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Mathematical Modeling of the Prediction Mechanism of Sensory Processing in The Context of a Bayes Filter

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***Abstract*— Prediction is a very important element of human intelligence and plays a major role in human behavior, perception, and learning. This paper presents the development of a mathematical model of the prediction mechanism in the context of a Bayes filter, which is the predominant schema used for integrating temporal data in the field of robot mapping and localization problems. We propose a generalized anticipatory Bayes filter that uses revised sensor values obtained from the prediction process at the measurement-update step to enhance the performance of the sensor model. The development of a generalized anticipatory Bayes filter is not only an extension of the original Bayes filter, but also a mathematical model of the human prediction mechanism of sensory processing. This work was verified by experiments using observed data.**

I. Introduction

Prediction is a very important element of human intelligence, with major roles in human behavior, perception, and learning. Here, *Prediction* is defined as a process based on past experience, which takes the present situation as input to estimate future consequences. In addition to the various benefits of prediction, this paper will concentrate on the benefits of prediction for perception.

In human perception, the results of the prediction of future consequences serve as reference signals that can be used in perceptual processing to avoid system instabilities due to delayed or missing sensory feedback [1]. As future information can be predicted and thus be made available before actual sensory information arrives, system control and stability can be optimized by incorporating predicted feedback information.

In the field of robot mapping and localization problems, Bayes filters are single dominating schemas for integrating temporal data. Bayes filters are closely related to Kalman filters, hidden Markov models, dynamic Bayes networks, and partially observable Markov decision processes [2]. Bayes filters extend Bayes' rule to temporal estimation problems and use a recursive estimator to compute a sequence of posterior probability distributions over quantities that cannot be observed directly.

A Bayes filter algorithm usually has two essential steps: a prediction step and a measurement-update step [3]. The prediction step calculates the belief regarding the newly predicted state based on the prior belief for the state at a previous time point and the control information. In the measurement-update step, the Bayes filter multiplies the belief for the predicted state by the probability that the new measurement may have been observed. In this step, the probability provided by robotic measurement is called a sensor model.

The sensor model in the Bayes filter provides a suitable starting point for integrating a prediction mechanism with the Bayes filter. After calculation of the belief regarding the predicted state from the prediction step, we can predict the sensory inputs of a mobile robot based on this newly obtained information. The predicted sensor value is then compared with the actual measured sensor value; if a mismatch is observed between these two sensor values, the actual sensor value will be mapped to a revised sensor value that represents internal acceptance of the mobile robot over the actually measured value. In the measurement-update step, we should consider both the actual sensor value and the revised sensor value concurrently.

Integration of a prediction mechanism resulted in a new version of the Bayes filter, which we call a generalized anticipatory Bayes filter. The development of a generalized anticipatory Bayes filter is not only an extension of the original Bayes filter, but also represents mathematical modeling of the human prediction mechanism of sensory processing; this is the main contribution of our research.

This paper is structured as follows. Section II presents a brief discussion of related work. In Section III, we discuss the development of a generalized anticipatory Bayes filter incorporating a prediction mechanism of sensory processing. Experiments using an actual robot platform and conclusions are presented in Sections IV and V, respectively.

II. Related Work

The topic of prediction has been studied by a number of researchers in different disciplines ranging from biology, psychology, and physiology, to engineering and artificial intelligence. In research on mobile robotics, several methods have been suggested for utilizing the prediction mechanism in a variety of cognitive and behavioral systems.

Rosen reported that the functionality of an anticipatory system: (a) does nothing if the model predicts that the target system is likely to stay in a "desirable" course, or (b) activates the effector to correct the "trajectory" of the target system if the model warns that an undesirable outcome is imminent [4].

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Mind RACES [5] was a three-year EC-funded project that focused mainly on the concept of anticipatory systems related to the behavior, perception, learning, and emotion of intelligent agents. This outstanding and systematic research regarding the specialized topic of anticipation produced many important publications.

Hoffmann proposed a conceptual framework for anticipatory behavioral control (ABC), which assumes that behavioral competence emerges through the acquisition of action-effect rather than stimulus-response associations [6]. That is, body movements become determined by anticipations of their own sensory consequences.

Butz *et al.* discussed how anticipatory mechanisms may be beneficial for the process of sensory processing [7]. The prediction of future states, and thus the prediction of future stimuli, influences stimulus processing. To be able to make predictions, the agent must use a predictive model of its environment. That is, sensory anticipation is strongly related to preparatory attention in psychology, in which top-down processes such as task-related expectations influence sensory processing. Sensory anticipatory behavior results in a predisposition for processing sensory input. For example, the agent may become more susceptible to specific sensory input while ignoring other sensory input.

Few studies have been dedicated to the application of prediction in robot mapping and localization. In one experimental study two robots had the goal of exchanging places while navigating through an area with or without obstacles [8]. In another study, an episodic memory-based approach was proposed for computing anticipatory robot behavior in a partially observable environment, and the results of a robot navigation experiment were presented [9].

Zhang and Suh proposed an anticipatory Bayes filter that integrates a prediction mechanism with a sensor model [10]. In their study the sensor model was conditioned on both the predicted state and the experience of the robots, and modeled the latter term as an inhibition weight for computational convenience. The authors dealt with the inhibition weight as a static value obtained from the offline learning process.

III. Generalized Anticipatory Bayes Filter

As noted above, a Bayes filter includes prediction and measurement-update steps as presented in equations (1) and (2), respectively. Compared to our proposed Bayes filter, we call this the original Bayes filter. Before developing a generalized anticipatory Bayes filter, it is necessary to introduce the concept of sensory transformation.

The robot must simultaneously process multi-dimensional feature data according to the various sensor data obtained from different types of apparatus, and a variety of methods are available to process these raw data. To apply a revision procedure to actual sensor values, it is necessary to transform these multi-dimensional data to an abstract data type that shares the same value range. We take the form $f \rightarrow z$ to denote the proposed transformation. Here, f is the measured sensor value and z is the transformed value for f, where $z \in [0, 1]$. The details of the transformation

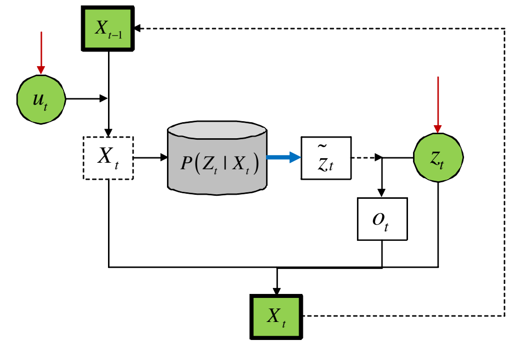


Fig. 1. Structure of the generalized anticipatory Bayes filter.

methods are dependent on the data type that belongs to a different dimension, and the properties of that data type and the magnitude of the transformed value will follow the importance of the data type and the actual measured value. We attach the subscript $_t$ for z_t to reflect that the sensor values are measured at consecutive time points.

A. Original Bayes Filter

In the Bayes filter equation, we follow the common notation using X, u, and z to refer to state, control, and measurement, respectively, and can therefore present the Bayes filter as:

$$\overline{bel}(X_t) = \sum_{X_{t-1}} P(X_t|u_t, X_{t-1}) bel(X_{t-1}) \quad (1)$$

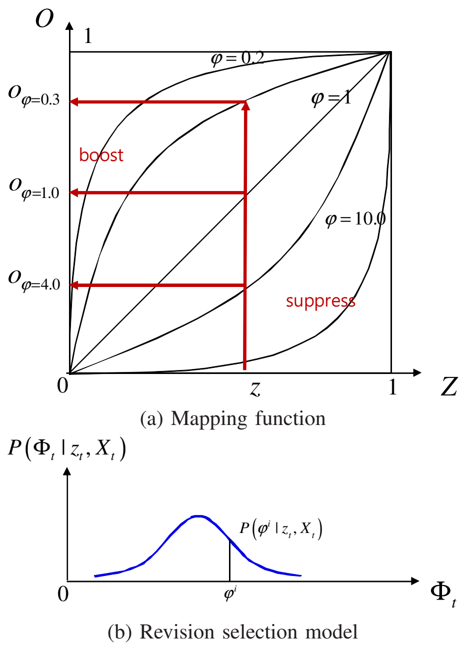
$$bel(X_t) = \eta P(z_t|X_t)\overline{bel}(X_t) \quad (2)$$

where $\overline{bel}(X_t)$ is the belief that the robot assigns to state X_t after the prediction step, the probability $P(X_t|u_t, X_{t-1})$ is used to model the motion of robots and is called the motion model, the probability $P(z_t|X_t)$ is used to model the measurement of the robots and is called the sensor model, and η is a normalizing constant. As output, the Bayes filter gives the belief $bel(X_t)$ at state X_t based on the control and measurement of the robot. To allow a distinction between this and the new sensor model described later, the probability $P(z_t|X_t)$ is hereafter called the original sensor model. In this paper, we adopt the notation using upper and lower case letters to represent random variables and known values, respectively. Given two steps of Bayes filter equations, our objective is to extend the original sensor model, so a prediction mechanism should be included in this model.

B. Prediction and Revision of Sensor Value

As mentioned briefly in the Introduction, we introduce two new types of sensor value, predicted sensor values and revised sensor values, which play key roles in our generalized anticipatory Bayes filter.

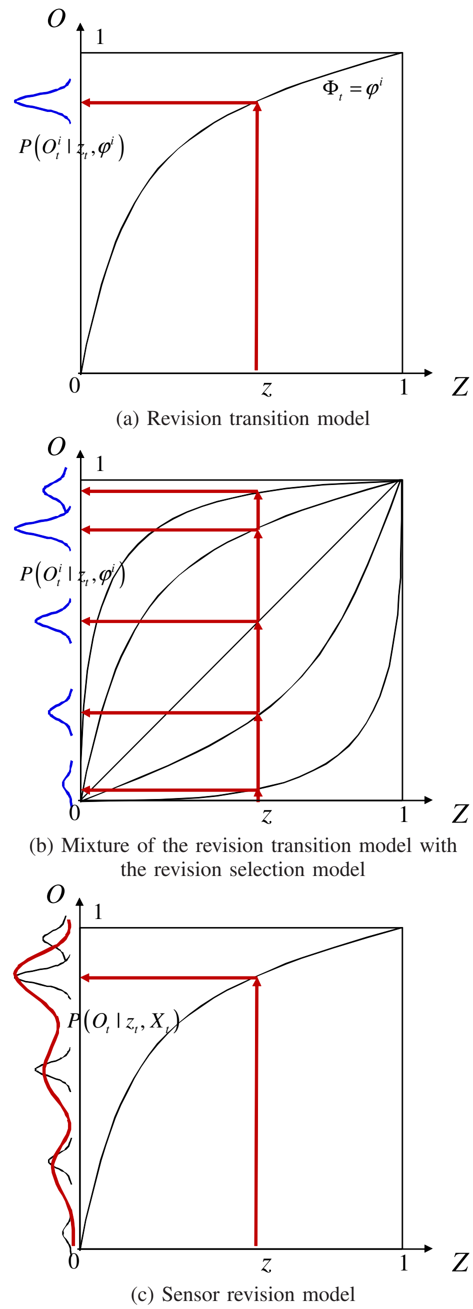
Figure 1 shows the overall structure of the generalized anticipatory Bayes filter. The variables X_{t-1} and X_t in the shaded boxes represent the states of the robot at times $t-1$ and t, respectively. The shaded circles show the control value u_t and the actually measured sensor value z_t. Following



(a) Mapping function

(b) Revision selection model

Fig. 2. Mapping function.



(a) Revision transition model

(b) Mixture of the revision transition model with the revision selection model

(c) Sensor revision model

Fig. 3. Illustration of the stochastic revision process.

the prediction step in the Bayes filter, we can calculate the belief regarding the predicted state X_t from X_{t-1} and u_t by equation (1). Based on this state value X_t, we can predict the sensor value $\tilde{z}_t$ using the knowledge of the environment in which the robot resides. In the Bayesian approach, the sensory knowledge of the environment is abstracted in the form of a probabilistic model, $P(Z_t|X_t)$. Affected by the predicted sensor value $\tilde{z}_t$, the measured sensor value z_t is revised to a new value o_t that represents internal acceptance of the mobile robot against the actually measured sensor value z_t. In our generalized anticipatory Bayes filter, the actual sensor value z_t and the revised sensor value o_t are both considered in the sensor model.

The dotted arrow to the right side of $\tilde{z}_t$ in Fig. 1 indicates that the predicted sensor value is used as a reference signal to affect the revision process for actual sensor value. The arrow pointing from X_t to X_{t-1} in Fig. 1 represents the recursive process of the Bayes filter, implying that the state value X_t will be used as X_{t-1} in the next computation cycle.

C. Stochastic Revision Process

The revision of the actual sensor value z_t to the revised sensor value o_t follows a stochastic revision process. At the beginning of this section, we introduced a method to transform variant dimensional sensor data to an abstract data type that shares the same value range. Such a transformation on real sensor value greatly facilitates the revision process for the abstracted sensor value. We define a mapping function:

$$O_t = c z_t^{\Phi_t} \tag{3}$$

where O_t is the revised sensor variable, Φ_t is the revision variable, and c is a constant used to control the magnitude of revision. Here, we only consider the case where $c = 1$.

The revision variable Φ_t takes a non-negative real number as a value. As shown in Fig. 2(a), an actually measured sensor value z_t can be mapped to different revised sensor values o_t according to different values of Φ_t from 0 to $+\infty$. When revision variable $\Phi_t = 1$, the actual sensor value is not revised; if Φ_t takes a value less than 1, then o_t is larger than z_t, and we say that z_t is boosted; in contrast, if Φ_t is larger than 1, then we say z_t is suppressed.

Selection of the revision value φ^i is governed by a revision selection model $P(\Phi_t|z_t, X_t)$, as shown in Fig. 2 (b). The shape of the revision selection model is mainly controlled by the predicted sensor value $\tilde{z}_t$ and the degree of mismatch between z_t and $\tilde{z}_t$ because the robot expects to observe the predicted sensor value $\tilde{z}_t$ whenever it is resident in a familiar environment.

The stochastic revision process is conducted as shown in Fig. 3(a). First, select a revision variable φ^i according to the revision selection model $P(\Phi_t|z_t, X_t)$. After a fixed revision value φ^i is chosen, the actual sensor value z_t is then mapped to the revised sensor variable O_t. Here, the revised sensor variable is no longer a fixed value but an unknown variable that forms a probabilistic distribution $P(O_t^i|\varphi^i, z_t)$, we call it the revision transition model. The superscript ^{i} above O_t^i indicates that this probabilistic distribution is formed through a transition path caused by revision value φ^i.

As the choice of φ^i is governed by the revision selection model $P(\Phi_t|z_t, X_t)$, the probability value of $P(\varphi^i|z_t, X_t)$ functions as a bandwidth on the revision transition path. By taking the mixture of each revision transition model $P(O_t^i|\varphi^i, z_t)$ with the value of $P(\varphi^i|z_t, X_t)$, some of the distributions are emphasized and the rest are weakened according to the value of $P(\varphi^i|z_t, X_t)$. These effects are shown in Fig. 3 (b).

When the revision transition model $P(O_t^i|\varphi^i, z_t)$ is weighted by the probability value $P(\varphi^i|z_t, X_t)$, it is no longer strictly a probability distribution. Therefore, we should sum all these revision value-dependent distributions $P(O_t^i|\varphi^i, z_t)$ along the coordinate of O_t to reconstruct a new form of probabilistic distribution, which we call the sensor revision model. This process is shown in Fig. 3 (c). In the revision process from X_t, z_t to O_t the revision selection variable Φ_t functions as an intermediate variable and is collapsed at the final summation operation. Therefore, the sensor revision model can be expressed in the form $P(O_t|z_t, X_t)$; this will be clarified in the following subsection through mathematical derivation.

In summary, the whole stochastic revision process described above involves marginalization of the revision value φ^i to finally obtain a probabilistic distribution over the revised sensor variable O_t. Once we have obtained a probabilistic model over the revised sensor variable O_t, then we can determine a specific value o_t from the sampling process.

D. Generalized Anticipatory Bayes Filter

To mathematically integrate the original Bayes filter with the prediction mechanism as described in section III (B) and (C), it is necessary to include not only the actually measured sensor value z_t, but also the revised sensor value o_t in the sensor model. When the revised sensor value is included in the sensor model that becomes a new type of sensor model, this is named the predictive sensor model and is represented as $P(O_t, z_t|X_t)$. The predictive sensor model can be expressed as a product of the sensor revision model and the original sensor model:

$$P(O_t, z_t|X_t) \quad = \quad P(O_t|z_t, X_t)P(z_t|X_t) \qquad (4)$$

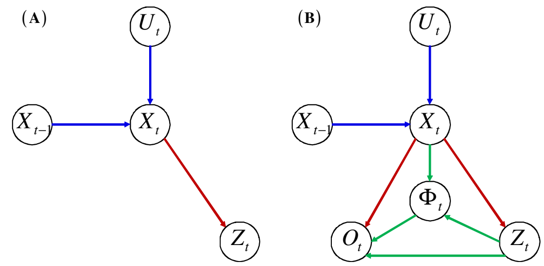


Fig. 4. Dynamic Bayesian networks for two different Bayes filters. (A) Original Bayes filter. (B) Generalized anticipatory Bayes filter.

The sensor revision model is generated by marginalization of the revised sensor variable Φ_t, which can be derived as follows:

$$\begin{aligned} P(O_t|z_t, X_t) &= \sum_{\varphi^i} P(O_t, \varphi^i|z_t, X_t) \\ &= \sum_{\varphi^i} P(O_t, |\varphi^i, z_t, X_t)P(\varphi^i|z_t, X_t) \\ &= \sum_{\varphi^i} P(O_t, |\varphi^i, z_t)P(\varphi^i|z_t, X_t) \\ &= \sum_{\varphi^i} P(O_t^i, |\varphi^i, z_t)P(\varphi^i|z_t, X_t) \qquad (5) \end{aligned}$$

The first line involves marginalization of the revision variable Φ_t; in the second line, we have applied the product rule and divided it into two terms, with the right term being the revision selection model; on the third line, X_t is eliminated, because when the actual sensor value z_t and the revision value φ^i are given, then the revised sensor variable O_t is independent of the state variable X_t; the change in the fourth line relative to the third line is that there is a superscript ^{i} on the variable O_t^i, because the revision variable φ^i does not contribute directly to the distribution of O_t but does so indirectly through O_t^i.

We derived the predictive sensor model through equations (4) and (5), but it cannot be directly used in the measurement-update step because $P(O_t, z_t|X_t)$ is a probabilistic distribution and not a value. Therefore one more step is needed to extract a fixed probability value, $P(o_t, z_t|X_t)$. This was done by sampling a certain value o_t from the predictive sensor model $P(O_t, z_t|X_t)$, obtainig the probability value at that point, and finally substituting this sampled value $P(o_t, z_t|X_t)$ into the measurement-update equation.

Therefore, we completed the process of developing a generalized anticipatory Bayes filter, reorganized as follows:

$$\overline{bel}(X_t) = \sum_{X_{t-1}} P(X_t|u_t, X_{t-1})bel(X_{t-1}) \qquad (6)$$

$$P(O_t, z_t|X_t) = P(O_t|z_t, X_t)P(z_t|X_t) \qquad (7)$$

$$o_t \sim P(O_t, z_t|X_t) \qquad (8)$$

$$bel(X_t) = \eta P(o_t, z_t|X_t)\overline{bel}(X_t) \qquad (9)$$

Equation (6) and (7) are equal to equations (1) and (4), *Applying The Predictive Sensor Model*

1. Calculate $\overline{bel}(X_t)$ for each predicted state
2. Calculate the winner state $\widehat{X}_t$
3. If $\overline{bel}(\widehat{X}_t) < \theta$ then apply $P(z_t|\widehat{X}_t)$ and return to 1
4. Apply $P(o_t, z_t|\widehat{X}_t)$ according to the result of $Draw(\overline{bel}(\widehat{X}_t))$
5. Continue loop 1 through 4.

Fig. 5. Algorithm for applying the Predictive Sensor Model

respectively, but the marginalization process on φ^i was hidden for concise representation. In equation (8), the symbol "$\sim$" indicates the operation from which a certain value of o_t was sampled. In equation (9), the value $P(o_t, z_t|X_t)$ was taken from the predictive sensor model $P(O_t, z_t|X_t)$ at point o_t and used to weight the belief regarding predicted state X_t to complete the measurement-update step. The dynamic Bayesian networks for the original Bayes filter and the generalized anticipatory Bayes filter are shown in Fig. 4 for comparison.

Application of the predictive sensor model in the generalized anticipatory Bayes filter should be performed carefully. If the predicted state is proven to be correct, then the predictive sensor model can improve the performance of the measurement-update step as intended. However, if the predicted state is shown to be incorrect, this can cause an even worse result. We propose the stochastic algorithm shown in Fig. 5 to avoid this problem. In this algorithm, the winner state is defined as follows:

$$\widehat{X}_t = \underset{X_t \in X}{\operatorname{argmax}} \sum_{X_{t-1}} P(X_t|u_t, X_{t-1}) bel(X_{t-1}) \qquad (10)$$

It is reasonable to apply the predictive sensor model for the winner state $\widehat{X}_t$ which has the maximum belief at time t. The function $Draw(\overline{bel}(\widehat{X}_t))$ determines whether $P(o_t, z_t|\widehat{X}_t)$ is performed, according to the results of sampling over probability $\overline{bel}(\widehat{X}_t)$.

IV. EXPERIMENTS

In this paper, we have proposed a generalized anticipatory Bayes filter that includes a revised sensor value in the measurement-update step to improve the performance of the Bayes filter.

A. Experimental Environment

We evaluated our approaches through actual robot experiments in an indoor environment, as shown in Fig. 6. Pioneer 3-AT was used as the real robot platform in the experiments, with three Web cameras mounted on top of the robot (Fig. 7). These cameras were placed on the same plane and at a height of 100cm from the floor; one faced leftward, one forward, and one rightward. Images were collected at a resolution of 320×240 pixels from the three cameras in turn at a frame rate of 10 fps. However, we used only the images from the leftward- and rightward-facing cameras in the experiments.

The experimental environment was represented as a topological map, and the performance of the generalized anticipatory Bayes filter was evaluated in the context of a

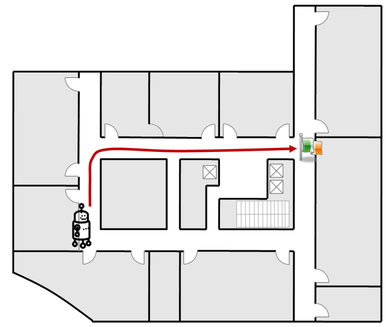


Fig. 6. Experimental environment. The start and finish points are indicated with the robot and flag icons respectively.

localization problem. Topological maps are graph-like spatial representations in which nodes represent states in the agent's configuration space and edges represent the system trajectories that take the agent from one state to another. The meanings of nodes and edges in a topological map vary according to the application as well as the algorithms used to build them [11]. We represented the nodes of a topological map as distinct places and edges as transitions made by a robot as it moved from one place to another. When the robot navigates a path, positions that share similar sensory features are grouped together. Our experimental environment was divided into 13 distinct places using the method described previously [10].

B. Visual Data Encoding

A number of methods are available for encoding vision data, and the choice of method depends mainly on the intended application. In corridor-like environments, few visual features can be uniquely identified. However, interior moldings, doors, and vanishing lines in the environment form a number of straight lines; therefore, in processing vision data in our experiments, we decided to adopt only linear features.

First, all line segments in an image were extracted into one of four groups according to the angle formed between the line and the horizontal plane. The four groups were divided according to the quantized angles of 0^o, 45^o, 90^o and 135^o. After allocating a line to a group, all line lengths in the same group were added to yield a four-element histogram. As we used images taken from the left and right cameras, we had eight values at the end of each image acquisition step. These data were put into one vector and divided by the length of that vector, which yielded a normalized vector of length 1. By inspecting this vector value, we obtained an understanding of the presentation of line components and their quantitative tendencies in the measurement data.

C. Implementation of The Generalized Anticipatory Bayes Filter

In Fig. 1 the symbol $\tilde{z}_t$ indicates the predicted sensor value. The problem of how to predict a sensor value when the related state is given can be solved by two approaches: sampling from the sensor model $P(z_t|X_t)$, or using the expected sensor value obtained from the sensor model. In this experiment, we simply adopted the latter approach.

The revision selection model $P(\Phi_t|z_t, X_t)$ is the most important element in the generalized anticipatory Bayes filter, and so this model should be chosen carefully. Controlling the shape of the sensor revision model $P(O_t|z_t, X_t)$ directly affects the performance of the generalized anticipatory Bayes filter. In general, when the revision sensor model is evenly distributed, the revision ability is weakened. In contrast, when it is concentrated on a given area, the revision ability is strengthened.

Figure 3(b) shows an example of revision transition model $P(O_t^i|z_t, \varphi^i)$. When the values for z_t and φ^i are given, the mean of this model should be located around a certain point:

$$o_t^i = (z_t)^{\varphi^i} \tag{11}$$

and the variances are the same for all different φ^i. Keeping the same shape for all revision transition models can have benefits with regard to canceling these terms from computations.

The third step in Fig. 5 introduces the parameter θ to determine whether to apply the predictive sensor model in the measurement-update step. This value controls the degree of belief that the robot has for the present state; a high value of θ indicates that a higher degree of belief is necessary for application of the predictive sensor model.

D. Experimental Results

The purpose of this experiment was to assess the total enhancement attributable to the predictive sensor model. We compared the performance of the original Bayes filter and the generalized anticipatory Bayes filter based on the resulting belief score. The main difference between the original sensor model and the predictive sensor model is that the latter utilizes the information of the predicted state, to filter out the ambiguity caused by outlier sensor data.

Our experiment demonstrated an improvement in performance using the generalized anticipatory Bayes filter. The scores using the original Bayes filter and the Anticipatory Bayes filter were 52.34% and 73.36%, respectively, indicating a performance improvement of 28.65%. This enhancement of the Bayes filter was mainly due to integration of the prediction mechanism with the sensor model, allowing the outlier sensor data to be effectively filtered out by our proposed predictive sensor model. In pattern recognition, outlier data represent small numbers of samples that may be clustered as members of a certain group but that contradict the main property of that group, and so tend to lead to incorrect conclusions. The resulted scores were relatively low, because in the localization process the odometer data were not taken into account. We focused on the improvement of localization performance of the Anticipatory Bayes filter against the original Bayes filter, when all parameters and the experimental conditions were the same. A related demonstration video is available [12].

Fig. 7. Real robot platform. A Pioneer 3-AT mounted with three web cameras for image gathering.

V. Conclusions and Future Work

In this paper, we proposed a predictive sensor model to improve the efficiency of the measurement-update step in the Bayes filter. We proposed a method that includes the revised sensor value in the ordinary sensor model. Thus, we developed a generalized anticipatory Bayes filter to mimic the human prediction mechanism. Our experimental results showed that applying the prediction mechanism to the Bayes filter affects the perceptual process of sensory input and provides a higher degree of accuracy than the original sensor model.

In future work, we will evaluate our method applied to large-scale problems, and problems other than robot localization. Further studies are also required for modeling the human prediction mechanism in behavior control systems.

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