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Mutual Localization in a Multi-Robot System with Anonymous Relative Position Measures

Antonio Franchi Giuseppe Oriolo Paolo Stegagno

Abstract— **We address the mutual localization problem for a multi-robot system, under the assumption that each robot is equipped with a sensor that provides a measure of the relative position of nearby robots without their identity. Anonymity generates a combinatorial ambiguity in the inversion of the measure equations, leading to a multiplicity of admissible relative pose hypotheses. To solve the problem, we propose a two-stage localization system based on MultiReg, an innovative algorithm that computes on-line all the possible relative pose hypotheses, whose output is processed by a data associator and a multiple EKF to isolate and refine the best estimates. The performance of the mutual localization system is analyzed through experiments, proving the effectiveness of the method and, in particular, its robustness with respect to false positives (objects that look like robots) and false negatives (robots that are not detected) of the measure process.**

I. Introduction

This paper deals with *mutual localization* (ML) in multi-robot systems. ML problems are of great importance in performing decentralized tasks that require data fusion, such as cooperative map-building and formation control. Clearly, the accuracy of the localization can significantly affect the quality of the task execution.

In a multi-robot system, we refer to *relative mutual localization* (RML) as the problem of estimating the relative poses (position and orientation) among the moving frames attached to the robots. Assuming that each robot also has its own fixed frame, one can in addition define *absolute mutual localization* (AML) as the problem of estimating the relative poses among the various fixed frames. If each robot is self-localized with respect to its own fixed frame, the solution of RML can be obtained in principle from the solution of AML (and vice versa) by simple changes of coordinates.

To the best of our knowledge, no researchers have so far investigated the AML problem directly. Previous works have addressed either the RML problem or the problem of cooperative localization (CL) of a multi-robot system in a common[1] fixed frame. Roughly speaking, two different classes of approaches emerge: *filter-based* and *geometry-based*. The former use Extended Kalman (EK) or particle filters to estimate relative poses from measures, while the latter perform an instantaneous inversion of the mapping between relative poses and measures.

In most of the early filter-based approaches, such as [1], [2], [3], [4], the RML problem is solved by filtering out the noise from the output of a vision-based sensor that directly measures the relative poses between robots; at the same time, the filter provides a solution to the CL problem. In other works, the filter was also used to reconstruct the non-measured part of the change of coordinates; examples include [5], where relative range-only measures obtained by a combined RF/ultrasonic sensor are used; [6], where an extension of [1] is presented for different sensing equipments; and [7], where a detailed analysis is performed for range-only measurement.

As for geometry-based approaches, the problem of estimating the relative positions of robots in a formation by range-only measures or bearing-only measures has been investigated in [8], [9] and [10]. In the case of position (bearing plus range) relative measures, it is possible to obtain the relative pose of a robot respect to another by simply processing two bearing and one range measure [11].

A possible limitation of all the above methods is the assumption that the relative measures also provide the identity of the robots. In fact, interesting situations that may arise in practice are: *i)* the identities of the detected robots is not known (anonymous measures), *ii)* false positives (false detected robots) and *iii)* false negatives (undetected robots) occur in the relative position measurement process. The first and the second situation fit, for example, robot measurement systems based on a feature extraction module that looks for characteristics that are common to all robots and may also be found in other objects: for example, this happens when the robots and some obstacle in the environment have the same size, color, or shape, either by chance or by hostile camouflage. The third situation accounts for the fact that robots within the sensing range may not be detected, e.g., due to occlusions.

A pioneering work that addresses the anonymous RML problem is [12], in which an algorithm based on geometrical arguments is proposed to obtain relative pose estimates from anonymous bearing measurements. However, the method does not take into account false positives or false negatives, preventing its application to the aforementioned situations.

In this paper, we address RML and AML problems with anonymous measures affected by false positives and negatives, as formalized in Sect. II. The proposed two-stage localization system is described in Sect. III. The first stage

A. Franchi, G. Oriolo and P. Stegagno are with the Dipartimento di Informatica e Sistemistica, Università di Roma “La Sapienza”, Via Ariosto 25, 00185 Roma, Italy. E-mail: {franchi,oriolo,stegagno}@dis.uniroma1.it

[1]The idea of a common fixed frame presumes a certain degree of centralization, because it is necessary that robots share some information at the beginning of the task.

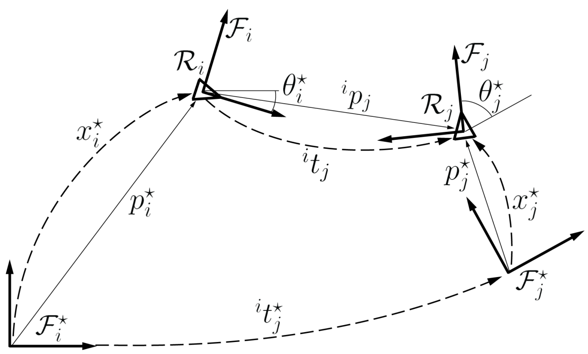

Fig. 1. Geometric setting for mutual localization problems. Triangles are robots, solid arrows are position vectors and dashed arrows are pose vectors.

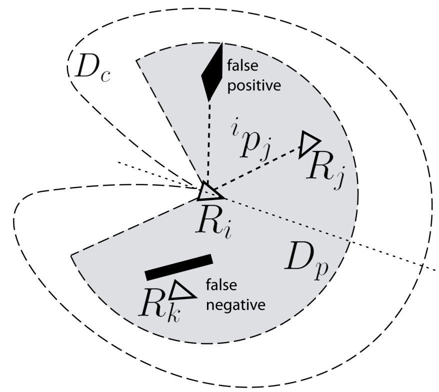

Fig. 2. Robot detection and communication. Triangles are robots, black polygons are occluding objects, and the grey region is the perception set.

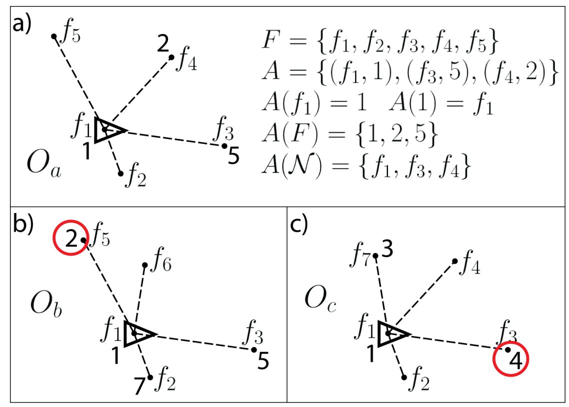

Fig. 3. The structure of an observation (triangles are robots, points are features): a) An example observation O_a by $\mathcal{R}_1$; b) O_b is irreconcilable with O_a because robot $\mathcal{R}_2$ is associated to a different feature; c) O_c is irreconcilable with O_a because feature f_3 is associated to a different robot.

is a multiple registration algorithm that generates on-line all the geometrically admissible RML solutions (Sect. IV). With the aid of self-localization data, these are used to solve the AML problem via data association and multiple EK filtering (Sect. V). Experimental results are presented in Sect. VI.

II. Problem Formulation

We take the following assumptions (refer to Figs. 1 and 2).

1) The multi-robot system includes n robots $\mathcal{R}_1, \ldots, \mathcal{R}_n$, where $n \geq 2$ is unknown. The robots move in $\mathbb{R}^2$. Let $\mathcal{N} = \{1, \ldots, n\}$ be the robot index set.
2) Each $\mathcal{R}_i$ ($i \in \mathcal{N}$) has two associated frames: a *fixed* frame $\mathcal{F}_i^\star$ and a *moving* frame $\mathcal{F}_i$ (see Fig. 1). The latter is rigidly attached to a representative point of the robot. Given $i, j \in \mathcal{N}$, denote by ${}^it_j$ and ${}^it_j^\star$ the 3-vectors describing the position and orientation (*pose*) respectively of $\mathcal{F}_j$ with respect to $\mathcal{F}_i$, and of $\mathcal{F}_j^\star$ with respect to $\mathcal{F}_i^\star$. Given ${}^it_j$, it is immediate to build the change of coordinates ${}^iT_j$ from $\mathcal{F}_j$ to $\mathcal{F}_i$. The configuration of robot $\mathcal{R}_i$ is the pose of $\mathcal{F}_i$ with respect to $\mathcal{F}_i^\star$ and is indicated by $x_i^\star = (p_i^{\star T}\, \theta_i^\star)^T$ (see Fig. 1).
3) Each $\mathcal{R}_i$ comes with an independent *self-localization module* that provides an estimate $\hat{x}_i^\star$ of $x_i^\star$, i.e., the pose of the robot $\mathcal{R}_i$ in the frame $\mathcal{F}_i^\star$.
4) Each $\mathcal{R}_i$ is equipped with a *robot detector*, a sensor device that measures the *relative position* ${}^ip_j$ of other robots, provided that they fall in a perception set D_p that is rigidly attached to $\mathcal{F}_i$ (see Fig. 2). Note that no assumption is taken on the shape of D_p, in particular the robot detector does not need to be omnidirectional.
5) Each $\mathcal{R}_i$ has a *communication module* that can send/receive data to/from any other robot $\mathcal{R}_j$ contained in a communication set D_c (see Fig. 2). We assume that $D_p \subseteq D_c$, so that if $\mathcal{R}_i$ can detect $\mathcal{R}_j$ it can also communicate with it. Each message sent by $\mathcal{R}_i$ contains: (1) the robot index i (2) the estimate $\hat{x}_i^\star$ as provided by the self-localization module (3) the measures coming from the robot detector.

The relative position measures provided by the robot detector are *anonymous*, in the sense that they do not include the index j of the detected robot. This is true, for example, when the detection process relies on features that are common to all the robots. A consequence of anonymity is the existence of *ambiguous* situations (such as that in Fig. 6a), where the same set of measures is obtained for different configurations of the multi-robot system. As shown in Fig. 2, the robot detector is also prone to *false positives* (it can be deceived by objects that look like robots) and *false negatives* (robots belonging to D_p which are not detected, e.g., due to line-of-sight occlusions). Hence, the measures coming from the robot detector will be generically referred to as *features* – they might or not represent actual robots. False negatives (robot belonging to D_c that do not receive messages) may also affect the communication, whereas false positives may be easily avoided by appropriate message coding.

Our objective is to solve the *absolute mutual localization* problem for the generic i-th robot, i.e., to estimate ${}^it_j^\star$, for $j \neq i$. As a byproduct, this will also solve the *relative mutual localization* problem, i.e, provide an estimate of ${}^it_j$, for $j \neq i$. Note that, if the anonymity assumption is removed, the geometric computation of ${}^it_j^\star$ from $x_i^\star$, $x_j^\star$ (estimated by the self-localization modules) and ${}^ip_j$, ${}^jp_i$ (measured by the robot detectors) becomes a simple exercise.

A. Observations

To clarify the structure of the information coming from a robot detector, we introduce the concept of *observation*, i.e., a pair $O := (F, A)$, where F is a set of features expressed in

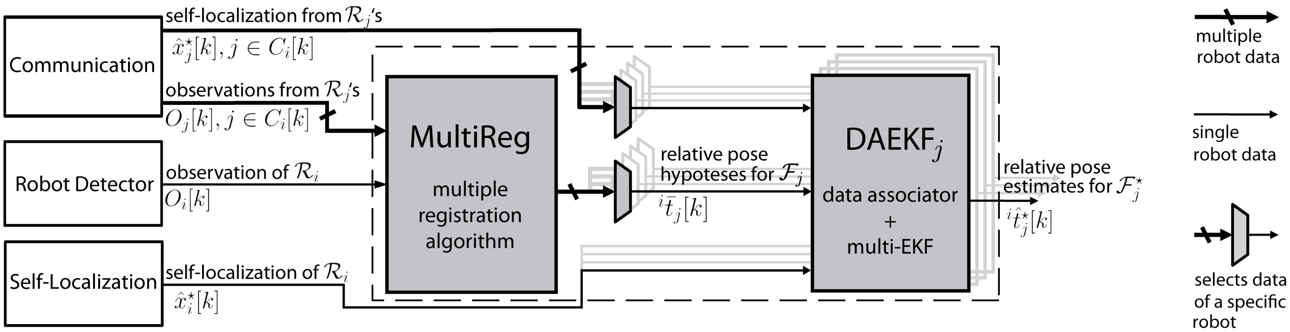

Fig. 4. A scheme of the mutual localization system that runs on $\mathcal{R}_i$.

the moving frame attached to the robot, and $A \subset F \times \mathcal{N}$ is a *functional relation* on $F \times \mathcal{N}$ (this means that $|A(f)| \leq 1$, where $|\cdot|$ denotes the cardinality of a set). In particular, given $f \in F$, we denote with $A(f) := \{i \in \mathcal{N} :\ (f, i) \in A\}$ the robot index (if any) associated to the feature f; given $i \in \mathcal{N}$, we denote with $A(i) := \{f \in F :\ (f, i) \in A\}$ the feature (if any) associated to the i-th robot (see Fig. 3a). Also, denote by $A(F) := \cup_{f \in F} A(f)$ the set of robot indexes appearing in O and by $A(\mathcal{N}) := \cup_{i \in \mathcal{N}} A(i)$ the set of features of O that are associated to some robot. A feature f is called *anonymous* when $A(f) = \emptyset$, i.e., $f \notin A(\mathcal{N})$. Two observations $O_1 = (F_1, A_1)$ and $O_2 = (F_2, A_2)$ expressed in the same robot frame are said to be *irreconcilable* if:

a) two different features are associated to the same robot, i.e., $\exists f_1 \in A_1(\mathcal{N}), \exists f_2 \in A_2(\mathcal{N})$, with $f_1 \neq f_2$, such that $A_1(f_1) = A_2(f_2)$ (see Fig. 3b), or if
b) two different robots are associated to the same feature, i.e., $\exists f \in A_1(\mathcal{N}) \cap A_2(\mathcal{N})$ such that $A_1(f) \neq A_2(f)$ (see Fig. 3c).

Each robot detector provides an observation in which all features are anonymous, except for the feature at the origin which is associated to the index of the measuring robot; this is called a *raw observation*.

III. The Mutual Localization System

The mutual localization system running on $\mathcal{R}_i$ is composed by a cascade of two subsystems, as shown in Fig. 4. The first subsystem is a memoryless registration algorithm called *MultiReg*. Denote by $C_i[k] \subset \mathcal{N}$ the set of (indexes of) robots from which $\mathcal{R}_i$ receives data in the time interval $[t_{k-1}, t_k)$. At each step k, the inputs of MultiReg are a set of observations: one is provided directly by the robot detector, while the others come from the robots in $C_i[k]$ through the communication module. The output of MultiReg is a set ${}^i\bar{t}_j$ of hypotheses on the relative poses ${}^it_j$, for each $j \in C_i[k]$.

The second subsystem, called *DAEKF*, is a variable-size array of components, DAEKF$_j$, $j \in \cup_{h=1}^k C_i[h]$, each consisting of a *data associator* and a *multi-EKF*. The input of DAEKF$_j$, at step k, is the set ${}^i\bar{t}_j$ of current hypotheses on ${}^it_j$. At the same time, each DAEKF$_j$ also receives the estimates $\hat{x}_i^\star$ and $\hat{x}_j^\star$, $j \in C_i[k]$, respectively from the self-localization and the communication module. The output of DAEKF$_j$ is a set ${}^i\hat{t}_j^\star$ of estimates of ${}^it_j^\star$, for each $j \in \cup_{h=1}^k C_i[h]$. In the following, we detail the structure of MultiReg and DAEKF.

IV. Multiple Registration with MultiReg

At each step k, the generic robot $\mathcal{R}_i$ executes the MultiReg algorithm to perform a memoryless *multiple registration* among the observations coming from $\mathcal{R}_i$ and all the $\mathcal{R}_j$'s, $j \in C_i[k]$; in our context, this means computing all the possible relative poses ${}^it_j$ of the frames attached to the $\mathcal{R}_j$'s using purely geometric arguments.

A. Binary registration

MultiReg uses binary registration as the basic tool. Given two observations $O_1 = (F_1, A_1)$ and $O_2 = (F_2, A_2)$ such that $A_1(F_1) \cap A_2(F_2) = \emptyset$ (always satisfied in MultiReg, see Sect. IV-B), consider a candidate change of coordinates T between the two associated frames. Letting $T(F) = \{q \in \mathbb{R}^2 | \exists f \in F :\ T(f) = q\}$, a binary relation $B \subset F_1 \times F_2$ is associated to T as follows: $(f_1, f_2) \in B \Leftrightarrow \|f_1 - T(f_2)\| \leq \delta$, where δ is a given *fitting threshold*. The elements of $B(F_1)$ and $B(F_2)$ are the *inliers* of F_2 and F_1, respectively. The cardinality of B, denoted by $|B|$, is the *number of inliers*.

Given $\delta > 0$ and $\mu > 0$, performing a *binary registration* of O_1, O_2 means finding a change of coordinates T such that the associated B is left- and right-unique, and satisfies: *i)* $|B| \geq \mu$ and *ii)* $|A(f_1) \cup A_2(f_2)| \in \{0, 1\}\ \forall (f_1, f_2) \in B$. The first condition is a constraint on the *minimum number of inliers* (note that $|B| = |B(F_1)| = |B(F_2)|$). The second requires that, for any pair of features (f_1, f_2) that are related by B, either f_1 or f_2 (or both) must be anonymous. In fact, being $A_1(F_1) \cap A_2(F_2) = \emptyset$, a 'double' assignment would certainly represent a conflict.

Once T has been determined, a new observation $O_{12} = (F_{12}, A_{12})$ is generated, where $F_{12} = F_1 \cup T(F_2)$ and $A_{12} \subset F_{12} \times \mathcal{N}$ is such that for any $f \in F_{12}$ it is

$$A_{12}(f) = \begin{cases} A_1(f) & \text{if } f \in A_1(\mathcal{N}) \\ A_2(f^*) & \text{if } f \in T(A_2(\mathcal{N})) \\ A_2(B(f)) & \text{if } f \in B(F_2) \backslash A_1(\mathcal{N}) \\ A_1(B(f^*)) & \text{if } f \in T(B(F_1)) \backslash T(A_2(\mathcal{N})) \\ \emptyset & \text{otherwise} \end{cases}$$

where $f^* := T^{-1}(f)$ (see Fig. 5). For our purposes, the output of the binary registration (called *solution* in the following) is the triple $r(O_1, O_2) = (T, O_{12}, |B|)$. Clearly, for a given pair of observations there may exist multiple changes of coordinates that satisfy the above conditions, and therefore multiple solutions. We call two solutions *irreconcilable* if

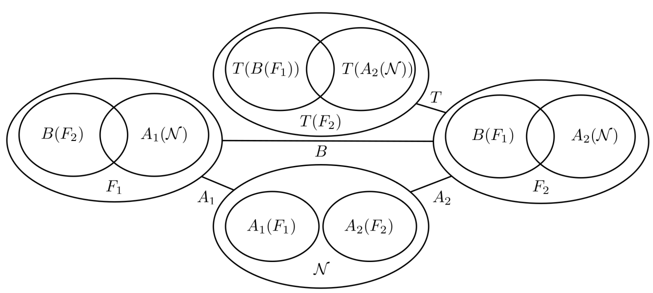

Fig. 5. Sets of indexes/features involved in a binary registration with the associated relations.

their corresponding observations are irreconcilable. In the following, we assume that the binary registration algorithm returns a finite set of irreconcilable solutions.

The combinatorial essence of the problem suggests the use of probabilistic techniques, while the presence of outliers (i.e., features observed by only one robot) calls for a robust estimation paradigm. We chose RANSAC [13] for binary registration because it has both this properties. Our implementation follows from the algorithm presented in [14] for a binary lidar scan registration and can be found in [15].

B. Multiple registration

At each step k, $\mathcal{R}_i$ executes MultiReg on the set Ω made by its own raw observation $O_i = \{F_i, A_i\}$ and the raw observations $O_j = \{F_j, A_j\}$, for $j \in C_i[k]$. Let $\mathcal{N}_\Omega$ be the set of the indexes of the robots whose observations are in Ω. Since MultiReg is a memoryless algorithm, we drop k in the following. As stated before, it is $|A_j(F_j)| = 1$, $\forall j \in \mathcal{N}_\Omega$, and $\cap_{j\in\mathcal{N}_\Omega} A_j(F_j) = \emptyset$. We set $\Omega_j = \Omega \backslash O_j$ and, for any $\bar{\mathcal{N}} \subseteq \mathcal{N}_\Omega$, we set $\Omega_{\bar{\mathcal{N}}} = \Omega \backslash \{O_j : j \in \bar{\mathcal{N}}\}$. The output is the set $X(\Omega) = \{{}^i\bar{t}_j,\}_{j\in\mathcal{N}_\Omega}$ where ${}^i\bar{t}_j = \{\ldots, {}^i\bar{t}_{jh}, \ldots\}_{j\in\mathcal{N}_\Omega}$, whose generic element ${}^i\bar{t}_{jh}$ is a estimate of ${}^it_j$.

A pseudocode description of MultiReg is given in Table I. MultiReg executes $|C_i|$ iterations. Step 1 initializes the first iteration. At step 2a, during the l-th iteration, $|C_i| - l$ binary registrations $r(\tilde{O}_l, O)$ are performed between the observation so far obtained, $\tilde{O}_l$, and the raw observations $O \in \Omega_{U_l}$. Their solutions γ are stored in Γ. At step 2b, the solutions with the maximum number of inliers are stored in Γ^*, and at step 2c $\tilde{\Gamma}$ is computed as a maximal subset of irreconcilable solutions of Γ^*. At step 2d, the estimated change of coordinates T_γ in γ is tuned, for each $\gamma \in \tilde{\Gamma}$, by minimizing the mean square error of the inliers pairs using the algorithm in [16]. At step 2e, MultiReg forks in $|\tilde{\Gamma}|$ branches, one for each $\gamma \in \tilde{\Gamma}$. If $\gamma = \{T_\gamma, O_\gamma, |B_\gamma|\} \in \tilde{\Gamma}$ is a solution given by the registration of $\tilde{O}_l$ with O_s, the new iteration of the branch starting from γ is initialized with $\tilde{O}_{l+1} = O_\gamma$ as partial registered observation and $U_{l+1} = U_l \backslash \{s\}$ as set of indexes of unregistered observations. A branch is terminated with no solution if the set Γ becomes empty. Otherwise, at the $|C_i|$-th iteration the branch contains a solution that is irreconcilable with the solutions of all the other branches. The algorithm returns as output the set of all solutions found in all branches.

An example of MultiReg execution is shown in Fig. 6 for a simple ambiguous configuration (Fig. 6a). The objective

TABLE I

MULTIREG ALGORITHM FOR THE i-TH ROBOT

inputs
$\Omega = \{\ldots, O_j, \ldots\}$, with $O_j = (F_j, A_j)$ — $|C_i| + 1$ raw observations

variables
${}^i\bar{t}_{jl} = (\ldots, {}^i\bar{t}_{jhl}, \ldots)$ — partial solution at the l-th iteration
$\tilde{O}_l$ — partial registered observation at the l-th iteration
$U_l \subseteq \mathcal{N}_\Omega$ — indexes of the unregistered observations at the l-th iteration

output
$X(\Omega) = \{\ldots, {}^i\bar{t}_{js}, \ldots\}$ — set of solutions (in shared memory)

algorithm
1. ${}^i\bar{t}_{j1} = (\mathbf{0}_3, \ldots, \mathbf{0}_3)$, $\tilde{O}_1 = O_i$, $U_1 = \mathcal{N}_i$, $X(\Omega) = \emptyset$
2. **for** l=1 **to** $|C_i|$
   a. $\Gamma = \cup_{O\in\Omega_{U_l}} r(\tilde{O}_l, O)$
   b. $\Gamma^* = \{\gamma = (T_\gamma, O_\gamma, |B_\gamma|) \in \Gamma \mid |B_\gamma| = \max_{|B|} \Gamma\} \subseteq \Gamma$; this is the subset of Γ of elements that maximize the number of inliers
   c. compute a maximal subset of irreconcilable solutions $\tilde{\Gamma} \subseteq \Gamma^*$
   d. perform a least square estimation for every $\gamma \in \tilde{\Gamma}$, substituting T_γ with that minimizing the mean square error among the inliers and recomputing O_γ accordingly
   e. **fork** $\forall\gamma \in \tilde{\Gamma}$ with
      i. $\tilde{O}_{l+1} = O_\gamma$;
      ii. ${}^i\bar{t}_{jh} = ({}^i\bar{t}_{2(l-1)}, \ldots, {}^i\bar{t}_{|C_i|(l-1)})$
      iii. ${}^i\bar{t}_{hl} = t_\gamma$
      iv. $U_{l+1} = U_l \backslash \{s\}$ where s is the index of the raw observation added in γ
3. the solution of each branch is put in the set of solutions $X(\Omega)$

of the algorithm is the multiple registration of the raw observations O_1, O_2, O_3 (Fig. 6b). The MultiReg instance on $\mathcal{R}_1$ performs $|C_1| = 2$ iterations. At first iteration, the raw observation O_1 is chosen as partial registered observation $\tilde{O}_1$, and the indexes of the other observations are put in the set of the unregistered observations U_1. Then, two binary registrations are performed between $\tilde{O}_1$ and O_2, O_3 respectively (Fig. 6c). The results are put in $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$, and $\Gamma^* = \Gamma$ is selected as the subset of elements of Γ that maximize the number of inliers. Then, a maximal subset $\{\gamma_1, \gamma_3\} = \tilde{\Gamma} \subseteq \Gamma^*$ of irreconcilable solutions in Γ^* is computed (Fig. 6d). In the second iteration, for each $\gamma_i \in \tilde{\Gamma}$ ($i = 1, 3$) the algorithm forks, initializing a new branch with $\tilde{O}_2 = O_{\gamma_i}$, and deleting the index of the registered robot from U_1: in particular in the first branch we set $U_2 = \{3\}$ and in the second $U_2 = \{2\}$. The first branch executes the binary registration between $\tilde{O}_2$ and O_3 (Fig. 6e1), finding the solution γ_5, and the second executes a binary registration between $\tilde{O}_2$ and O_2 (Fig. 6e2), finding the solution γ_6.

V. DATA ASSOCIATION AND EK FILTERING

At the k-th step, the DAEKF subsystem running on robot $\mathcal{R}_i$ is composed by an array of $|\cup_{h=1}^k C_i[h]|$ components (see Fig. 4). Each component is associated to a robot $\mathcal{R}_j \in \cup_{h=1}^k C_i[h]$ and provides estimates of ${}^it_j^\star$. At step k its inputs are:

1) The estimates provided by the self-localization modules

$$\begin{aligned}\hat{x}_j^\star[k] &= x_j^\star[k] + w_j^\star[k] \\ \hat{x}_i^\star[k] &= x_i^\star[k] + w_i^\star[k]\,,\end{aligned}$$

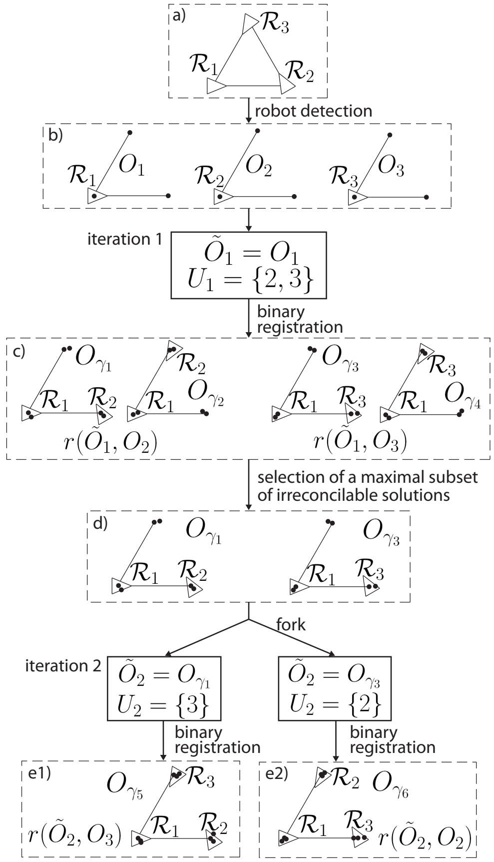

Fig. 6. An example of MultiReg execution in a simple ambiguous situation: a) actual configuration b) raw observations of the robots c) results of the binary registrations between $\tilde{O}_1$ and O_2, O_3 d) selection of a maximal subset of irreconcilable solutions of Γ^* e1) result of the binary registration for the first branch e2) result of the binary registration for the second branch.

where $w_j^\star[k]$ and $w_i^\star[k]$ are gaussian noises with zero mean and covariances $\widehat{Q}_j^\star[k]$ and $\widehat{Q}_i^\star[k]$. Note that $\hat{x}_j^\star[k]$ is available provided that $\mathcal{R}_j \in C_i[k]$;

2) The set of hypotheses about the relative pose ${}^i t_j[k]$

$$ {}^i\bar{t}_j[k] = \{\ldots, {}^i\bar{t}_{jh}[k], \ldots\}, $$

provided by MultiReg. Here, ${}^i\bar{t}_{jh}[k]$ is a gaussian random variable with unknown mean and covariance ${}^i\bar{Q}_{jh}[k]$.

Each DAEKF_j (see Fig. 7) is composed by a *data associator* (DA_j) and a variable-size *multi-EKF* ($\text{EKF}_j = \{\ldots, \text{EKF}_{jl}, \ldots\}$). The data associator is a 'nearest neighbor-like' memoryless algorithm [17] in charge of dispatching each relative pose hypothesis ${}^i\bar{t}_{jh}[k]$ produced by MultiReg to the appropriate EKF_{jl} of the array. This EKF_{jl}, taking $\hat{x}_j^\star[k]$, $\hat{x}_i^\star[k]$ and ${}^i\bar{t}_{jh}[k]$ as inputs, produces an estimate ${}^i\hat{t}_{jl}^\star[k]$ of ${}^i t_j^\star$, together with its covariance matrix.

At step k, DA_j converts each ${}^i\bar{t}_{jh}[k]$ to one hypothesis on ${}^j t_i^\star$, using $\hat{x}_j^\star[k]$, $\hat{x}_i^\star[k]$ and geometric computation. Then, the covariance-weighted distance of each hypothesis from the estimate of each EKF_{jl} is computed. Each hypothesis is used as input for the filter with the closest estimate, provided that the distance is smaller than a *maximum distance* $d_{\max}$. For each hypothesis ${}^i\bar{t}_{jm}[k]$ which is not associated to any

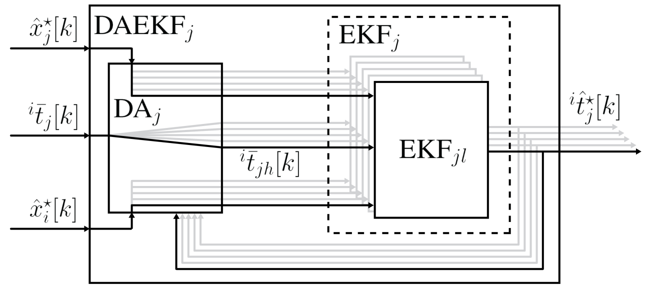

Fig. 7. A scheme of the DAEKF_j which estimates ${}^i t_j^\star$.

EKF_{jl}, a new filter is added to EKF_j, initialized with the triple $\{{}^i\bar{t}_{jm}[k], \hat{x}_i^\star[k], \hat{x}_j^\star[k]\}$.

At each step, a *mark* is associated to each EKF_{jl}, given by the number of *hits* (steps in which a hypothesis is associated to that filter) in the last $L[k]$ steps, where $L[k]$ is the *backward horizon*. The EKF_{jl} with the highest mark provides the best current estimate of ${}^i t_j^\star$. An EKF_{jl} whose mark goes below a certain threshold $\mu_{\min}$ is removed from the EKF_j array. The model equation of the generic EKF_{jl}, used to estimate the constant parameter ${}^i t_j^\star$, is ${}^i t_j^\star[k] = {}^i t_j^\star[k-1]$. The measurement equation, the Jacobian matrix and the expression of the Kalman gain can be found in [15].

VI. Experiments

The proposed ML method has been implemented and validated using the MIP experimental software, described in [18] and available at `http://www.dis.uniroma1.it/labrob/software/MIP`. In particular, we have simulated the robots with Player/Stage in the testing phase, and used an actual team of 5 Khepera III robots in the experimental phase (see Fig. 8). Each robot is equipped with a Hokuyo URG-04LX laser range finder, that has a 240° angular range and a linear range artificially limited to 2 m.

The robot detector is a simple feature extraction algorithm that inspects the laser scan searching for the indentations made by the vertical cardboard squares mounted atop each robot in the shadow zone of the range finder. Since each square can give $1-12$ cm wide indentations of the laser scan, depending on the relative orientation between the measuring and the measured robot, the detector cannot distinguish among robots and obstacles whose size is in the same range. Moreover, the squares are identical, and therefore the features are anonymous. Accurate measures of the ${}^i t_j^\star$ to be used as ground truth are taken in advance by a human operator. Self-localization is obtained by simple dead reckoning.

Fig. 9 refers to the early steps of a 5 min experiment involving five robots (numbered 0–4) and four deceiving obstacles. The results shown are those produced by the mutual localization system running on robot $\mathcal{R}_4$ at 10Hz, which is the same frequency of the laser range finder. Note that the initial configuration of the team is highly symmetrical, and therefore very ambiguous for MultiReg; moreover, it contains several occlusions. At the beginning the best available estimates for $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$ are wrong, due to occlusions and symmetry; however, as the experiment proceeds, correct estimates are quickly identified and prevail.

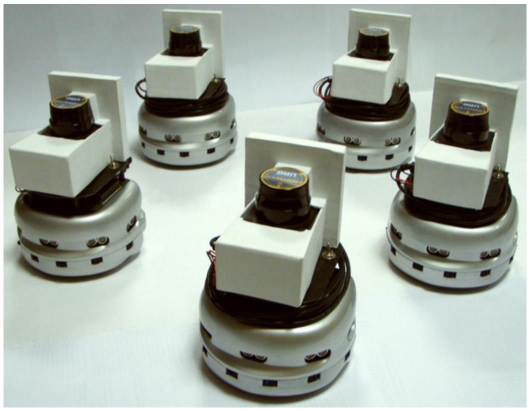

Fig. 8. The 5 Khepera III used in our experiments. A cardboard square is placed in the shadow zone of the URG-04LX to allow robot detection.

Fig. 9. Above: stroboscopic motion in the early steps of the experiment (robots are numbered 0–4). Below: the best estimates for the same steps (lighter impressions indicates older estimates) and the measured features measured by the robot detectors (small dots).

A clip of this experiment, including step-by-step comments, is contained in the video accompanying the paper.

Fig. 10 summarizes the result of the experiment in terms of estimation errors (cartesian and angular) and marks assigned by DAEKF_1 to the available hypotheses on the relative pose of the $\mathcal{R}_1$ fixed frame with respect to that of $\mathcal{R}_4$. The timescale is 150 sec. Note in particular how the best estimate, easily identifiable by the three (darker) lines that achieve the smaller errors and the higher mark, appears only 5 sec (approximately) after the beginning of the experiment.

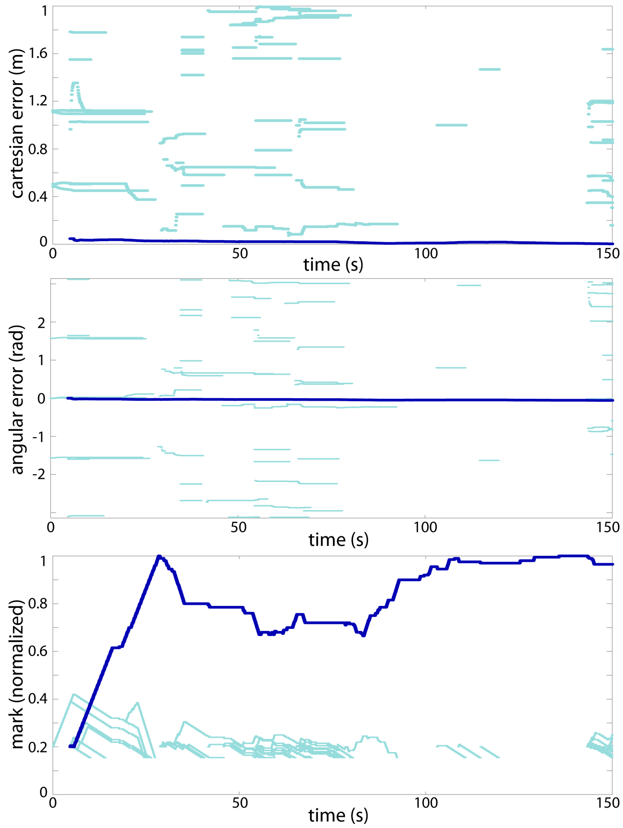

Fig. 10. Errors and mark for the best estimate on the pose of the fixed frame of $\mathcal{R}_1$ with respect to that of $\mathcal{R}_4$. The light lines in the background refer to all other hypotheses on the estimate. Each light line represents the life of an estimate. Plotting is interrupted for estimates whose normalized mark goes below 0.15.

Another experiment, in which two robots are used as deceiving mobile obstacles (they do not communicate their measures), is shown in the accompanying video. See also `http://www.dis.uniroma1.it/labrob/research/mutLoc.html` for more details and other experiments.

A. *MultiReg running time*

The running time of MultiReg, which accounts for most of the cycle time of our mutual localization system, depends on the number $|\Omega|$ of raw observations it receives in input, as well as on the ambiguity of the multi-robot system configuration. Note that $\max|\Omega| = n$. In unambiguous situations, $|\Omega|(|\Omega|-1)/2$ binary registrations are needed to produce a solution; since each binary registration requires constant time in the worst case, the time complexity of MultiReg in this case is $o(n^2)$. In ambiguous situations, as many as $(n-1)!$ irreconcilable solutions may exist, leading to an $o(n!)$ time complexity.

Fig. 11 reports some statistics on the running time of MultiReg as a function of $|\Omega|$ and of the number of solutions it finds. The first plot shows that the upper bound increases exponentially, the lower bound is constant and the mean value time has an approximately quadratic increasing rate. These results are consistent with the above theoretical prediction. In the second plot, the mean value increases linearly whereas upper bounds are higher for small numbers

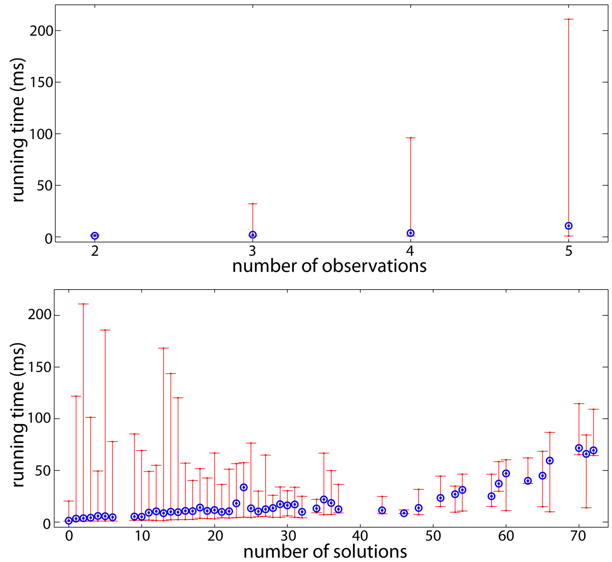

Fig. 11. Max-min (horizontal ticks) and average (circles) values of the running times (ms) of MultiReg with respect to the number of input observations and the number of solutions. Data based on 63898 executions of MultiReg during 38 real robot experiments.

of solutions. This is due to the fact that small solution numbers were much more frequent (about 25000 samples against a few dozens). All things considered, our experiments indicate that the proposed mutual localization system can easily run at 10 Hz for a team of five robots.

VII. Conclusions

In this paper, we have presented a novel method for mutual localization in multi-robot systems. In particular, our technique allows to estimate the changes of coordinates among the various robot frames using relative position measures that are anonymous as well as affected by false positives and negatives. The data available to each robot are processed by the MultiReg algorithm to obtain a set of hypotheses on the relative pose of the other robots of the team. The anonymity hypothesis causes an ambiguity in the inversion of the observations, that is solved using a multi-hypotheses filter. Satisfactory performance has been obtained both in simulations and in real robot experiments, showing that the proposed localization system is applicable in practice.

One possible problem with the proposed approach is that the running time of MultiReg may increase considerably if the number of its solutions grows. However, the case with factorial number of different solutions is obtained only in particular configurations in which a subset of the observations are roughly the same. We are currently developing a theoretical study of the ambiguity introduced by the anonymity hypothesis, aimed at reducing the number of MultiReg solutions by generating in linear time a single representative for each class of equivalent solutions. Another possible technique to reduce the complexity might be the use of some threshold on the number of the registered observations. Another improvement would be obtained by considering only solutions that are sufficiently close to the currently available estimates, so as to introduce a feedback mechanism from the DAEKF subsystem to MultiReg. Moreover, the 'nearest neighbor' policy of the data association can be avoided by resorting to a particle filter that samples also on data association, such as that developed in [19]. Work in progress also deals with the application of the developed system to decentralized tasks, such as formation control and cooperative exploration.

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