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Shape Control of a Deformable Object by Multiple Manipulators

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Abstract—**Shape control of a deformable object by a robotic system is a challenging problem because of the difficulty of imposing shape change by a finite number actuation points to an essentially infinite dimensional object. In this paper, a new approach to shape changing of deformable objects by a system of manipulators is presented. First, an integrated dynamic equation of motion for a system of multiple manipulators handling a deformable object is developed. The initial and the final shapes of the deformable object are specified by curves that represent the boundary of the object. We design an optimization-based planner that minimizes an energy-like criterion to determine the locations of the contact points on the desired curve representing the final shape of the object. The motion of each manipulator is controlled independently without any communication between them. The simulation results demonstrate the efficacy of the proposed method.**

I. Introduction

DEFORMABLE objects are used in many economically important industries such as food, automobiles, aerospace, leather and packaging. Most of the tasks involving handling of deformable objects are done manually which make them labor intensive and time consuming requiring fast and accurate manipulation of material by skilled human operators. Automatic handling of rigid objects is readily available and can be reprogrammed to perform different tasks when needed. Application of such multiple robot systems for handling deformable objects will reduce the time associated with manual handling and increase productivity by lowering the cost and increasing the precision. However, this a difficult and challenging task primarily due to low stiffness of the deformable objects that makes them easily susceptible to large deformations. As a result, deformable objects may exhibit deformation by changing in shape and volume under the applied forces during handling. Besides, there are large variations of stiffness and damping properties of different deformable objects which are further influenced by environmental conditions. The aforesaid properties and behavior of deformable objects pose many problems when interacting with robots to automate their handling.

Deformable objects are used extensively in a wide range of industrial applications including the manufacturing and assembly of garment and footwear products, the packing industry and aircraft manufacturing. Manipulation of deformable parts such as inserting an elastic bar into a hole of rigid body and vice versa for precise assembly is a major industrial problem. It can be seen from a survey [1] that although it was identified as a high priority area, very less work has been done on the automated handling of deformable objects. The reason is the complexity of modeling and control of larger deformations of deformable objects in real time. This research addresses the issue related to shape control of a deformable object by controlling its boundary such that an initial shape is transformed to the desired one. Shape change task can be found in forming operations of bread dough, pizza dough in food industry and in formation of clay products of desired shapes in pottery work where a large deformation of the object is required. For instance, the forming process of clay product requires stretching and shrinking operations by multiple fingers. Molding can be used in automatic shape control of deformable objects for batch production. However, there are operations where molding can not be applied and hence developing an automated process to control the shape of the deformable objects could be beneficial.

In this paper, we focus on the problem of effecting shape change on a deformable object given the initial actuation points. That is, we design a controller for deforming the object from an initial shape to a final one if we know how many actuation points are available and where they are located. Also note that the nature and capabilities of the end-effectors of the manipulators are important for effecting shape change. For example, the nature of shape control will be different if the end-effectors can only apply push or pull or both to deform the object. We then develop the coupled dynamics of the multiple manipulators along with the deformable object. We develop a control law for shape changing based on the inverse dynamics method. We design a planner based on an optimization technique to compute the desired final contact locations.

The remainder of this paper is organized as follows: in Section II, we discuss the existing literature on deformable object modeling and shape control. A mathematical description of the problem is given in Section III. Section IV outlines the procedure to derive the mathematical model of the deformable object. We also develop the integrated dynamics involving the object dynamics and the manipulator dynamics in this section. The control method is discussed in Section V. The effectiveness of the derived control law is demonstrated by simulation in Section VI.

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Finally, the conclusions and the future work are discussed in Section VII.

II. Existing Literature on Deformable Object Modeling and Shape Control

A considerable amount of work has been performed on multiple robotic systems during the last few decades [2-8]. Mostly, the position and/or force control of multiple manipulators handling a rigid object were studied in [2-4]. However, there were some works on handling deformable object by multiple manipulators presented in [5-8]. Sun *et al*. [5] presented a cooperation task of controlling the reference motion and the deformation when handling a deformable object by two manipulators. In [6] the control of position/orientation and the vibration suppression of each contact points of a flexible object by two manipulators were presented. Zheng *et al*. [7] studied the assembly operation of inserting a flexible beam into a rigid hole. Hirai *et al*. [8] developed a robust control law for manipulation of 2D deformable parts using tactile and vision feedback to control the motion of the deformable object with respect to the position of selected reference points. However, to the best of our knowledge there is no such work on controlling the overall shape of a deformable object by multiple manipulators.

A wide variety of modeling approaches have been presented in the literature dealing with computer simulation of deformable objects [9]. These are mainly derived from physically-based models to produce physically valid behaviors. Mass spring models are one of the most common forms of deformable objects. A general mass-spring model consists of a set of point masses connected to its neighbors by massless springs. Mass-spring models have been used extensively in facial animation [10], cloth motion [11] and surgical simulation [12]. Tokumoto *et al.* [13] proposed a modeling method of viscoelastic objects for deformation control. Many researchers have used finite element methods to model the deformable objects. Finite element models have been used in the computer simulation to model facial tissue and predict surgical outcomes [14, 15]. An important issue of modeling deformable objects such as food (e.g., dough) and tissue (e.g., organs) is that they have both elasticity and viscosity properties. These objects are called rheological objects. A comprehensive study on rheological objects based on three main deformation properties such as residual deformation, bouncing displacement and vibration decrease was performed by Tokumoto *et al.* [16]. It is concluded that a three element model i.e., a spring and a damper in parallel (Voigt model) with another damping element in series gives an appropriate choice of deformable object modeling, which we choose in this work.

Considerable effort has been put into the design of robust and optimal control methods for shape control of 2-D flexible structure. Various approaches for shape control algorithm were presented in [17-19]. A robust shape control of a flexible structure against parameter variation was presented in [17]. Kashiwase *et al*. [18] proposed a simple zero P-ID controller by using many sensors and actuators. Dang *et al.* [19] developed a shape control algorithm of a flexible structure that was used for shape morphing. These works are important and have many useful applications such as morphing of aircraft wings. In all of these cases, microactuators are used for shape morphing of flexible structures that are embedded or bonded on the surface. Thus these works focus on small deformation (i.e., micro actuation) and are not geared towards producing macro-level shape change of the object. In our intended application, for example changing the shape of a dough, we will need large deformation. Also note that, in our framework, it is possible to apply pulling, pushing, stretching etc. at the boundary contact points to create the intended shape.

III. Problem Statement

Consider a planar deformable object that moves and changes in time defining a new shape when subjected to applied forces. We define that the object is a compact and closed set in $\Re^2$. The boundary of such a set is represented by a closed two dimensional curve. This closed curve is described parametrically as

$$\mathbf{c}(\sigma) = [x(\sigma), y(\sigma)] \tag{1}$$

where σ is the normalized curve parameter, $0 \le \sigma \le 1$. The motion of a deformable object can be described by translation, rotation and deformation. Translation and rotation can be described by the translation of the center of mass of the object and a rotation about an axis passing through the center of mass, while the deformation can be analyzed by the dynamics of shape change.

Thus, the shape error can be defined as

$$D = \sum_{k=1}^{\infty} \| \mathbf{c}(\sigma_k) - \hat{\mathbf{c}}(\sigma_k) \| \tag{2}$$

where, $\mathbf{c}(\sigma)$ is the actual shape and $\hat{\mathbf{c}}(\sigma)$ is the desired shape. In this paper, we investigate the shape change problem in the following form:

Problem: Given a desired shape $\hat{\mathbf{c}}(\sigma)$ and a current shape $\mathbf{c}(\sigma)$ of the planar object, find a control action on $\mathbf{c}(\sigma)$, such that the shape error $D(\mathbf{c}(\sigma), \hat{\mathbf{c}}(\sigma))$ defined according to some norm is minimum.

However, since the boundary of the object, i.e., the current shape will be actuated by a finite number of actuation points, say n, the above problem is restated as follows:

Define all end effectors positions that are also the contact points on the deformable objects $\mathbf{p}(\mathbf{q}) \in \Re^{2n}$ as

$$\mathbf{p}(\mathbf{q}) = \begin{bmatrix} \mathbf{p}_1(\mathbf{q}) \\ \vdots \\ \mathbf{p}_n(\mathbf{q}) \end{bmatrix} \tag{3}$$

where, $\mathbf{q}$ is the joint variable of the manipulators. Define also the location of these contact points on the desired curve $\mathbf{p}_d(\boldsymbol{\sigma}) \in \Re^{2n}$ as

$$\mathbf{p}_d(\boldsymbol{\sigma}) = \begin{bmatrix} \hat{\mathbf{c}}(\sigma_1) \\ \vdots \\ \hat{\mathbf{c}}(\sigma_n) \end{bmatrix} \tag{4}$$

where, $\sigma_i \in \Re$ $(i = 1, \cdots, n)$ is a curve parameter corresponding to the position of the i-th manipulator. Then define the shape error at the actuation points, $\mathbf{e}(\mathbf{q},\boldsymbol{\sigma}) \in \Re^{2n}$, as

$$\mathbf{e}(\mathbf{q},\boldsymbol{\sigma}) = \mathbf{p}(\mathbf{q}) - \mathbf{p}_d(\boldsymbol{\sigma}) \tag{5}$$

The overall shape error of the object is defined as the summation of the shape error at each actuation point expressed as

$$d = \sum_{i=1}^{n} \| \mathbf{e}_i(\mathbf{q}_i, \sigma_i) \| \tag{6}$$

Note that the above definition of the shape error assumes the discrepancy in each contact point location between the initial shape and the final desired shape. It is a discrete representation of the shape error. Thus a controller that reduces this error to zero does not guarantee that all points on the curve will exactly match the desired curve. However, it will be shown in Section VI that such discrete definition, which is motivated by practicality (e.g., a limited number of actuation points), produce acceptable shape change results. It will also be shown that as the number of contact points increases, the convergence to the desired shape improves.

Thus the problem becomes: Given a desired shape and a current shape of the planar object, and also given the number of actuation points, find a control action such that $\mathbf{e}(\mathbf{q},\boldsymbol{\sigma}) \to \mathbf{0}$.

IV. Modeling of System Dynamics

Consider a schematic in Fig. 1 where n manipulators are performing a cooperative task on a deformable object changing shapes from $\mathbf{c}(\sigma)$ to $\hat{\mathbf{c}}(\sigma)$. We assume that all the manipulator grippers are firmly attached to the deformable body. Knowing the positions of the manipulator tips at the moment when the contact with the manipulated object is made, we derive the mathematical model of the whole system such that all the manipulators move in a coordinated manner to induce the desired motion.

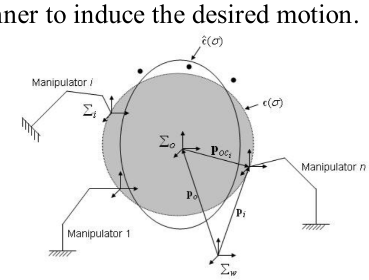


Fig. 1: Schematic of manipulators handling a deformable object.

A. Deformable Object Dynamics

We model the deformable body as a rheological object using discrete networks of mass-spring-damper system. The point masses are located at the nodal points and a three element model is inserted between them. Fig. 2 shows a single layer of the deformable body. Each element is labeled as E_j for $j = 1,2,\cdots,NE$, where NE is total number of elements in a single layer. Let m_i be the mass point at the i-th node of the system. The dynamic equation representing each point mass can be written in the standard form as,

$$\mathbf{M}_{oi}\ddot{\mathbf{z}}_i + \mathbf{C}_{oi}\dot{\mathbf{z}} + \mathbf{G}_{oi} = \mathbf{F}_i + \mathbf{F}_i^{\text{int}} \tag{7}$$

for $i = 1,2,\cdots,N$ where N is the total number mass point in a layer. $\mathbf{z}_i \in \Re^2$ is the position vector at the i-th mass point represented as $\mathbf{z}_i = [x_i, y_i]^T$, $\mathbf{M}_{oi} \in \Re^{2\times2}$ is the symmetric positive definite mass matrix, $\mathbf{C}_{oi} \in \Re^{2\times2}$ is the coriolis matrix, $\mathbf{G}_{oi} \in \Re^2$ is denoted the terms containing stiffness, spring lengths etc. and $\mathbf{F}_i \in \Re^2$ is the external force vector given by $\mathbf{F}_i = [F_{ix}, F_{iy}]^T$. In this paper, $\mathbf{F}_i$ represents the contact force exerted by the manipulators. $\mathbf{F}_i^{\text{int}} \in \Re^2$ is the internal reaction force vector generated due to the interaction among the mass points in the network.

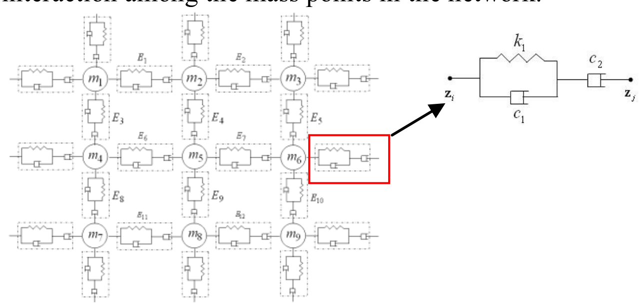


Fig. 2: Model of a rheological object with interconnected mass-spring-damper. k_1 is the spring stiffness, c_1 and c_2 are the damping coefficient of non-residual and residual deformation part, respectively.

B. Manipulator Dynamics

Consider n planar manipulators with 2 DOF are handling a deformable object. The Lagrange equation of motion of the i-th manipulator in the Cartesian space is given by

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{p}}_i + \mathbf{C}(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{p}}_i + \mathbf{G}(\mathbf{q}_i) = \mathbf{u}_i - \mathbf{F}_i \tag{8}$$

where $\mathbf{q}_i \in \Re^2$ is the joint variables, $\mathbf{p}_i \in \Re^2$ is the position vector of the i-th end-effector, $\mathbf{M}_i(\mathbf{q}_i) \in \Re^{2\times2}$ denotes the symmetric positive definite inertia matrix, $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \Re^{2\times2}$ accounts for the coriolis effects, $\mathbf{G}_i(\mathbf{q}_i) \in \Re^2$ represents the elastic effects due to spring, bending stiffness and gravity. $\mathbf{u}_i \in \Re^2$ is the control input and $\mathbf{F}_i \in \Re^2$ is the contact force that the end-effectors exerts on the object.

C. Coupled Dynamics

In order to integrate the dynamics of the multiple manipulators with the dynamics of the deformable object, we first rewrite the dynamics of the deformable objects in terms of two classes of mass points, one set represents the n number of mass points that are in contact with the manipulators and the rest of the mass points (i.e., $N-n$) that constitute the object model. The dynamics of the first set of mass points are written by replacing $\mathbf{z}$ by $\mathbf{p}$ as

$$\mathbf{M}_{oi}\ddot{\mathbf{p}}_i + \mathbf{C}_{oi}\dot{\mathbf{p}}_i + \mathbf{G}_{oi} = \mathbf{F}_i + \mathbf{F}_i^{\text{int}}, \quad i = 1,2,3,\dots,n \tag{9}$$

And the second set of dynamics equations are written as

$$\mathbf{M}_{oj}\ddot{\mathbf{z}}_j + \mathbf{C}_{oj}\dot{\mathbf{z}}_j + \mathbf{G}_{oj} = \mathbf{F}_j^{\text{int}}, \quad j = n+1,\dots,N \tag{10}$$

By eliminating $\mathbf{F}_i$ from the manipulator dynamics (8) and the object dynamics of the first set (9), the coupled dynamics between the manipulators and the contact mass points are obtained as follows:

$$\overline{\mathbf{M}}_i\ddot{\mathbf{p}}_i + \overline{\mathbf{C}}_i\dot{\mathbf{p}}_i + \overline{\mathbf{G}}_i = \mathbf{u}_i \tag{11}$$

where, $\overline{\mathbf{M}}_i = \mathbf{M}_i + \mathbf{M}_{oi}$, $\overline{\mathbf{C}}_i = \mathbf{C}_i + \mathbf{C}_{oi}$, and $\overline{\mathbf{G}}_i = \mathbf{G}_i + \mathbf{G}_{oi} - \mathbf{F}_i^{\text{int}}$

The coupled dynamics can be rewritten more concisely for all contact mass points as follows:

$$\overline{\mathbf{M}}\ddot{\mathbf{p}} + \overline{\mathbf{C}}\dot{\mathbf{p}} + \overline{\mathbf{G}} = \mathbf{u} \tag{12}$$

where, $\mathbf{p} = (\mathbf{p}_1^T \cdots \mathbf{p}_n^T)^T$, $\overline{\mathbf{M}} = blockdiag(\overline{\mathbf{M}}_1 \cdots \overline{\mathbf{M}}_n)$, $\overline{\mathbf{C}} = blockdiag(\overline{\mathbf{C}}_1 \cdots \overline{\mathbf{C}}_n)$, $\overline{\mathbf{G}} = (\overline{\mathbf{G}}_1^T \cdots \overline{\mathbf{G}}_n^T)^T$, and $\mathbf{u} = (\mathbf{u}_1^T \dots \mathbf{u}_n^T)^T$.

Eq. (12) can be used to design the control law for the manipulators and Eq. (10) can be integrated to find the position of the internal mass points and the boundary mass points those are not interacted by the manipulators. The motions of these mass points are affected by the reaction forces generated due to the interaction.

V. Design of Shape Controller

We develop an optimization technique that generates the motion plans of each individual manipulator, i.e., $\mathbf{p}_d(\sigma_i)$, $i = 1,2,\cdots,n$, such that the initial shape is transformed to the desired one by minimizing a given energy-like criterion of the whole body. The energy-like function is given by:

$$\mathbf{\Pi} = \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i(\boldsymbol{\sigma})^T \mathbf{W}_i \boldsymbol{\varepsilon}_i(\boldsymbol{\sigma}) \tag{13}$$

where $\boldsymbol{\varepsilon}_i(\boldsymbol{\sigma}) = \mathbf{p}_i(\mathbf{q}) - \mathbf{p}_d(\boldsymbol{\sigma})$, $i = 1,2,\dots,n$, $\mathbf{W}_i \in \Re^{2\times 2}$ are diagonal weight matrices. The optimal values of the curve parameters will be the solution of:

$$\min_{\sigma_i \in \boldsymbol{\sigma}} \mathbf{\Pi} \tag{14}$$

The planner generates the desired reference locations for the chosen contact points. All manipulators in the system are controlled to execute the desired motion at the contact points using the following control law

$$\mathbf{u} = \overline{\mathbf{M}}\mathbf{v} + \overline{\mathbf{C}}\dot{\mathbf{p}} + \overline{\mathbf{G}} \tag{15}$$

$$\mathbf{v} = \mathbf{J}_{\mathbf{M}}^{-1}\{\ddot{\mathbf{p}}_d - \dot{\mathbf{J}}_{\mathbf{M}}\dot{\mathbf{q}} - \mathbf{K}_p\mathbf{e} - \mathbf{K}_d\dot{\mathbf{e}}\} \tag{16}$$

where $\mathbf{J}_{\mathbf{M}} \in \Re^{2n\times 2n}$ is the Jacobian of the manipulator, $\mathbf{K}_p$, and $\mathbf{K}_d \in \Re^{2n\times 2n}$ are symmetric and positive definite matrices, and $\ddot{\mathbf{p}}_d \in \Re^{2n}$ is desired acceleration which is zero in our present case as we assume that the final contact locations for each manipulators are constant in the time of interest.

Substituting the control law (15) and (16), into the integrated system (12), we obtain the following error system.

$$\ddot{\mathbf{e}} + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} = 0 \tag{17}$$

which shows that $\mathbf{e}$ and $\dot{\mathbf{e}}$ converge to zero exponentially when $\mathbf{K}_p$, and $\mathbf{K}_d$ are symmetric and positive definite. The schematic of the proposed controller is depicted in Fig. 3.

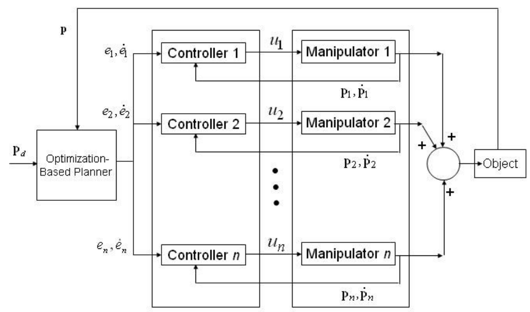


Fig. 3: Schematic of the proposed controller.

VI. Simulation and Discussion

In this section, a shape control task by multiple manipulators is presented using the developed control law. We choose a circular deformable object of diameter 0.08 m as an initial shape in 2D. The goal is to deform it to obtain two different shapes: i) an ellipse, and ii) a square. The desired shapes can be represented as:

i) For the ellipse, $\hat{\mathbf{c}}(\sigma) = [a\cos(2\pi\sigma), b\sin(2\pi\sigma)]$, where $a = 0.05$ m, $b = 0.032$ m, and $\sigma \in [0\,1]$.

ii) For the square, the vertices are (a,0), (0,a), (-a,0), (0,-a), where, $a = 0.05$ m. In this work, we represent the square using a B-spline to get a continuous representation of the boundary.

We discretize the circle with 145 elements of mass-spring-damper. We assume that the total mass remains constant during the whole deformation process. Referring to [16] for a rheological object model of wheat dough we take m=0.006 kg, k_1=460 N/m, c_1=2452 Ns/m , and c_2=4904 Ns/m. In this simulation, we use $\mathbf{K}_p$=$k_p\mathbf{I}$, and $\mathbf{K}_d$=$k_d\mathbf{I}$, where k_p=500, k_d=50 and $\mathbf{I} \in \Re^{2\times 2}$ unity matrix. We present four different simulations based on the number of actuation points placed on the periphery of the object. In particular, we choose 12, 18, 24, and 36 actuation points placed equidistantly in all cases as our initial contact locations. The desired final locations for these contact points are determined by the optimization-based planner (eq. (14)), which are then given to the controller (eq. (15)) as the reference final locations to be achieved.

The initial, the desired and the final shapes of the deformable object when controlling shape from the circle to the ellipse and from the circle to the square are shown in Figs. 4 and 5. In these figures we are showing only the boundary points to represent the shapes. These figures show the feasibility of obtaining the desired shapes by using the presented control law. As expected, the performance increases with increasing the number of contact points. The time responses of the root mean square (RMS) of the shape error at the actuation points, $\mathbf{e}$, are depicted in Figs. 6 and 7. The figures show their convergence to zero. The forces exerted on the object to change the shape from the circle to the ellipse and from the circle to the square are shown in Figs. 8 and 9 for case IV of two different contact locations. It can be seen that the forces are high initially and decrease with time as the shapes converge to the desired one. Forces of all other contact points also reduce with time when the shape converges to the desired one. Force values reach to zeros when final shapes are achieved.

VII. Conclusions and Future Work

We develop a new framework to achieve shape control of deformable objects by robotic manipulators. The methodology presented here describes the modeling techniques of deformable rheological objects subjected to continuous deformation. To control the shape of the deformable object, a simulation based on a desired planar curve is presented and the boundary is regulated subject to continuous deformation. An optimization-based motion planning scheme for the object is introduced to determine the desired curve parameter. The planner generates the desired inputs to the system according to the computed desired curve parameter. The motion of each manipulator is controlled independently without communication among them.

Future work includes testing the controller with more complex shapes of the deformable object and verifying the methodology by experiments. The feasibility of implementing a large number of actuators in real time could be a difficult task. Motivated by the work presented in [20] and the reference therein we are investigating how many actuation points and their locations are required to efficiently effect the desired shape change operation.

References

[1] M. Saadat and P. Nan, "Industrial applications of automatic manipulation of flexible materials," *Industrial Robot: An International Journal*, vol. 29, no. 5, pp. 434-442, 2002.

[2] Y. F. Zheng and J. Y. S. Luh, "Control of two coordinated robots in motion," *Proc. of 24th IEEE Conf. on Decision and Control*, vol. 24, pp. 1761-1766, 1985.

[3] S. Hayati, "Hybrid position/force control of multi-arm cooperating robots," *Proc. of IEEE Int. Conf. on Robotics and Automation*, vol. 3, pp. 82-89, 1986.

[4] J. M. Tao and J. Y. S. Luh, "Robust position and force control for a system of multiple redundant robots," *Proc. of IEEE Int. Conf. on Robotics and Automation*, vol. 3, pp. 2211-2216, 1992.

[5] D. Sun, X. Shi and Y. Liu, "Modeling and cooperation of two-arm robotic system manipulating a deformable object," *Proc. of IEEE Int. Conf. on Robotics and Automation*, vol. 3, pp. 2346-2351, 1996.

[6] D. Sun, Y. Liu and J. K. Mills, "Cooperative control of a two-manipulator system handling a general flexible object," *Proc. of IEEE Int. Conf. on Intelligent Robots and Systems*, vol. 1, pp. 5-10, 1997.

[7] Y. F. Zheng, R. Pei and C. Chen, "Strategies for automatic assembly of deformable objects," *Proc. of IEEE Int. Conf. on Robotics and Automation*, vol.3, pp. 2598-2603, 1991.

[8] S. Hirai, T. Tsuboi, and T. Wada, "Robust grasping manipulation of deformable objects," *Proc. of IEEE Symp. on Assembly and Task Planning*, pp. 411-416, 2001.

[9] S. F. F. Gibson and B. Mirtich, "A survey of deformable modeling in computer graphics," *MERL Technical Report,* TR97-19, 1997.

[10] D. Terzopoulos and K. Waters, "Physically-based facial modeling, analysis, and animation," *The Journal of Visualization and Computer Animation*, vol. 1, no. 2, pp. 73-80, 1990.

[11] D. Baraff and A. Witkin, "Large steps in cloth simulation," *In ACM SIGGRAPH 98 Conf. Proc.*, pp. 43-54, 1998.

[12] A. Joukhadar and C. Laugier, "Dynamic simulation: model, basic algorithms, and optimization," In J.-P. Laumond and M. Overmars, editors, *Algorithms For Robotic Motion and Manipulation*, pp. 419-434, A. K. Peters Ltd. 1997.

[13] S. Tokumoto, Y. Fujita, and S. Hirai, "Deformation modeling of viscoelastic objects for their shape control," *Proc. of the 1999 IEEE Int. Conf. on Robotics and Automation,* pp. 767-772, May 1999.

[14] R. M. Koch, M. H. Gross, F. R. Carls, D. F. Von Buren, G. Fankhauser, Y. I. H. parish. "Simulating facial surgery using finite element models," *In ACM SIGGRAPH 96 Conf. Proc.*, pp. 421-428, 1996.

[15] S. D. Pieper, D. R. Laub, and J. M. Rosen. "A finite element facial model for simulating plastic surgery," *Plastic and Reconstructive Surgery*, 96(5), pp. 1100-1105, Oct. 1995.

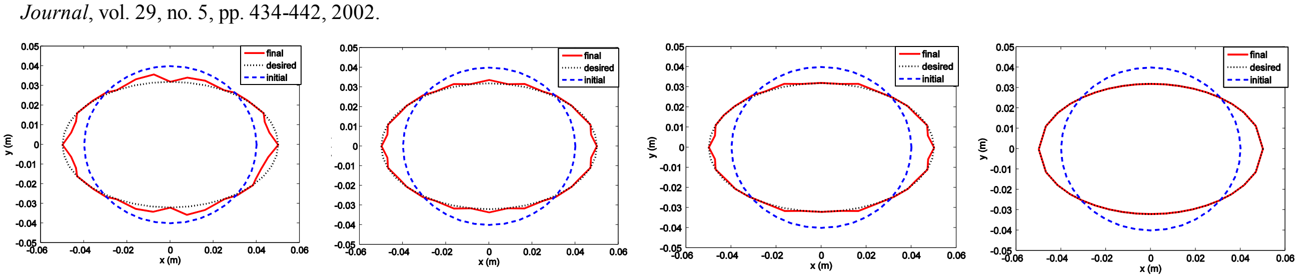
[16] S. Tokumoto, S. Hirai, and H. Tanaka, "Constructing virtual rheological objects," *Proc. World Multiconference on Systemics, Cybernetics and Informatics,* pp. 106-111, Auland 2001.

[17] T. Shima, T. Fukunaga, K. Tsujita and K. Tsuchiya, "Robust shape control of a flexible structure," *Acta Astronautica*, vol. 41, no. 1, pp. 7-14, 1997.

[18] T. Kashiwase, M. Tabata, K. Tsuchiya and S. Akishita, "Shape control of flexible structures," *Journal of Intelligent Material Systems and Structures*, vol. 2, no. 1, pp. 110-125, 1991.

[19] P. Dang, F. L. Lewis, K. Subbarao, H. Stephanou, "Shape control of flexible structure using potential field method," *17th IEEE Int. Conf. on Control Applications*, Texas, USA, pp. 540-546, 2008.

[20] Y. Q. Chen, K. Moore, and Z. Song, "Diffusion boundary determination and zone control via mobile actuator-sensor networks (MAS-net)-challenges and opportunities," *Intelligent Computing: Theory and Applications II, SPIE Defense and Security Sym*. 2004



(I) (II) (III) (IV)

Fig. 4: Initial (blue dashed), desired (black dotted) and final (red solid) shapes for four different cases based on number of actuation points when controlling shape from the circle to the ellipse: (I) 12, (II) 18, (III) 24, and (IV) 36.

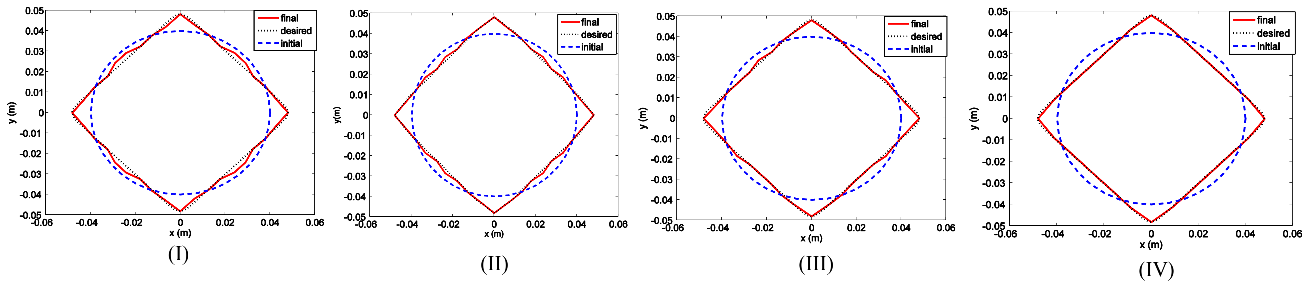


Fig. 5: Initial (blue dashed), desired (black dotted) and final (red solid) shapes for four different cases based on number of actuation points when controlling shape from the circle to the square: (I) 12, (II) 18, (III) 24, and (IV) 36.

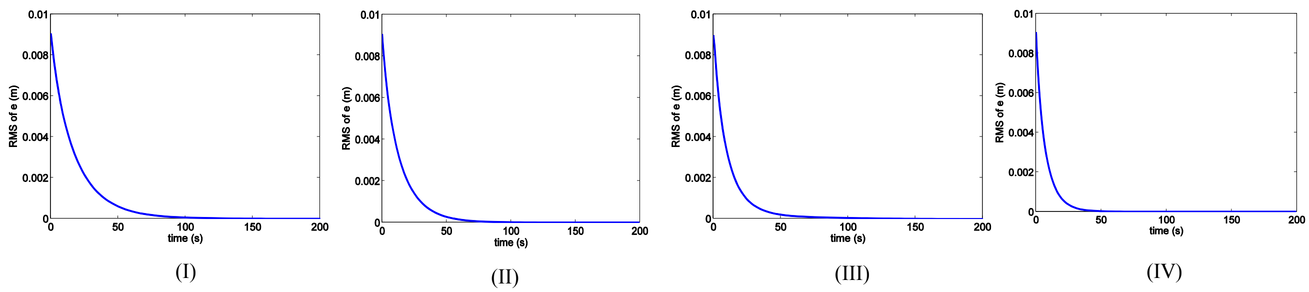


Fig. 6: RMS of shape error, e, for four different cases based on number of actuation points when controlling shape from the circle to the ellipse: (I) 12, (II) 18, (III) 24, and (IV) 36.

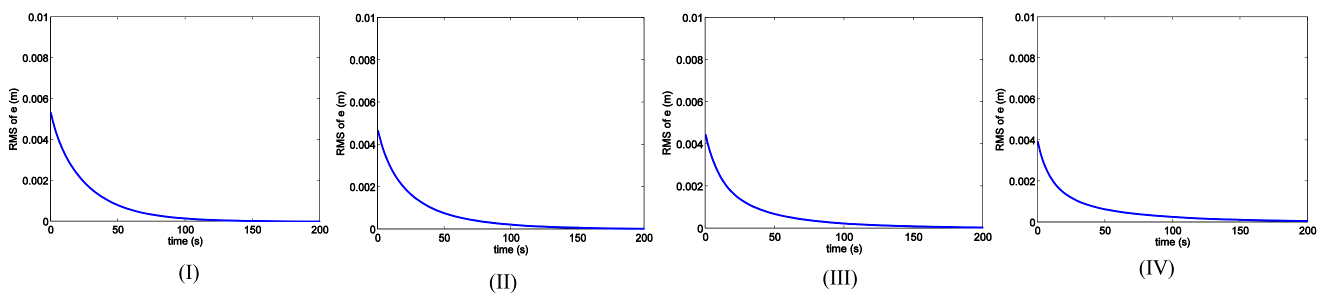


Fig. 7: RMS of shape error, e, for four different cases based on number of actuation points when controlling shape from the circle to the square: (I) 12, (II) 18, (III) 24, and (IV) 36.

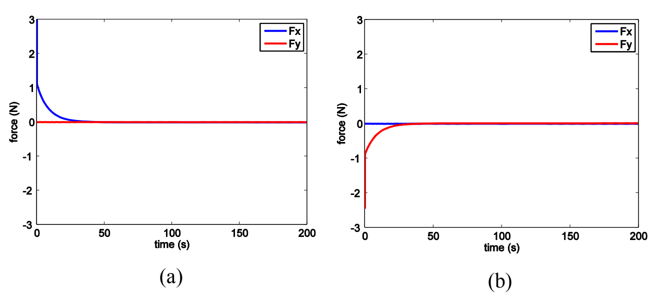


Fig. 8: Contact forces when controlling shape from the circle to the ellipse for case IV at two contact locations: (a) 0 and (b) 90 degrees with respect to *x*-axis, respectively.

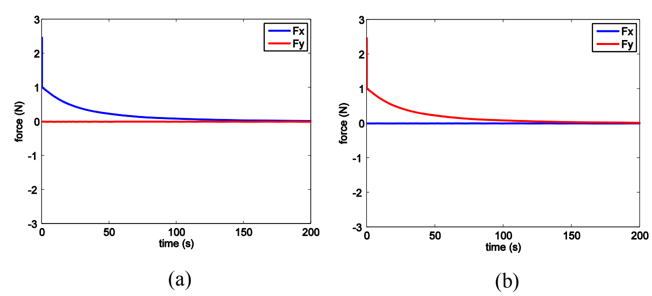


Fig. 9: Contact forces when controlling shape from the circle to the square for case IV at two contact locations: (a) 0 and (b) 90 degrees with respect to *x*-axis, respectively.