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Optimal trajectory design for parametric excitation walking

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***Abstract*— Parametric excitation walking is one of methods that realize a passive dynamic like walking on the level ground. In parametric excitation walking, up-and-down motion of the center of mass restores mechanical energy and sustainable gait is generated. Walking ability and walking performance strongly depend on the reference trajectory of the center of mass. In this paper, we propose an optimization method for the reference trajectory of parametric excitation walking. There are two problems for optimization. One is that search space of a reference trajectory is inherently infinite dimensional. Another is that it takes long simulation time to generate steady gait for a given reference trajectory. Therefore, the proposing optimization method adopts the following strategy. For the former, we confine the reference trajectory to the quartic spline curve and take the parameter of spline curve as decision variables. For the latter, we discretize the search space and adopt a local search method usually used in combinational optimization problems. We apply the proposed method to a kneed biped robot, and optimize the reference trajectory of its swing leg.**

I. INTRODUCTION

Parametric excitation increases mechanical energy of a system by up-and-down movement of the center of mass. A children's swing is an example using parametric excitation principle. Asano et al. [4] proposed a gait generation method using the parametric excitation principle on level ground. In the method, the center of mass moves up-and-down with telescopic-leg and mechanical energy is restored. Harata et al. [6], [7] applied the parametric excitation method to a kneed biped robot, in which up-and-down movement of the center of mass was realized by bending and stretching knee, and showed that sustainable gait was generated by knee torque only. Harata et al. [9], [10] also proposed parametric excitation based inverse bending walking in which the knee was bent in inverse direction to human.

To apply the parametric excitation principle, the center of mass is moved along appropriate trajectory. In a pendulum, Lavrovskii and Formal'skii [5] proved the optimal trajectory, A→B→C→D→E, shown in Fig. 1, along which the increase of total mechanical energy was maximized, supposed that the length of a pendulum, l, was changed instantaneously. According to the above optimal trajectory, Asano et al. [4] used the cube of sine curve to construct a reference trajectory of the swing-leg, because the trajectory was smooth enough. On the same reason, Harata et al. [6], [7] also used the cube of sine curve for the reference trajectory of the swing-leg knee angle.

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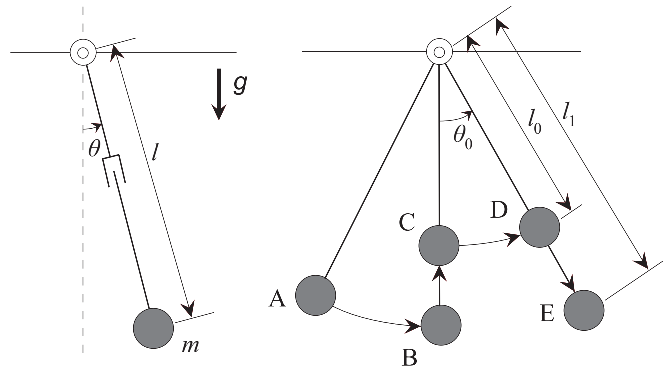

Fig. 1. Optimal trajectory of pendulum for parametric excitation

The optimal trajectory shown in Fig. 1 has nothing but theoretical meaning because the center of mass cannot be moved instantaneously in real machines. Moreover there is significant difference between a pendulum and a biped robot in that the hip joint of a biped robot is movable while the supporting point of a pendulum is fixed at a ceiling, and hence, the trajectory of Fig. 1 may not be optimal for biped robots.

In this paper, we propose an optimization method for a reference trajectory of the kneed biped robot proposed by Harata et al. [6], [7]. In the model, knee torque is designed to track completely a reference trajectory for the knee angle of swing leg. Hence, not only walking performance but also walking possibility strongly depend on a reference trajectory. However, there are two problems to optimize a reference trajectory. One is that search space of a reference trajectory is inherently infinite dimensional. Another is that it takes long simulation time to generate a steady gait for the given reference trajectory.

Therefore, the proposing optimization method adopts the following strategy. For the former, we confine the reference trajectory to the quartic spline curve and set the parameters of spline curve as decision variables. The reason of using a quartic spline curve is that the angular velocity equals to zero at the beginning of bending and the end of stretching. This has the advantage that the energy loss caused by knee strike is almost negligible in stretching the knee to the straight posture, like the cube of sine curve. For the latter, we first generate a steady gait for a certain reference trajectory and then generate a steady gait corresponding to the slightly perturbed trajectory. Moreover, we discretize the search space and utilize a local search method usually used in combinational optimization problems.

We apply the proposed method to the reference trajectory of a knee angle of the kneed biped robot [6], [7]. Our numerical experiments show that the trajectories constructed by quartic spline are more flexible and efficient than those by the cube of sine curve used in the previous papers [6], [7]. Moreover, the optimal trajectory show in Fig. 1 for a pendulum is shown to be also very close to optimal for the biped robot with respect to walking speed.

This paper is organized as following: Section II explains the model of biped robot treated in this paper and parametric excitation walking. In section III, we first introduce the reference trajectory using the quartic spline curve and then propose the optimization method for the reference trajectory. Section IV shows the simulation results of the proposed method. Finally in Section V, we conclude this paper.

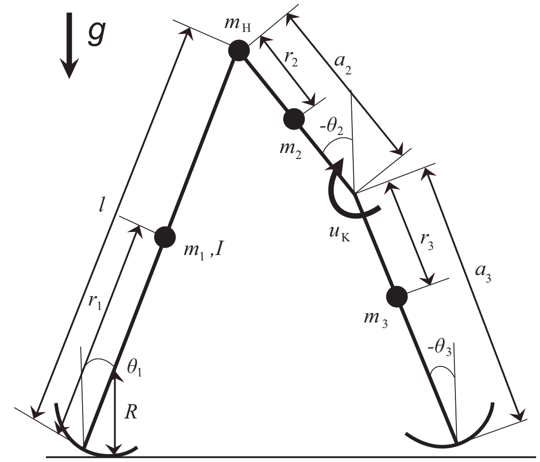

Fig. 2. Model of planar kneed biped robot with semi-circular feet

II. PARAMETRIC EXCITATION WALKING

A. Model of planer biped robot with semicircular feet

Fig. 2 illustrates the biped robot treated in this paper. The robot has four point mass and three degrees of freedom, and has semicircular feet whose centers are on each leg. Since there are two mass on a leg, the support-leg has inertia moment. The dynamic equation during single support phase takes the form

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{S}u_K - \boldsymbol{J}^{\mathrm{T}}\lambda, \qquad (1)$$

where $\boldsymbol{\theta} = [\ \theta_1 \quad \theta_2 \quad \theta_3\ ]^{\mathrm{T}}$ is the generalized coordinate vector, $\boldsymbol{M}$ is the inertia matrix, $\boldsymbol{C}$ is the Coriolis force and the centrifugal force, and $\boldsymbol{g}$ is the gravity vector. The matrix $\boldsymbol{J} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$ is a Jacobian derived from a knee constraint, $\theta_2 = \theta_3$, and $\lambda \in \mathbb{R}$ is knee binding force. Control input vector $\boldsymbol{S}u_K$ in Eq. (1) is given by

$$\boldsymbol{S}u_K = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} u_K. \qquad (2)$$

In the robot, two types of collision occur at a knee and at the ground. The robot gait consists of the following three phases.

- The first phase (Single support phase I): The support-leg rotates around the contact point between a semicircular foot and ground, and the knee of swing-leg is not fixed, that is, knee binding force λ equals to zero, and hence, the knee angle of swing-leg can be controlled by input torque.
- The second phase (Single support phase II): The support-leg rotates around the contact point and the knee of swing-leg is locked in a straight posture by knee binding force. When the first phase changes to the second phase, a completely inelastic collision occurs at a knee.
- The third phase (Double support phase): This phase occurs instantaneously, and the support-leg and the swing-leg are exchanged after the collision at the ground.

B. Parametric excitation walking

In this subsection, we explain the parametric excitation walking with knees [6], [7]. In parametric excitation method, up-and-down motion of the center of mass restores total mechanical energy lost by heel strike. Up-and-down motion is realized by bending and stretching a swing-leg knee. Harata et al. [6], [7] have used the reference trajectory for the relative knee-joint angle as

$$\begin{aligned} \tilde{h}(t) &= (\theta_2 - \theta_3)_d \\ &= \begin{cases} A_m \sin^3\left(\frac{\pi}{T_{\mathrm{set}} - \delta}(t-\delta)\right) & (\delta \le t \le T_{\mathrm{set}}) \\ 0 & (\text{otherwise}), \end{cases} \end{aligned} \qquad (3)$$

where $\delta > 0$ is bending delay, A_m is the maximum bending angle and T_{set} is the desired settling-time which is the period during bending and stretching. In this trajectory, the angular velocity equals to zero when the beginning of bending and end of stretching. From this fact, energy loss caused by knee strike is almost negligible.

We make swing-leg knee angle to track completely the reference trajectory by partial feedback linearization method [6], [7] and the control input is given by

$$\begin{aligned} u_K = &\left[\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}\left[\boldsymbol{M}^{-1}\boldsymbol{S}\right]\right]^{-1}\begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \\ &\times \boldsymbol{M}^{-1}(\boldsymbol{M}\begin{bmatrix} 0 & 0 & -\ddot{h} \end{bmatrix} + \boldsymbol{C}\dot{\boldsymbol{\theta}} + \boldsymbol{g}) \end{aligned} \qquad (4)$$

III. OPTIMIZATION METHOD FOR REFERENCE TRAJECTORY

In this section, we first introduce the reference trajectory consists of quartic spline and then propose an optimization method for the reference trajectory.

A. Reference trajectory using quartic spline curve

A quartic spline curve is given by

$$s(t) = a + bt + ct^2 + dt^3 + et^4, \qquad (5)$$

where a, b, c, d and e are parameters, which are determined uniquely by five constraints. In this paper, a reference trajectory for the knee angle of swing leg $\theta_2 - \theta_3$ is designed

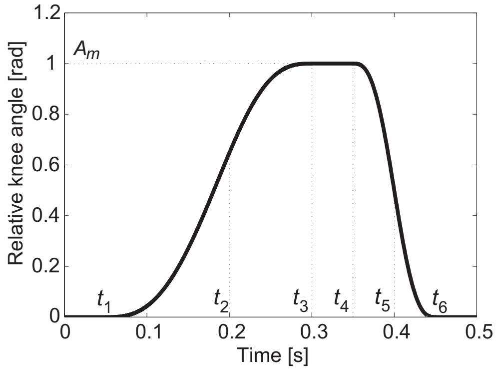

Fig. 3. Example of reference trajectory using quartic spline curve

as following. Suppose that the instance just after the third phase as the initial time of cycle, $t = 0$s, and set six time instances $0 \leq t_1 < t_2 < t_3 \leq t_4 < t_5 < t_6$. t_1 is the beginning of bending a knee, t_3 is the end of bending a knee, t_4 is the beginning of stretching and t_6 is the end of stretching. During the interval $[t_3, t_4]$, a knee of swing leg is maintained to be the maximum bending angle A_m. In addition, t_2 and t_5 are the connecting point. Four intervals $[t_1, t_2]$,$[t_2, t_3]$,$[t_4, t_5]$,$[t_5, t_6]$ are connected by spline curves. Consequently, the reference trajectory h is given by

$$
\begin{aligned}
h(t) &= (\theta_2 - \theta_3)_d \\
&= \begin{cases}
0 & \text{if } 0 \leq t < t_1, t_6 < t \\
a_1 + b_1 t + c_1 t^2 + d_1 t^3 + e_1 t^4 & \\
 & \text{if } t_1 \leq t \leq t_2 \\
a_2 + b_2 t + c_2 t^2 + d_2 t^3 + e_2 t^4 & \\
 & \text{if } t_2 \leq t \leq t_3 \\
A_m & \text{if } t_3 \leq t \leq t_4 \\
a_4 + b_4 t + c_4 t^2 + d_4 t^3 + e_4 t^4 & \\
 & \text{if } t_4 \leq t \leq t_5 \\
a_5 + b_5 t + c_5 t^2 + d_5 t^3 + e_5 t^4 & \\
 & \text{if } t_5 \leq t \leq t_6.
\end{cases}
\end{aligned}
\tag{6}
$$

The number of parameters used to define h is twenty. These are uniquely determined by fixing times $t_1 \sim t_6$ and the maximum bending angle A_m, and imposing additional constraints such that the first and second derivatives at $t = t_1, t_3, t_4, t_6$ equal zero, and that the first, second and the third derivatives at connecting point t_2, t_5 are continuous.

B. Optimization method for reference trajectory

As stated in the previous subsection, a reference trajectory for a knee joint angle of swing leg are uniquely and completely determined when the time parameters $t_1 \sim t_6$ and the maximum bending angle A_m are determined. Since the dynamic equation of our model is nonlinear, the steady gaits for a given reference trajectory is in general difficult to obtain analytically. Hence, we usually rely on numerical simulation for generating a steady gait. The numerical simulation starts from a certain initial conditions and converges to a steady gait gradually by solving the dynamic equation and the impact equation numerically. However, the initial condition is itself strongly depend on the reference trajectory and is difficult to determined a priori. Moreover, it takes long time (e.g. a handled steps or more) to converge to a steady gait from an initial condition. These issues are significant drawback in optimizing a reference trajectory.

Therefore, the optimization method proposed in this paper adopts the following strategy. We first generate a steady gait corresponding to a reference trajectory which is known to generate sustainable walking, and evaluate a walking performance index value for the steady gait. Next we perturb slightly the reference trajectory of the current steady gait, and generate the steady gait corresponding to the perturbed trajectory form the current gait as initial gait. In this time, if the deviation of these trajectories is small, it is expected to take not so much time to converge to a steady gait. When the walking performance index value of the perturbed trajectory is better than that of original trajectory, we update the trajectory and the corresponding gait, and repeat the above procedure again.

To implement the above strategy, there is one significant issue that the search space for time parameters $t_1 \sim t_6$ is inherently continuous. But in numerical simulation is in general implemented in discrete time spaces on a computer, and hence, we discretize the search space. The proposed optimization method is summarized as follows.

- Step 0 (Initialization) The search space for time instances $t_1 \sim t_6$ is discretized appropriately. Choose initial parameters $t_1^0 \sim t_6^0$ from the discretized search space corresponding to a known steady gait w_0 with the trajectory. Evaluate the walking performance index p_0 for the gait w_0. Set the index set $I_0 = \{1, \cdots, 6\}$ and set $k = 0$.
- Step 1 If $I_k = \phi$, stop. The current gait w_k is locally optimal. Otherwise, choose $i \in I_k$ and $I_k = I_k \backslash \{i\}$.
- Step 2 Generate the steady gaits w_{k+} and w_{k-} corresponding to the trajectories of t_i^k replaced with $t_i^k + 1\text{unit}$ and $t_i^k - 1\text{unit}$, respectively.
- Step 3 Evaluate walking performance indices p_{k+} and p_{k-} for the steady gaits w_{k+} and w_{k-}, respectively. Set $\bar{p} = \min\{p_k, p_{k+}, p_{k-}\}$.
- Step 4 If $\bar{p} = p_{k\pm}$, then $t_i^{k+1} = t_i \pm 1\text{unit}$, $w_{k+1} = w_{k\pm}$, $I_k = \{1, \ldots, 6\}$ and $k = k + 1$. Go to Step 1. Otherwise, go to Step 1.

In the above algorithm, examples of walking performance indices are average walking speed, foot clearance, the maximum input torque and specific resistance. Among them, the specific resistance [8] is defined by

$$\mu = \frac{\int_{0^+}^{T^-} |u_K(\dot{\theta}_2 - \dot{\theta}_3)| \mathrm{d}t / T}{M_g g \overline{V}}, \tag{7}$$

and represents energy efficiency. The smaller a specific resistance value is, the more efficient a walking is. In Eq. (7), 0^+ and T^- represent the time just after and before collision at the ground, respectively, M_g is the total mass of a biped robot and $\overline{V}$ is walking speed of one step.

TABLE I

PHYSICAL PARAMETERS OF THE KNEED BIPED ROBOT

a_3	0.18	m	R	0.15	m
a_2	0.12	m	m_2	0.4	kg
r_3	0.09	m	m_3	1.6	kg
r_2	0.06	m	m_H	2.2	kg
l	0.30	m	I	0.0072	kgm^2

We remark that the proposed algorithm is one of local search methods usually used in combinatorial optimization problems, and hence, the method in general can find a local optimum only. In particular, local optima obtained by the algorithm depend on an initial steady gait and the selection rule of the index in Step 1.

IV. SIMULATION RESULTS

In this section, we show simulation results of the optimization method proposed in the previous section for reference trajectory of the biped robot (Fig. 2), whose parameters are shown in Table I.

Simulation conditions are described as follows. Search range of $t_1 \sim t_5$ was $0 \sim 0.45$s and discretized by the 0.01s. We fixed t_6 as $t_6 = T_{set} = 0.45$s. A selection rule of index at Step 1 is $1, 2, 3, 4, 5$ in order. We choose the initial steady gait corresponding to the reference trajectory h with $A_m = 1.00$rad, $t_1 = 0.14$s, $t_2 = 0.19$s, $t_3 = 0.30$s, $t_4 = 0.30$s and $t_5 = 0.44$s.

Once, the local optimal solution is obtained, we reduce the maximum bending angle A_m by 0.01rad, then apply the optimization method again whose initial gait in the previous local optimal gait. We repeat this procedure until steady gait generation fails.

For the comparison purpose, we optimize the reference trajectory $\tilde{h}$ using the cube of sine curve in a similar way. We search the optimal trajectory for $\tilde{h}$ under condition that $T_{set} = 0.45$s and bending delay δ between 0 and 0.45s.

Fig. 4 shows the simulation results with respect to specific resistance for each maximum bending angle, and Fig. 5 shows the corresponding walking speed. From Fig. 4, it is observed that the reference trajectory h using quartic spline curve is more efficient than reference trajectory $\tilde{h}$ using the cube of sine curve. In addition, the reference trajectory h can generate the sustainable gait in the region $A_m \geq 0.63$rad, while the reference trajectory $\tilde{h}$ can generate the gait in the region $A_m \geq 0.76$rad. Fig. 4 shows that the optimal bending angle for the reference trajectory using quartic spline curve is 0.71rad and that for the reference trajectory using the cube of sine curve is 0.82rad with respect to specific resistance.

Fig. 6 illustrates optimal trajectory for the maximum bending angle $A_m = 0.82$rad. From Fig. 6, it is observed that the swing leg knee is kept in the maximum bending angle for a short time in the result of the reference trajectory using quartic spline curve. Table II shows specific resistance, walking speed, maximum torque and foot clearance of the simulation result shown in Fig. 6. From table II, walking

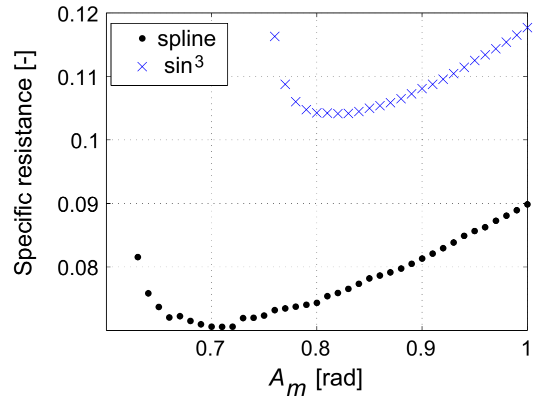

Fig. 4. Comparison of optimal trajectories with respect to specific resistance

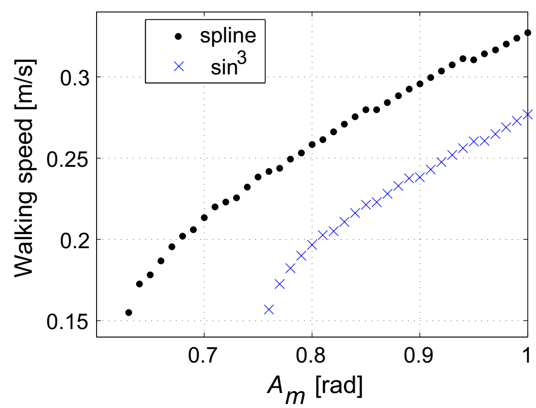

Fig. 5. Comparison of optimal trajectories with respect to walking speed

speed and foot clearance of the reference trajectory using quartic spline curve are larger than those of the reference trajectory using the cube of sine curve. The maximum torque of reference trajectory using the cube of sine curve is smaller than that of the reference trajectory using quartic spline curve, because the reference trajectory using the cube of sine curve is smoother than that using quartic spline curve. Table III shows the optimal parameters of the reference trajectory using quartic spline curve when $A_m = 0.82$rad. Note that the bending delay δ of the reference trajectory using the cube of sine curve is 0.18s.

We compare the reference trajectory using quartic spline curve with the reference trajectory using the cube of sine curve in the case of the similar walking speed. Table IV shows specific resistance, walking speed, maximum torque and foot clearance of the simulation result. From Table IV, it is observed that the reference trajectory using quartic spline curve is more efficient than the reference trajectory using the cube of sine curve. Foot clearance of the reference trajectory using the cube of sine curve is larger than that of the reference trajectory using quartic spline curve, because the maximum bending angle A_m of the reference trajectory using the cube of sine curve is larger than that of the reference trajectory using quartic spline curve. Fig. 7 illustrate the simulation result. Fig. 7(a) shows angular position of the

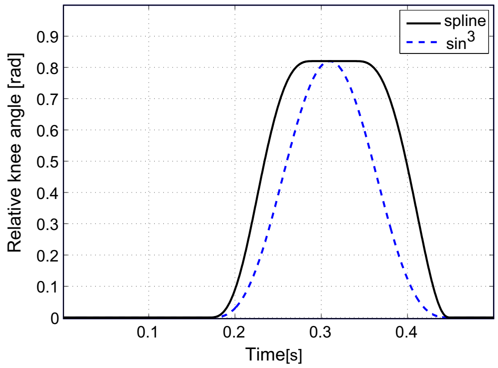

Fig. 6. Optimal trajectory for $A_m = 0.82$rad

TABLE II
SIMULATION RESULTS

	$\sin^3$	spline
Specific resistance [-]	0.1041	0.0759
Walking speed [m/s]	0.2050	0.2662
Maximum torque [Nm]	0.6440	0.7518
Foot clearance[mm]	0.075	0.338

reference trajectory h using quartic spline curve, Fig. 7(b) shows mechanical energy of the reference trajectory h, Fig. 7(c) shows angular position of the reference trajectory $\tilde{h}$ using the cube of sine curve and Fig. 7(d) shows mechanical energy of the reference trajectory $\tilde{h}$. From Fig. 7(d) and Fig. 7(b), it is observed that variation of mechanical energy of the reference trajectory using quartic spline curve is smaller than that using the cube of sine curve, and then the reference trajectory using quartic spline curve is more efficient. Fig. 7(b) shows that energy is constant during the keeping interval.

Next, we optimize the reference trajectory using quartic spline curve for other indices, such as foot clearance, walking speed and maximum input torque. Here, we fix the maximum bending angle as 0.90rad. We note that the optimization with respect to foot clearance and walking speed is maximizing and the optimization with respect to torque is minimizing.

Table V shows the optimization results and their parameters of reference trajectory are shown in Table VI. Fig. 9 illustrates the optimal trajectories with respect to each index.

From Table V, it is observed that maximum torque is large in the case of foot clearance and walking velocity. Optimization for walking speed, particularly, needs 39Nm and then, specific resistance is the largest. From Fig. 9 and Table VI, it is observed that the beginning of bending t_1 is early in the case of foot clearance. In the case of maximum torque, there is no keeping interval. From Fig. 9, it is observed that knee is rapidly bent and stretched in the case of walking speed, and then, large torque is needed. This trajectory is very close to the optimal trajectory shown by Fig. 1. Fig. 9 and Table VI show that foot clearance becomes large as the beginning of bending is early and the beginning of stretching is late. On the other hand, walking speed becomes large as the beginning of bending is late.

TABLE III
OPTIMAL PARAMETERS

t_1	t_2	t_3	t_4	t_5	t_6
0.17	0.22	0.29	0.34	0.44	0.45

TABLE IV
RESULTS OF OPTIMIZATION

	$\sin^3$	spline
A_m[rad]	0.97	0.82
Specific resistance [-]	0.1143	0.0759
Walking speed [m/s]	0.2648	0.2662
Maximum torque [Nm]	0.8191	0.7518
Foot clearance[mm]	0.386	0.338

V. CONCLUSION AND FUTURE WORKS

In this paper, we propose the optimization method for parametric excitation walking. We utilized the quartic spline curve instead of cube of sine curve and optimize the parameters of spline curve with respect to some indices. The proposed method improves energy efficiency, walking speed, foot clearance and maximum torque, respectively.

The proposed method optimized only the reference trajectory. To grow in efficiency, we think that optimization of the physical parameters of the biped robot is needed, for example mass and link length. Another method to grow in efficiency is the parametric excitation based inverse bending walking proposed by Harata et al. [9], [10]. We will apply the proposed method to inverse bending walking and then gait become more efficiency.

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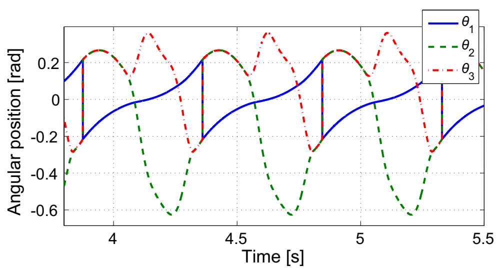
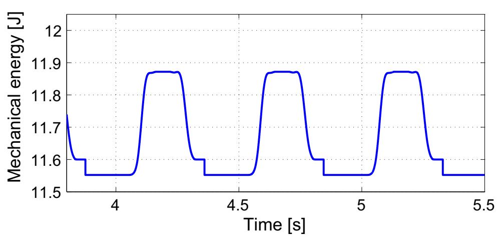
TABLE V

RESULTS OF OPTIMIZATION WITH RESPECT TO SEVERAL INDICES

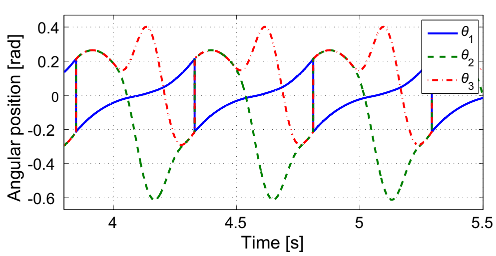
Index	Waling speed [m/s]	Foot clearance [mm]	Maximum torque [Nm]	Specific resistance [-]
Walking speed	0.3350	0.409	39.1941	1.2922
Foot clearance	0.2396	1.504	4.4147	0.2664
Maximum torque	0.2758	0.758	0.7071	0.0942
Specific resistance	0.2957	0.488	0.8391	0.0813



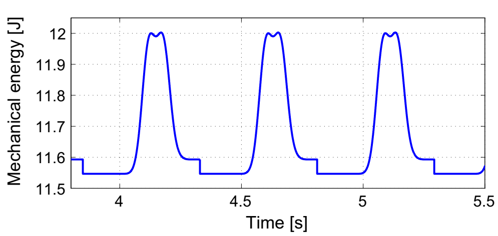
(a) Angular position



(b) Mechanical energy



(c) Angular position



(d) Mechanical energy

Fig. 7. Simulation results of same walking speed

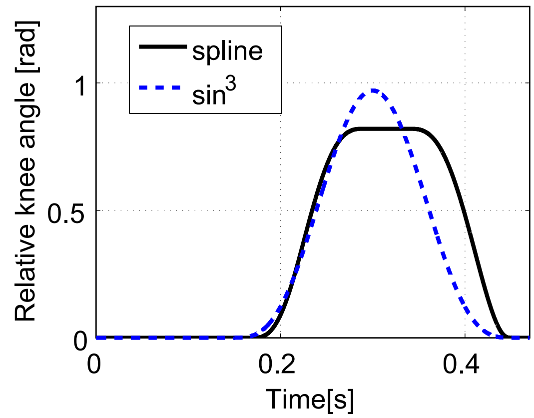


Fig. 8. Reference trajectory in the case of the similar walking speed

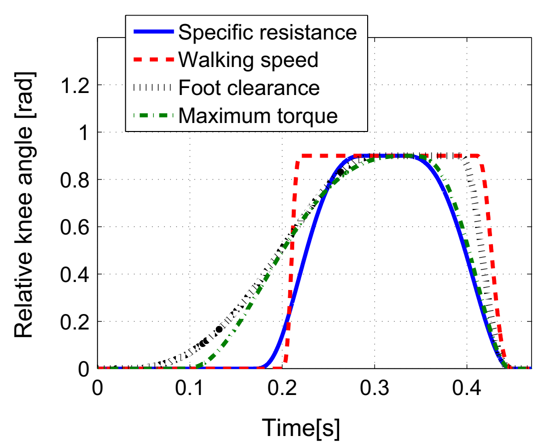


Fig. 9. Optimal trajectories for each index

TABLE VI

PARAMETERS OF OPTIMAL TRAJECTORIES

Index	t_1	t_2	t_3	t_4	t_5	t_6
Specific resistance	0.17	0.20	0.29	0.33	0.44	0.45
Foot clearance	0.02	0.27	0.31	0.39	0.41	0.45
Walking speed	0.20	0.21	0.22	0.41	0.42	0.45
Maximum torque	0.09	0.12	0.34	0.34	0.44	0.45