Abstract—In this paper, we study the problem of door-opening by using modular reconfigurable robot (MRR). Based on multiple working mode control of the joint modules of the MRR, we first propose an online mode-switch strategy for all joint modules so that each joint module can be easily determined when it should be switched between active working mode and passive working mode during the door-opening process. Based on the proposed mode-switch strategy, a hybrid control scheme is proposed for door-opening. Simulation results are used to demonstrate the validity and efficiency of the proposed mode-switch strategy and the hybrid control scheme.

Index Terms - Modular reconfigurable robot, door-opening, mode switch, hybrid control.

I. INTRODUCTION

Modular reconfigurable robots (MRRs) have been extensively investigated in the robotics field [1], [2], [3]. From the mechanism point of view, an MRR consists of a set of similar or identical standardized modules. Through assembling these modules in various configurations, MRRs can perform different tasks flexibly with significant application potential [4]. As a result, control of MRRs becomes a promising research area in robotics. For robot practical applications, door-opening control, as many other control tasks for robots working in uncontrolled environments or human environments [5], is still challenging. In the literature of door-opening control, much research effort has been made, and most of which is focused on mobile robot manipulators. Yuta and Nagatani presented general approaches of the door-opening strategy [6], [7], where they applied the concept of action primitives to door-opening. Later, Petersson et al. proposed a high-level control approach, which used the off-the-shelf algorithms of force/torque control, for door-opening by mobile robots [9]. Kahtib [10] and Hanebeck et al. [11] proposed simultaneous control of both the mobile base and the robot arm. Ulyanov et al. [12] proposed fuzzy neural network strategy FNN and fuzzy control approaches for an intelligent mobile robot based on GA. For traditional serial robots, Slotine et al. proposed a control method of following the path of least resistance [8] to solve the problem of door-opening with a simple control algorithm. This control method, however, requires high resolution joint velocity measurements. Compared with literatures applied to either mobile robots with a fixed-configurable robotic arm, only a few researchers have studied the door-opening cases of using reconfigurable robots [13]. MRRs are particularly suitable for mobile ground and space applications as they can be conveniently mounted on any mobile platform and allow on-site changes of configuration, etc. The combination of mobile platforms with MRRs will lead to enhanced adaptability, flexibility and reconfigurability of the integrated mobile manipulators.

In this paper, with respect to the multi-mode feature of MRRs’ joint modules, we first propose a mode switch strategy. Based on this strategy, a hybrid control scheme is subsequently proposed, in which three control approaches are applied to joint modules working in three different modes. Specifically, for the joint modules working in the passive mode, a feed forward torque control approach is applied to compensate the friction of the joints to ensure they move freely; for the joint modules working in the defined current-active mode, a decomposition-based control approach is applied; and for the joint modules working in the defined post-active mode, a feedback position control approach is applied. With the proposed control scheme, the position errors of each active joint and the tracking errors of the door can be guaranteed to asymptotically converge to zero.

The rest of paper is organized as follows. Section II briefly introduces the structure of the MRR, and its dynamic model. Section III addresses the working-mode-switch strategy and the hybrid control methodology. Section IV shows simulation results, and Section V offers conclusions.

II. DYNAMIC MODEL OF THE MRR

A picture of one MRR joint module developed in our laboratory is depicted on Fig. 1. Each joint module consists of a brushless DC motor, an encoder, a brake, homing and limit sensors and a harmonic drive with an integrated torque sensor and amplifier [15]. For the door-opening control problem studied in this paper, we assume that the MRR’s end-effector has already grasped the door knob. Hence, the MRR is constrained during door-opening process. Referring
to [14] and [17], the dynamic equation of an MRR with n-joint modules can be derived as:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + \cdots + \Gamma^{-1} \tau_i = \tau + f \]  

(1)

where \( q = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^n \) denotes generalized coordinates; \( q_i, \dot{q}_i, \ddot{q}_i \) represent the rotation angle, angular velocity and angular acceleration of the \( i \)-th joint, respectively; \( M \in \mathbb{R}^{n \times n} \) denotes the inertia matrix; \( C(q, \dot{q}) \in \mathbb{R}^n \) is a vector containing Coriolis, centrifugal forces; \( f_\text{d}(q, \dot{q}) \in \mathbb{R}^n \) is a vector representing frictional force; \( \tau_i \equiv [\tau_{1i}, \tau_{2i}, \ldots, \tau_{ni}]^T \in \mathbb{R}^n \) and \( \tau_{si} \) denotes the coupling torque at the \( i \)-th joint location; \( \Gamma \equiv \text{diag} \{ \gamma_1, \gamma_2, \ldots, \gamma_n \} \in \mathbb{R}^{n \times n} \) and \( \gamma_i \) denotes the reduction ratio of the \( i \)-th speed reducer (\( \gamma_i \geq 1 \)); \( \tau \in \mathbb{R}^n \) is the actuation input; and \( f \in \mathbb{R}^n \) is the vector of constraint forces in the joint space. Here, referring to [14], \( f_\text{d} \) has the following expression,

\[ f_{\text{d}i} \triangleq b_{mi}\gamma_i\dot{q}_i + (f_{ci} + f_{si}\exp(-f_{si}\dot{q}_i^2))\text{sgn}(\dot{q}_i) \]  

(2)

where \( b_{mi} \) denotes the moment of inertia of the \( i \)-th rotor about the axis of rotation; \( b_{mi}, f_{ci}, f_{si}, f_{di} \) denote the viscous frictional coefficient, the Coulomb friction-related parameter, the static friction-related parameter, a positive parameter corresponding to the Stribeck effect, respectively. The sign function is defined as

\[ \text{sgn}(\dot{q}_i) = \begin{cases} 1 & \text{for } \dot{q}_i > 0 \\ 0 & \text{for } \dot{q}_i = 0 \\ -1 & \text{for } \dot{q}_i < 0 \end{cases} \]  

(3)

Let \( \phi(q) \in \mathbb{R}^m \) represent the constraint function, which include a set of \( m \) independent equations, we have

\[ \phi(q) = 0, \quad \frac{\partial \phi}{\partial \dot{q}} = J_c(q)\dot{q} = 0 \]  

(4)

The function \( \phi(q) \) is twice continuous differentiable [18] with a Jacobian matrix denoted by \( J_c(q) \in \mathbb{R}^{m \times n} \). The constraint force \( f \) can be expressed in terms of a generalized multiplier \( \lambda \in \mathbb{R}^m \) by the following equation.

\[ f = J_c^T(q)\lambda \]  

(5)

As a result of accumulated research efforts [20], it has been recognized that there exists a proper partition \( q^1 \in \mathbb{R}^{n-m} \), and \( q^2 \in \mathbb{R}^m \), such that \( q = [q^1 \ q^2]^T \). From (4),

\[ J_c(q) = \begin{bmatrix} J_{c1}(q) & J_{c2}(q) \end{bmatrix} = \begin{bmatrix} \frac{\partial \phi(q)}{\partial q^1} & \frac{\partial \phi(q)}{\partial q^2} \end{bmatrix} \]  

(6)

From the implicit function theorem, the constraint equation (4) can always be expressed explicitly as in [21]

\[ q^2 = \sigma(q^1) \]  

(7)

This enables one to write

\[ \dot{q} = L(q)q^1 \]  

(8)

\[ L(q) = \begin{bmatrix} I_{n-m} & J_{c1}^T(q)J_{c1}(q) \\ -J_{c2}^T(q)J_{c2}(q) \end{bmatrix} \]  

(9)

where \( I_{n-m} \in \mathbb{R}^{(n-m) \times (n-m)} \) is an identity matrix. It is then easy to derive

\[ L^T(q)J_c^T(q) = 0 \]  

(10)

Substituting (5) and (8) into (1), we have

\[ M(q)L(q)q^1 + M(q)L(q)q^2 + C(q, \dot{q})L(q)q^1 + f_\text{d}(q, \dot{q}) + \Gamma^{-1} \tau_i = \tau + J_c^T(q)\lambda \]  

(11)

Left multiplying \( L^T(q) \) at both sides of (11) yields

\[ M_1(q^1)q^1 + C_1(q^1, q^2)q^1 + L^T(q)f_\text{d}(q, \dot{q}) + \Gamma_1^{-1} \tau_i = L^T(q)\tau \]  

(12)

where

\[ M_1(q^1) \triangleq L^T(q)M(q)L(q) \]  

(13)

\[ C_1(q^1, q^2) \triangleq L^T(q)C(q, \dot{q})L(q) + L^T(q)C(q, \dot{q})L(q) \]  

(14)

\[ \Gamma_1^{-1} \triangleq L^T(q)\Gamma^{-1} \]  

(15)

### III. CONTROL DESIGN

For the door-opening task performed by an MRR consisting only of rotary joint modules, the mode-switch strategy for each joint module appears significantly critical because it determines the success and efficiency of door-opening control to a great extent. In this section, we first address the problem of unknown parameter estimation, which is necessary for the subsequent mode-switch strategy. We then propose a mode-switch strategy and the control laws used in the proposed hybrid control scheme, and finally provide the corresponding stability proof.

#### A. Unknown Parameter Estimation

In order to determine when the working mode of each joint should be switched from passive to active or from active to passive, a mode switch strategy is required. And we need to estimate the door radius, \( r \), the base position of the MRR, \( (x_1, y_1) \), and the knob height with respect to the base of the MRR, \( h \). Fig. 2 shows the initial configuration of the MRR, i.e., \( C_1 \), where \( \theta_{(j)} \) denote the rotation angle of the \( i \)-th joint at the \( j \)-th configuration. The reference frame is set as shown in Fig. 2. The origin of the reference frame is set as the intersection point of the hinge and the horizontal plane that crosses the rotation axis of the first joint. Since the MRR end-effector has already firmly grasped the knob. In the estimation process, we apply a small torque only to the 2nd joint to keep the door closed until the unknown parameters

![Fig. 2. The initial configuration of the MRR (5-joint case)](image-url)
are estimated. At the same time, the other joints are set in the passive working mode.

Let \((x_1e, y_1e, z_1e)\) denote the tip position of the end-effector in the reference frame, \(l_i\) denote the length of the \(i^{th}\) link, \(q_i\) denote the rotation angle of the \(i^{th}\) joint. From Fig. 2, with respect to the door radius, \(r\), we have
\[
x_e(t)^2 + y_e(t)^2 = r^2
\]
\[
z_e(t) = h
\] (16) (17)

With respect to the reference frame shown in Fig. 2 and coordinate transfer, we derive
\[
x_e(t) = x_1 - \left[ \sum_{i=2}^{n-1} l_i \sin \left( \sum_{j=2}^{i} \theta_j(t) \right) \right] \cos \theta_1(t)
- l_n \sin \left( \sum_{i=2}^{n-1} \theta_j(t) \right) \cos (\theta_1(t) + \theta_n(t))
\] (18)
\[
y_e(t) = y_1 - l_1 + \sum_{i=2}^{n-1} l_i \cos \left( \sum_{j=2}^{i} \theta_j(t) \right)
\] (19)
\[
z_e(t) = - \left[ \sum_{i=2}^{n-1} l_i \sin \left( \sum_{j=2}^{i} \theta_j(t) \right) \right] \sin \theta_1(t)
- \sum_{i=2}^{n-2} l_i \sin \theta_i(t) \sin (\theta_1(t) + \theta_n(t))
\] (20)

For simplicity, let us introduce the two function definitions \(L_x(t)\) and \(L_y(t)\).

\[
L_x(t) \triangleq - \left[ \sum_{i=2}^{n-1} l_i \sin \left( \sum_{j=2}^{i} \theta_j(t) \right) \right] \cos \theta_1(t)
- \sum_{i=2}^{n-1} l_i \cos \left( \sum_{j=2}^{i} \theta_j(t) \right)
\]
\[
L_y(t) \triangleq - l_1 + \sum_{i=2}^{n-1} l_i \cos \left( \sum_{j=2}^{i} \theta_j(t) \right)
\]

Substituting \(L_x(t), L_y(t)\) into (18),(19) and the resulted equations into (16) and rearranging each term, we have
\[
L_x^2(t) + L_y^2(t) = x_1^2 - x_1^2 - 2x_1L_x(t) - 2y_1L_y(t)
\] (21)

Let us define
\[
P = \begin{pmatrix} 1 & 2L_x(t) & 2L_y(t) \\ : & : & : \\ r^2 - x_1^2 - y_1^2 & -x_1 & -y_1 \end{pmatrix}, \quad W = \begin{pmatrix} L_x^2(t) + L_y^2(t) \\ : & : & : \end{pmatrix},
\]
\[
\lambda = \begin{pmatrix} r^2 - x_1^2 - y_1^2 \\ -x_1 \\ -y_1 \end{pmatrix}
\]

Equation (21) can be re-written as
\[
W = P\lambda
\] (22)

A straightforward least squares approximation is then performed.
\[
\lambda = (P^T P)^{-1} P^T W
\] (23)

where \(\lambda\) is used to solve for the estimated parameters \(r, x_1, y_1\). As long as \(x_1\) and \(h\) are estimated, the desired rotation angle of the 1st joint can be calculated as shown in Fig. 2.
\[
\theta_1(c) = \tan^{-1} \left( \frac{h}{x_e} \right)
\]

At that moment, all the links of the MRR are located in the same plane.

B. Mode-Switch Strategy

Based on the parameters estimated, we propose a mode-switch strategy starting from the configuration at the end of parameter estimation, with two assumptions: (i) the door-opening direction is known (right-side open or left-side open) and; (ii) the robot base is located within the applicable door-opening area, which can be calculated from (18) and (19). The mode-switch strategy of an n-joint MRR can be described as follows: (i) activate and rotate the 1st joint while keeping the other joints passive, until \(1(c)\) is reached, as shown in Fig. 2; (ii) keep the rotation angle of the 1st joint unchanged, activate and rotate the \(i^{th}\) joint (i starts from 2) while keeping the \(j^{th}\) joint (\(n \geq j > i\) passive, until the hinge, the end-effector and the \(i^{th}\) joint are located in the same straight line, as shown in Fig. 3; (iii) activate and rotate the next joint (\(i + 1)^{th}\), keep the rotation angle of the \(i^{th}\) joint unchanged while setting the following joints passive, until the hinge, the end-effector and the \((i + 1)^{th}\) joint are located in the same straight line; (iv) repeat step (iii), until the door is opened. Following this strategy, after step (i), all the joints and links are located in the same plane and the moment of each joint being switch from passive to active is easy to be calculated. This is the unique advantage of the proposed mode-switch strategy. Here we give the solution to a 5-joint case. For the 2nd joint, when the 1st joint reaches the rotation angle of \(\theta_1(c)\), it is set in the active mode. For the 3rd joint, when the hinge, the end-effector and the 2nd joint are located in the same straight line, i.e., \(C_2\) of Fig. 3, the 3rd joint is set in the active mode. At that moment, from Fig. 3, we have
\[
x_2 = x_1, \quad y_2 = y_1 - l_1,
\]
\[
tan\alpha_1 = \frac{x_1}{y_1 - l_1}
\] (24)
\[ \theta_{3(a)} = \cos^{-1} \left( \frac{\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c)}{2(l_1 + l_4)/l_2} \right) \]

Let \( \theta_{2(c)} \) denotes the rotation angle of the 2\(^{nd}\) joint at the above moment, we have

\[
\theta_{2(c)} = \pi - \tan^{-1} \frac{x_2}{y_2} + \pi - \theta_1 - (\pi - \theta_{3(a)}) = \pi + \theta_{3(a)} - \tan^{-1} \frac{x_2}{y_2} - \beta_1
\]

where

\[
\beta_1 = \cos^{-1} \left( \frac{l_1 + l_4 - l_2^2 + \left(\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c)\right)^2}{2(l_1 + l_4)(\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c))} \right)
\]

Following similar procedure, we can calculate \( \theta_{4(a)} \) and \( \theta_{3(c)} \).

\[
\theta_{4(a)} = \cos^{-1} \left( \frac{l_2^2 + l_4^2 - \left(\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c)\right)^2}{2l_1l_4} \right)
\]

\[
\theta_{3(c)} = \pi - (\theta_{4(a)} - \beta_2) - \gamma_1
\]

where

\[
\gamma_1 = \cos^{-1} \left( \frac{x_2^2 + y_2^2 + l_2^2 - x_3^2 - y_3^2}{2l_2\sqrt{x_2^2 + y_2^2}} \right)
\]

\[
\beta_2 = \cos^{-1} \left( \frac{l_2^2 + \left(\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c)\right)^2 - l_3^2}{2l_4 \left(\sqrt{x_2^2 + y_2^2} - r / \cos\theta_1(c)\right)^2} \right)
\]

The basic idea of this approach is to keep as many links as possible aligned and set the joints in the active mode one by one in sequence so that the torque required at each active joint is much smaller and the door opening angle, \( \alpha_3 \), is much bigger than those achieved in [13].

**C. Control Design**

From the proposed mode-switch strategy, we may categorize the modes of MRR joints in three types: passive mode, current-active mode and post-active mode. Here the passive mode refers to the mode in which a joint rotates freely with friction compensation; the current-active mode refers to the mode, which starts from the joint being activated until the following joint being activated; and the post-active mode refers to the mode, in which the rotation angle of the joint keeps unchanged. With respect to these three modes, a hybrid control scheme is proposed. For the joints work in the passive mode, friction must be compensated so that the output shaft of the joints can be moved freely. Referring to [13], based on the motion trend and the angular velocity of the passive joints, a feedforward torque can be applied to compensate the friction. Hence, the control law for the joints working in the passive mode can be expressed as follows.

\[
\tau_i = -f_{ni} \exp(-f_{si} q_i^2) \text{sgn}(q_i) - b_{mi} q_i \quad i = 2, \ldots, n
\]

where \( f_{ni} \) represents the constant part of the friction and is less than the static friction \( f_{si} \). Since the magnitude of constant friction part often dominates the overall magnitude of the total friction at lower speed, by applying \( \tau_i \) expressed in (30) can thus substantially compensate the friction. For the joints working in the post-active mode, we employ the technique of position control, for example, a PD feedback control method. Define the position and velocity errors as

\[
e = q - q^d, \quad \dot{e} = \dot{q} - \dot{q}^d
\]

The control law is

\[
\tau_i = k_i e_i + k_d \dot{e}_i \quad i = 1 \ldots n
\]
where \( k_i \) and \( k_d \) are constant proportional and derivative control gains. For the joints working in the current-active mode, the objective of the control is that given a desired joint trajectory \( q^d \) to determine a control law such that \( q \to q^d \) as \( t \to \infty \). Two vectors used in the control design are defined as

\[
\begin{align*}
u &= q^d - \Lambda e \\
r &= \dot{e} + \Lambda e
\end{align*}
\]

where \( \Lambda \in R^{n \times n} \) is a positive constant matrix. Let \( B = \text{diag}\{b_m y\} \), \( f_c = [f_{c_1} \ldots f_{c_n}]^T \), \( f_s = [f_{s_1} \ldots f_{s_n}]^T \), \( f_\tau = [f_{\tau_1} \ldots f_{\tau_n}]^T \). \( \dot{B} \), \( \dot{f}_c \), \( \dot{f}_s \), \( \dot{f}_\tau \) denote the nominal values of \( B \), \( f_c \), \( f_s \), \( f_\tau \), respectively. Define

\[
D(\dot{q}) = \begin{bmatrix} \dot{q} & \text{sgn}(\dot{q}) & \rho \text{sgn}(\dot{q}) & -\dot{f}_c q^2 \rho \text{sgn}(\dot{q}) \end{bmatrix}
\]

where \( \rho \triangleq \exp(-f_c q^2) \).

In order to ensure the active joints follow their corresponding desired trajectories and satisfy the constraints, the control laws are defined as,

\[
\tau = \tau_0 + D(\dot{q})u_p + \Gamma^{-1} \tau_s - J^T_c(q)\lambda^d - K_r r
\]

where \( \lambda^d \) denotes the desired constraint force;

\[
\tau_0 = M(q)\dot{u} + C(q, \dot{q})u + B\dot{q} + (\dot{f}_c + \dot{f}_c \rho)\text{sgn}(\dot{q})
\]

\[
u_p = -k \int_0^t D(\dot{q})^T r dt
\]

Here \( u_p \) is designed to compensate for the effect of the constant parametric uncertainty \( \tilde{f} \). \( K_r \in R^{n \times n} \) is a constant gain matrix.

Substitute control law (37) into (1), we have the closed-loop equation as

\[
M(q)\ddot{r} + C(q, \dot{r})r = D(\dot{q})(\tilde{f} + u_p) + J^T_c(q)(\lambda - \lambda^d) - K_r r
\]

Left multiplying \( L^T(q) \) on both sides of (40), from (11), we derive the reduced equation, which is similar to (12),

\[
M(q)\dot{r}^1 + C_1(q^1, \dot{q}^1)r^1 = L^T(q)D(\dot{q})(\tilde{f} + u_p) - L^T(q)K_r L(q)r^1
\]

Theorem 1: Given the system (12), the tracking error asymptotically converges to zero under the control law defined by (37~39).

Proof: The Lyapunov function candidate is defined as

\[
V = \frac{1}{2} r^T M_1 r^1 + \frac{1}{2} k_\xi^T \xi
\]

where

\[
\xi = \frac{1}{k} \tilde{f} - \int_0^t D(\dot{q})^T r dt
\]

Since \( k \) and \( \tilde{f} \) are both constant, we have

\[
\dot{\xi} = -D(\dot{q})^T r
\]

Differentiating (42) yields,

\[
\dot{V} = \frac{1}{2} r^T M_1 r^1 + r^T M_1 r^1 + k_\xi^T \dot{\xi}
\]

Combining (41), (43), (44), we have

\[
\dot{V} = \frac{1}{2} r^T (L^T(q)D(\dot{q})(\tilde{f} + u_p) - L^T(q)K_r L(q)r^1 - k_\xi^T D(\dot{q})^T r
\]

Since \( \frac{1}{2} M_1 - C(q^1, \dot{q}^1) \) is a skew-symmetric matrix [22] and \( r = L(q)r^1 \), we have

\[
\dot{V} = \frac{1}{2} r^T (L^T(q)D(\dot{q})(\tilde{f} + u_p) - L^T(q)K_r L(q)r^1 - kr^T D(\dot{q}) \tilde{f}
\]

\[
- \int_0^t D(\dot{q})^T r dt - r^T L^T(q)K_r L(q)r^1
\]

Since \( V \leq 0 \), \( \dot{V} < 0 \), from (42) and (47), it is evident \( ||r^1|| \) converges exponentially to zero, i.e., \( e \to 0 \) as \( t \to \infty \). Also, \( q^{2d} = \sigma(q^{1d}) \), which implies \( \dot{q}^2 \to \dot{q}^{2d} \), if \( q^1 \to q^{1d} \).

Therefore, using control law (37), (38), (39), the closed-loop system is globally asymptotically stable in the sense that \( q \to q^d \), as \( t \to \infty \).

IV. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the validity and effectiveness of the proposed control scheme. This simulation is based on the MRR with four joint modules (the forth joint module has two degree-of-freedoms) that are designed and assembled in the Systems and Control Lab at Ryerson University. Fig. 4 shows the MRR joint angles trajectories, the desired trajectories, as well as the moments where the mode-switch happens. The MRR joints tracking errors are shown in Fig. 5. From these results, it is clear that the mode-switch strategy and the hybrid controller make the MRR track the desired position trajectories and satisfy the constraints.
V. CONCLUSIONS

Using the modular reconfigurable robot (MRR) to open a door is a challenging but meaningful task, because of the flexibility and versatility of MRRs. In this paper, for door-opening using MRRs, we propose a mode-switch strategy and a hybrid control method with respect to the joints working in different modes. The efficiency of the proposed mode-switch strategy and control methodology are demonstrated through simulation results.

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