overall additive measurement noise, and

$$\boldsymbol{v}_\Delta = \boldsymbol{v}_{p1} - \boldsymbol{v}_{p2}.$$

Assuming that the noises of the two GPS receivers are not corrected with the IMU noise, the covariance matrix of the measurement noise takes the form

$$\boldsymbol{R}_k = E[\boldsymbol{v}_k \boldsymbol{v}_k^T] = \begin{bmatrix} \boldsymbol{R}_{\Delta_k} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_a^2 \boldsymbol{1}_3 \end{bmatrix}. \tag{8}$$

Note that the discrete-time observation vector (7) is a nonlinear function of the quaternion. To linearize the observation vector, one also needs to derive the sensitivity of the nonlinear observation vector with respect to the system state vector. To this end, consider small orientation perturbations

$$\delta\boldsymbol{q} = \boldsymbol{q} \otimes \bar{\boldsymbol{q}}^*. \tag{9}$$

around a nominal quaternion $\bar{\boldsymbol{q}}$. As will be discussed in the following section, to take the decomposition rules of quaternion into account, the state vector to be estimated by the EKF is defined as

$$\boldsymbol{x} = \mathrm{col}(\delta\boldsymbol{q}_v, \boldsymbol{\rho}) \tag{10}$$

with $\boldsymbol{\rho} = \mathrm{col}(\boldsymbol{b}, \boldsymbol{\eta}) \in \mathbb{R}^5$, where vector $\boldsymbol{b}$ is the corresponding gyro bias as will be discussed in the following section.

Now, by virtue of $\boldsymbol{A}(\boldsymbol{q}) = \boldsymbol{A}(\delta\boldsymbol{q} \otimes \bar{\boldsymbol{q}})$, one can compute the observation vector (7) in terms of the perturbation $\delta\boldsymbol{q}$. Using the first order approximation of nonlinear matrix function $\boldsymbol{A}(\delta\boldsymbol{q})$ from expression (1) by assuming a small rotation $\delta\boldsymbol{q}$, i.e., $\|\delta\boldsymbol{q}_v\| \ll 1$ and $\delta q_0 \approx 1$, we will have

$$\boldsymbol{A}(\delta\boldsymbol{q}) \approx \boldsymbol{1}_3 + 2[\delta\boldsymbol{q}_v \times]. \tag{11}$$

Thus, the sensitivity matrix can be written as

$$\boldsymbol{H} = \left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\right) = \begin{bmatrix} -2\bar{\boldsymbol{A}}[(\boldsymbol{e}_1 - \boldsymbol{e}_2)\times] & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times2} \\ 2\bar{\boldsymbol{A}}^T[\boldsymbol{k}\times]\bar{\boldsymbol{A}} & \boldsymbol{0}_{3\times3} & \bar{\boldsymbol{A}}^T\boldsymbol{N} \end{bmatrix},$$

where $\bar{\boldsymbol{A}} = \boldsymbol{A}(\bar{\boldsymbol{q}})$ and

$$\boldsymbol{N} \triangleq \frac{\partial \boldsymbol{k}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} = \begin{bmatrix} \boldsymbol{1}_2 \\ \boldsymbol{\eta}^T/(1 - \|\boldsymbol{\eta}\|^2)^{1/2} \end{bmatrix}.$$

## III. Attitude Estimation

### A. Rate Gyro

The relation between the time-derivative of the quaternion and the angular velocity $\boldsymbol{\omega}$ can be readily expressed by

$$\dot{\boldsymbol{q}} = \frac{1}{2}\underline{\boldsymbol{\omega}} \otimes \boldsymbol{q} \qquad \text{where} \quad \underline{\boldsymbol{\omega}} \triangleq \begin{bmatrix} \boldsymbol{\omega} \\ 0 \end{bmatrix}. \tag{12}$$

The angular rate obtained from the gyro measurement is

$$\boldsymbol{\omega}_g = \boldsymbol{\omega} - \boldsymbol{b} - \boldsymbol{\epsilon}_g \tag{13}$$

where $\boldsymbol{b}$ is the corresponding bias vector; $\boldsymbol{\epsilon}_g$ is the angular random walk noises with covariances $E[\boldsymbol{\epsilon}_g \boldsymbol{\epsilon}_g^T] = \sigma_g^2 \boldsymbol{1}_3$. The gyro bias is traditionally modeled as [10]

$$\dot{\boldsymbol{b}} = \boldsymbol{\epsilon}_b, \tag{14}$$

where $\boldsymbol{\epsilon}_b$ is the random walk with covariances $E[\boldsymbol{\epsilon}_b \boldsymbol{\epsilon}_b^T] = \sigma_b^2 \boldsymbol{1}_3$. Then, adopting a linearization technique similar to [10], [11] one can linearize (12) about the nominal quaternion $\bar{\boldsymbol{q}}$ and nominal velocity $\bar{\boldsymbol{\omega}} = \boldsymbol{\omega}_g + \bar{\boldsymbol{b}}$, to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta\boldsymbol{q}_v = -\bar{\boldsymbol{\omega}} \times \delta\boldsymbol{q}_v + \frac{1}{2}\delta\boldsymbol{b} + \frac{1}{2}\boldsymbol{\epsilon}_g. \tag{15}$$

Note that, since $\delta q_o$ is not an independent variable and it has variations of only the second order, its time derivative can be ignored, as suggested in [11]. Then, setting the dynamics equations (15) and (14) in the standard state space form, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{\epsilon}, \tag{16}$$

where $\boldsymbol{\epsilon} = \mathrm{col}(\boldsymbol{\epsilon}_g, \boldsymbol{\epsilon}_b)$; and

$$\boldsymbol{F} = \begin{bmatrix} -[\bar{\boldsymbol{\omega}}\times] & \frac{1}{2}\boldsymbol{1}_3 & \boldsymbol{0}_{3\times2} \\ \boldsymbol{0}_{5\times3} & \boldsymbol{0}_{5\times3} & \boldsymbol{0}_{5\times3} \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} \frac{1}{2}\boldsymbol{1}_3 & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{1}_3 \\ \boldsymbol{0}_{2\times3} & \boldsymbol{0}_{2\times3} \end{bmatrix}$$

Note that the linear model (16), also known as *design model*, just serves to generate the filter.

### B. Discrete-Time Model

The equivalent discrete-time model of (16) is

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{x}_k + \boldsymbol{w}_k \tag{17}$$

where $\boldsymbol{\Phi}_k = \boldsymbol{\Phi}(t_k + t_\Delta, t_k)$ is the state transition matrix over time interval $t_\Delta$. In order to find a closed form solution for the state transition matrix, we need to know the nominal values too. The nominal values of the relevant states at interval $t_k < \tau \leq t_k + t_\Delta$ are given upon the latest estimate update becoming available, i.e., $\bar{\boldsymbol{q}}_k(\tau) = \hat{\boldsymbol{q}}_k$, $\bar{\boldsymbol{b}}_k(\tau) = \hat{\boldsymbol{b}}_k$. Similar to (5), the nominal value of the angular velocity is obtained by averaging through integration of the IMU signals at that interval between two consecutive GPS data acquisition, i.e.,

$$\bar{\boldsymbol{\omega}}_k = \hat{\boldsymbol{b}}_k + \frac{1}{t_\Delta}\int_{t_{k-1}}^{t_k} \boldsymbol{\omega}_g(\xi)d\xi. \tag{18}$$

Therefore, the state transition matrix can be obtained in closed form for constant angular velocity or for varying angular velocity which remains constant in direction. Let us define $\boldsymbol{\theta}_k(\tau) = \bar{\boldsymbol{\omega}}_k \tau$ and $\theta_k = \|\boldsymbol{\theta}_k\|$. Then, the state transition matrix $\boldsymbol{\Phi}_k(\tau) = \boldsymbol{\Phi}(\tau, t_k + \tau)$ takes on the form:

$$\boldsymbol{\Phi}_k(\tau) = \begin{bmatrix} \boldsymbol{\Psi}_{1_k}(\tau) & \frac{1}{2}\boldsymbol{\Psi}_{2_k}(\tau) & \mathbf{0} \\ \mathbf{0} & \mathbf{1}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1}_2 \end{bmatrix} \tag{19}$$

where the elements of the above matrix are given as

$$\boldsymbol{\Psi}_{1_k}(t_\Delta) = \mathbf{1}_3 - \frac{\sin\theta_k}{\theta_k}[\boldsymbol{\theta}_k \times] + \frac{1-\cos\theta_k}{\theta_k}[\boldsymbol{\theta}_k \times]^2 \tag{20}$$

$$\boldsymbol{\Psi}_{2_k}(t_\Delta) = \big(\mathbf{1}_3 + \frac{\cos\theta_k - 1}{\theta_k^2}[\boldsymbol{\theta}_k \times] + \frac{\theta_k - \sin\theta_k}{\theta_k^3}[\boldsymbol{\theta}_k \times]^2\big) t_\Delta,$$

Let $\boldsymbol{\Sigma}_\epsilon = \text{blockdiag}(\sigma_g^2 \mathbf{1}_3, \sigma_b^2 \mathbf{1}_3)$ be the continuous-time covariance matrix of the entire process noise. Then, the corresponding discrete-time covariance matrix is

$$\boldsymbol{Q}_k = E[\boldsymbol{w}_k \boldsymbol{w}_k^T] = \int_{t_k}^{t_k + t_{\Delta_k}} \boldsymbol{\Phi}(t) \boldsymbol{G} \boldsymbol{\Sigma}_\epsilon \boldsymbol{G}^T \boldsymbol{\Phi}^T(t) \mathrm{d}t, \tag{21}$$

which has the following structure

$$\boldsymbol{Q}_k = \begin{bmatrix} \boldsymbol{\Lambda}_{1_k} & \boldsymbol{\Lambda}_{2_k} & \mathbf{0} \\ \times & \sigma_b^2 t_\Delta \mathbf{1}_3 & \mathbf{0} \\ \times & \times & \mathbf{0} \end{bmatrix} \tag{22}$$

For small angle $\theta_k$, where $\sin\theta_k \approx \theta_k - \frac{1}{6}\theta_k^3$ and $\cos\theta_k \approx 1 - \frac{1}{2}\theta_k^2$, the elements of the state-transition matrix (20) can be effectively simplified. Then, upon substitution of the simplified state transition matrix into (21), we arrive at

$$\begin{aligned} \boldsymbol{\Lambda}_{1_k} &= \Big(\frac{\sigma_g^2 t_\Delta}{4} + \frac{\sigma_b^2 t_\Delta^3}{12}\Big)\mathbf{1}_3 + \frac{\sigma_b^2 t_\Delta^3}{240}[\boldsymbol{\theta}_k \times]^2 \\ &\quad + \Big(\frac{\sigma_g^2 t_\Delta}{80} + \frac{\sigma_b^2 t_\Delta^3}{1008}\Big)[\boldsymbol{\theta}_k \times]^4 \\ \boldsymbol{\Lambda}_{2_k} &= \sigma_b^2 t_\Delta^2 \Big(\frac{1}{4}\mathbf{1}_3 - \frac{1}{12}[\boldsymbol{\theta}_k \times] + \frac{1}{48}[\boldsymbol{\theta}_k \times]^2\Big). \end{aligned}$$

## IV. Estimator Design

### A. Quaternion Update in Innovation Step

Note that the state transition matrix (19) is used only for covariance propagation, while the system states have to be propagated separately by solving their own differential equations. Let us compose the redundant state ${}^a\boldsymbol{x} = \text{col}(\boldsymbol{q}, \boldsymbol{\rho})$, which contains the full quaternion $\boldsymbol{q}$ and parameters $\boldsymbol{\rho}$. Combining (12), (14) and $\dot{\boldsymbol{\rho}} = \mathbf{0}$, we then describe the state-space model of the system as

$${}^a\dot{\boldsymbol{x}} = \boldsymbol{f}({}^a\boldsymbol{x}, \boldsymbol{\epsilon})$$

The EKF-based observer for the associated noisy discrete system (17) is given in two steps: $(i)$ estimate correction

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \big(\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k\big)^{-1} \tag{23a}$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}\big(\boldsymbol{z}_k - \boldsymbol{h}(\hat{\chi}_k^-)\big) \tag{23b}$$

$$\boldsymbol{P}_k = \big(\mathbf{1}_{12} - \boldsymbol{K}_k \boldsymbol{H}_k\big)\boldsymbol{P}_k^- \tag{23c}$$

and $(ii)$ estimate propagation

$${}^a\hat{\boldsymbol{x}}_{k+1}^- = {}^a\hat{\boldsymbol{x}}_k + \int_{t_k}^{t_k + t_\Delta} \boldsymbol{f}({}^a\boldsymbol{x}(t), \mathbf{0})\, \mathrm{d}t \tag{24a}$$

$$\boldsymbol{P}_{k+1}^- = \boldsymbol{\Phi}_k \boldsymbol{P}_k \boldsymbol{\Phi}_k^T + \boldsymbol{Q}_k \tag{24b}$$

As mentioned before, the vector of discrete-time states, $\boldsymbol{x}_k$, contains only the variations $\delta\boldsymbol{q}_{v_k}$ not the quaternion $\boldsymbol{q}_k$. However, the full quaternion can be obtained from the former variables if the value of the nominal quaternion $\bar{\boldsymbol{q}}_k$ is given, that is

$$\delta\hat{\boldsymbol{q}}_k^- = \hat{\boldsymbol{q}}_k^- \otimes \bar{\boldsymbol{q}}_k^* \tag{25}$$

A natural choice for the nominal value of quaternion is its update estimate as $\bar{\boldsymbol{q}}(t_{k-1}) = \hat{\boldsymbol{q}}_{k-1}$. Since the nominal angular velocity $\bar{\boldsymbol{\omega}}_k$ is assumed constant at interval $t_{k-1} \leq t \leq t_k$, then, in view of (12), the nominal quaternion evolves from its initial value $\bar{\boldsymbol{q}}(t_{k-1})$ to its final value $\bar{\boldsymbol{q}}_k = \bar{\boldsymbol{q}}(t_k)$ according to:

$$\bar{\boldsymbol{q}}_k = e^{\frac{1}{2}[\underline{\boldsymbol{\theta}}_k \otimes]} \hat{\boldsymbol{q}}_{k-1}. \tag{26}$$

It can be shown that the above exponential matrix function has a closed-form expression so that the above equation can be written as

$$\begin{aligned} \bar{\boldsymbol{q}}_k =& \big(\cos\frac{\theta_k}{2} + \sin\frac{\theta_k}{2}\big)\hat{\boldsymbol{q}}_{k-1} \\ &+ \Big(\frac{2}{\theta_k}\sin\frac{\theta_k}{2} - \frac{1}{2}\cos\frac{\theta_k}{2}\Big)\underline{\boldsymbol{\theta}}_k \otimes \hat{\boldsymbol{q}}_{k-1} \end{aligned} \tag{27}$$

From (25), the innovation step of KF, i.e., (23b), can be written in terms of the full quaternion, $\boldsymbol{q}_k$ instead of its variation $\delta\boldsymbol{q}_{v_k}$, as

$$\begin{bmatrix} \delta\hat{\boldsymbol{q}}_{v_k} \\ \hat{\boldsymbol{\rho}}_k \end{bmatrix} = \begin{bmatrix} \text{vec}(\hat{\boldsymbol{q}}_k^- \otimes \bar{\boldsymbol{q}}_k^*) \\ \hat{\boldsymbol{\rho}}_k^- \end{bmatrix} + \boldsymbol{K}(\boldsymbol{z}_k - \boldsymbol{h}_k) \tag{28}$$

where $\bar{\boldsymbol{q}}_k$ is obtained from (27). Finally, assuming that $\|\delta\hat{\boldsymbol{q}}_{v_k}\| < 1$, a valid unit quaternion can be constructed from

$$\hat{\boldsymbol{q}}_k = \begin{bmatrix} \delta\hat{\boldsymbol{q}}_{v_k} \\ \sqrt{1 - \|\delta\hat{\boldsymbol{q}}_{v_k}\|^2} \end{bmatrix} \otimes \bar{\boldsymbol{q}}(t_k) \tag{29}$$

### B. Noise-Adaptive Filter

The IMU noises are not usually characterized by a time-invariant covariance. Therefore, $\sigma_a$ can be treated as a constant parameter, which can be either derived from the sensor specification or empirically tuned. However, the GPS measurement errors may vary from one point to the next, in which case the error depends on many factors such as satellite geometry, atmospheric condition, multipath areas, and shadow. Therefore, it is desirable to weight GPS measurement data in the GPS and IMU data fusion process heavily only when a "good" GPS measurement data is available.

In a noise-adaptive Kalman filter, the issue is that, in addition to the states, the covariance matrix of the measurement noise has to be estimated [12], [13]. From (2), we get

$$\Delta\boldsymbol{p} - \boldsymbol{A}(\boldsymbol{q})\Delta\boldsymbol{e} = \boldsymbol{v}_\Delta \tag{30}$$

Moreover, in view of quaternion error $\delta\tilde{\boldsymbol{q}}_v = \delta\boldsymbol{q}_v - \delta\hat{\boldsymbol{q}}_v$ and the first order approximation of a small rotation matrix (11), one can relate the actual rotation matrix to the nominal and estimated rotation matrices and the quaternion error by

$$\boldsymbol{A}(\boldsymbol{q}) \approx \hat{\boldsymbol{A}} + 2\bar{\boldsymbol{A}}[\delta\tilde{\boldsymbol{q}}\times], \tag{31}$$

where $\hat{\boldsymbol{A}} = \boldsymbol{A}(\hat{q})$ is the estimation of the rotation matrix. Let us define the residual error

$$\boldsymbol{\varrho} = \Delta\boldsymbol{p} - \hat{\boldsymbol{A}}\Delta\boldsymbol{e} = -\bar{\boldsymbol{A}}(\Delta\boldsymbol{e} \times \delta\tilde{\boldsymbol{q}}_v) + \boldsymbol{v}_{\scriptscriptstyle\triangle}, \tag{32}$$

where the RHS of (32) is obtained by using (31) in (30). It is apparent from (32) that $\boldsymbol{\varrho}$ is a linear combination of of $\tilde{\boldsymbol{q}}_v$ and $\boldsymbol{v}_{\scriptscriptstyle\triangle}$, and hence it, in fact, is a zero-mean, white noise sequence, i.e., $E[\boldsymbol{\varrho}] = \boldsymbol{0}$. On the other hand, since the Kalman filer gives the covariance matrix of all estimated states including the quaternion, one can get $\boldsymbol{P}_q = E[\delta\tilde{\boldsymbol{q}}_v \delta\tilde{\boldsymbol{q}}_v^T] \in \mathbb{R}^{3\times3}$ from the filter covariance matrix, i.e.,

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_q & \times \\ \times & \times \end{bmatrix} \in \mathbb{R}^{8\times8}.$$

With the exception of the DC component, the rate gyro provides much more accurate information about the attitude than the GPS meaning that the attitude error is mainly determined by the gyro noise. In other words, it is reasonable to assume that random variables $\delta\tilde{\boldsymbol{q}}_v$ and $\boldsymbol{v}_p$ are weakly correlated. Ignoring the cross correlation of these stochastic variables, one can obtain an estimate of the covariance matrix $\hat{\boldsymbol{R}}_{\scriptscriptstyle\triangle}$ from (32) as:

$$\hat{\boldsymbol{R}}_{\scriptscriptstyle\triangle k} = \boldsymbol{S}_k + 4\bar{\boldsymbol{A}}_k[\Delta\boldsymbol{e}\times]\boldsymbol{P}_{q_k}[\Delta\boldsymbol{e}\times]^T\bar{\boldsymbol{A}}_k^T \tag{33}$$

where

$$\boldsymbol{S}_k = E[\boldsymbol{\varrho}_k\boldsymbol{\varrho}_k^T] \approx \frac{1}{w}\sum_{i=k-w}^{k} \boldsymbol{\varrho}_i\boldsymbol{\varrho}_i^T \tag{34a}$$

$$= \boldsymbol{S}_{k-1} + \frac{1}{w}\Big(\boldsymbol{\varrho}_k\boldsymbol{\varrho}_k^T - \boldsymbol{\varrho}_{k-w}\boldsymbol{\varrho}_{k-w}^T\Big). \tag{34b}$$

Here, the term in the RHS of (34a) is an ergodic approximation of the covariance of the zero-mean residual $\boldsymbol{\varrho}$ in the sliding sampling window with length $w$. Despite the batch processing (34a) is equivalent to the recursive formulation (34b), the latter is more computationally efficient.

### C. Initialization of KF

For the first iteration of the EKF, an adequate guess of the initial states is required. The best guess for the parameters at $t = 0$ s is $\boldsymbol{\rho} = \boldsymbol{0}_{5\times1}$, while the initial orientation of the vehicle with respect to the inertial frame at $t = 0$ s has to be carefully selected so that our assumption $\|\delta\boldsymbol{q}_v\| \leq 1$ is satisfied in the innovation step (28)-(29); otherwise the error quaternion will not be unit norm. To prevent this from happening, it is important to keep the initial error in quaternion estimate small as much as possible based on the information available from the measurements.

Mathematically, the rotation matrix can be computed from two non-collinear unit vectors $\boldsymbol{k}$ and $\Delta\boldsymbol{p}/\|\Delta\boldsymbol{p}\|$ and their rotated versions $\boldsymbol{a}/\|\boldsymbol{a}\|$ and $\Delta\boldsymbol{e}/\|\Delta\boldsymbol{e}\|$. Similar to the methods for registration of 3-D laser scanning data [14], let us form the following matrices from the unit vectors as

$$\boldsymbol{D}_a = \begin{bmatrix}\boldsymbol{k} & \Delta\boldsymbol{p}/\|\Delta\boldsymbol{p}\| & \boldsymbol{k}\times\Delta\boldsymbol{p}/\|\boldsymbol{k}\times\Delta\boldsymbol{p}\|\end{bmatrix} \tag{35a}$$

$$\boldsymbol{D}_b = \begin{bmatrix}\boldsymbol{a}/\|\boldsymbol{a}\| & \Delta\boldsymbol{e}/\|\Delta\boldsymbol{e}\| & \boldsymbol{a}\times\Delta\boldsymbol{e}/\|\boldsymbol{a}\times\Delta\boldsymbol{e}\|\end{bmatrix} \tag{35b}$$

Note that the column vectors in (35) are normalized in order to avoid ill-conditioning of the matrices. Then, in the absence of measurement noise, i.e., $\boldsymbol{v} \equiv \boldsymbol{0}$, the above matrices are related by

$$\boldsymbol{D}_a = \boldsymbol{A}\boldsymbol{D}_b \tag{36}$$

Matrices $\boldsymbol{D}_a$ and $\boldsymbol{D}_b$ are non singular as long as $\boldsymbol{k}$ and $\Delta\boldsymbol{p}$ are not collinear, i.e., the line connecting the GPS antennas is not parallel to the gravity vector. Then, under this circumstance, the rotation matrix can be obtained from

$$\boldsymbol{A} = \boldsymbol{D}_a\boldsymbol{D}_b^{-1} \tag{37}$$

Solution (37) yields a valid rotation matrix $\boldsymbol{A}$ so that $\boldsymbol{A}^T\boldsymbol{A} = \boldsymbol{1}_3$ only if there is no error in the column vectors of matrices (35). This may not be the case in practice, however, due to the IMU and GPS noises. To correct this problem, one may observe that all singular values of any orthogonal matrix must be one. This means that the singular value decomposition of the RHS of (37) yields

$$\boldsymbol{D}_a\boldsymbol{D}_b^{-1} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T,$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal matrices and matrix $\boldsymbol{\Sigma} = \boldsymbol{1}_3 + \boldsymbol{\Delta}_\Sigma$ is expected to be close to the identity matrix, i.e., $\|\boldsymbol{\Delta}_\Sigma\| \ll 1$. Therefore, by ignoring small matrix $\boldsymbol{\Delta}_\Sigma$, a valid solution to the rotation matrix can be found as

$$\hat{\boldsymbol{A}} = \boldsymbol{U}\boldsymbol{V}^T, \tag{38}$$

which, then, can be used to obtain the equivalent quaternion at $t = 0$ s.

## V. Position Estimation

Now, with the estimation of attitude in hand, one can obtain an estimate of position $\boldsymbol{r}$ from either one of the equations in (1). Nevertheless, in order to improve the quality of the position estimate, it would be desirable to weight the GPS measurement with the "good" data more heavily than the one with "poor" data in the estimator; rather than all GPS measurements being given equal weights.

Assuming that the noises of the two GPS receivers are not mutually corrected, we will have

$$E[\boldsymbol{v}_{p_i}\boldsymbol{v}_{p_j}^T] = \begin{cases} \boldsymbol{R}_{p_i} & i = j \\ \boldsymbol{0} & i \neq j \end{cases}$$

that leads to

$$\boldsymbol{R}_{\scriptscriptstyle\triangle} = \boldsymbol{R}_{p_1} + \boldsymbol{R}_{p_2} \tag{39}$$

The objective is to estimate $\boldsymbol{R}_{p_1}$ and $\boldsymbol{R}_{p_2}$ from (39) upon real-time estimate of $\boldsymbol{R}_{\scriptscriptstyle\triangle}$ using (33). There is no unique solution to (39), unless the share of each GPS in the overall measurement uncertainty is known. To this end, one can take advantage of that most RTK GPS receivers return a real-time signal indicating the confidence on their position measurements. Now, assuming the covariance matrix of the $i$th GPS in (39) is proportional to its confidence measure $\mu_i$, one can assume

$$\boldsymbol{R}_{p_i} = \frac{\mu_i}{\mu_1 + \mu_2} \boldsymbol{R}_{\triangle} \qquad \forall i = 1, 2, \tag{40}$$

which trivially satisfies (39). On the other hand, setting equations (2) in the matrix form and then using (31) in the resultant equation, we arrive at

$$\boldsymbol{y} = \boldsymbol{L}\boldsymbol{r} + \underbrace{\boldsymbol{n}_p + \boldsymbol{n}_q}_{\text{noise}} \tag{41}$$

where

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{p}_1 - \hat{\boldsymbol{A}}\boldsymbol{e}_1 \\ \boldsymbol{p}_2 - \hat{\boldsymbol{A}}\boldsymbol{e}_2 \end{bmatrix}, \quad \boldsymbol{L} = \begin{bmatrix} \boldsymbol{1}_3 \\ \boldsymbol{1}_3 \end{bmatrix},$$

while the GPS measurement errors and attitude estimation errors are respectively represented by vectors

$$\boldsymbol{n}_p = \begin{bmatrix} \boldsymbol{v}_{p_1} \\ \boldsymbol{v}_{p_2} \end{bmatrix}, \qquad \boldsymbol{n}_q = -2\bar{\boldsymbol{A}} \begin{bmatrix} [\boldsymbol{e}_1 \times] \\ [\boldsymbol{e}_2 \times] \end{bmatrix} \delta\tilde{\boldsymbol{q}}_v.$$

Finally, using the argument that mutual correlation between $\boldsymbol{n}_p$ and $\boldsymbol{n}_q$ is negligible, we can write the covariance matrix of the entire noise in (41) as:

$$\boldsymbol{R}_r = E[\boldsymbol{n}_p \boldsymbol{n}_p^T] + E[\boldsymbol{n}_q \boldsymbol{n}_q^T] \tag{42}$$
$$= \begin{bmatrix} \boldsymbol{R}_{p_1} + 4\bar{\boldsymbol{A}}[\boldsymbol{e}_1\times]\boldsymbol{P}_q[\boldsymbol{e}_1\times]\bar{\boldsymbol{A}}^T & 4\bar{\boldsymbol{A}}[\boldsymbol{e}_1\times]\boldsymbol{P}_q[\boldsymbol{e}_2\times]\bar{\boldsymbol{A}}^T \\ 4\bar{\boldsymbol{A}}[\boldsymbol{e}_2\times]\boldsymbol{P}_q[\boldsymbol{e}_1\times]\bar{\boldsymbol{A}}^T & \boldsymbol{R}_{p_2} + 4\bar{\boldsymbol{A}}[\boldsymbol{e}_2\times]\boldsymbol{P}_q[\boldsymbol{e}_2\times]\bar{\boldsymbol{A}}^T \end{bmatrix}$$

Then, an optimal solution to (41), which minimizes the normalized measurement residual, can be obtained by the weighted pseudo-inverse where a suitable weighting matrix is the covariance matrix of the noise [15]. That is

$$\hat{\boldsymbol{r}} = (\boldsymbol{L}^T \boldsymbol{R}_r^{-1} \boldsymbol{L})^{-1} \boldsymbol{L}^T \boldsymbol{R}_r^{-1} \boldsymbol{y}, \tag{43}$$

If the orientation estimation error is sufficiently small, i.e., the second term in RHS of (42) is negligible, then (43) can be conveniently written as

$$\hat{\boldsymbol{r}} = \boldsymbol{W}_1(\boldsymbol{p}_1 - \boldsymbol{A}\boldsymbol{e}_1) + \boldsymbol{W}_2(\boldsymbol{p}_2 - \boldsymbol{A}\boldsymbol{e}_2), \quad \text{where} \tag{44}$$

$$\boldsymbol{W}_1 \triangleq \boldsymbol{R}_{p_2}\big(\boldsymbol{R}_{p_1} + \boldsymbol{R}_{p_2}\big)^{-1} \tag{45a}$$
$$\boldsymbol{W}_2 \triangleq \boldsymbol{R}_{p_1}\big(\boldsymbol{R}_{p_1} + \boldsymbol{R}_{p_2}\big)^{-1}. \tag{45b}$$

## VI. Pose Estimation From Three GPS Measurements

This section briefly describes a method to estimate the pose adequately from three independent GPS measurements and their corresponding estimated noise covariance matrices. Assume that $\boldsymbol{p}_3$, $\boldsymbol{v}_{p_3}$ and $\boldsymbol{e}_3$ denote the third GPS measurement, its noise, and location of its antenna on the vehicle, respectively. Also, denote $\Delta\boldsymbol{p}' = \boldsymbol{p}_1 - \boldsymbol{p}_3$ and $\Delta\boldsymbol{e}' = \boldsymbol{e}_1 - \boldsymbol{e}_3$. Then, in a development similar to (35) to (38), one can calculate the rotation matrix from the three GPS measurements by replacing vectors $\boldsymbol{k}$ and $\boldsymbol{a}$ in (35) by $\Delta\boldsymbol{p}'/\|\Delta\boldsymbol{p}'\|$ and $\Delta\boldsymbol{e}'$, respectively. Furthermore, the optimal position estimation can be obtained similar to the development from (41) to (45).

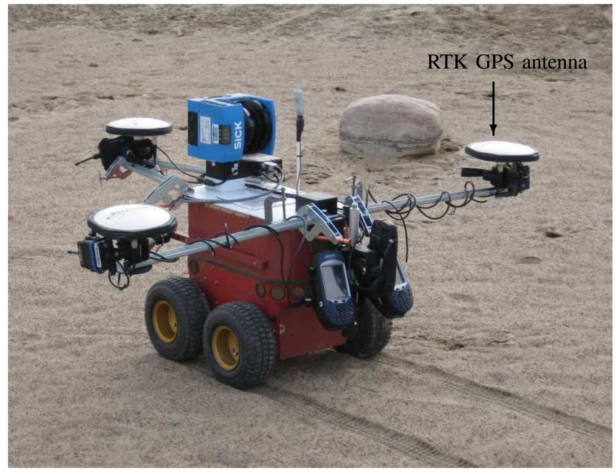


Fig. 2. The CSA *red rover* with three RTK GPS antennas.

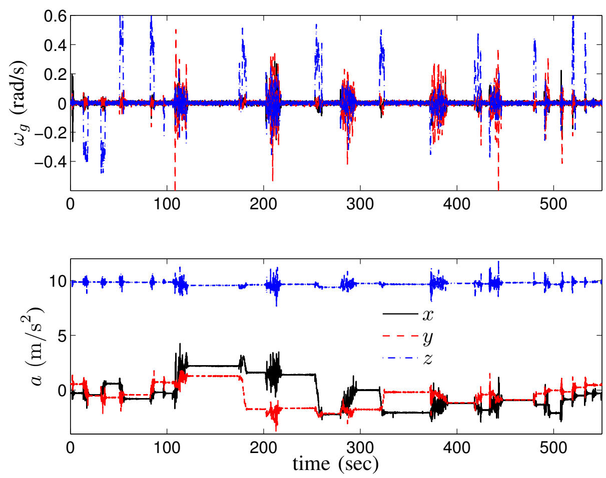


Fig. 3. IMU outputs.

## VII. Experiment

Fig. 2 shows the CSA *red rover*, which is equipped with three RTK GPS receivers along with satellite antennas and radio modems (model Promark3RTK from Magellan Navigation Inc.) in addition to an IMU device from Crossbow Technology, Inc. (model IMU300). Experiments were conducted on the 30 $\times$ 60 m Mars Emulation Terrain (MET) of the CSA .

An operator sent a pre-scheduled sequence of primitive commands to the mobile robot—e.g.,"move forward of a certain distance", "rotate clockwise by $45°$", etc—so that the robot follows a pre-planned path going through some specified via points. The GPS and IMU measurements are received at the rate of 1 Hz and 20 Hz, respectively, and the corresponding trajectories are depicted in Figs. 3 and 4. The variances of the IMU noise are set to $\sigma_g = 5\times10^{-5}$ rad/s and $\sigma_a = 0.3$ m/s$^2$, while the covariance matrix of the baseline vector measurement from the two GPS data is estimated in the online fashion as illustrated in Fig. 5. It is apparent from the figure that the covariances are not constant, rather they fluctuate significantly over time, e.g., the noises are

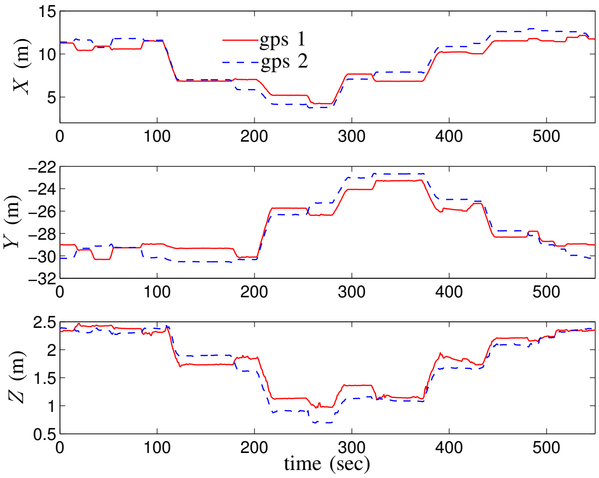


Fig. 4. RTK GPS outputs.

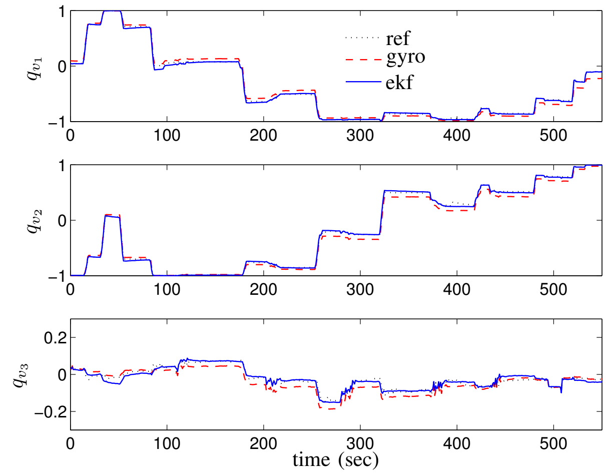


Fig. 7. Vehicle attitude.

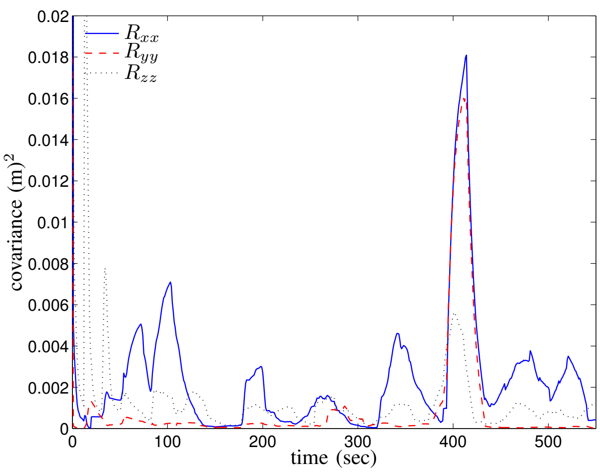


Fig. 5. Diagonal entries of the estimated covariance matrix of the baseline measurement errors $\hat{\boldsymbol{R}}_{\Delta}$.

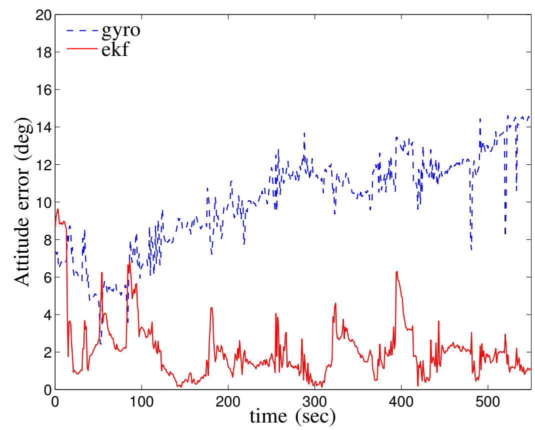


Fig. 8. Attitude errors.

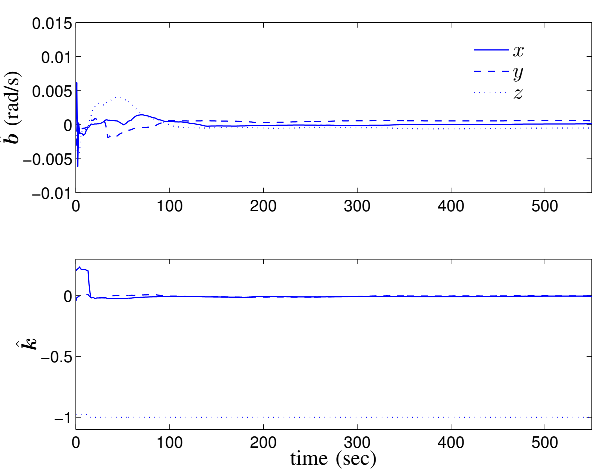


Fig. 6. Estimated parameters.

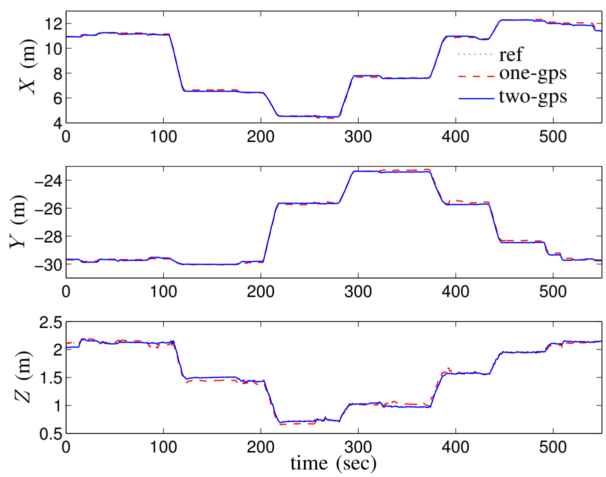


Fig. 9. Vehicle position.

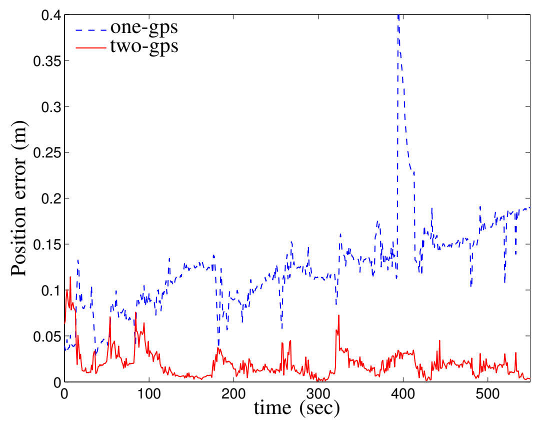


Fig. 10. Position errors.

particularly strong around $t = 400$ s. In our experiment, we compare the Two-GPS-IMU fusion method with One-GPS-IMU method. Fig. 6 illustrates that the KF can quickly identify the parameters, i.e., gyroscope bias and the direction cosines of the gravity vector. Trajectories of the estimated vehicle attitude (represented by quaternion) based on the Two-GPS-IMU fusion method are illustrated in Fig. 7. For a comparison, trajectories of the attitude obtained once from numerical integration of the gyro rate signals and then from the geometric pose estimation using three GPS measurements are also shown in the figure. Fig. 8 illustrates the time-history of the orientation errors calculated by

$$\text{Attitude Error} = 2\sin^{-1}\|\text{vec}(\hat{\boldsymbol{q}}^* \otimes \boldsymbol{q}_{\text{ref}})\|. \tag{46}$$

It is clearly evident from the figure that the attitude estimation from the adaptive KF exhibits no drift, whereas the orientation error using the gyro alone accumulates gradually due to its drift, i.e., a drift of $14^\circ$ over the 500 s. period.

The second part of the experiment involves estimating the vehicle position. It was already known from graphs of Fig. 5 that the GPS measurements are erroneous at the time around $t = 400$ s that is mainly attributed to GPS 1. However, since GPS 2 still provides "good" data during the faulty period of GPS 1, the stochastic estimator described in Section V should be still able to estimate the position. To demonstrate the robustness of the latter estimator, we will use the position estimate obtained from averaging of three GPS measurements as described in Section VI. Fig. 9 shows trajectories of the vehicle position based on three methods: i) One-GPS and gyo , ii) Two-GPS method and iii) Three-GPS method (reference trajectory). The errors between the reference and the other two position estimation methods are calculated and the results are plotted Fig. 10. The spike in the position error at the time around $t = 400$ s is due to the fact that the error margin of the GPS 1 receiver during that period is incidentally high.

## VIII. Conclusions

A method for driftless 3-dimensional attitude determination and reliable position estimation of a mobile robot by optimally fusing the information from IMU and two RTK GPS receivers in an adaptive KF has been developed. In the face of variation and uncertain GPS noise model, the covariance matrix of measurement noise was estimated upon information obtained in real time from the GPS measurements, so that the KF filter is continually "tuned" as well as possible. In addition to the vehicle attitude, the KF estimator is able to estimate all parameters required for attitude determination including the direction cosines of the gravity vector relieving the need for any calibration. Furthermore, the weighted least-squares estimator has been derived for a robust estimation of the vehicle position that weights the GPS measurement with "good" data more heavily that the one with "poor" data in the estimation process. Experiments conducted on the CSA red rover and the results have shown that the Two-GPS-IMU system can provide a driftless attitude determination and reliable localization of the vehicle.

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