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Neurosurgical robot design and interactive motion planning for resection task

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***Abstract*— This paper presents the modeling of a new mini-invasive neurosurgical resection robot. This robot aims to help to remove brain tumors and is incorporated into a multi-robot neurosurgical system. We focus especially on the resection task. The robot is composed of three serial bending modules actuated by wires (cables) and uses an additional translation. The redundancy of the robot (seven degrees of freedom) allows more dexterity for the resection task. A kinematic (geometric) model is built distinguishing the structure and the actuation models, and integrating the influence of the wires into the kinematic behaviour of the robot. A method for kinematic redundancy handling is defined and assigns different parts of the robot to different tasks. An interactive path planning based on arbitrary sequence of elementary procedures assembled by the neurosurgeon is proposed and the yielding procedures are computed from the kinematic model.**

I. INTRODUCTION

Since the beginning of surgical robotics in the 80's, surgical robots continue to find their place in daily practice. Semi autonomous robots were developed in order to position invasive tools like biopsy needles, or non-invasive tools like microscopes, for laparoscopic and cardiac surgeries or neurosurgery. Some of them compute tool paths for bone resection [1]. Master-slave robots help surgeons in mini-invasive interventions by scaling and filtering surgeon movements on a joystick and performing the operation via robotized slave arms ([2], [3]). Synergic robots help surgeons by preventing them from performing forbidden movements which are defined before the operation [1]. However, no robot deals with autonomous motion and volume resection with evolving free space on soft tissue (inside the body). This paper addresses the issue of an autonomous robot for mini-invasive neurosurgery, and particularly for brain tumor removal. The relevance of such a robot is to access tumors that are not attainable in manual operations, and to improve the quality and the safety of surgical tumor removal. In this paper we focus especially on the resection robot design, modeling and path planning.

In the following section we examine the background of the project and present the requirements for the surgical task. In section III the resection tool is chosen and the design of the robot is developped. In section IV the forward kinematics is defined. Thoughts about resection strategy are then conducted in section V using equations developped in section IV, and lead to first simulations of brain tumor removal.

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II. BACKGROUND

A. Neurosurgical system

This work is part of a project dedicated to the design of a mini-invasive neurosurgical robot, which aims to help surgeons to remove brain tumors [4],[5]. The surgical task would be performed automatically after pre-operative trajectory planning based on MRI images, while the surgeon would supervise the operation. The whole system is divided into four parts: a carrier robot, a skull clamping device, an access robot and a resection robot (see Fig. 1). The carrier robot positions the system with respect to the patient's head. The clamping device drills the skull, clamps itself onto it and prevents the access robot and resection robot from moving inside the brain unintentionally. The access robot gives the resection robot access to the tumor and enables vital or critical brain areas to be avoided. A study about the design of the access robot can be found in [4]. The present paper deals with the resection robot, which has to remove a tumor without damaging the brain. The resection robot and motion planning have to be designed to scan the tumor volume and at the same time remove tumorous cells.

B. Requirements for the resection robot

Due to the neurosurgical background, the resection robot must deal with obvious constraints such as safety and reliability, but furthermore has to cope with more specific

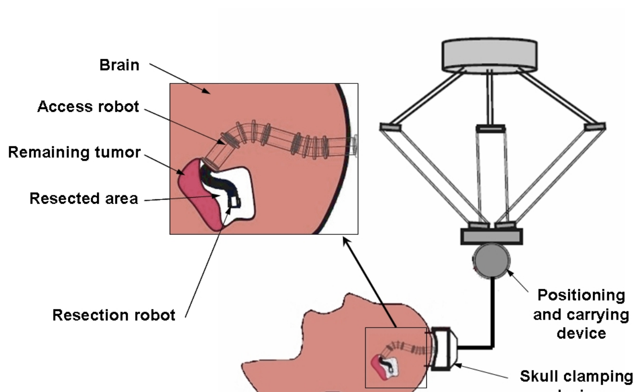

Fig. 1. Architecture of the whole neurosurgical system.

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difficulties. One important particularity of tumor resection is the high diversity of tumor shapes and sizes (length is from 3 cm to 8 cm). An example of brain tumor is given in Fig. 2. The adaptability of the robot to various tumors and different possible entry points is ensured by a small size (8mm wide maximum), high amplitude (90° of bending) and sufficient number of degrees of freedom (5 is the minimum, the sixth one is the axis of rotation of the resection tool, but more will allow redundancy and thus sane brain avoidance).

The second particularity of the resection task inside the brain is the very limited workspace. In fact, the resection robot has almost no workspace at all when it starts removing the tumor since all space is filled either with tumor tissue either with brain tissue, but as the robot removes tumor parts its workspace increases and evolves.

C. Relevant existing devices

Relevant devices able to fill or at least approach the specified requirements can be divided in two categories: systems with elastic joints, and systems with rigid joints (see Table 1 for relevant features of some presented systems).

Most systems with elastic joints are often constituted by shape memory alloys (SMA), like the vertebra system described in [6] for colonoscopy or the endoscope EMIL [7]. The active catheter presented in [8] uses three SMA tubes that can slide one into the other. The actuation of this system is the elastic energy embedded inside the tubes themselves: the angular amplitude is not sufficient for our application. Some systems are built with elastic joints but are actuated by tendons ([9], [10]). Finally, pneumatic systems can also be found ([11]). All these devices are dedicated to endoscopy rather than manipulation of organic tissue, because of the lack of rigidity.

Rigid joint systems have also been greatly employed for medical or surgical applications. An interesting direct drive system that uses piezoelectric motors is an articulated arm developed to perform a laparoscopic surgical operation, but is limited by its size and electric connections [12]. The Agend endoscope [13] seems particularly relevant for resection because of its high number of degrees of freedom afforded by many vertebras, but actuation is performed by SMA springs and prevents large amplitude. Tools as Da Vinci® surgical master-slave system [14] have been developed to give surgeons additional degrees of freedom. They are actuated by wires and have a vertebra structure. Instead of vertebras, some systems use ball joints [15] and are actuated by tendons as well. Yamashita and colleagues imagined a system using connecting rods [16], but the number of degrees of freedom is limited. Gears have also been employed [17].

None of the presented systems entirely fill the requirements, because of their size, their joints angular amplitude, or their too small or too large number of degrees of freedom. Thus a new specific system for surgical tumor resection is designed.

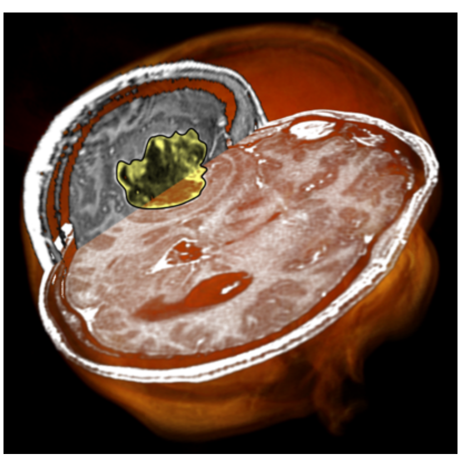


Fig. 2. Left temporal malignant glioma (yellow), outlined on axial and sagittal images (embedded in the head, orange).

III. RESECTION ROBOT

A. Resection robot design

As described in section II-B, the chosen resection robot has a minimum of 6 degrees of freedom. Its structure is based on the micromanipulator developped by Harada et al. [15] with high dexterity and only one joint with two degrees of freedom. High rigidity and simplicity are associated to a very small size. To reach 90° of amplitude along two directions, the manipulator presented in [15] is composed of two serial ball joints and is actuated by four wires. The concept for the resection robot is to integrate several of these modules to constitute the robot structure (Fig. 3). A total of three modules is chosen to provide the required six degrees of freedom. Each module is actuated by three wires instead of four to allow the integration of three modules (hence nine wires go through the first module). The balls are drilled to create an internal space through the robot [15].

A seventh degree of freedom in translation is provided to the resection robot by the access robot, described in [4]. The redundancy will allow more dexterity and collision avoidance ability for tumor removal.

B. Resection tool choice and design

Common tools used by neurosurgeons during tumor removal are retractors, tissue forceps and dissector, hemostatic systems, tumor resector and navigation instruments.

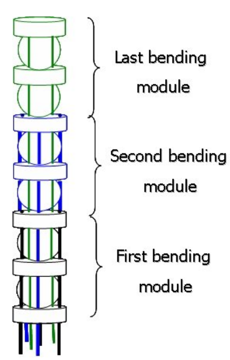
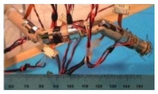
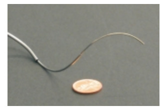
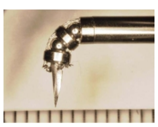
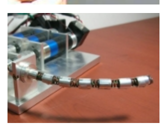


Fig. 3. The resection robot, composed of three modules (the translation system is not represented here)

TABLE I

CHARACTERISTICS OF SOME RELEVANT EXISTING DEVICES

	Rigid joint systems					Elastic joint systems				
	System	Ref.	Diameter or width (mm)	Dof	Joint amplitude	System	Ref.	Diameter or width (mm)	Dof	Joint amplitude
Direct drive systems		[12]	10	5			[6]	15	2	±15°
		[13]	7	2	±10°		[8]	7	2 per tube	
Remote motorization systems		[14]	5	2	±40°		[9]	5	2	±90°
		[16]	3.5	2	±90°		[7]	24	2	±180°
		[17]	10	2	±90°		[11]	17	2	±120°
		[15]	2.5	2	±90°		[10]	8	3	

For hemostasis, chemical (glue, foam…) or mechanical (clips…) systems, and electro (mono or bipolar) or photo (laser) coagulation are used. Tumor removal alternates blunt and delicate dissections, with suction devices and ultrasonic surgical knife (USK). In case of firm tissue, electrical rings, providing section plus coagulation, with or without the help of micro scissors, are used. The evacuation of resected tissue is done manually (piece by piece with forceps) or through suction tube (tumor cell outflow; isolated or embedded in USK).

For the moment, the USK are not miniaturized yet; a rotating wire ring associated to a suction tube is chosen for tumor removal, although this kind of device must be tested and validated on real tumorous tissue or similar matter. As the wire ring is rotating, the resected area can be modeled as a sphere (like a ball end cutter). The ring is actuated via a rotating wire passing through the resection robot structure. The integration of the suction tube is not yet adressed in the paper, but will be realized at the external periphery of the robot because small size and curvature of an internal tube may cause the coagulation of evacuated tissues.

IV. RESECTION ROBOT MODELING

The resection robot involves a mixed architecture: as the structure is a serial one (the ball joints are serially assembled), the actuation is parallel. From now the structure and actuation kinematic models will be distinguished.

A. Module kinematic diagram

If only half a module is considered, the corresponding ball joint enables three rotations but the wires prevent the torsion between the two plates (we assume that torsional torques are balanced with tension in wires, but a gap can remain according to the robot configuration and must be considered in a static model). To integrate this specificity into the kinematic model, we choose a three parameters model with a dependancy relation that reproduces the real behaviour of half a module. The three parameters correspond to three rotating joints. The axis of two of them are parallel to the robot axis of symmetry (see Fig. 4), the other one (middle joint of angle β) is the bending joint. The dependancy relation is chosen so that the model acts as if the axis of the middle joint has been rotated by an angle α. A complete module is assumed to be constituted of two ball joints that behave exactly in the same way. After simplification (see Fig. 4), a four rotating joints model is obtained, with two independant parameters α and β.

B. Forward kinematics of a module

1) Serial structure model: let us denote T_{01} the transformation matrix of the end frame fixed to the superior plate with respect to the reference frame fixed to the base plate (see Fig. 5), and $2h$ the distance between two ball centers (h is different from the ball radius, see figure 6). The direct kinematic model of a module is then described by T_{01} in

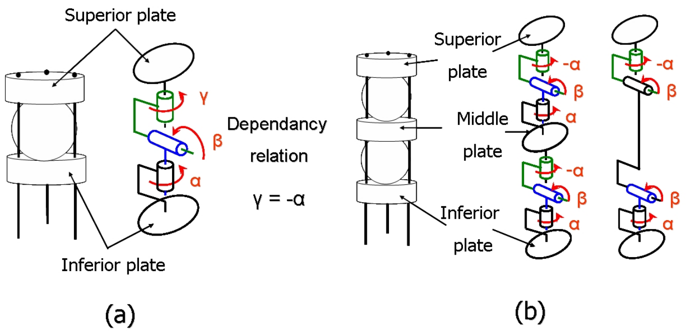


Fig. 4. Kinematic modeling by integrating the wires and the corresponding dependancy relation for a ball joint (a), and modeling of a complete module with the resulting simplification (b).

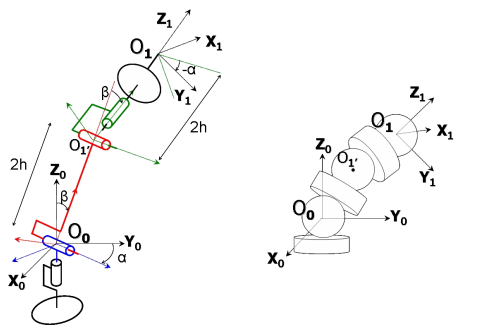


Fig. 5. Definition of the end frame (O_1,$\mathbf{X_1}$,$\mathbf{Y_1}$,$\mathbf{Z_1}$) fixed to the superior plate and the reference frame (O_0,$\mathbf{X_0}$,$\mathbf{Y_0}$,$\mathbf{Z_0}$) fixed to the base plate.

(1), which is a non-linear function of the two independant parameters α and β:

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4hc\alpha s\beta c\beta + 2hc\alpha s\beta & c\alpha^2\left(c\beta^2 - s\beta^2\right) + s\alpha^2 & c\alpha s\alpha\left(c\beta^2 - s\beta^2\right) - c\alpha s\alpha & 2c\alpha s\beta c\beta \\ 4hs\alpha s\beta c\beta + 2hs\alpha s\beta & c\alpha s\alpha\left(c\beta^2 - s\beta^2\right) - c\alpha s\alpha & s\alpha^2\left(c\beta^2 - s\beta^2\right) + c\alpha^2 & 2s\alpha s\beta c\beta \\ 2hc\beta + 2h\left(-s\beta^2 + c\beta^2\right) & -2c\alpha s\beta c\beta & -2s\alpha s\beta c\beta & c\beta^2 - s\beta^2 \end{bmatrix}, \quad (1)$$

where $c\alpha = \cos(\alpha)$, $s\alpha = \sin(\alpha)$, $c\beta = \cos(\beta)$ and $s\beta = \sin(\beta)$.

2) Actuation model: as the actuation is parallel, the inverse kinematic model (i.e. the wire lengths are function of parameters α and β) is first necessary. The direct model is then infered from it. We denote R the ball radius, e the plate thickness. If we assume that:

- wires radii are ignored,
- wires are located on the plates along a circle with a radius R equal to the ball radius,
- wires are straight inside the plates,
- wires in contact with the balls adopt its curvature, otherwise they are straight,
- between the plates, wires are located in the plane defined by the ball center and the two hanging points P_i and P'_i (see Fig. 6),

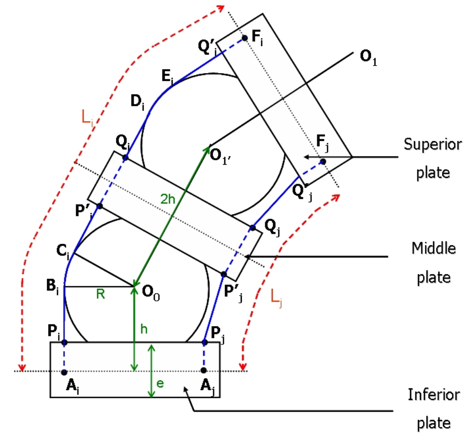


Fig. 6. Definition of the wires length. The wire i is in contact with the ball, the corresponding length L_i is the sum of lengths AP, PB, CP', $P'Q$, QD, EQ', $Q'F$ and the arcs $\widehat{BC}$ and $\widehat{DE}$. The wire j is not in contact, its length is the sum of AP, PP', $P'Q$, QQ', $Q'F$.

kinematic reasoning gives relations between the wire length L_i and the norm $\|\overrightarrow{P_iP'_i}\|$:

- if there is not contact between the wire and the ball ($\|\overrightarrow{P_iP'_i}\| \leq 2h - e$)

$$L_i = 2\left\|\overrightarrow{P_iP'_i}\right\| + 2e \quad (2)$$

- if there is contact ($\|\overrightarrow{P_iP'_i}\| \geq 2h - e$)

$$L_i = 4h + 4R\arcsin\left(\frac{\|\overrightarrow{P_iP'_i}\|}{2\sqrt{R^2 + \left(h - \frac{e}{2}\right)^2}}\right) \dots - 4R\arctan\left(\frac{h - \frac{e}{2}}{R}\right) \quad (3)$$

Due to actuation redundancy (three wires actuate a module although two parameters are independant), we choose only two equations among the three relations that express the wire lengths, and infer the norms $\|\overrightarrow{P_iP'_i}\|$ as a function of the length L_i from (2) and (3). Since we can express the norms with α and β, we obtain two nonlinear equations in trigonometrical function of α and β, which can be written as polynomial equations. The system with two unknowns has been solved using dialytic elimination method [18] and the parameters α and β have been calculated from the lengths of the module wires.

C. Direct kinematic model of the robot

The direct kinematic model is computed in two steps. First, the direct actuation model gives the parameters α_1, β_1, α_2, β_2, α_3 and β_3 of the three modules as functions of the wires lengths L_i. As the L_i of a module depend on the previous module actuation, α_1 and β_1 are first computed and useful lengths for the second module are calculated for the computation of α_2 and β_2. α_3 and β_3 are infered with the same reasoning. The second step concerns the direct structure model, which is a product of transformation matrices depending on α_1, β_1, α_2, β_2, α_3, β_3 and the translation parameter.

V. PRELIMINARY SIMULATIONS OF TUMOR REMOVAL

A. Kinematic redundancy handling

In case of kinematic redundancy, the inverse kinematic model has an infinite number of solutions. Usually the path of the end-effector of redundant robots is already computed in the workspace according to the task, and the joint path can be calculated by adding kinematic constraints to the Jacobian matrix, or by using its pseudo-inverse with null-space projection methods [19]. In the later method, several kinematic criteria can be integrated in an optimisation such as obstacle or joint limits avoidance [12]. However, these techniques lead to solutions only when the range of motion is small. A technique adressing global redundancy resolution is presented in [20], but requires an arbitrary choice concerning key path points. Methods for path planning dedicated to redundant robots have also been developped. The former ones set redundant joints to specified values according to the task and robot constraints, or consists in inserting additional relations [21], but these methods are limited for complex tasks. Recent methods use cell decomposition, roadmaps (e.g. [22]) or potential fields (e.g. [23]). Though, all these methods compute path in static workspace between an initial and a goal configurations, which implies that the last one is already known.

In our case, the workspace available for the robot is not only evolving, but depends on the robot path. None of the mentionned methods carries out this kind of issue. The difficulty is here to define a feasible path for the redundant resection robot that would scan the tumor volume while integrating tool accessibility and collision avoidance with brain and tumor tissue. Previous work concerning trajectory planning for tumor resection [5] integrates tool accessibility but does not integrate the joint limits of the robot. To adapt this strategy to the robot limitations and ensure tool accessibility, we propose to scan the tumor by elementary volumes reachable by the robot. To this aim, the robot is uncoupled by devoting its last module for scanning an elementary volume and the other degrees of freedom for positioning it. Hence, the positioning part of the robot is no longer redundant and the related inverse kinematic model can be computed. We call intermediate pose the pose of the frame fixed to the end of the positioning part of the robot, composed by the translation module, the first and second bending modules.

B. Interactive path planning

Let us remind that an elementary volume is defined by the resected (or scanned) area when only the two degrees of freedom of the last module are varying. It is then defined by the last module workspace for a given intermediate pose (see Fig. 7). With the previously described method, collision avoidance is ensured by an arbitrary choice of the sequence of positions of resected elementary volumes.

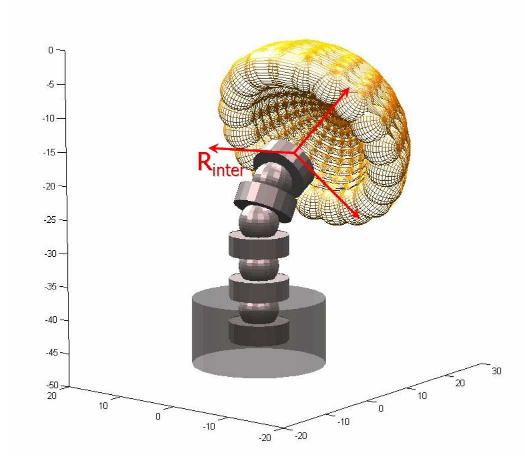


Fig. 7. Elementary volume (in yellow) resected by the last module with respect to the fixed intermediate pose R_{inter} (the translation module and the two first modules are fixed). The parameters α and β, defined on Fig. 4 respectively vary in $[0, 2\pi]$ and $\left[0, \frac{\pi}{4}\right]$. The last module neither the resection tool are represented here.

These choices would be realized by neurosurgeons offline and possibly online. But as an elementary volume is small with respect to tumor sizes (that is to say that a high number of volumes would be necessary to choose), we define elementary procedures that merge several elementary volumes. The position, orientation, size of these procedures is let to neurosurgeons' mind, whereas the system verifies and validates collision avoidance and accessibility of the robot for the chosen sequence. This path planning is therefore interactive as the neurosurgeon chooses himself the sequence of procedures and this choice is limited by the system. Two elementary procedures are defined hereafter (entry procedure and basic procedure).

1) Definition of the entry procedure: the orientation of the entry procedure is given by the orientation of the access robot when it comes in contact with the tumor, and the position is given by the contact point. The entry procedure aims at making free space for the robot ; it is arbitrarily chosen very simple in such a way that only the forward kinematics defined in section IV is necessary to determine the end-effector path. While the last module scans elementary volumes the rest of the robot is translated along the given orientation, and the first and second modules parameters are set to zero. Only the length (stroke of the translation) and the radius (stroke of the last module) of the procedure are chosen by the neurosurgeon according to the tumor and the limits of the robot (i.e. maximum length and extremum radii).

2) Definition of an elementary procedure: the other elementary procedure has the same shape as the entry procedure, but can be located on any point inside the tumor according to the available space and robot joints limits. Its position and orientation are chosen by the neurosurgeon. Whereas the first and second modules were fixed for the entry procedure, the intermediate pose varies and the unique solution of the intermediate inverse kinematics must be computed for each elementary volume pose. The inverse

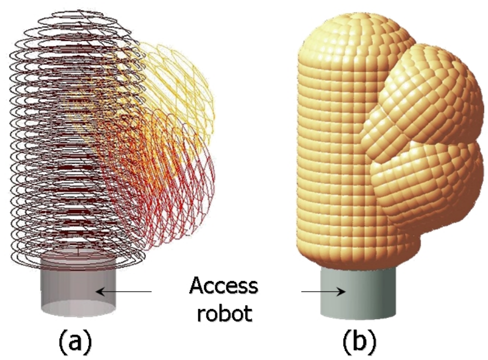


Fig. 8. Example of resection path planning (a) (the trajectory of the end-effector is composed of the black entry procedure and the red and yellow elementary procedures), and the resected area (b).

kinematic model of the structure is computed numerically, and the inverse kinematic model of the actuation is computed analytically from (2) and (3).

3) First simulation of tumor removal: given a brain tumor and given the entry point into the tumor and the access robot orientation, a relevant choice of sequence of procedures defines the resection robot path and deals with robot redundancy and collision avoidance. An example of such a sequence (an entry procedure and two other elementary procedures) for a given tumor is presented in Fig. 8. The interactive path planning must be implemented so that neurosurgeons can access to which procedures are feasible at any time.

VI. CONCLUSIONS AND FUTURE WORKS

This paper addressed the design of a mini-invasive neurosurgical robot dedicated to tumor resection. This robot is composed of three bending modules and one translation module, with seven degrees of freedom. The corresponding kinematic model was elaborated, and allowed computation of the direct kinematic model. Yet, it must be validated according to the real behaviour of the robot. An interactive path planning was designed, which deals with kinematic redundancy, tool accessibility and collision avoidance thanks to a relevant choice of procedures sequence. Future works will concern limitations of choices for procedures sequence according to workspace availability and tumor shape, as well as simulations, static modeling and realization of a demonstrator for validation.

VII. ACKNOWLEDGMENTS

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