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A Time-Varying Wave Impedance Approach for Transparency Compensation in Bilateral Teleoperation

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Abstract— **Among the still existing issues in bilateral teleoperation, there is the inability by force-feedback control schemes to guarantee delay-independent stability and achieve both position coordination and force reflection independently of the remote environmental dynamics. Particularly, most bilateral control frameworks fail to address position coordination when interacting with rigid environments. In this paper we present a novel control strategy that aims to passively compensate for position errors that arise during contact tasks and, in general, achieve stability and transparency when alternating between unobstructed (free) and obstructed (contact) environments. The proposed control framework exploits the wave impedance independent passivity property of the scattering transformation to guarantee stability and transparency by gradually switching between a low wave impedance, ideal for free motion, and a sufficiently large impedance, suitable for contact tasks. The validity of the control framework is verified through simulations and experiments on a pair of nonlinear robots.**

I. Introduction

In principle, a teleoperation system is a dual robotic set that enables a human operator to manipulate, sense, and physically interact with a distant environment. In such system, the desired manipulation or task is performed remotely by a slave robot which tracks the motion of a locally human-controlled master robot. The master and slave robot are coupled through a communication channel that, ideally, should be *transparent* to the operator, meaning that he or she should feel as if being directly active in the remote location [1]. This is generally achieved by transmitting remote slave information (e.g., position, velocity, and force) to the master robot in what is called a bilateral connection. Unfortunately, bilateral configurations can potentially yield a teleoperation system unstable due to unreliabilities (e.g., delays [2] and data losses [3]) experienced on the communication channel.

Stability issues induced by time delays on bilateral teleoperators have been studied since the mid 1960s [2]. However, it was not until the late 1980s that passivity-based control and scattering theory, derived from network theory, combined to guarantee the stability of force feedback teleoperators independently of any sized constant delay [4]. Ever since, the scattering and passivity formulation, extended later with the notion of wave variables [5], has arguably become one of the fundamental control approaches for stabilizing bilateral teleoperators (refer to [6] for a review on bilateral control frameworks).

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Aside from stability issues, time delays are also known to affect transparency. According to [1], transparency is achieved when the transmitted impedance to the operator equals the environmental impedance. An alternative interpretation is given in [7], where a system is said to be transparent if the position of the slave equals the master's position and the human force is equal to the net environmental force. Based on either formulation, considerable research efforts have been aimed to conciliate transparency-based objectives while still enforcing time delay independent stability (see [1], [7]–[12] for examples and further discussion).

Despite the fact that most force-feedback frameworks for bilateral teleoperation are designed to achieve both stability and transparency, their results generally depends on the dynamics of the environment which more than often is unknown or, at least, variable. Precisely, one of the still prevalent issues in bilateral frameworks is the failure to adjust transparency when transitioning from an unobstructed (free movement) to an obstructed (rigid contact) environment, and vice versa. For instance, in wave-based approaches, transparency highly depends on the wave impedance: a control parameter specified by the designer [13]. For free motion, the ideal wave impedance should be infinitesimal such that the increase of inertia induced by the delay is barely perceived by the operator; whereas for rigid environments, the desired wave impedance should be infinitely large such that a stiff environment is felt by the operator [14]. Compromising the value of the wave impedance to best satisfy both scenarios would lead to a system that feels rather sluggish in free motion with substantial position errors (also referenced as position drifts [15]) when interacting with rigid environments. A similar behavior is also experienced when tuning traditional proportional-derivative (PD) architectures where, in general, control gains are limited by stability constrains and consequently, position errors arise while in contact motion [11], [15], [16].

Time-varying compensation of position errors during contact tasks has been previously addressed in [17] via a wave-based scheme that introduces the notion of a variable rest length. The role of the variable rest length is to modify the desired target position according to the position drift and applied forces such that the error between the master and slave position converges to zero. A similar approach based on the variable rest length is presented in [18], where an energy tank replaces the dissipative element in the wave scattering transformation for impedance matching such that the energy is stored rather than dissipated. The stored energy is then used to adequately change the variable rest length without relying on the operator's energy as in [17]. In both of the above methods, the communication delay must be known in order to perform the position compensation.

This paper presents now a novel control strategy for position compensation during contact tasks where the wave impedance independent passivity property in the scattering transformation is exploited. The proposed control framework builds on the wave-based approach in [19] and introduces a time-varying wave impedance for transparency compensation when transitioning between unobstructed and obstructed environments. An akin strategy has been previously presented in [20], where the wave impedance alternates between two discrete values according to the current task, given that the mechanical and control systems dissipate enough energy to perform the transition and preserve passivity. In contrast, our proposed control framework gradually changes the wave impedance, allowing for passive and smooth switches between arbitrary small impedances (suitable for free environments) and sufficiently large impedances (ideal for rigid contact). Simulation and experimental results in a pair of nonlinear robots validate the proposed control and compensation strategy.

II. Problem Formulation

A. Modeling the Teleoperators

We address the task of remotely controlling an n-degree-of-freedom (DOF) slave robot coupled bilaterally to an n-DOF master robot through a time delayed communication channel. The master and slave teleoperator have nonlinear Euler-Lagrangian dynamics given by[1]

$$\begin{aligned} M_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + C_m(\mathbf{q}_m, \dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) &= \mathbf{f}_m + \bar{\boldsymbol{\tau}}_m \\ M_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + C_s(\mathbf{q}_s, \dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{g}_s(\mathbf{q}_s) &= \mathbf{f}_s + \bar{\boldsymbol{\tau}}_s \end{aligned} \quad (1)$$

where $\mathbf{q}_i = \mathbf{q}_i(t) \in \Re^n$ are the generalized coordinates, $M_i(\mathbf{q}_i) \in \Re^{n\times n}$ are the positive definite inertia matrices, $C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \Re^{n\times n}$ are the centrifugal and Coriolis matrices, $\mathbf{g}_i(\mathbf{q}_i)$ are the gravitational forces, $\mathbf{f}_i = \mathbf{f}_i(t) \in \Re^n$ are the human and environmental forces, and $\bar{\boldsymbol{\tau}}_i = \bar{\boldsymbol{\tau}}_i(t) \in \Re^n$ are the control inputs for the master ($i = m$) and slave robots ($i = s$). Due to its Euler-Lagrangian dynamic structure, the jk^{th} element of $C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is given by

$$C_i^{jk}(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \sum_{l=1}^{n} \frac{1}{2}\left[\frac{\partial M_i^{jk}}{\partial q_i^l} + \frac{\partial M_i^{jl}}{\partial q_i^k} - \frac{\partial M_i^{kl}}{\partial q_i^l}\right]\dot{q}_i^l \quad (2)$$

and therefore, (1) satisfies the well known passivity property

$$\dot{M}_i(\mathbf{q}_i) = C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + C_i^T(\mathbf{q}_i, \dot{\mathbf{q}}_i). \quad (3)$$

B. Control Objectives

Our control goal is to design the inputs $\bar{\boldsymbol{\tau}}_i$ such that stability and transparency of the close-loop system in (1) are achieved. Explicitly, we would like $\bar{\boldsymbol{\tau}}_i$ to enforce position coordination for finite delays, i.e.,

$$\mathbf{q}_m(t) - \mathbf{q}_s(t) \rightarrow 0 \quad (4)$$

and static force reflection, i.e.,

$$\mathbf{f}_m(t) \rightarrow -\mathbf{f}_s(t) \quad (5)$$

as $\dot{\mathbf{q}}_i \rightarrow 0$; independently of the structure of the remote environment. Furthermore, we would like the operator to perceive low and high impedances when interacting with free and rigid environments, respectively.

C. Assumptions

In the following analysis we make the assumption that delays on the transmission lines from master to slave, T_m, and from slave to master, T_s, are constant but not necessarily equal. Furthermore, we assume that the slave robot is equipped with force/torque sensors[2] and that it is able to communicate contact information to the master robot.

III. Control Framework

In this section we proceed to develop the bilateral control framework. First, we address the problem of stability through the passivity formalism since, in general, passivity is a sufficient condition for stability. Then, we proceed to guarantee transparency-based objectives.

Definition 3.1: [21] A system with input $\mathbf{x}$ and output $\mathbf{y}$ is said to be passive if

$$\int_0^t \mathbf{x}^T\mathbf{y}d\theta \geq -\kappa^2 + \nu^2\int_0^t \mathbf{x}^T\mathbf{x}d\theta + \rho^2\int_0^t \mathbf{y}^T\mathbf{y}d\theta \quad (6)$$

for some $\kappa, \nu, \rho \in \Re$. Moreover, it is said to be lossless if equality persists and $\nu = \rho = 0$, input strictly passive if $\nu \neq 0$, and output strictly passive if $\rho \neq 0$.

In order to passivize and hence stabilize the teleoperators, we propose the design of the control inputs as

$$\bar{\boldsymbol{\tau}}_i = -M_i(\mathbf{q}_i)\Lambda\dot{\mathbf{q}}_i - C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\Lambda\mathbf{q}_i + \mathbf{g}_i(\mathbf{q}_i) + \boldsymbol{\tau}_i \quad (7)$$

where $\Lambda \in \Re^{n\times n}$ is, without loss of generality, a diagonal positive definite constant matrix and $\boldsymbol{\tau}_i = \boldsymbol{\tau}_i(t) \in \Re^n$ are the coordination control inputs to be designed. Then, the dynamic equations of the system in (1) reduce to

$$\begin{aligned} M_m(\mathbf{q}_m)\dot{\mathbf{r}}_m + C_m(\mathbf{q}_m, \dot{\mathbf{q}}_m)\mathbf{r}_m &= \mathbf{f}_m + \boldsymbol{\tau}_m \\ M_s(\mathbf{q}_s)\dot{\mathbf{r}}_s + C_s(\mathbf{q}_s, \dot{\mathbf{q}}_s)\mathbf{r}_s &= \mathbf{f}_s + \boldsymbol{\tau}_s \end{aligned} \quad (8)$$

where

$$\mathbf{r}_i(t) = \dot{\mathbf{q}}_i(t) + \Lambda\mathbf{q}_i(t). \quad (9)$$

The control law in (7) is a passivity-based control method [22], which means that the master and slave teleoperators, with reduced dynamics (8), are passive with respect to the input $\mathbf{f}_i + \boldsymbol{\tau}_i$ and output $\mathbf{r}_i$. Mathematically,

$$\begin{aligned} \int_0^t (\mathbf{f}_i + \boldsymbol{\tau}_i)^T\mathbf{r}_i d\theta &= \mathbf{r}_i^T(\theta)M_i(\mathbf{q}_i(\theta))\mathbf{r}_i(\theta)\Big|_{\theta=0}^{\theta=t} \\ &\geq -\mathbf{r}_i^T(0)M_i(\mathbf{q}_i(0))\mathbf{r}_i(0). \end{aligned}$$

Remark 3.1: Note that the control law in (7) assumes complete knowledge of the dynamics of the master and slave robot. In [19], [22], a passivity-based adaptive law is

[1] In what follows, we will omit time dependence of signals when necessary to avoid cluttering of equations.

[2] This assumption can be replaced with any other contact/proximity sensor.

suggested for the case where the parameters are unknown. Such approach can be easily extended to our proposed control framework without altering the passivity and position convergence results presented in this paper.

Now, we are left to design the control inputs $\boldsymbol{\tau}_i$ such that the communication channel is passivized independently of the delay and that position and force tracking of the teleoperators are guaranteed. With this in mind, we propose the use of the scattering transformation and wave variables $\mathbf{u}_i$ and $\mathbf{v}_i$ [4], [5]. For the master side, the outputs of the scattering transformation are computed as

$$\mathbf{u}_m(t) = (2B_m(t))^{-\frac{1}{2}}(B_m(t)\mathbf{r}_{md}(t) - \boldsymbol{\tau}_m(t)) \quad (10)$$

$$\mathbf{r}_{md}(t) = (2B_m^{-1}(t))^{\frac{1}{2}}\mathbf{v}_m(t) - B_m^{-1}(t)\boldsymbol{\tau}_m(t) \quad (11)$$

where $B_m(t) \in \Re^{n\times n}$, the wave impedance, is a time-varying, positive definite matrix that will be designed under transparency concerns; and $\mathbf{v}_m(t) = \mathbf{v}_s(t - T_s)$ is the upcoming, delayed wave variable from the slave's scattering transformation. Then, the coordination control input can be given as

$$\boldsymbol{\tau}_m(t) = B_m(t)(\mathbf{r}_{md}(t) - \mathbf{r}_m(t)). \quad (12)$$

Likewise, for the slave side, the outputs of the scattering transformation are computed as

$$\mathbf{v}_s(t) = (2B_s(t))^{-\frac{1}{2}}(B_s(t)\mathbf{r}_{sd}(t) - \boldsymbol{\tau}_s(t)) \quad (13)$$

$$\mathbf{r}_{sd}(t) = (2B_s^{-1}(t))^{\frac{1}{2}}\mathbf{u}_s(t) - B_s^{-1}(t)\boldsymbol{\tau}_s(t) \quad (14)$$

where $B_s(t) = B_m(t - T_m)$ and $\mathbf{u}_s(t) = \mathbf{u}_m(t - T_m)$. Similar to the master case,

$$\boldsymbol{\tau}_s(t) = B_s(t)(\mathbf{r}_{sd}(t) - \mathbf{r}_s(t)). \quad (15)$$

We now show that passivity of the communication channel is achieved independently of delays and variance of the wave impedance. Manipulating (10) to (14), we obtain that

$$\begin{aligned}&\boldsymbol{\tau}_m^T\mathbf{r}_{md} + \boldsymbol{\tau}_s^T\mathbf{r}_{sd}\\ &\quad = -2B_m^{-1}B_m\tfrac{\mathbf{u}_m-\mathbf{v}_m}{2}\tfrac{\mathbf{u}_m+\mathbf{v}_m}{2} - 2B_s^{-1}B_s\tfrac{\mathbf{v}_s-\mathbf{u}_s}{2}\tfrac{\mathbf{v}_s+\mathbf{u}_s}{2}\\ &\quad = -\tfrac{1}{2}\left(\mathbf{u}_m^2 - \mathbf{v}_m^2 + \mathbf{v}_s^2 - \mathbf{u}_s^2\right)\end{aligned}$$

and integrating with respect to time,

$$\begin{aligned}&-\textstyle\int_0^t(\boldsymbol{\tau}_m^T\mathbf{r}_{md} + \boldsymbol{\tau}_s^T\mathbf{r}_{sd})d\theta\\ &\qquad = \tfrac{1}{2}\textstyle\int_{t-T_m}^t \mathbf{u}_m^2 d\theta + \tfrac{1}{2}\int_{t-T_s}^t \mathbf{v}_s^2 d\theta \geq 0\end{aligned} \quad (16)$$

where the negative sign in front of the integral is owed to the power inflow. The lower bound in (16) implies that the energy is temporary stored in the transmission lines and therefore, the communication channel is passive independently of delays. In addition, the reader can easily verify that using the scattering transformation and the coordination control inputs (12) and (15), (11) and (14) reduce to

$$\mathbf{r}_{md}(t) = \frac{1}{2}\left(\Gamma(t)\mathbf{r}_s(t - T_s) - \mathbf{r}_m(t)\right) \quad (17)$$

$$\mathbf{r}_{sd}(t) = \frac{1}{2}\left(\mathbf{r}_m(t - T_m) - \mathbf{r}_s(t)\right) \quad (18)$$

where $\Gamma(t) = B_m(t - T_m - T_s)^{\frac{1}{2}}B_m(t)^{-\frac{1}{2}}$. As we will show in section IV, the above proposed control law guarantees position convergence and force reflection of the teleoperators in the sense of (4) and (5).

Remark 3.2: The control law in (7), (12), and (15), in conjunction with the wave scattering formalism, resembles the control framework proposed in [19]. The difference lies on the use of a wave impedance that is time-varying rather than constant. This property, as will be shown in section IV, will avail the proposed control framework to compensate for position errors during contact tasks.

Up to now, we have designed the control inputs $\bar{\boldsymbol{\tau}}_i$ based on passivity and position coordination. We are yet to tune the control law such that transparency is achieved for both free and restricted environments. This task is left for the next subsection.

A. Tuning the Wave Impedance

Transparency in wave-based control frameworks, as previously discussed in section I, highly depends on the wave impedance, B_i. Ideally, we would like the wave impedance to alternate from a small value, B_{free}, when the slave is free to move; to a large value, B_{cont}, as soon as the slave robot makes contact with a rigid surface. For sake of simplicity, we will assume that $B_i(t) > 0$ are diagonal matrices.

We propose the update law for the diagonal jj^{th} entry of the wave impedance matrix to be given as

$$\dot{B}_m^{jj}(t) = \begin{cases} \min\{\overline{\beta}^j(t), \hat{\Lambda}^{jj}B_m(t)\}, & \text{if } \left\|f_s^j(t - T_s)\right\| > 0\\ -\underline{\beta}^j(t), & \text{otherwise}\end{cases}$$

$$B_s^{jj}(t) = B_m^{jj}(t - T_m) \quad (19)$$

where f_s^j is the j^{th} component of $\mathbf{f}_s$, $\hat{\Lambda} < \Lambda$ is a $n \times n$ diagonal positive definite matrix with entries $\hat{\Lambda}^{jj}$, and $\overline{\beta}^j$ and $\underline{\beta}^j$ are nonnegative scalar functions that drive B_m^{jj} to B_{cont}^{jj} and B_{free}^{jj}, respectively.

IV. Stability and Transparency Analysis

As aforementioned, the two foremost goals in bilateral teleoperation are stability and transparency. In this section we demonstrate that the proposed control framework achieves both objectives. We first evaluate the standard case where the human and environment are modeled as passive systems. Then, we prove our original claims as we relax this assumption on the operator.

Theorem 4.1: Consider the teleoperation system in (1) with control law (7), (12), and (15). Suppose that the human and remote environment are passive with respect to (9), i.e., $\exists\ \kappa_i \in \Re$ such that

$$-\int_0^t \mathbf{f}_i^T\mathbf{r}_i d\theta \geq -\kappa_i^2, \qquad \text{for } i = \{m, s\}. \quad (20)$$

Then, for all arbitrary initial conditions, the closed-loop teleoperation system is stable, all signals are bounded, and the system achieves position coordination and static force reflection in the sense of (4) and (5)[3].

[3]This is a modified version of Theorem 4.1 in [19]. Here we show that $\mathbf{f}_m \to -\mathbf{f}_e$ as $(\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i) \to 0$ and prove stability and position convergence for non-constant wave impedances.

Proof: Define the slave's coordination error as

$$\mathbf{e}_s(t) = \mathbf{q}_s(t) - \mathbf{q}_m(t - T_m) \tag{21}$$

and consider the following Lyapunov candidate function

$$V = \frac{1}{2}(\mathbf{r}_m^T M_m \mathbf{r}_m + \mathbf{r}_s^T M_s \mathbf{r}_s) + \frac{1}{4}\mathbf{e}_s^T \Lambda B_s \mathbf{e}_s + \kappa_m^2 + \kappa_s^2 - \int_0^t (\mathbf{f}_m^T \mathbf{r}_m + \mathbf{f}_s^T \mathbf{r}_s) d\theta - \int_0^t (\boldsymbol{\tau}_m^T \mathbf{r}_{md} + \boldsymbol{\tau}_s^T \mathbf{r}_{sd}) d\theta. \tag{22}$$

Taking the derivative of V with respect to time and applying the skew symmetric property (3), we have

$$\begin{aligned}\dot{V} =& \boldsymbol{\tau}_m^T \mathbf{r}_m + \boldsymbol{\tau}_s^T \mathbf{r}_s + \frac{1}{2}\dot{\mathbf{e}}_s^T \Lambda B_s \mathbf{e}_s + \frac{1}{4}\mathbf{e}_s^T \Lambda \dot{B}_s \mathbf{e}_s \\ &- \boldsymbol{\tau}_m^T \mathbf{r}_{md} - \boldsymbol{\tau}_s^T \mathbf{r}_{sd} \\ \leq& \frac{1}{2}\dot{\mathbf{e}}_s^T \Lambda B_s \mathbf{e}_s + \frac{1}{4}\mathbf{e}_s^T \Lambda \dot{B}_s \mathbf{e}_s - (\mathbf{r}_{sd} - \mathbf{r}_s)^T B_s (\mathbf{r}_{sd} - \mathbf{r}_s) \\ &- (\mathbf{r}_{md} - \mathbf{r}_m)^T B_m (\mathbf{r}_{md} - \mathbf{r}_m).\end{aligned}$$

Now, using (18) and the fact that $\dot{B}_s(t) \leq \hat{\Lambda} B_s(t)\ \forall t \geq 0$ and $\mathbf{r}_s(t) - \mathbf{r}_m(t - T_m) = \dot{\mathbf{e}}_s + \Lambda \mathbf{e}_s$ we obtain

$$\begin{aligned}\dot{V} \leq& -(\mathbf{r}_{md} - \mathbf{r}_m)^T B_m (\mathbf{r}_{md} - \mathbf{r}_m) \\ &- \frac{1}{4}\dot{\mathbf{e}}_s^T B_s \dot{\mathbf{e}}_s - \frac{1}{4}\mathbf{e}_s^T \Lambda \tilde{\Lambda} B_s \mathbf{e}_s\end{aligned} \tag{23}$$

where $\tilde{\Lambda} = \Lambda - \hat{\Lambda} > 0$. Since $\dot{V}$ is negative semi-definite, we conclude that the teleoperation system is close-loop stable in the sense of Lyapunov. Furthermore, we can also show that (22) is bounded and so $\mathbf{r}_i$ are also bounded. Using the Comparison Lemma [23], we can conclude that $\dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathcal{L}_\infty$, and therefore, the coordination error $\mathbf{q}_m - \mathbf{q}_s$ and its velocity are also bounded. Thus we are left to prove (4) and (5).

Invoking LaSalle's Principle [23], we have that $\dot{V} = 0 \Rightarrow (\dot{\mathbf{e}}_s, \mathbf{e}_s) \to 0$. Therefore, $\boldsymbol{\tau}_s \to 0$ and the slave's dynamics reduces to $M_s \dot{\mathbf{r}}_s + C_s \mathbf{r}_s = \mathbf{f}_s$. Consider now the following positive definite function

$$V_s = \mathbf{r}_s^T M_s \mathbf{r}_s - \int_0^t \mathbf{f}_s^T \mathbf{r}_s d\theta. \tag{24}$$

Then, we can show by taking its time-derivative that $\dot{V}_s(t) = 0$. Thus, $V_s(t) = V_s(0)\ \forall t \geq 0$ which implies that

$$\begin{aligned}V_s =& \mathbf{r}_s^T M_s \mathbf{r}_s - \int_0^t (\dot{\mathbf{r}}_s^T M_s \mathbf{r}_s + \mathbf{r}_s^T C_s \mathbf{r}_s) d\theta \\ =& \int_0^t (2\mathbf{r}_s^T C_s \mathbf{r}_s + \dot{\mathbf{r}}_s^T M_s \mathbf{r}_s) d\theta - \int_0^t \mathbf{r}_s^T C_s \mathbf{r}_s d\theta \\ =& \int_0^t \mathbf{f}_s^T \mathbf{r}_s d\theta = V_s(0).\end{aligned}$$

By boundedness of the inertia matrix [22], we can show that $\mathbf{r}_s^T M_s \mathbf{r}_s = 2V_s(0) \Rightarrow (\dot{\mathbf{q}}_s, \mathbf{q}_s) \to (0, \bar{\mathbf{q}}_s)$, where $\bar{\mathbf{q}}_s$ is a constant vector. Then, using the fact that $\mathbf{e}_s \to 0$ we have

$$\begin{aligned}&\mathbf{q}_s(t) - \mathbf{q}_m(t - T_m) = \\ &\quad \mathbf{q}_s(t - T_m) - \mathbf{q}_m(t - T_m) + \textstyle\int_{(t-T_m)}^t \dot{\mathbf{q}}_s d\theta \to 0\end{aligned}$$

which for finite T_m, gives $\mathbf{q}_s(t) - \mathbf{q}_m(t) \to 0$ and position convergence is established.

Now, consider the system under steady-state conditions, i.e., $(\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i) = 0$ and $B_s = B_m$. Then, (8) simplifies to

$$2\mathbf{f}_m = -B_m \Lambda (\mathbf{q}_s - \mathbf{q}_m), \quad 2\mathbf{f}_s = -B_s \Lambda (\mathbf{q}_m - \mathbf{q}_s)$$

and it is easy to see that $\mathbf{f}_m(t) = -\mathbf{f}_s(t)$, which completes the proof. ■

We just showed that, when the human and environment are modeled as passive systems, position and force tracking are enforced. We now relax this passivity assumption on the human operator and suppose that the environment is output strictly passive. This emulates the scenario in which the slave interacts with a rigid environment. We will show that the position error is bounded and that indeed, $\mathbf{q}_m - \mathbf{q}_s \to 0$ as $\|B_i\| \to \infty$ even for contact tasks.

Theorem 4.2: Consider the teleoperation system in (1) with control law (7), (12), and (15). Suppose that 1) the environment is output strictly passive, i.e., $\exists\ \kappa_s, \rho_s \in \Re, \rho_s \neq 0$ such that

$$-\int_0^t \mathbf{f}_s^T \mathbf{r}_s d\theta \geq -\kappa_s^2 + \rho_s^2 \int_0^t \mathbf{r}_s^T \mathbf{r}_s d\theta \tag{25}$$

and 2) the human force is bounded, i.e., $\|\mathbf{f}_m\| < \eta$ for some $\eta \in (0, \infty)$. Then, for all arbitrary initial conditions, the closed-loop teleoperation system is stable, static force reflection is achieved, and the slave's coordination error is uniformly ultimately bounded with ultimate bound inversely proportional to the wave impedance.

The proof for static force reflection follows similar to Theorem 4.1, therefore, it will be omitted. We now proceed to demonstrate closed-loop stability and boundedness of the coordination error.

Proof: Consider the following Lyapunov candidate function

$$\begin{aligned}\tilde{V} =& \frac{1}{2}(\mathbf{r}_m^T M_m \mathbf{r}_m + \mathbf{r}_s^T M_s \mathbf{r}_s) + \frac{1}{4}\mathbf{e}_s^T \Lambda B_s \mathbf{e}_s - \int_0^t \mathbf{f}_s^T \mathbf{r}_s d\theta \\ &+ \kappa_s^2 - \rho_s^2 \int_0^{t-T_s} \mathbf{r}_s^T \mathbf{r}_s d\theta - \int_0^t (\boldsymbol{\tau}_m^T \mathbf{r}_{md} + \boldsymbol{\tau}_s^T \mathbf{r}_{sd}) d\theta.\end{aligned}$$

Taking its derivative with respect to time and using (17), (18), and the fact that $\dot{B}_s(t) \leq \hat{\Lambda} B_s(t)\ \forall t \geq 0$ and $\mathbf{r}_s(t) - \mathbf{r}_m(t - T_m) = \dot{\mathbf{e}}_s + \Lambda \mathbf{e}_s$ we obtain

$$\begin{aligned}\dot{\tilde{V}} \leq& -\frac{1}{4}\dot{\mathbf{e}}_s^T B_s \dot{\mathbf{e}}_s - \frac{1}{4}\mathbf{e}_s^T \Lambda \tilde{\Lambda} B_s(t) \mathbf{e}_s - \rho_s^2 \|\mathbf{r}_s(t - T_s)\|^2 \\ &- \frac{1}{4}(\mathbf{r}_m - \Gamma \mathbf{r}_s(t - T_s))^T B_m (\mathbf{r}_m - \Gamma \mathbf{r}_s(t - T_s)) + \mathbf{f}_m^T \mathbf{r}_m.\end{aligned}$$

Now, let us define $\mathbf{e}_r(t) = \mathbf{r}_m(t) - \Gamma(t)\mathbf{r}_s(t - T_s)$. Notice that $\mathbf{f}_m^T \mathbf{r}_m = \mathbf{f}_m^T \mathbf{e}_r + \mathbf{f}_m^T \Gamma \mathbf{r}_s(t - T)$, where we can upper bound $\mathbf{f}_m^T \Gamma \mathbf{r}_s(t - T)$ by $\frac{1}{4\rho^2}\|\Gamma \mathbf{f}_m\|^2 + \rho^2 \|\mathbf{r}_s(t - T_s)\|^2$. Then,

$$\begin{aligned}\dot{\tilde{V}} \leq& \mathbf{f}_m^T \mathbf{e}_r - \frac{1}{4}\mathbf{e}_r^T B_m \mathbf{e}_r + \frac{1}{4\rho^2}\|\Gamma \mathbf{f}_m\|^2 - \frac{1}{4}\dot{\mathbf{e}}_s^T B_s \dot{\mathbf{e}}_s \\ &- \frac{1}{4}\mathbf{e}_s^T \Lambda \tilde{\Lambda} B_s \mathbf{e}_s.\end{aligned}$$

If we now denote $\underline{\sigma}(A)$ as the minimum eigenvalue of the matrix A, let $\epsilon \in (0, 1)$ be a constant, and recall that $\|\mathbf{f}_m\| \leq \eta$, then we can show that

$$\dot{\tilde{V}} \leq \eta \left\| \mathbf{e}_r \right\| - \frac{\epsilon}{4}\underline{\sigma}(B_m) \left\| \mathbf{e}_r \right\|^2 - \frac{1}{4}(1-\epsilon)\underline{\sigma}(B_m) \left\| \mathbf{e}_r \right\|^2$$
$$- \frac{1}{4}\underline{\sigma}(B_s) \left\| \dot{\mathbf{e}}_s \right\|^2 - \frac{1}{4}(1-\epsilon^2)\underline{\sigma}(B_s)\underline{\sigma}(\Lambda\tilde{\Lambda}) \left\| \mathbf{e}_s \right\|^2$$
$$- \frac{\epsilon^2}{4}\underline{\sigma}(B_s)\underline{\sigma}(\Lambda\tilde{\Lambda}) \left\| \mathbf{e}_s \right\|^2 + \frac{\eta^2}{4\rho^2} \left\| \Gamma \right\|^2$$

and consequently,

$$\dot{\tilde{V}} \leq -\frac{1}{4}(1-\epsilon)\underline{\sigma}(B_m) \left\| \mathbf{e}_r \right\|^2 - \frac{1}{4}\underline{\sigma}(B_s) \left\| \dot{\mathbf{e}}_s \right\|^2$$
$$- \frac{1}{4}(1-\epsilon^2)\underline{\sigma}(B_s)\underline{\sigma}(\Lambda\tilde{\Lambda}) \left\| \mathbf{e}_s \right\|^2 < 0$$

for $\|\mathbf{e}_s\| > \frac{\eta}{\epsilon}\delta(B_m)$, where

$$\delta(B_m) = \left(\frac{4}{\underline{\sigma}(B_m)^2} + \frac{\|\Gamma\|^2}{\rho_s^2 \underline{\sigma}(B_m(t-T_m))\underline{\sigma}(\Lambda\tilde{\Lambda})} \right)^{\frac{1}{2}}.$$

Since $\dot{\tilde{V}} < 0$ for sufficiently large $\|\mathbf{e}_s\|$, we conclude that the system is closed-loop stable and the coordination error is uniformly ultimately bounded with ultimate bound given by $\frac{\eta}{\epsilon}\delta(B_m)$. ■

In general, the above theorem states that the slave's coordination error converges to a ball of radius $\frac{\eta}{\epsilon}\delta(B_m)$. Therefore, $||\mathbf{e}_s|| \to 0$ as either $\underline{\sigma}(B_m(t)) \to \infty$ or $\eta \to 0$. We can even formulate a more precise bound under steady state conditions. For instance, suppose that $\dot{\mathbf{q}}_i \to 0$, $\mathbf{q}_i(t - T_i) \to \mathbf{q}_i(t)$, and $\Gamma(t) \to 1$. Then, the master dynamics (8) simplifies to $2\mathbf{f}_m = -B_m\Lambda(\mathbf{q}_s - \mathbf{q}_m)$, which implies that

$$\|\mathbf{q}_m - \mathbf{q}_s\| = 2\left\|(B_m\Lambda)^{-1}\mathbf{f}_m\right\| \leq 2\eta\left\|B_m\Lambda\right\|^{-1}.$$

Thus, it is easy to note that by increasing B_m, the error effectively goes to zero.

V. Simulations

As a mean of validation, we simulated the response of two 1-DOF teleoperators with the proposed controller. Both master and slave robots have identical linear dynamics with $M_i = 1[kg]$, $C_i = 0[kg/s]$, and $g_i = 0[N]$ and are coupled through an asymmetric time-delayed communication channel with $T_m = 0.6[s]$ and $T_s = 0.4[s]$. The environment is modeled as a stiff wall located at $q_s = 4[m]$ with a reaction force given by

$$f_s = \begin{cases} -10\dot{q}_s - 500(q_s - 4)[N], & \text{if } q_s \geq 4[m] \\ 0[N], & \text{otherwise} \end{cases}$$

while the human is modeled as a constant force source for the first $30[s]$ and then as a PD-type controller, i.e.,

$$f_m = \begin{cases} 12[N], & \text{if } 0 \leq t \leq 30[s] \\ -20\dot{q}_m - 25q_m[N], & \text{if } t > 30[s]. \end{cases}$$

The control parameters and update law for the wave impedance are $\Lambda = 5[1/s]$ and

$$\overline{\beta}(t) = \begin{cases} 2, & \text{if } B_m(t) < B_{cont} \\ 0, & \text{if } B_m(t) = B_{cont} \end{cases}$$

$$\underline{\beta}(t) = \begin{cases} 2\left(5 - 4\frac{B_m(t) - B_{free}}{B_{cont}}\right), & \text{if } B_m(t) > B_{free} \\ 0, & \text{if } B_m(t) = B_{free} \end{cases}$$

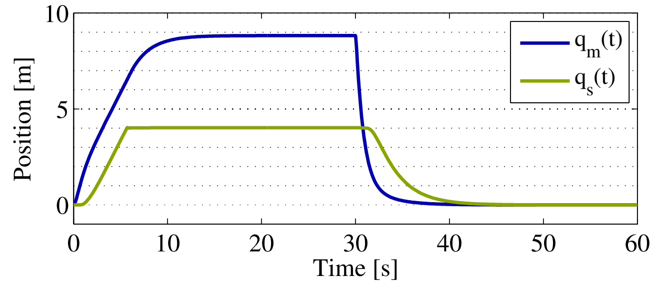

Fig. 1. Position of master and slave robot with constant $B_m(t) = B_{free}$.

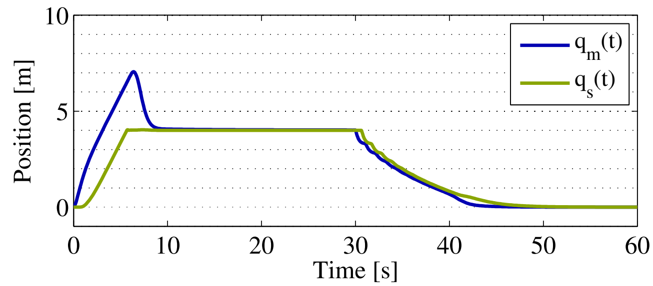

Fig. 2. Position of master and slave robot with time-varying $B_m(t)$.

where $B_{free} = 1[Ns/m]$ and $B_{cont} = 50[Ns/m]$.

We first simulated the system with a constant wave impedance tuned for free motion, i.e., $B_m(t) = B_{free}\ \forall t \geq 0$. The response is illustrated in Fig. 1. Notice that the position error remains bounded and that position tracking is achieved once the slave retrieves from the wall. However, when the slave robot is in contact with the environment, a constant position error (i.e., position drift) of $4.800[m]$ arises, which may mislead the remote perception of the operator.

The same conditions were then simulated employing the proposed control framework and the results are plotted in Fig. 2. Notice that the position error during the contact task is drastically reduced to nearly 0.33% ($0.016[m]$), which represents a subtantial improvement on position tracking from the previous case. Once the slave ceases contact with the wall, both master and slave positions converge to zero.

Note also that the settling time is slightly larger for the proposed controller. This is mainly caused by the transition from B_{cont} to B_{free} as governed by $\underline{\beta}(t)$. In general, faster transitions will allow for shorter settling time as $\Gamma(t) \to 1$ in a small time. However, to do so may produce higher transient oscillations due to ephemeral large values of $\Gamma(t)$ in the control law. In practice, as it will be shown through experiments, these transitions seem to have no drastic effect in the coordination error or settling time when compare to the constant wave impedance's case.

VI. Experiments

Besides simulations, we conducted experiments on a pair of 2-DOF identical planar-revolute-joint robots coupled through constant communication channels with delays $T_m = 0.4[s]$ and $T_s = 0.3[s]$. Both master and slave robots are equipped with a pair of optical encoders that measure link's angular position and velocity (via digital estimation), and a force-torque sensor, located at the end-effector, that measures forces sensed/exerted by the operator/environment. The nonlinear dynamics of the teleoperators (1), with the gravitational torques neglected due to the system's planar configuration, is given by

$$M_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \gamma \end{bmatrix} \tag{26}$$

$$C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} \delta_i \dot{q}_i^2 & \delta_i(\dot{q}_i^1 + \dot{q}_i^2) \\ -\delta_i \dot{q}_i^1 & 0 \end{bmatrix} \tag{27}$$

where $\alpha_i = 52.72 + 5.85\cos(q_i^2)$ $[10^{-2}Nm^2]$, $\beta_i = 3.27 + 2.91\cos(q_i^2)$ $[10^{-2}Nm^2]$, $\gamma = 3.27$ $[10^{-2}Nm^2]$, and $\delta_i = -8.16\sin(q_i^2)$ $[10^{-4}kgm^3]$. For more details on the experimental setup, consult [24].

The desired trajectory (or task) performed by the operator was the following: first, to displace the master teleoperator from the initial position $\mathbf{q}_0 = [0, -\pi]^T$ $[rad]$ to $\mathbf{q}_c = [-0.4\pi, 0]^T$ $[rad]$; then, to hold the master's position around $\mathbf{q}_c$ for nearly $20[s]$; and finally, to return to the initial configuration $\mathbf{q}_0$. At the slave's environment, we placed an aluminum v-shaped wall at $\mathbf{q}_w = [-0.70, -1.52]^T$ $[rad]$ in order to obstruct and lock the motion of the slave robot.

We first conducted the experiment with a constant wave impedance of $(B_{free}^{11}, B_{free}^{22}) = (0.8, 0.6)[Nsm]$, tuned for free motion. The response of the system, with control parameters $\Lambda^{11} = 10[1/s]$ and $\Lambda^{22} = 8[1/s]$, is reported in Fig. 3. Despite the fact that the position error nearly converges to zero during free motion, a large position error arises for both links when the slave robot is in contact with the wall.

We then performed the same experiment for a time-varying wave impedance with $(B_{free}^{11}, B_{free}^{22}) = (0.8, 0.6)[Nsm]$, $(B_{cont}^{11}, B_{cont}^{22}) = (12, 9)[Nsm]$, and update law given by

$$\overline{\beta}^j(t) = \|\mathbf{f}_e(t - T_s)\| \frac{B_{cont}^{jj} - B_m^{jj}(t)}{10}, \qquad \hat{\Lambda}^{jj} = \frac{\Lambda^{jj}}{1.01}$$

$$\underline{\beta}^j(t) = 1.6\left(B_m^{jj}(t) - B_{free}^{jj}(t)\right)$$

where $\mathbf{f}_e(t) \in \Re^2$ is the environmental reaction force sensed at the tip of the slave's end-effector and is related to the $\mathbf{f}_s(t)$ through the Jacobian matrix $J_s(t)$ as $\mathbf{f}_s(t) = J_s^T(t)\mathbf{f}_e(t)$ [22]. The system response is shown in Fig. 4. The position error between master and slave is considerably attenuated during contact with the wall and approaches zero when the slave is free to move.

Fig. 5 contrasts the $\mathcal{L}_2$ norm of the coordination error (i.e., $||\mathbf{q}_m(t) - \mathbf{q}_s(t)||$) for the cases of a constant and a time-varying wave impedance. Notice that the steady-state error during contact is decreased from $0.79[rad]$ to $0.12[rad]$ when employing the time-varying wave impedance approach. When the slave retrieves from the wall, the behavior becomes similar in both cases with the position error approaching zero. The fact that the coordination error does not converge exactly to zero in neither case, while in free motion, can be attributed to considerable bearing friction suffered by the teleoperators as reported in [11], [25].

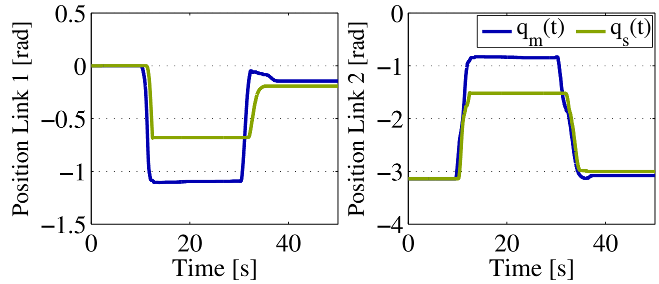

Fig. 3. Position of master and slave robot with constant $B_m(t) = B_{free}$.

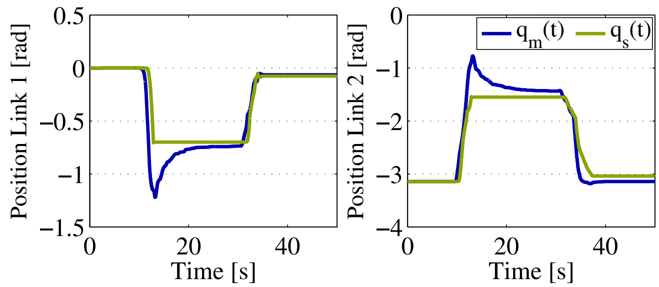

Fig. 4. Position of master and slave robot with time-varying $B_m(t)$.

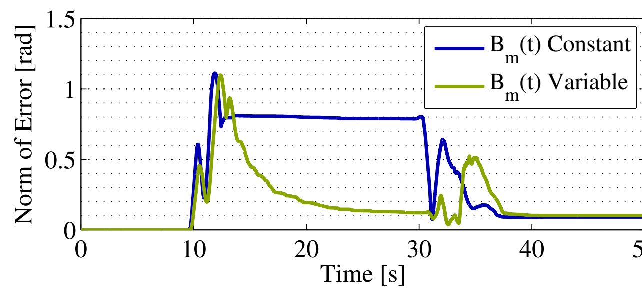

Fig. 5. $\mathcal{L}_2$ norm of the position error.

Similarly, Fig. 6 compares the force applied at the master and slave's end-effectors, i.e., $||\mathbf{f}_h||$ and $||\mathbf{f}_e||$, for both controllers. Observe that in the case of a time-varying wave impedance, the human applied a larger effort trying to retain the master position at $\mathbf{q}_c$. Despite of the operator's larger effort, the coordination error was substantially attenuated. Notice also that, as the system achieves a steady-state, $||\mathbf{f}_h(t)||$ approaches $||\mathbf{f}_e(t)||$[4].

Finally, Fig. 6 also evidences a temporary difference on the force reflection by the time-varying wave impedance approach that lasted for several seconds while the slave was in contact with the wall. This contrast between the high-magnitude force perceived by the operator, which can also be interpreted as an indicator of the characteristic high-valued stiffness/impedance of the environment being remotely con-

[4]It should be mentioned that, in general, $||\mathbf{f}_h||$ should not necessarily converge to $||\mathbf{f}_e||$, since the proposed control framework only guarantees force reflection with respect to $\mathbf{f}_m$ and $\mathbf{f}_s$, or equivalently, $||J_m^T\mathbf{f}_h(t)|| \to ||J_s^T\mathbf{f}_e(t)||$. However, due to the fact that both master and slave have identical dynamics and that the coordination error becomes sufficiently small, the Jacobian matrix of both robots are nearly equal, i.e., $J_m \approx J_s$. Therefore, $||\mathbf{f}_h(t)|| \approx ||\mathbf{f}_e(t)||$.

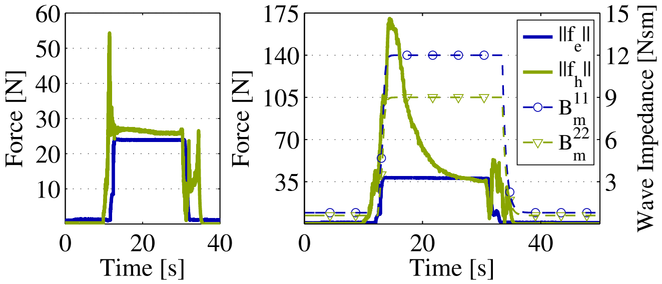

Fig. 6. Force tracking and wave impedance. To the left, the human and environmental force are plotted for the case of a constant wave impedance. To the right, the case of a time varying-wave impedance is illustrated.

tacted, and the moderate sensed environmental force on the slave robot can be better explained by mathematically examining the net forces acting on the system. First, in the case of the slave robot, we have that its position is locked around $\mathbf{q}_w$. Therefore, $\dot{\mathbf{q}}_s = 0$. Using then (8), (26) and (27), we can easily show that $J_s^T \mathbf{f}_e = \mathbf{f}_s = -\boldsymbol{\tau}_s$. On the other hand, in the case of the master teleoperator, $\dot{\mathbf{q}}_m \neq 0$ and consequently,

$$J_m^T \mathbf{f}_h = \mathbf{f}_m = M_m(\mathbf{q}_m)\dot{\mathbf{r}}_m + C_i(\mathbf{q}_m, \dot{\mathbf{q}}_m)\mathbf{r}_m - \boldsymbol{\tau}_m. \quad (28)$$

Thus, even though the wave impedance stabilizes at a constant value B_{cont} fast enough (as reported in Fig. 6) and the control $\boldsymbol{\tau}_m \approx \boldsymbol{\tau}_s$; the force perceived by the operator $\mathbf{f}_m$ is still affected by the first two terms in (28); which in turn, exclusively depend on the velocities and accelerations of the master teleoperator. This means that the strong forces sensed at the operator's site are mostly owed to the slow attenuation of the position error. In fact, it is not until the system achieves an equilibrium (i.e., $(\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i) \rightarrow 0$) that the first two terms in (28) vanish and force reflection (i.e., static) is ultimately achieved.

VII. Conclusions

By exploiting the independent-passivity and dependent-transparency properties of the wave impedance on wave-based teleoperation, we proposed a novel control framework that achieves stability and transparency when transitioning between unobstructed and obstructed environments. The proposed control framework builds on [19] and introduces a time-varying wave impedance that passively changes from an arbitrary small value, suitable for free motion, to a sufficiently large value, ideally for stiff environments. The control strategy is proved to achieve closed-loop stability of the teleoperation system and to enforce smooth position tracking and static force reflection independently of delays, even for a non-passive human model. We finally showed the validity of the control strategy through simulations and experiments on a pair of nonlinear teleoperators.

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