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Regressor–Free Force/Position Control of Fixed–Base Exoskeletons for Rehabilitation Tasks

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Abstract— **The required tasks in fixed-base exoskeletons demand a fast position/force controller; yet robust against unknown disturbances due to the application itself is tightly coupled with a human in a wide range of operational conditions, which give rise to human-exoskeleton interaction dynamics, high nonlinear uncertain exoskeleton dynamics, noisy sensors and other parametric uncertainties, such as environmental contacts. These factors do not allow to account on a precise dynamical model, thus model-based (regressor-based) controllers are difficult to implement. This paper deals with a regressor-free smooth PID-like fast force/position controller which guarantees finite-time convergence within second order sliding modes, thus ensuring inherent robustness. Experimental platform allows assessing its performance for rehabilitation tasks, which validates its functionality in practical implementation.**

I. INTRODUCTION

A. Background

Arguably, it is well–known that exoskeletons are wearable devices, either fixed inertial base (FIB) or mobile inertial base (MIB, that is non-inertial) for upper or lower limbs[13]. There is a vast literature devoted to discuss diverse aspects on design, analysis, synthesis, control and applications of such exoskeletons [4], [10], [16]. Nowadays, exoskeletons can be treated as complex biomechatronic machines, which have evolved alongside the technological stream and along the trends of human rehabilitation therapies, [12]. Exoskeletons are in general light-weight robots with inherent control problems introduced by flexibility, human–machine interaction, input noisy signals, complex friction as well as remote power transmission [10], the control problem is a significant issue, yet the precision, accuracy and robustness of the closed-loop systems are still a problem due to (1) user signals can vary from user to user; and (2) the heuristic controller is not robust against parametric disturbances and depends whatsoever to each user.

A major concern in this realm is that the user is a patient, that is, a human with disabilities, thus it is not reasonable to expect a certain good level of command by the user, consequently it is of interest to develop advanced control techniques to relieve the patient of stringent operational requirements. Although there are these distinct control problems, some of them can be diminished or neglected according to the operational regime via careful path planning of the rehabilitation task. However, rehabilitation tasks are by nature a constrained task, that is a force/position task, which can be treated either as impedance or explicit or implicit force/position control problem, however to our best knowledge, implementations of advanced nonlinear control schemes of constrained exoskeletons for rehabilitations tasks are unknown.

B. Control Issues

Despite these impressive developments, it can be said that exoskeletons for rehabilitation tasks are in their early research stage because it is not yet well understood how to exploit better the biomechanical nature of exoskeletons, which are in fact composed not only by the exoskeleton itself, but also it includes an impaired human driving the exoskeleton. So far, most of the research has been developed by biomechanical and neuroscience groups, who have somehow been more concerned to the mechanical design issues, according to rehabilitation needs, using simple output feedback control strategies with heuristic EMG or EEG human-based driving signals, see [5], [13]. The benefits of more powerful control strategies might contribute to deal with the kinesthetic coupling of human-exoskeleton as well as the parametric and model uncertainties. Simple position or impedance regulators might not suffice the stringent requirements even of a simple therapy because, intrinsically, the motion regimen of such therapy stands for dynamical force-position tracking tasks, not position regulation tasks. Then the control problem is how to design dynamical force-position tracking control, whose closed-loop exhibit fast and robust tracking.

C. The Nature of Constrained Rehabilitation Tasks

Fixed–base exoskeletons for rehabilitation tasks require environmental interaction with certain precision because the electromechanical exoskeleton is constrained by the environment, in this case the patient. Strictly speaking this system is modeled by highly coupled nonlinear Differential Algebraic Equations (DAE Index-2), requiring to comply with the holonomic constraint (a given constrained profile). However assuming that the patient suffers motor disabilities, which may introduce trembling and unmodeled dynamics, a robust DAE-based controller is required. Then, such constrained tasks demand fast response to match the given operational frequency, with given accuracy and precision imposed by the therapy, as well as robustness against unknown perturbations

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and endogenous dynamics to fulfill at every instant the holonomic constraint. Additionally notice that for rehabilitation purposes, it is important to convey quickly, precisely and smoothly the desired contact force, similar to the performance of a physician or therapist on the patient. This stands for smooth, robust and fast convergence of tracking errors.

It is worth to remark that force control requires high sampling rates because the contact force variable is a high–bandwidth variable by nature. Another stringent requirement is security, because the mechanical robot exoskeleton is mechanically in contact to a patient, then a deterministic passive closed-loop behavior is a must, which in turns requires also a real-time deterministic system to handle all kind of involved signals timely.

Finally, we call again the attention that the complex biomechatronic nature of exoskeletons makes unreasonable to assume knowledge of the full exact nonlinear model, thus model-based controllers are not an option despite well-proven model-based nonlinear smooth control techniques, such as adaptive control schemes [14], [9]. It is the unavailability of the exact knowledge of the regressor, rather than the computational cost involved in computing the regressor, which deprives us to rely on model-based control schemes. Last but not least, a tracking regime is preferred instead of regulation one to be able to reproduce continuously bounded time-varying desired position/force trajectories, designed according to the therapist protocol.

D. The Problem Statement

Choosing properly the control scheme requires to find out the structural properties of the full nonlinear model to be able to guarantee any given performance in closed-loop, in particular guaranteing a passive energetic closed-loop coupling. It is argued in this paper that although rehabilitation tasks are low performance tasks, in comparison to industrial or research robots, the requirement of guaranteing a passive behavior in closed-loop when the patient is carrying the exoskeleton needs conservative semi-global nonlinear controllers. All these discussions lead us to formulate the following control problem[1]:

Design a robust and fast smooth force/position control scheme which ensures tracking of fixed–base exoskeletons subject to unknown induced endogenous and exogenous bounded dynamics, assuming that just position and force measurements as well as exact knowledge of the holonomic constraint are available.

E. Contribution

A viable force/position tracking controller for FIB wearable exoskeletons is proposed to yield a fast and robust chattering–free second order sliding mode controller for the full nonlinear constrained DAE model, without explicit knowledge of the regressor. The closed-loop accounts for robust strictly passive performance, when operated reasonably at low frequencies typical of rehabilitation tasks[2]. Additionally, to achieve perceivable zero lag or delay by the patient, time-base generator are introduced in the sliding surfaces to yield finite time convergence so as to the trajectory might converge before the next human trajectory is generated. This last characteristic ensures not only precise tracking in the space axis, but also in the time axis. Preliminary experiments validate the proposed approach.

F. Organization

The organization is five–fold: Section II shows the main assumptions about the dynamical structure of the wearable exoskeletons. The proposed force/controller satisfying the holonomic constraints and its stability analysis are developed into Section III. Section IV outlines remarks about the controller and the characteristics of the closed-loop. The Light–Exoskeleton as experimental system is used for demonstrating the proposed control's performance; its physical structure and kinematics of the system are shown in Section V. The fast and robust orthogonal force/position real time performance of the *light–Exoskeleton* is presented in Section VI, whereby finite–time convergence on the applied force is demonstrated through preliminary therapist experimental protocol. Finally conclusions and immediate future work are given in Section VII.

II. THE CLASS OF CONSTRAINED FIXED–BASE EXOSKELETON ROBOTS

Consider the class of rigid[3] wearable fully actuated constrained mechanical systems modeled by the Euler–Lagrange formalism[2]. Let the constrained lagrangian be, [1],

$$\mathscr{L} = K(q,\dot{q}) + P(q) + \varphi^T(q)\lambda \tag{1}$$

where scalars $K(q,\dot{q})$ and $P(q)$ stand for the kinetic and potential energies, respectively, $\varphi(q) \in \mathbb{R}^m$ represents the holonomic constraint and $\lambda \in \mathbb{R}^m$ stands for the Lagrangian multiplier, being $(q,\dot{q})$ the generalized coordinates. The resulting system is a set of differential algebraic equations (DAE)[11] of index–2 over the entire domain of the Euclidian space in $\mathbb{R}$ compliance with the holonomic constraint $\varphi = 0$ for all time. The Lagrange multiplier $\lambda \in \mathbb{R}$ steams due to calculus of variations and physical principles and stands as the exerted force by the physical user–exoskeleton interaction. The resulting nonlinear dynamical equations and its structural properties are presented in this section.

A. Holonomic Exoskeleton Constraints

During force/position motion, the exoskeleton is constrained in operational space $\varphi(\mathscr{X}) = 0$, for $\mathscr{X} = (x,y,x,\alpha,\beta,\gamma) \in \mathbb{R}^m$, for $m = 6$ in the general case, being the first three elements of $\mathscr{X}$ the cartesian position, while the last three entries of $\mathscr{X}$ denote the Euler angles. Forward kinematics $\mathscr{X} = f(q)$ produces a holonomic constraint in

[1]Hereafter, "model–free" term can be used indistinctly with "regressor–free" as long as the control scheme is derived in joint space.

[2]This reasonable assumption avoids excitation of eigenmodes associated to flexible link or flexible joint.

[3]Rigid term suggests that exoskeleton kinematics can be straightforward computed using constant parameters.

terms of generalized coordinates $\varphi(q) \doteq \varphi(f(q)) = 0$ as follows

$$\varphi(q) = 0 \Rightarrow J_\varphi(q)\dot{q} = 0 \mapsto J_\varphi(q)\ddot{q} + \dot{J}_\varphi(q)\dot{q} = 0 \quad (2)$$

In this context, notice that the holonomic constraint is modeled in generalized coordinates, which suggests that the time derivative of (2) yields the velocity vector $\dot{q}$ orthogonal orthogonal to $J_\varphi(q)$. This fact guarantees the open loop passive behavior of the DAE system.

B. Nonlinear FIB Exoskeleton Dynamics

The dynamical model of a rigid exoskeleton of n–links in joint space, applying the constrained Lagrangian yields the following DAE system,

$$\begin{aligned} H(q)\ddot{q} + \{C(q,\dot{q}) + B_0\}\dot{q} + g(q) &= \tau + \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}\lambda \\ &+ \tau_u \\ \varphi(x,y,z) &= 0 \end{aligned} \quad (3)$$

where $(q,\dot{q}) \in \mathbb{R}^{2n}$, are the dynamic states, position and velocity, respectively; $H(q) \in \mathbb{R}^{n\times n}$, stands as inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ stands as the coriolis matrix, the linear positive definite matrix composed of damping coefficients is given by $B_0 \in \mathbb{R}^{n\times n}$, $g(q) \in \mathbb{R}^n$ stands as the gravitational torque vector, and $\tau \in \mathbb{R}^n$ is the control input functions. $\tau_u(t)\mathbb{R}^n$ stands as the unknown perturbation torque imposed by the user–exoskeleton interaction whilst the haptic guidance is being carried out. Finally, $J_\varphi \in \mathbb{R}^n$ denotes the standard jacobian of the holonomic constraint $\varphi(x,y,z) \in \mathbb{R}$, which is available since it is assumed $\varphi(x,y,z) = 0$ is known.

C. Validation of Dynamic Model Properties

Accordingly, the Euler–Lagrange Formalism, L–Exos dynamics (3) exhibits the following properties, which are fundamental to design the controller and its subsequent stability analysis,

$$\begin{aligned} ||H(q)|| &\geq \lambda_{Min}(H(q)) > 0, \\ ||H(q)|| &\leq \lambda_{Max}(H(q)) < \infty, \\ ||C(q,\dot{q})|| &\leq \beta_2||\dot{q}||, \\ ||G(q)|| &\leq \beta_3 < \infty, \\ ||B_0|| &\geq \beta_4 > 0, \\ ||\tau_u|| &\leq \beta_5||\ddot{q}||, \\ |\varphi(x,y,z)| &\approx 0_a, \end{aligned} \quad (4)$$

where $\lambda_{Max}(E) \leq \beta_0 < \infty$, $\lambda_{Min}(E) \geq \beta_1 > 0$ are the maximum a minimum eigenvalues of $E \in \mathbb{R}^{n\times n}$, 0_a denotes the physically achievable zero. Furthermore, the set of positive scalars $\beta_0 \cdots \beta_5 \in \mathbb{R}$ can be easily computed.

D. Open–Loop Error Dynamics

Taking into account that (3) is linearly parameterizable by a regressor $Y = Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times m}$, which is composed of known nonlinear functions, times a vector of unknown constant parameters $\Theta \in \mathbb{R}^m$[14], there arises the parameterization of (3) in terms of a nominal reference $\dot{q}_r \in \mathbb{C}^1$ and its derivative as follows

$$Y_r\Theta = H(q)\ddot{q}_r + \{C(q,\dot{q}) + B_0\}\dot{q}_r + g(q) \quad (5)$$

Then substituting (5) into dynamics (3) yields the following open–loop error dynamics

$$\begin{aligned} H(q)\dot{S}_r + \{C(q,\dot{q}) + B_0\}S_r &= \tau - Y_r\Theta \\ &+ \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}\lambda + \tau_u \end{aligned} \quad (6)$$

where the error coordinates manifold $S_r \in \mathbb{K}^n$ are given by

$$S_r = \dot{q} - \dot{q}_r \quad (7)$$

Manifold S_r parameterizes the error space, similar to [14], although in this case for constrained dynamics, thus, it is possible to develop a similar controller as proposed by [9], such that the dynamical model DAE (3) tracks the desired position and force simultaneously, in spite of unknown exogenous dynamics and unknown intrinsic parameters of the exoskeleton.

III. Finite-time Convergent Force/Position Controller in Joint Space

Exoskeletons are complex and light mechanical systems, whose precise dynamical model is hardly known. Then, there are few control schemes which fulfills the requirements of fast, robust and model-free with formal stability results. It seems that soft-computed-based schemes are computing intensive and rely on heuristical tuning approaches, which is risky because the main aim of the controller must be to ensure a passive energetic behavior of the closed-loop coupled system [6]. Another option is based on [9], which produces passive energetic coupling, which is then preferred.

Tracking controller yields generally either asymptotic or exponential convergence of the *spatial attributes* of tracking errors, however, it is not possible to guarantee spatial tracking *at any given* desired time, because simply there is not any *desired time* variable. However, for rehabilitation robotic tasks, it is relevant to achieve control not only of the spatial attributes of tracking errors but also it is important to control the time attributes of tracking errors. This observation arises in clinical protocols, wherein the therapist induces a given rehabilitation tasks at a given time, not at any arbitrary time. We can achieve control of spatial coordinates as well as time coordinate with a novel mechanism called *Finite Time Convergence*, which is nothing but a well-posed accelerator of the convergence time of tracking errors, regardless of the initial condition of the system and feedback gain of the controller. In this paper, and similar to our previous result, we propose a finite-time attractors into the invariant sliding surfaces so as to produce simultaneous spatial and time tracking of position and force errors.

A. Orthogonalized Sliding Surface

Consider the following nominal reference $\dot{q}_r$,

$$\begin{aligned}\dot{q}_r &= Q\{\dot{q}_d - \alpha_1\Delta_q + S_{dp} - \gamma_1\int_{t_0}^{t} sgn(S_{qp}(\xi))d\xi\} \\ &\quad -\beta J_\varphi^T\{S_{qF} + \gamma_2\int_{t_0}^{t} sgn(S_{qF}(\xi))d\xi\} \qquad (8)\end{aligned}$$

where position and force errors are given by $\Delta_q = q - q_d$, respectively, $\Delta_{F_2} = \int_{t_1}^{t}\int_{t_0}^{t}(\lambda-\lambda_d)(\xi)(\zeta)d\xi d\zeta$ and $\Delta_F = \int_{t_0}^{t}(\lambda-\lambda_d)(\xi)d\xi$, the desired references for position and force, respectively, are $q_d(t) \in \mathbb{R}^n$ and $\lambda_d(t) \in \mathbb{R}$, function $sgn(\cdot)$ represents the signum function of the its vector argument. Positive feedback gains[4] are defined as $K_d, \alpha_1, \gamma_1 \in \Lambda_+^{n\times n}$ and $\alpha_2, \gamma_2, \eta, \beta \in \mathbb{R}_+$. The orthogonalized sliding surface[1] of position/force S_r is defined by,

$$S_r = Q(q)S_{vp} - \beta J_\varphi^T S_{vF} \qquad (9)$$

where each attractive stable manifold of the extended orthogonal position and force S_{vp} and S_{vF} are written as follows,

$$S_{vp} = S_{qp} + \gamma_1\int_{t_0}^{t} sgn(S_{qp}(\xi))d\xi \qquad (10)$$

$$S_{vF} = S_{qF} + \gamma_2\int_{t_0}^{t} sgn(S_{qF}(\xi))d\xi \qquad (11)$$

where

$$S_{qp} = S_p - S_{dp}, S_p = \dot{\Delta}_q + \alpha_1\Delta_q \qquad (12)$$

$$S_{dp} = S_p(t_0)e^{-\rho_0(t-t_0)} \qquad (13)$$

$$S_{qF} = S_F - S_{dF}, S_F = \Delta_F + \alpha_2\Delta_{F_2} \qquad (14)$$

$$S_{dF} = S_F(t_0)e^{-\rho_1(t-t_0)} \qquad (15)$$

with $\rho_0, \rho_1 \in \mathbb{R}_+$ as real positive constant values.

B. Control Design and Closed-Loop System

Let the model–free control law τ be

$$\begin{aligned}\tau &= -K_dS_r + \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}\big(-\lambda_d + \eta S v_F + \eta S_{dF} \\ &\quad + \eta\tanh(\mu S_{qF})\big) \\ &= -K_dS_r + \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}\{-\lambda_d + \eta\Delta_F + \eta\alpha_2\Delta_{F_2} \qquad (16) \\ &\quad + \eta\tanh(\mu S_{qF}) + \eta\gamma_2\int_{t_0}^{t} sgn(S_{qF}(\xi))d\xi\}\end{aligned}$$

with $\eta, \mu \in \mathbb{R}_+$ and $K_d = K_+^T$. For stability purposes, notice that equations (3), (6) and (16) produce the following closed-loop error equation

$$\begin{aligned}H(q)\dot{S}_r &= -\{C(q,\dot{q}) + B_0 + K_d\}S_r - Y_r\Theta - K_dS_r \\ &\quad + \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}Z_1 + \frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}Z_2 + \tau_u \qquad (17) \\ Z_1 &= \{\Delta_\lambda + \eta\Delta_F + \eta\alpha_2\Delta_{F_2}\}, \Delta_\lambda = \lambda - \lambda_d \\ Z_2 &= \eta\tanh(\mu S_{qF}) + \eta\gamma_2\int_{t_0}^{t} sgn(S_{qF}(\xi))d\xi\end{aligned}$$

whose stability properties are analyzed in the following section.

[4] $\Lambda_+^{n\times n}$ denotes the diagonal definite positive matrix subspace.

C. Stability Analysis

The regressor-free controller (16) guarantees the following stability properties

Theorem 3.1: Considering the fixed–base exoskeleton dynamics (3) in closed-loop with the force/position second order sliding PD controller (16). Then, the closed–loop dynamics (6) induces chattering-free second order sliding mode regime for all time, with local convergence for the force and position tracking errors, by choosing accordingly the feedback gains $K_d, \alpha_1, \alpha_2, \mu, \eta, \gamma_1, \gamma_2, \beta, \rho_1, \rho_2$, including bounded coupling perturbation due to human, modeled by $\tau_u(t)$.

Proof: A short sketch of the proof is as follows, [8]. Passivity-based analysis $\langle S_r, \tau\rangle$ leads to the following Lyapunov candidate function,

$$V = \frac{1}{2}(S_r^T H S_r + \varsigma S_{vF}^T S_{vF}) \qquad (18)$$

with $\varsigma \in \mathbb{R}_+$. If we define $K = K_d + B_0$ and $Z = Z_1 + Z_2$, then time derivative of the Lyapunov candidate function along its solution leads to,

$$\begin{aligned}\dot{V} &= -S_r^T(K_d + B_0)S_r + S_r^T Y_r\Theta + S_r^T\tau_u \\ &\quad + S_r^T\sigma\frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}(Z_1 + Z_2) - \varsigma S_{vF}^T S_{vF} \\ &\leq -S_r^T K S_r + ||S_r||||Y_r\Theta|| + ||S_r||\max\{|\tau_u||\} \\ &\quad + ||S_r||\sigma\left\|\frac{J_\varphi^T}{||J_\varphi J_\varphi^T||}\right\|||Z|| \qquad (19) \\ &\leq -S_r^T K_d S_r - \varsigma S_{vF}^T S_{vF} + ||S_r||||\delta||\end{aligned}$$

If K_d is large enough, it establishing the boundedness of all closed-loop signals. With result at hand, the derivative of the orthogonalized sliding surface S_r gives rise to two forced sliding surfaces, then if feedback gains σ, ς are large enough then second order sliding modes arises at each velocity- and force-subspace. Notices that this implies the invariance of S_{qp} and S_{qF}, in turn, the exponential convergence of position, velocity and force tracking errors. Additionally, if α_1 is modeled according to a *Time-Base Generator* [8], then position-velocity tracking errors converge to zero at the desired finite time $t_b > 0$. ■

IV. REMARKS ON PRACTICAL ISSUES

Implementation of nonlinear controllers are always difficult, since Lyapunov provides only conservative bounds of feedback gains, thus a precise and intuitive tuning procedure is not easy. In this context, we provide additional insight into the nature of the control system, keeping in mind discussions of subsection I.B. These remarks are useful to implement the controller and obtain such robust and fast force/position convergence of these class of fixed–base exoskeletons.

***Remark** 4.1:* **: Regressor-free Control.** It is worth to mention that the proposed controller does not require the regressor, it requires only a conservative values of bounds to choose the controller gains large enough in order to establish a sliding regime complying with equation (19). Notice that the full nonlinear DAE-2 system is considered for the design of the control and in its in the stability analysis, but not in the controller.

***Remark** 4.2:* **: Feedback Gains and Stability Domain.** Semi-global stability is obtained as long as the feedback gains are tuned accordingly to the proof, however the stability domain is not enlarged when feedback gains are tuned larger than established but it achieves global tracking. Is it straightforward to see that the sort of controller gains are $K_d, \alpha_1, \gamma_1 \in \Lambda_+^{n\times n}$ and $\alpha_2, \gamma_2, \rho_1, \rho_2, \eta, \beta \in \mathbb{R}_+$. With an abuse of the mathematical notation we suggest the following tuning procedure: $K_{d_i} = 2\sqrt{\alpha_{1_i}}$, $|Kd_i| \gg |\gamma_1|$, $\rho_1 \approx \rho_2 := [20, 300]$, $|\alpha_2| > |\eta| > |\beta| \gg |\gamma_2|$.

***Remark** 4.3:* **: Computational Cost.** One advantage of implementing this control relies precisely in the fact of its regressor-free structure, thus the associated computational–cost is very low, since the controller is a nonlinear PID-like controller. This reduces considerably the burden of the software integration and real-time considerations.

***Remark** 4.4:* **: Endogenous/exogenous Disturbances.** Notice that many fixed–base exoskeletons [7], [10] estimates indirectly the model parameters and use them, for instance for gravity compensation, however it is prone to errors since such parameters are an approximation of the real ones, let alone it cannot deal with bounded unknown disturbances. In contrast, our proposal is able to compensate any bounded disturbances like those produced by human–exoskeleton interaction, induced vibrations, as well as other merged exogenous unknown dynamics into the closed–loop. Such endogenous/exogenous disturbances can be characterized within bounds, since motion capabilities and neurophysiological behaviors of patients are available off-line.

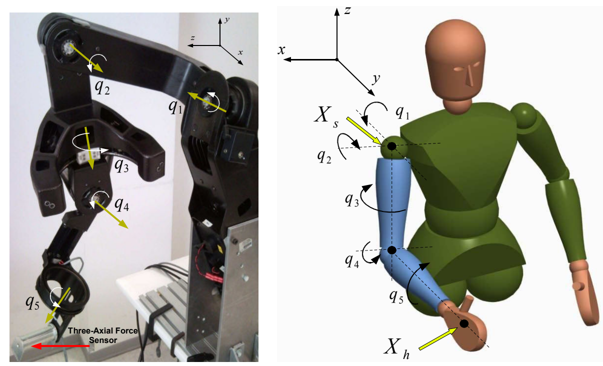
***Remark** 4.5:* **: Very Fast Tracking.** The finite–time convergent can be induced using a time–generator base (TBG) to shape the time-varying gains α_1, α_2 automatically, see[8], guaranteing very fasts convergence, even faster than exponential one.

***Remark** 4.6:* **: Noise Sensitivity.** The double–integrator included into controller design, see equation(14) acts as a "filter" of the zero–mean noisy components of the force sensor, those components can be derived from the sensitive level of force sensor, introduced because of the haptic coupling between user and exoskeleton. Other sensor noise or electronic noise are dealt with well–established filtering techniques

***Remark** 4.7:* **: Parametric Invariance.** It is well known that sliding modes, either first or high order, produces order reduction and invariance to model system variations. That is, notice that there arises two first–order differential equations out of open-loop second order system, which are independent of the exoskeleton dynamics, those are $S_{vp}(t) = 0$ and $S_{vF}(t) = 0$. This means that the controller compensate parametric disturbances and unmodeled dynamics of any parametric uncertainties because $S_{vp}(t) = 0$ and $S_{vF}(t) = 0$ are independent of these dynamics, guaranteing an exceptional robust behavior, as long as the sliding mode condition is preserved.

V. The Experimental System

The Light–Exoskeleton, or L-Exos for short [3], is composed of five degrees-of-freedom, which four of them are fully actuated and the last one is used for measuring the wrist pronation/supination motion. L–Exos is intended for



(a) L-Exos mechanical design (b) Human arm basic movements

Fig. 1. Absolute angular coordinates corresponding with the basic arm L–Exos movements.

providing a haptic biomechanical force/position kinesthetic coupling in three dimensional workspace to the user right–arm. Notice that the basic task resembles a human arm motion: (1) Adduction/ abduction; driven by joint q_1. (2) Flexion/extension;driven by joint q_2 and q_4. (3) Internal/external rotation;driven by joint q_3. (4) Pronation/supination; given by q_5, see figure 1(b).

A. Kinematics of L–Exos

For this contribution the kinematics of the L–Exos plays an important role because of the accuracy of computing the holonomic constraints depends on the forward and inverse kinematic analysis.

In one hand, the Denavith–Hartenberg (D–H) parameters [2] are shown in table I.

TABLE I
Denavith–Hartenberg Parameters

Link (i)	d_i	a_i	α_i	q_i
0	0	0	$-\alpha_0$	0
1	d_1	0	$-\frac{\pi}{2}$	$q_1 - \frac{\pi}{2}$
2	0	0	$\frac{\pi}{2}$	$q_2 + \frac{\pi}{2}$
3	d_3	0	$-\frac{\pi}{2}$	q_3
4	0	0	$\frac{\pi}{2}$	q_4
5	d_5	0	0	q_5

d_i stands as a constant distance for $i = 1,3,5$.
α_0 is the inclination angle of the inertial fixed frame.

Therefore, the forward kinematics based on (D–H) convention, is straightforward computed as follows,

$$\begin{aligned} T_b^i &= f(d_i, a_i, \alpha_i, q_i), i = 0, 1, \dots 5 \\ &\triangleq \begin{bmatrix} R_b^5 & d_b^5 \\ 000 & 1 \end{bmatrix} \end{aligned} \tag{20}$$

where $R_b^5 \in \mathbb{R}^{3\times 3}$ and $d_b^5 = (x, y, z)^T \in \mathbb{R}^3$ stand as the L–Exos rotation matrix and cartesian position with respect to fixed inertial base b.

In the other hand, the inverse kinematics problem is solved using either closed–form or iterative solution, but keeping in mind the capabilities of providing access to properly drive the generalized coordinates for each patient in rehabilitation tasks.[5].

B. Hardware and Software Integration Issues

Consider the L–Exos platform, which is worn by a healthy user in Fig. 2, L–Exos consists of four servo–motors

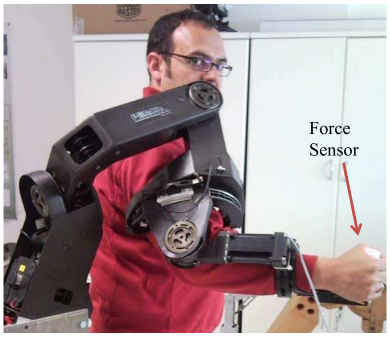


Fig. 2. L–Exos platform is worn by user

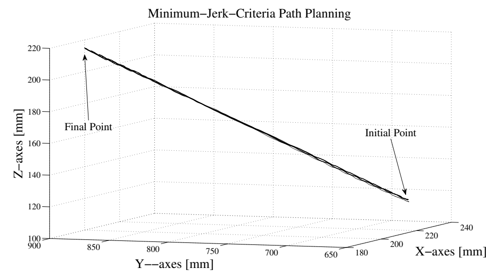
controlled by PWM, remote power transmission based on tendons–pulleys; the three–axial high precision force sensor at the end-effector provides measurement of contact forces with 2048 CPR encoders provides robust reading of joint displacements. A PC-based Pentimu IV at 500 Mhz implants a XPC Real–Time Windows Target of Matlab® stands for the processing unit, with a 14–bit ISA acquisition system board, running at $0.3\,ms$ sample-rate. The proposed joint controller requires a transformation of the cartesian task into its corresponding joint task, using the kinematics approach, i.e, $f^{-1}:\mathscr{K}^3 \mapsto \mathbb{R}^n$ where $\mathscr{K}^3$ is the admissible L–Exos workspace. The reaching task complies with the holonomic constraint $\varphi \in \mathscr{K}$,i.e,

$$\varphi(x,y,z) \triangleq Ax(t)+By(t)+Cz(t)-D \approxeq 0 \tag{21}$$

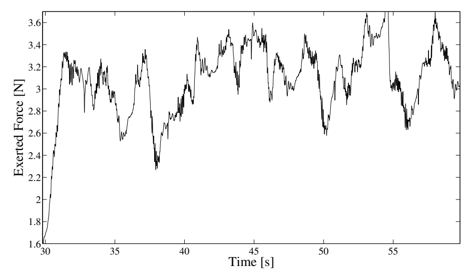
VI. Experimental Results

A. Diagnostic Exercise: Getting $\lambda_d(t)$ without force control

Reaching task is one of the most interesting protocols for rehabilitation due to exercise forearm muscle involved in adduction/adbuction, extension/flexion, internal/external rotation an so forth. Reaching is established between two points by tracking an smooth polynomial function which connects them complying with the minimum–jerk–criteria (smoothness of the second derivative on time of the velocity)[15], but normally it has been done by using conventional controllers in the operational space[7]. Notice that it is worth to know *a priori* the force values range to avoid any injures or damages to user arm, because its proper value may vary and depend on patient to patient. Indeed, the reaching task consists on tracking a defined line (21) which connects two points, in this case $X(t_0) = (225,663,117)$mm and $X(T) = (188,882,219)$mm. The time interval for each trial is $T = 5$s, whereby the exercise is done back and forth for a number of times $m \in \mathbb{R}$, in this case $m = 10$, guaranteing that forearm external rotation and flexion/extension are properly treated[6]. Taking into account just position control, the feedback control gains are $K_d = 10$, $(\alpha_1,\alpha_2) = (25,4)$, $\rho_1 = \rho_2 := 200$, $\eta = 3.1$ and $\beta = 0.8$, obtaining the behavior depicted in Fig. 3. Reaching task is carried out by a healthy user at this stage



(a) Reaching task into operational L–Exos space



(b) Exerted force by user during the haptic guidance

Fig. 3. Reaching task without force control

of this project, without force–control, which leads to choose a maximum force of 3.75N, i.e, $\max|\lambda_d(t))| \leq 3.75$N, see Fig. 3(b).

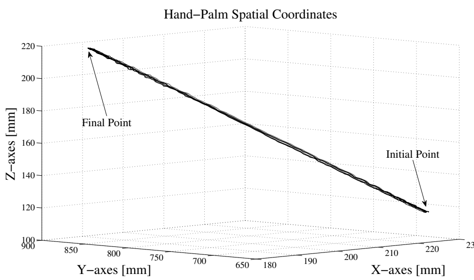
B. Force/Position Control of the Reaching Task

According to previous sub–section, the desired force has chosen taking into account the maximum exerted force by the user, i.e, $\lambda_d = 2.5 + 0.23\sin(\omega t)$N with $\omega = \frac{2\pi}{T_f}$ and $T_f = 6$ s. The closed-loop experimental results are shown in figures 4-5, while the generalized coordinates, and the respective holonomic constraint, are given in figure 5. Our proposal achieves a "physically achievable tracking error of zero", while complying to the holonomic constraint, with convergence to the desired force[7], it suggests that depending on each user we need to define the minimum bounded tracking error in practice such that error manifold S_{vF} is verified. As we can see in the Fig. 5(a), the flexion/extension of the arm and forearm have been smoothly controlled as well as the internal/external rotation with any impulsive jerk.

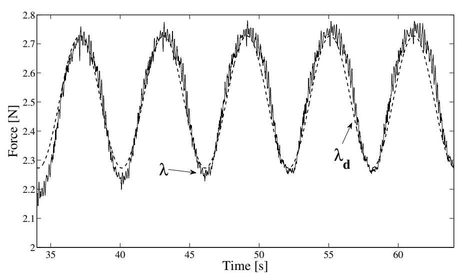
[5]This matter deserves more attention, hence into this contributions for avoiding cumbersome notation the inverse kinematics solution is not presented.

[6]This matter deserves more attention, though the medical assessment and protocol are not the objectives of this paper.

[7]For rehabilitation sake, it can be said that this is successful. Notice that we use the same controller gains.

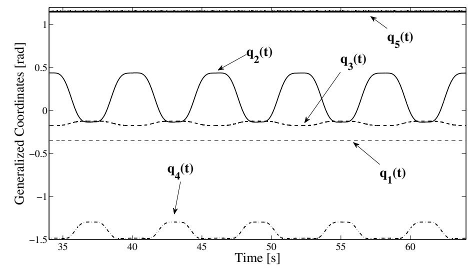


(a) Hand–palm spatial coordinates

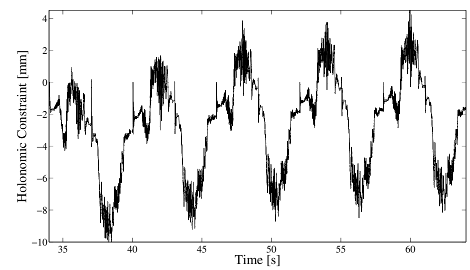


(b) Controlled force during the motion

Fig. 4. Rehabilitation task with force/position control.



(a) Joint space controlled variables



(b) Computed Holonomic Constraint $\varphi(q)$ given in mm

Fig. 5. Generalized Coordinates and Compliant Holonomic Constraint

VII. Conclusions and Future Work

The nature and requirements of fixed-base exoskeletons for rehabilitations tasks are discussed to design a more convenient controller. In this realm, the full nonlinear DAE-2 constrained model is considered and a regressor-free smooth PID-like fast and robust force/position controller is proposed and tested in real time. The closed-loop guarantees finite-time convergence within second order sliding modes, with a rather low computational cost, though there are involved several feedback gains, as expected for this complex system. Experiments on the L–Exos assess the closed-loop performance which validates its functionality in practical implementation on a healthy user, however, different threshold and feedback gains may vary for real patients, but similar performance is expected. Our proposed controller offers the capability of tracking force signals whilst simultaneously tracks a smooth position profile, even in the presence of human–machine interaction and unknown parametric and unmodeled uncertainties. This resembles to the task of a real physician on the patient. This proposal has been implemented in conjunction with Virtual Environments at the Cisanello Hospital in Pisa, Italy, as part of the clinical pilot testing protocol for arm rehabilitation.

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