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Optimal Tightening Forces for Multi-Fingered Robust Manipulation

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Abstract— **We address the problem of choosing adequate contact forces to ensure the stability of a multi-fingered grasp in face of external disturbances of unknown intensity. The contact forces we look for are tightening, pre-strain forces rather than active, direction-related blocking forces. We compute such tightening forces, together with blocking forces and a lower-bound approximation of an existing grasp quality measure, in only one linear programming problem. The tightening forces ensure robustness to the largest-minimum resisted disturbance wrench, but a variety of optimal contact forces may be further computed from the desired robustness by solving a quadratic programming problem.**

Keywords— **Multi-fingered hand, disturbances, robustness, grasp quality measure, force optimization problem, linear programming, quadratic programming.**

I. Introduction

A. Statement of the problem

Nowadays still uncommon, humanoid robots are meant to play an important part in our future daily lives. Yet our everyday environment is particularly unfit to robots as we know them. Complex, unstructured, and changing, it is the cause of most of the problems robotics research has to tackle in a variety of areas. In particular, in the domain of dextrous, multi-fingered manipulation, it results in the requirement of a certain amount of robustness in the robots' prehensile abilities. Safe interacting with human-beings means indeed that keeping hold of objects in face of potential disturbances is necessary: unexpected forces may happen, or estimates of the object characteristics may be wrong.

Human grasps provide this level of robustness through enveloping grasps or tightening contact forces that squeeze the manipulated object in case a disturbance happens. These tightening, pre-strain forces depend on the characteristics of the disturbances that are expected to happen (directions and intensities). In this work, we provide a method for computing such contact forces. The problem that is investigated is different from, though related to, what are known as the force optimization problem and the problem of measuring the quality of a grasp.

B. Related work

The force optimization problem is the problem of finding optimal contact forces against one or several known external wrenches, for instance gravity or a specified box in wrench space (defined by lower and upper bounds on the force and torque components). If there are several wrenches, the usual approach is to solve several force optimization problems.

The force optimization problem has been extensively studied, and solved by a variety of techniques. [1], [2] discretize the contact cones and use linear programming to compute optimal contact forces. [3], [4] also use discretized contact cones and linear programming but compute optimal contact primitive wrenches. [5], [6] use discretized cones and quadratic programming: a quadratic criteria on the contact force components has a better physical interpretation than a linear one as it represents a force intensity. [7]–[9] use exact contact cones and formulate the problem as a convex optimization problem subject to linear matrix inequality constraints (LMI). [10], [11] use the dual of the force optimization problem; [10] also proposes a method for solving efficiently a family of related force optimization problems.

Grasp quality measures are also numerous [12]. The most common one is the criteria of the residual ball, or largest ball [13]–[17], which was recently thoroughly explored by the exact, analytical study of [18]. It measures the size of the grasp wrench space GWS, i.e. the size of the set of all possible resultant wrenches produced by the fingers on the object, the magnitude of the contact forces being somehow limited. This criteria states that the quality of the grasp is the residual radius of the grasp wrench space, that is to say, the radius of the largest origin-centered L_2-ball of the wrench space $se_3^*(\mathbb{R})$ that is fully contained in GWS. This index measures the largest-minimum wrench that the grasp can produce on the object. Equivalently, it measures the largest-minimum disturbance wrench that the grasp can resist. By *largest-minimum* we mean the largest produced or resisted wrench in the direction of the wrench space for which this wrench is the smallest (also known as the *worst* direction): $\min_{w \text{ direction}} \max_{\rho \text{ intensity}}(\rho w)$, see also [12, page 10].

There are several variations of the criteria of the largest ball. One of them may be called the criteria of the largest disturbance polytope and was introduced by [19], [20]. This criteria uses a convex set of $se_3^*(\mathbb{R})$, including the origin, instead of a ball. This set represents the expected disturbances, or at least the disturbances the hand is supposed to resist. The quality index is the largest scale factor that makes this set of expected disturbances fully contained in the opposite of the grasp wrench space, $-GWS$. It measures the largest-minimum disturbance wrench that the grasp can resist in the directions of the vertices of the convex set.

[20] gives an algorithm to compute this quality measure. This algorithm consists of a set of linear or non-linear programming problems, depending on whether the constraints

are linear or not (they are linear when the contacts are frictionless, when the grasp is two-dimensional or when the contact cones are linearized). There are as many programming problems as disturbance directions in the convex set used in place of the ball, which explains why this convex set is a polytope in computer implementations.

C. Contribution

To ensure the grasp is stable in face of unknown disturbances, we yield adequate tightening forces from the set of expected disturbance directions in only one linear programming problem with a variable in $\mathbb{R}^{3n_c+1}$, n_c being the number of contacts (i.e. the number of fingers). This linear program is obtained from a larger one with a variable in $(\mathbb{R}^{3n_c})^{n_d+1}$, n_d being the number of investigated disturbance directions, by considering the worst-case components of the disturbances. This dimension reduction technique is an alternative to solving a force optimization problem for each disturbance direction. It should be noted that the dimension of the resulting linear program does not depend on the number of disturbances, which means that a large number of disturbance directions may be considered simultaneously.

The linear program also computes a grasp quality measure, avoiding the need to solve another as many optimization problems as disturbance directions. This measure is both different from and related to the quality measure of [20]. In fact, it is a lower-bound approximation of it.

The contact forces we compute are pre-strain forces relative to all the investigated disturbance directions, rather than active blocking forces in each direction. They ensure robustness in face of the largest-minimum acceptable disturbance, however a variety of tightening forces may be further computed from the desired robustness by solving a quadratic program in $\mathbb{R}^{3n_c}$. The tightening forces are to be applied in the absence of any disturbance, not only during the disturbance. We also compute the force variations that turn this tightening into disturbance-blocking forces.

This is a difference to underline: in the classical formulation of the force optimization problem, there is no notion of pre-strain. Since the external wrench to resist is known, only blocking forces against it are needed. On the contrary, in our force optimization problem, there are also unknown disturbances. In this case, tightening becomes very valuable for the grasp to withstand a disturbance more easily. It also reflects what many physiologists have observed, e.g. [21]: grip forces increase before a disturbance, not only after.

In other words, optimality in the classical formulation of the force optimization problem is a minimality: forces have just the required magnitude to resist a given external wrench. Our optimal forces have more than this required magnitude, but they act as pre-strain and provide a kind of passive robustness for the grasp, whereas the classical formulation provides only active robustness where no tightening forces are applied prior to any disturbance.

D. Outline of the paper

The rest of this paper is as follows. Notations and definitions are given in section II. In section III, we describe the problem of grasp robustness and how to get tightening forces against specified disturbances. In this introductory section, the disturbances are supposed entirely known. In section IV, the disturbance intensities are unknown and we solve the problem of grasp robustness to the largest-minimum disturbance wrench. We get robust tightening forces and an index of grasp quality with respect to the disturbances it can resist. Section V gives a simulation example and section VI concludes the paper.

II. Notations and definitions

A. Basic notations

We let n_c denote the number of contacts in the grasp and i denote the index of a contact: $i \in [|1, n_c|]$. The different frames are illustrated on figure 1: ref is an inertial reference frame, obj is the frame of the object being manipulated, and c_i denotes both the contact point between the object and the i-th finger and the contact frame $(\vec{t}_i^1, \vec{t}_i^2, \vec{n}_i)$, with $\vec{n}_i$ outward and normal to the object's surface.

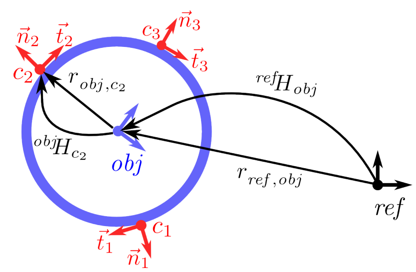

Fig. 1. Frames and homogeneous transforms

Frames are related through homogeneous transforms, e.g.:

$$ {}^{ref}H_{obj} = \begin{pmatrix} {}^{ref}R_{obj} & r^{ref}_{ref,obj} \\ 0_{1,3} & 1 \end{pmatrix} \in SE_3(\mathbb{R}) $$

locates the object relatively to ref through the rotation ${}^{ref}R_{obj} \in SO_3(\mathbb{R})$ between the bases of the frames and the translation $r^{ref}_{ref,obj} \in \mathbb{R}^3$ from the origin of ref to the origin of obj, this vector being written in ref coordinates.

B. Contact modeling

Contacts between the fingers and the object are modeled as rigid point contacts with friction. The forces $f_1, \ldots, f_{n_c} \in \mathbb{R}^3$ applied by the fingers on the object are assumed written in their respective contact frames $c_1, \ldots, c_{n_c}$.

In sections III and IV we will take into account two assumptions on the contact forces. The first one is that they are unilateral (from the finger to the object): $(f_i)_n \leq 0$, $\forall\, i \in [|1, n_c|]$. The notation $()_n$ is for the normal component; $()_t$ will denote the tangential one. The second assumption results from the Coulomb friction conditions: no sliding of the contact i occurs if $||(f_i)_t|| \leq \mu ||(f_i)_n||$ (figure 2). μ denotes the dry friction coefficient.

It is well-known that the Coulomb non-sliding condition may be linearized by approximating the contact cone with a multi-faceted cone, and that the resulting linearized equations also accounts for unilaterality. That is, if we define $f = (f_1, \ldots, f_{n_c})^T$ the column vector of the contact forces,

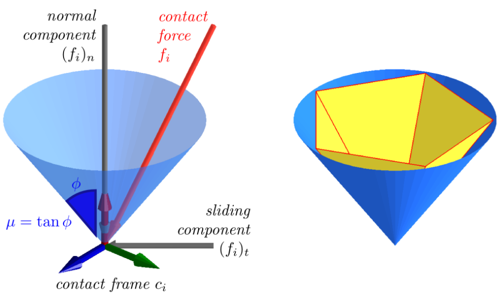

Fig. 2. A non-sliding contact and its exact and linearized contact cones

we may find matrices C and d such that non-sliding and unilaterality conditions read:

$$Cf + d \leq 0_{n_c \times n_e, 1} \tag{1}$$

C is $(n_c \times n_e, 3n_c)$ in size, n_e being the number of edges in the cone discretization; d is a vector with $n_c \times n_e$ lines.

C. Object dynamics and grasp map

We let V_{obj} denote the absolute twist of the object and $W_{f \to obj}$ denote the wrench applied on the object by the fingers, both written in the frame of the object:

$$V_{obj} = \begin{pmatrix} v_{obj} = v^{obj}_{obj/ref} \\ \omega_{obj} = \omega^{obj}_{obj/ref} \end{pmatrix} \qquad W_{f \to obj} = \begin{pmatrix} f_f = f^{obj}_{f \to obj} \\ m_f = m^{obj}_{f \to obj} \end{pmatrix}$$

v_{obj} is the velocity of the object's center of mass, ω_k is its angular velocity, both velocities are relative to the reference frame. f_f is the resultant force applied by the fingers and m_f is the moment of this force at the object's center of mass. All four quantities are written in the object frame.

We let m_{obj} denote the mass of the object and $[I]_{obj}$ denote its inertia tensor, written in the frame obj. Both quantities are arranged into the object's generalized mass matrix: $M_{obj} = \text{diag}\left(m_{obj} I_3, [I]_{obj}\right)$, at the center of mass of the object.

The object dynamics is as follows:

$$M_{obj}\left(\dot{V}_{obj} - g\right) + N_{obj} V_{obj} = W_{dp \to obj} \tag{2}$$

where $N_{obj} V_{obj}$ are the Coriolis forces, and all quantities are written in the object frame. In particular, the gravity wrench $M_{obj}\, g$ and the resultant wrench applied by the fingers are:

$$M_{obj}\, g \overset{\text{def}}{=} \begin{pmatrix} m_{obj} I_3 & 0_{3,3} \\ 0_{3,3} & [I]_{obj} \end{pmatrix} \begin{pmatrix} {}^{obj}R_{ref} \begin{pmatrix} 0 \\ 0 \\ -9.81 \end{pmatrix} \\ 0_{3,1} \end{pmatrix} = m_{obj}\, g \tag{3}$$

$$W_{f \to obj} = \sum_{i=1}^{n_c} \begin{pmatrix} {}^{obj}R_{c_i} & 0_{3,3} \\ \hat{r}^{obj}_{obj,c_i}\, {}^{obj}R_{c_i} & {}^{obj}R_{c_i} \end{pmatrix} \begin{pmatrix} f_i \\ 0_{3,1} \end{pmatrix} \tag{4}$$

where $\begin{pmatrix} f_i \\ 0_{3,1} \end{pmatrix}$ is the wrench applied by finger i, and $\hat{}$ denotes the operation that returns a skew-symmetric matrix for cross-product by the input vector: $\hat{\vec{r}}\vec{u} = \vec{r} \times \vec{u}$. In the end, we get:

$$W_{f \to obj} = \begin{pmatrix} {}^{obj}R_{c_1} & \dots & {}^{obj}R_{c_{n_c}} \\ \hat{r}^{obj}_{obj,c_1}\, {}^{obj}R_{c_1} & \dots & \hat{r}^{obj}_{obj,c_{n_c}}\, {}^{obj}R_{c_{n_c}} \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_{n_c} \end{pmatrix}$$

$$\overset{\text{def}}{=} G\, f \tag{5}$$

This matrix G is called the *grasp map* of the grip.

D. Disturbances

There may also be disturbances involved in the right-hand side of the object's equations of motion (2). We let $W_{dist} \in se_3^*(\mathbb{R})$ denote a disturbance on the object, written in the object frame obj, and $W^1_{dist}, \dots, W^{n_d}_{dist} \in se_3^*(\mathbb{R})$ denote a set of such disturbances, that may happen and that we want the grasp to be able to withstand. We remind that the wrench space $se_3^*(\mathbb{R})$ is the dual space of the twist space $se_3(\mathbb{R})$. j denotes the index of a disturbance: $j \in [|1, n_d|]$.

III. The Problem of Grasp Robustness

A. Problem statement

The problem of grasp robustness against the disturbances $W^1_{dist}, \dots, W^{n_d}_{dist} \in se_3^*(\mathbb{R})$ may be stated as follows: find contact forces $f^1_0, \dots, f^{n_c}_0 \in \mathbb{R}^3$ such that:

1) In the absence of any disturbance:
   - The object's equations of motion are satisfied.
   - The contacts are non-sliding.
   - The contact forces $f^1_0, \dots, f^{n_c}_0$ are unilateral.
   - Their intensities remain below a certain admissible threshold.
2) $\forall\, j \in [|1, n_d|]$, $\exists\, \delta f^1_j, \dots, \delta f^{n_c}_j \in \mathbb{R}^3$ such that, when the object is subject to the contact forces $f^1_0 + \delta f^1_j, \dots, f^{n_c}_0 + \delta f^{n_c}_j$ and to the disturbance W^j_{dist}:
   - Its equations of motion are still satisfied.
   - The contacts are still non-sliding.
   - The forces $f^1_0 + \delta f^1_j, \dots, f^{n_c}_0 + \delta f^{n_c}_j$ are unilateral.
   - Their intensities remain below the same admissible threshold.

The force variations $\delta f^1_j, \dots, \delta f^{n_c}_j$ enable the grasp to withstand the disturbance j; however they are *not* part of the tightening forces $f^1_0, \dots, f^{n_c}_0$. We define:

$$f_0 = \begin{pmatrix} f^1_0 \\ \vdots \\ f^{n_c}_0 \end{pmatrix} \in \mathbb{R}^{3n_c} \qquad \delta f_j = \begin{pmatrix} \delta f^1_j \\ \vdots \\ \delta f^{n_c}_j \end{pmatrix} \quad \forall\, j \in [|1, n_d|]$$

We can formulate the problem of grasp robustness against $W^1_{dist}, \dots, W^{n_d}_{dist}$, $n_d \geq 1$, as follows:

Find f_0 s.t. $\exists\, \delta f_1, \dots, \delta f_{n_d}$ s.t.

$$\left\{ \begin{array}{l} \text{equations of motion:} \\ \quad G f_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \\ \quad G(f_0 + \delta f_j) + W^j_{dist} = \\ \qquad M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \quad \forall j \in [|1, n_d|] \\ \text{non-sliding contacts (friction cones):} \\ \quad ||(f_0)_t|| \leq \mu\, ||(f_0)_n|| \\ \quad ||(f_0 + \delta f_j)_t|| \leq \mu\, ||(f_0 + \delta f_j)_n|| \quad \forall j \in [|1, n_d|] \\ \text{unilateral contact forces:} \\ \quad (f_0)_n \leq 0_{n_c,1} \\ \quad (f_0 + \delta f_j)_n \leq 0_{n_c,1} \quad \forall j \in [|1, n_d|] \\ \text{bounded contact forces:} \\ \quad ||f_0|| \leq f_{max} \\ \quad ||f_0 + \delta f_j|| \leq f_{max} \quad \forall j \in [|1, n_d|] \end{array} \right. \tag{6}$$

The $|| \; ||$ notation means:

$$||f_0|| = \begin{pmatrix} ||f_0^1|| \\ \vdots \\ ||f_0^{n_c}|| \end{pmatrix} \in \mathbb{R}^{n_c} \quad ||(f_0)_{n,t}|| = \begin{pmatrix} ||(f_0^1)_{n,t}|| \\ \vdots \\ ||(f_0^{n_c})_{n,t}|| \end{pmatrix} \in \mathbb{R}^{n_c}$$

In a similar way, f_{max} is a vector of n_c positive elements.

Studies about the force optimization problem usually assume that the object is in static equilibrium: $V_{obj} = 0_{6,1}$, $\dot{V}_{obj} = 0_{6,1}$. It is however possible to take explicitly the object's motion into account, merely by writing the equations of motion rather than the equations of static equilibrium. Another possibility is to consider that $M_{obj}\dot{V}_{obj}$ and $N_{obj}V_{obj}$ are negligible with respect to gravity and disturbances (which is most often the case). In this case, the right-hand sides of the equations of motion are reduced to $-M_{obj}g = -m_{obj}g$ (see (3)), and we fall back to the equations of equilibrium.

B. Linearization of the grasp robustness problem

The linearization of the contact cones yields (see (1)):

non-sliding contacts and unilateral forces:

$$\begin{aligned} & Cf_0 + d \leq 0_{n_c \times n_e,1} \\ & C(f_0 + \delta f_j) + d \leq 0_{n_c \times n_e,1} \quad \forall\, j \in [|1, n_d|] \end{aligned} \tag{7}$$

As for the "bounded forces" constraints, we may write $||f_0|| \approx ||(f_0)_n|| = -(f_0)_n$. This approximation is correct when the friction cones are not too wide (the friction coefficient is not too large); it means that they are truncated by a plane at f_{max}-level. We define the $(n_c, 3n_c)$-matrix $E = \text{diag}\left(\left(\begin{smallmatrix}0 & 0 & 1\end{smallmatrix}\right), \ldots, \left(\begin{smallmatrix}0 & 0 & 1\end{smallmatrix}\right)\right)$, it is such that $Ef_0 = (f_0)_n$. The linearized "bounded forces" constraints are then:

bounded forces:

$$\begin{aligned} & -Ef_0 \leq f_{max} \\ & -E(f_0 + \delta f_j) \leq f_{max} \quad \forall\, j \in [|1, n_d|] \end{aligned} \tag{8}$$

After these linearizations, the grasp robustness problem may be rewritten:

Find f_0 s.t. $\exists\, \delta f_1, \ldots, \delta f_{n_d}$ s.t.

$$\begin{cases} \text{equations of motion:} \\ \quad Gf_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj}V_{obj} \\ \quad G\delta f_j = -W_{dist}^j \quad \forall\, j \in [|1, n_d|] \\ \text{non-sliding contacts and unilateral forces:} \\ \quad Cf_0 \leq -d \\ \quad C(f_0 + \delta f_j) \leq -d \quad \forall\, j \in [|1, n_d|] \\ \text{bounded forces:} \\ \quad -Ef_0 \leq f_{max} \\ \quad -E(f_0 + \delta f_j) \leq f_{max} \quad \forall\, j \in [|1, n_d|] \end{cases} \tag{9}$$

We have (9) $\Rightarrow$ (6). Let us denote $x = (f_0, \delta f_1, \ldots, \delta f_{n_d})^T$ the unknown of (9). This problem is a system of linear equations and inequations in x. The dimension of the unknown may be quite large, though not untractable: $x \in (\mathbb{R}^{3n_c})^{n_d+1}$. We will see in section IV-A, (20), that it can be much reduced. The system to be solved is:

$$\begin{cases} \overline{G}x = \overline{M}_{obj}(\dot{\overline{V}}_{obj} - \overline{g}) + \overline{N}_{obj}\overline{V}_{obj} - \overline{W}_{dist} \\ \overline{C}x \leq -\overline{d} \\ -\overline{E}x \leq \overline{f}_{max} \end{cases} \tag{10}$$

$$\overline{G} = \begin{pmatrix} G & & & \\ & G & & \\ & & \ddots & \\ & & & G \end{pmatrix} \quad \overline{g} = \begin{pmatrix} g \\ 0_{6,1} \\ \vdots \\ 0_{6,1} \end{pmatrix} \quad \overline{W}_{dist} = \begin{pmatrix} 0_{6,1} \\ W_{dist}^1 \\ \vdots \\ W_{dist}^{n_d} \end{pmatrix}$$

$$\overline{M}_{obj} = \begin{pmatrix} M_{obj} & & & \\ & 0_{6,6} & & \\ & & \ddots & \\ & & & 0_{6,6} \end{pmatrix} \quad \overline{N}_{obj} = \begin{pmatrix} N_{obj} & & & \\ & 0_{6,6} & & \\ & & \ddots & \\ & & & 0_{6,6} \end{pmatrix}$$

$$\overline{V}_{obj} = \begin{pmatrix} V_{obj} \\ 0_{6,1} \\ \vdots \\ 0_{6,1} \end{pmatrix} \quad \overline{C} = \begin{pmatrix} C & & & \\ C & C & & \\ \vdots & & \ddots & \\ C & & & C \end{pmatrix} \quad \overline{d} = \begin{pmatrix} d \\ d \\ \vdots \\ d \end{pmatrix}$$

$$\overline{E} = \begin{pmatrix} E & & & \\ E & E & & \\ \vdots & & \ddots & \\ E & & & E \end{pmatrix} \quad \overline{f}_{max} = \begin{pmatrix} f_{max} \\ f_{max} \\ \vdots \\ f_{max} \end{pmatrix}$$

C. Least-effort robust grasp, part one

As the equation in system (10) is under-determined, we add an objective function to choose among its possible solutions. It is physically sensible to minimize the (possibly weighted) L_2-norms of the contact forces:

$$Q_{l.e.} = \text{diag}\left(Q_0, Q_1, \ldots, Q_{n_d}\right)$$

$$\begin{aligned} ||x||^2_{Q_{l.e.}} &= x^T Q_{l.e.} x \\ &= f_0^T Q_0 f_0 + \delta f_1^T Q_1 \delta f_1 + \cdots + \delta f_{n_d}^T Q_{n_d} \delta f_{n_d} \\ &= ||f_0||^2_{Q_0} + ||\delta f_1||^2_{Q_1} + \cdots + ||\delta f_{n_d}||^2_{Q_{n_d}} \end{aligned}$$

Eventually we get the following problem:

$$\begin{cases} \min\limits_x \dfrac{1}{2} x^T Q_{l.e.} x \\ \overline{G}x = \overline{M}_{obj}(\dot{\overline{V}}_{obj} - \overline{g}) + \overline{N}_{obj}\overline{V}_{obj} - \overline{W}_{dist} \\ \overline{C}x \leq -\overline{d} \\ -\overline{E}x \leq \overline{f}_{max} \end{cases} \tag{11}$$

When we solve this quadratic program, we find the tightening forces f_0 needed to realize a *least-effort robust grasp* against the disturbances $W_{dist}^1, \ldots, W_{dist}^{n_d}$.

IV. Robustness to the Largest-Minimum Resisted Disturbance Wrench

We remind that by *largest-minimum* we mean the largest resisted wrench in the direction of the wrench space for which this wrench is the smallest (also known as the *worst* direction): $\min_{w \text{ direction}} \max_{\rho \text{ intensity}} (\rho w)$.

A. Largest-minimum-disturbance robust grasp

Now we suppose that we have a unitary disturbance W_{dist}, indicating only a direction of disturbance. We are interested in knowing the maximum disturbance λW_{dist}, $\lambda \geq 0$ (i.e. in the same direction as W_{dist}), such that the grasp is robust. Or, we have a set of unitary disturbances $W_{dist}^1, \ldots, W_{dist}^{n_d}$, defining a polytope of directions in wrench space, and we are interested in knowing the largest scale factor $\lambda \geq 0$ such that the grasp is robust to all the disturbances in the scaled polytope. This is the problem of grasp robustness to the largest-minimum resisted disturbance wrench:

$$\text{Find } f_0 \text{ and maximum } \lambda \geq 0 \text{ s.t. } \exists\, \delta f_1, \dots, \delta f_{n_d} \text{ s.t.}$$

$$\begin{cases} \text{equations of motion:} \\ \quad G f_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \\ \quad G(f_0 + \delta f_j) + \lambda W^j_{dist} = \\ \qquad M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \quad \forall j \in [|1, n_d|] \\ \text{non-sliding contacts:} \\ \quad ||(f_0)_t|| \leq \mu\, ||(f_0)_n|| \\ \quad ||(f_0+\delta f_j)_t|| \leq \mu\, ||(f_0+\delta f_j)_n|| \quad \forall j \in [|1, n_d|] \\ \text{unilateral forces:} \\ \quad (f_0)_n \leq 0_{n_c,1} \\ \quad (f_0 + \delta f_j)_n \leq 0_{n_c,1} \quad \forall j \in [|1, n_d|] \\ \text{bounded forces:} \\ \quad ||f_0|| \leq f_{max} \\ \quad ||f_0 + \delta f_j|| \leq f_{max} \quad \forall j \in [|1, n_d|] \end{cases} \tag{12}$$

This is basically the same as (6), except that the disturbances are no longer fixed in intensity and that the scale parameter λ is looked for together with the robust forces.

It must be noted that the disturbances $W^1_{dist}, \dots, W^{n_d}_{dist}$ need actually not be unitary, and the polytope they describe needs not be regular. On the contrary, if a specific task to realize makes some disturbance directions and/or intensities more relevant than others, it is judicious to adapt the shape of the polytope according to the shape of this task wrench space: it provides better exploration of the relevant directions of the wrench space during the scaling of the polytope, and results in task-oriented robustness.

The optimization problem (12) may be linearized in the same way as we did for (6). We get:

$$\max_{f_0, \delta f_j, \lambda}(\lambda) \text{ subject to the constraints:}$$

$$\begin{cases} \text{equations of motion:} \\ \quad G f_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \\ \quad G \delta f_j = -\lambda W^j_{dist} \quad \forall\, j \in [|1, n_d|] \\ \text{non-sliding contacts and unilateral forces:} \\ \quad C f_0 \leq -d \\ \quad C(f_0 + \delta f_j) \leq -d \quad \forall\, j \in [|1, n_d|] \\ \text{bounded forces:} \\ \quad -E f_0 \leq f_{max} \\ \quad -E(f_0 + \delta f_j) \leq f_{max} \quad \forall\, j \in [|1, n_d|] \\ \text{and also:} \\ \quad \lambda \geq 0 \end{cases} \tag{13}$$

The unknowns of this problem are $f_0, \delta f_1, \dots, \delta f_{n_d}$ and λ. We reduce the dimension of the problem by considering that the variables $\delta f_1, \dots, \delta f_{n_d}$ are those of minimum norm that meet the "equations of motion" conditions. That is to say, we first solve the following n_d auxiliary quadratic programming problems:

$$\begin{cases} \min_{\delta f_j} \frac{1}{2} \delta f_j^T Q_j \delta f_j \\ G \delta f_j = -\lambda W^j_{dist} \end{cases} \tag{14}$$

The physical interpretation of this assumption is that since the hand is already squeezing the object with the tightening forces f_0, the force variations δf_j that will make the hand resist the disturbance λW^j_{dist} are likely to be minimal and just the required amount to compensate for the disturbance.

As for the Q_j matrices, it is possible to understand them as compliance matrices, accounting for a certain amount of compliance in the fingers at their contact points with the object. However, this interpretation is still work in progress and needs further investigation.

It is easy to prove from the usual first-order optimality conditions of (14) (gradient of the lagrangian with respect to δf_j and equality constraint) that the solution of (14) is :

$$\begin{aligned} &\delta f_j = -\lambda G^*_j W^j_{dist} \\ &\text{with } G^*_j = Q_j^{-1} G^T [G Q_j^{-1} G^T]^{-1} \end{aligned} \tag{15}$$

The expression of G^*_j is to be compared with the expression of the pseudo-inverse of the grasp map, $G^+ = G^T[GG^T]^{-1}$. It appears that G^*_j is a weighted pseudo-inverse, or a generalized inverse, of G.

We use (15) to make (13) become:

$$\max_{f_0, \lambda}(\lambda) \text{ subject to the constraints:}$$

$$\begin{cases} \text{equation of motion:} \\ \quad G f_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \\ \text{non-sliding contacts and unilateral forces:} \\ \quad C f_0 \leq -d \\ \quad C f_0 - C G^*_j W^j_{dist} \lambda \leq -d \quad \forall\, j \in [|1, n_d|] \\ \text{bounded forces:} \\ \quad -E f_0 \leq f_{max} \\ \quad -E f_0 + E G^*_j W^j_{dist} \lambda \leq f_{max} \quad \forall\, j \in [|1, n_d|] \\ \text{and also:} \\ \quad \lambda \geq 0 \end{cases} \tag{16}$$

Then, we compact the $n_d + 1$ constraints "non-sliding contacts and unilateral forces":

$$\begin{cases} C f_0 \leq -d \\ C f_0 - C G^*_j W^j_{dist} \lambda \leq -d \quad \forall\, j \in [|1, n_d|] \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} C f_0 + \max(0_{n_c \times n_e,1}, -C G^*_1 W^1_{dist} \lambda, \dots, -C G^*_{n_d} W^{n_d}_{dist} \lambda) \\ \leq -d \end{bmatrix}$$

$$\Leftrightarrow C f_0 + \min(0_{n_c \times n_e,1}, C G^*_1 W^1_{dist}, \dots, C G^*_{n_d} W^{n_d}_{dist}) \lambda \leq -d$$

where $\max$ (respectively $\min$) is a vector whose k-th line is the maximum (respectively minimum) element of the k-th lines of its argument vectors.

We may similarly compact the "bounded forces" constraints, and in the end, if we define:

$$S_1 = \min(0_{n_c \times n_e,1}, CG_1^* W_{dist}^1, \ldots, CG_{n_d}^* W_{dist}^{n_d}) \quad (17)$$
$$S_2 = \max(0_{n_c,1}, EG_1^* W_{dist}^1, \ldots, EG_{n_d}^* W_{dist}^{n_d}) \quad (18)$$

we may write the following simplified problem from (16), (17) and (18):

$$\max_{f_0,\lambda}(\lambda) \text{ subject to the constraints:}$$

$$\begin{cases} \text{equation of motion:} \\ \quad Gf_0 = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj} \\ \text{non-sliding contacts and unilateral forces:} \\ \quad Cf_0 + S_1 \lambda \leq -d \\ \text{bounded forces:} \\ \quad -Ef_0 + S_2 \lambda \leq f_{max} \\ \text{and also:} \\ \quad \lambda \geq 0 \end{cases} \quad (19)$$

The reductions (17) and (18) are "worst-case" reductions. We have: (19) $\Rightarrow$ (16) $\Rightarrow$ (13) $\Rightarrow$ (12), and none of these implications is an equivalence.

The dimension of the unknown has been much reduced; we define this new unknown, $x \in \mathbb{R}^{3n_c+1}$ and a companion vector c:

$$x = \begin{pmatrix} f_0 \\ \lambda \end{pmatrix} \qquad c = \begin{pmatrix} 0_{3n_c,1} \\ 1 \end{pmatrix} \qquad \lambda = c^T x \quad (20)$$

Eventually, we rewrite (19) as the following linear programming problem:

$$\begin{cases} \min_x(-c^T x) \\ A_{eq}\, x = b_{eq} \\ A_{neq}\, x \leq b_{neq} \end{cases} \quad (21)$$

$$A_{eq} = \begin{pmatrix} G & 0_{6,1} \end{pmatrix} \qquad b_{eq} = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj}$$
$$A_{neq} = \begin{pmatrix} C & -S_1 \\ -E & S_2 \\ & -c^T \end{pmatrix} \quad b_{neq} = \begin{pmatrix} -d \\ f_{max} \\ 0 \end{pmatrix}$$

This problem can be solved efficiently by a variety of algorithms. The solution λ^{sol} is a size index of the set of disturbances the grasp is able to withstand; it measures the largest-minimum resisted wrench in the investigated directions. The solution f_0^{sol} is the tightening forces to apply on the object to realize a robust grasp against all the disturbances in the scaled-by-λ polytope of disturbance directions.

The quality measure λ^{sol} looks similar to the one of [20]: it is also a largest scale factor ρ that makes a convex compact set of expected disturbance directions $\mathcal{D}$ fully contained in the grasp wrench space GWS: $\max_{\rho\mathcal{D} \subset GWS, \rho \geq 0}(\rho)$. As we reviewed earlier, the quality measure in [20] is computed with as many linear programs as disturbance directions in $\mathcal{D}$, often a polytope. Contact forces are also computed as by-products of those linear programs. Besides the fact that we solve a different problem (we look for tightening contact forces), our method is different in the following three points.

First, we only need one linear programming problem of reduced dimension, in $\mathbb{R}^{3n_c+1}$, to get our quality measure. It is worth noting that:

1) Because of the successive linearizations (13), dimension reduction (14 to 16) and constraint combinations (17 to 19), this measure is not exactly the measure itself as we would have found if we had solved (12) rather than (21), but an approximation of it.
2) The dimension of our linear program does not depend on the number of disturbances, n_d. Therefore, the polytope of disturbance directions may have a complex shape, with a lot of vertices, without impairing the computational cost other than what is needed for the computation of S_1 and S_2 (17 and 18). In contrast, the usual approach is to solve n_d linear programs in $\mathbb{R}^{3n_c+1}$, which may pose a problem if n_d is too large.

Second, we take both known wrenches (gravity, but other external loads may also be considered along with g) and unknown wrenches (disturbances) into account.

Third, we distinguish between the forces that are applied in the absence of any disturbance (f_0) and those that are applied during a disturbance ($f_0 + \delta f_j$). This is an important distinction because it enables us to compute both tightening forces, that act as pre-strain passive forces, and active blocking forces. In contrast, the forces computed by [20] and in the classical formulation of the force optimization problem are active ones, there is no notion of pre-strain. Despite the similarities, our problem is different: our primary goal is to compute tightening forces.

Because we make this difference, our robustness/quality problem (12) has more constraints than the one of [20]. Namely, we have extra constraints, those on f_0 alone. As a result, our problem is conservative with respect to theirs, and the quality index we find is a lower bound of theirs: the more constraints, the more underestimated the grasp ability to resist disturbances.

B. Least-effort robust grasp, part two

Once we have solved (21), we are able to go back to (11) and find contact forces for a least-effort robust grasp, with an unknown of much smaller dimension. We merely modify (21) with another criteria, set λ in $[0, \lambda^{sol}]$ indicating a desired level of robustness, and have f_0 the only unknown. We get the following quadratic programming problem:

$$\begin{cases} \min_{f_0} \frac{1}{2} f_0^T Q_0 f_0 \\ A'_{eq}\, f_0 = b'_{eq} \\ A'_{neq}\, f_0 \leq b'_{neq} \end{cases} \quad (22)$$

$$A'_{eq} = G \qquad b'_{eq} = M_{obj}(\dot{V}_{obj} - g) + N_{obj} V_{obj}$$
$$A'_{neq} = \begin{pmatrix} C \\ -E \end{pmatrix} \quad b'_{neq} = \begin{pmatrix} -d + \lambda S_1 \\ f_{max} - \lambda S_2 \end{pmatrix}$$

C. Integration in control frameworks

The integration of tightening abilities into already existing controls of multi-fingered hands is easy. Computed-torque controls, for instance, typically use the object dynamics and the pseudo-inverse of the grasp map to get desired contact forces from the desired motion of the object: $f^{[d]} = G^{+}(M_{obj}(\dot{V}_{obj}^{[d]} - g) + N_{obj}V_{obj}^{[d]}) + f_I^{[d]}$, where the $[d]$ superscript means *desired* and f_I denotes the internal contact force, that cannot produce any motion since in $\ker G$, but describes how hard the object is squeezed. Taking tightening into account merely bounds to $f_I^{[d]} = f_0^{sol}$, with f_0^{sol} either the contact forces of the largest-minimum-disturbance robust grasp (section IV-A, (21)) or the contact forces of a least-effort robust grasp (section IV-B, (22)), *with the right-hand sides of the "equations of motion" constraints set to* $0_{6,1}$. Indeed, this results in $Gf_0^{sol} = 0_{6,1}$, and in that way we are sure that f_0^{sol} is an internal force.

In [22], we describe an optimization-based control of dextrous manipulation that offers the advantage of trade-off between many different desired values and constraints. In particular, it enables easy addition of desired values: we simply define $f^{[d]} = f_0^{sol}$ and an associated priority matrix. The optimization basis of our control combines this desired value with all the others, in particular with the one on f that is due to the desired motion of the object.

V. A SIMULATION EXAMPLE

In this section, we demonstrate the tightening abilities our robustness study brings to the optimization-based control we describe in [22]. The example is a four-fingered hand keeping hold of an object in the presence of disturbances. It is simulated with ARBORIS, an open-source dynamical engine for articulated rigid body mechanics, written in MATLAB programming language at CEA/LIST and UPMC/ISIR [23].

A. Robustness at equilibrium

The robustness objective is as follows: the grasp should be able to withstand disturbances in any of the six force directions along the x, y and z axes of the reference frame, up to 75% of the largest-minimum resisted wrench. That is to say, we consider the following six disturbance directions, written in ref:

$$W^{\pm x} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad W^{\pm y} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad W^{\pm z} = \begin{pmatrix} 0 \\ 0 \\ \pm 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

We solve (21), section IV-A: we get the quality measure λ^{sol} and appropriate contact forces for the largest-minimum-disturbance robust grasp. Then we solve (22), section IV-B, for $\lambda = 0.75\,\lambda^{sol}$: we get appropriate optimal contact forces f_0^{sol}. Eventually, we set the desired tightening forces at $f^{[d]} = f_0^{sol}$, as explained in section IV-C.

The desired object motion is to remain at rest at the object's initial position. Gravity is set to zero and we use eight-faces contact cones and a friction coefficient $\mu = 0.8$. Equations (21) and (22) are solved with a constraint $f_{max} = 2$ N for each finger force norm.

Disturbances are applied on the object successively in each of the six directions. Their intensity is about 75% of the intensity of the largest-minimum resisted wrench, that is to say, $||W_{dist}^{\pm x,y,z}|| \lessapprox 0.75\,\lambda^{sol}$. They last 0.1 s each. The grasp withstands all six disturbances; two of them are illustrated on figure 3.

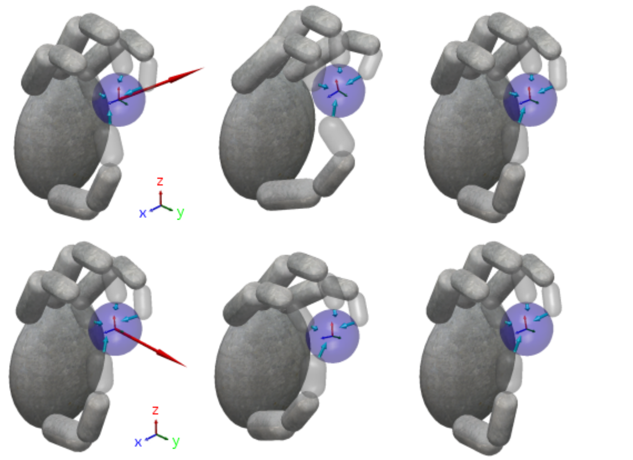

Fig. 3. Two disturbances (red arrows, left) are withstood by the hand (center), which then returns to equilibrium (right)

On the contrary, if the same disturbances happen when the control of the hand does not provide any robustness, it is no wonder that they result in the hand losing grip, figure 4. In this figure, the same manipulation was executed with the tightening objective emulated with desired normal contact forces accounting for some light tightening: $(f_i)_n^{[d]} = 0.5\text{ N}\,\forall i \in [|1,4|]$. It is not possible to increase this objective very much because it is not neutral with respect to the static equilibrium of the object: a larger objective provides more tightening but hinders static equilibrium. In contrast, the robustness objective we design in this paper offers the advantage of taking into account the object's equation of motion (or static equilibrium, in this case).

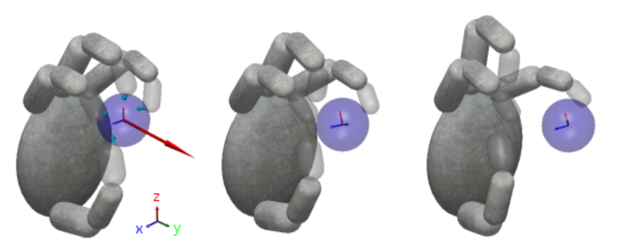

Fig. 4. No robustness objective results in poor grasp in face of disturbances

B. Robustness in motion

Now the same four-fingered hand is subject to gravity and supposed to translate the object 2 cm backwards, along $-y$, from $t = 1$ s to $t = 3$ s. This desired object motion is plotted in black on figure 5. During the motion, a disturbance along $+x$ happens at $t = 2$ s. Except the motion and gravity, everything remains the same as previously, in particular the robustness objective and the intensity and duration of the disturbance. The grasp withstands the disturbance and completes successfully its motion objective. Figure 5 plots the position of the center of the object during the disturbed motion.

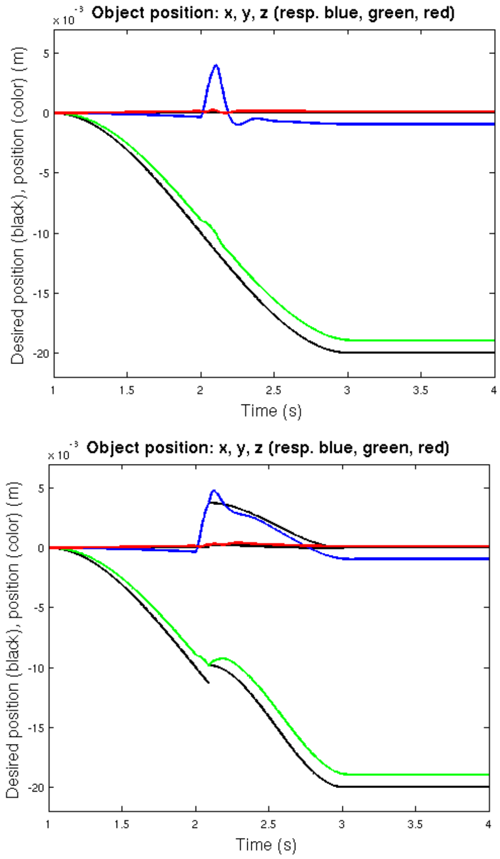

Fig. 5. The motion of the grasped object is disturbed along $+x$ but the robustness objective ensures that the hand does not lose its grip on the object, enabling it to complete the motion. On the bottom plot, the object desired trajectory is regenerated after the disturbance is detected, giving the hand a more human-like response time than on the top plot.

VI. Conclusion

In this paper, we compute pre-strain tightening forces providing direction-independent robustness for a multi-fingered grasp, by merging the problems of grasp quality measure and contact force optimization. We deal with these problems by solving a linear programming problem and a consecutive quadratic programming problem, both of reduced dimensions and with no dependence on the number of expected disturbance directions. We are also able to find blocking contact forces against the disturbances, and a lower-bound approximation of the quality measure of [20].

A limitation of our robustness study is that it implicitly assumes that the grasp is infinitely rigid, with fixed contacts and fingers so stiff that we do not have to take them into account. However, during a disturbance, it is definitively not the case. Future work should model the compliance of the fingers and of the grasp. In particular, the part played by the compliance matrices of (14) and (15) should be investigated.

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