A Deceleration Control Method of Automobile for Collision Avoidance based on Driver's Perceptual Risk

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Abstract—To reduce rear-end crash of automobiles, it is important to judge necessity of deceleration assistance as earlier as possible and initiate the assistance naturally. On the other hand, we have derived a mathematical model of driver's perceptual risk of proximity in car following situation and successfully derived driver deceleration model to describe deceleration patterns and brake initiation timing of expert driver. In this research, an automatic braking system for collision avoidance will be proposed based on the formulated brake profile model and brake initiation model of expert driver to realize smooth, secure brake assistance naturally. It will be shown that the proposed control method can generate smooth profile for various conditions. In addition, experimental results using a driving simulator will show validity of the proposed system based on subjective evaluation.

I. INTRODUCTION

Driver assistance systems such as warning system and pre-crash safety system have been developed to reduce and mitigate crashes in road traffic. In the view point of preventive safety, deceleration assistance control is effective when collision risk is high and it is difficult for the driver to avoid it. On the other hand, driver can feel anxiety or nuisance against the system if the initiation timing of automatic brake and/or deceleration profile is not appropriate and it may make the system inefficient. Thus, in order to realize an acceptable and efficient system, it is important to know characteristics of comfortable deceleration behavior and apply them to deceleration assistance system.

As a pioneer in deceleration behavior of car driver, Lee [1] developed theoretical framework of drivers longitudinal control based on TTC (Time-To-Collision) associated with visual information. Kondoh et al. investigate the risk perception and showed that it can be represented by TTC and THW(Time-Headway) [2]. Isaji et al. [3] and Wada et al. [4] have proposed a performance index of approach and alienation, KdB as a model of driver’s perceptual risk of a preceding vehicle and its another version, KdB_c based on area change of preceding vehicle on driver’s retina. These indices have been applied to modeling of braking behaviors of expert drivers [5]. Kitajima et al. have surveyed such evaluation indices concerning rear-end collision risk [6].

On the other hand, there are researches concerning design and evaluation of collision avoidance system and ACC(Adaptive Cruise Control) system. For example, Goodrich et al. [7] characterized the behavior in a phase plane of TTC vs. THW. Baret et al. have evaluated efficacy of ACC based on Gipps model that is a car-following model in traffic engineering [8]. Hiraoka et al. derived car-following model for realizing comfortable ACC system by applying concept of minimum jerk model to longitudinal vehicle behavior [9]. Suzuki et al. proposed a method to estimate driver status in car following situation and its application to driver assistance system [10].

It is important to introduce driver’s perceptual risk described before into its design of such driver assistance systems in order to realize comfortable and secure system. To implement this concept, we have proposed a deceleration control method of automobile based on the perceptual risk [11].

In this paper, a brake assistance system will be proposed for preventing rear-end crash based on an expert driver's deceleration model derived from driver's perceptual risk. Initiation timing of brake assistance will be determined by driver’s brake initiation timing model. Final target status of two vehicles, say, convergence distance by the braking system will be determined based on driver’s risk model. Finally, deceleration profile connecting the brake initiation timing and final target status will be determined by driver’s deceleration pattern model. Validity of the proposed control method will be shown by the experiments using a driving simulator.

II. PROBLEM DESCRIPTION

Suppose that a driver follows a car in the same lane as shown in Fig.1. Rear-end crash occurs when the driver does not notice the approach of the preceding car due to driver’s errors etc. In such situation, rear-end crashes can be reduced by detecting dangerous situation as earlier as possible and starting to decelerate automatically. To avoid collision, assistance system need to start decelerates earlier than emergency avoidance system such as so-called pre-crash safety system. One of difficulties to realize such system is how to prevent driver’s annoyance.

![Fig.1 Car Following Situation](image-url)
In order to realize such system, the following three questions need to be solved at least.

Q1) When to initiate automatic deceleration control
Q2) How to determine deceleration pattern of the system
Q3) How to determine final target status of two vehicles

On the other hand, status of the two vehicles can be described by gap between two vehicle $D$, velocity of following vehicle $V_o$, and velocity of preceding vehicle $V_p$. Thus, the three questions above need to be solved by specifying a set of state variable $[D, V_p, V_o]$ for each question. For example, we need to solve how to define danger status $\Omega_{\text{danger}}$ to start deceleration assist and how to define target convergence status $\Omega_{\text{conv}}$(Fig.2). In this paper, we address the questions by introducing driver’s perceptual risk to reduce driver’s annoyance and anxiety against the system. For this purpose, driver’s perceptual risk also needs to be represented by state variable $[D, V_p, V_o]$.

![Fig.2 Schematic Image of Trajectory of State Variable of two vehicles in Deceleration Assistance System](image)

III. DECELERATION BEHAVIOR MODEL OF EXPERT DRIVERS BASED ON PERCEPTUAL RISK

A. Index of Perceptual Risk of Proximity

Suppose that a car follows a preceding car in the same lane as shown in Fig.1. In such situation, the driver evaluates the risk against approach of the preceding car appropriately and realizes safe driving by operating pedals and a steering wheel based on the perceived results. So far, we have hypothesized that drivers detect the approach of the preceding car and recognize the risk by its area changes on the retina as shown in Fig.3 and determine the operation of deceleration based on it and a perceptual risk index $K_{dB}$ has been derived as eq.(1) [3],[4].

$$K_{dB}(a) = \begin{cases} 
10\log_{10}(\frac{4 \times 10^7 \times V_r}{D^3})\sgn(-V_r + aV_p) & (|4 \times 10^7 \times V_r / D^3| \geq 1) \\
0 & (|4 \times 10^7 \times V_r / D^3| < 1)
\end{cases}$$

(1)

where $D$ denotes gap between two vehicles. Relative velocity $V_r$ is defined as eq.(2) using velocity of the preceding vehicle $V_p$, and velocity of the following vehicle $V_o$.

$$V_r(t) = V_p(t) - V_o(t) = \frac{d}{dt} D(t)$$

(2)

We call this variable $K_{dB}$ “performance index for approach and alienation” at the moment of the driver’s operation such as deceleration and acceleration. Index $K_{dB}$ is increased when the preceding car is approaching relatively to the following car as similar as increase of the driver’s visual input. $K_{dB}$ is increased when the driver does not react to this regardless of cause of risky conditions such as low arousal level, inattentive or other reasons depending on driver’s status. It has been shown that $K_{dB}$ can discriminate between braking behaviors of normal safe driving and those in crash accidents that are extracted from micro data of crashes [3], [4]. In addition, it has been also shown that $K_{dB}$ can be a trigger to transit following mode to deceleration mode in the case that driver’s following behavior is modeled by mode transition model based on hybrid dynamical system [10].

In addition, another perceptual risk index $K_{dB,e}$ has been proposed as eq.(3) by introducing effect of changes of perceptual risk by preceding vehicle’s velocity into account and it has been shown that it can formulate brake initiation timing [5].

$$K_{dB,e}(a) = \begin{cases} 
10\log_{10}(\frac{4 \times 10^7 \times V_r + aV_p}{D^3})\sgn(-V_r + aV_p) & (|4 \times 10^7 \times (-V_r + aV_p) / D^3| \geq 1) \\
0 & (|4 \times 10^7 \times (-V_r + aV_p) / D^3| < 1)
\end{cases}$$

(3)

B. Model of Expert Driver’s Deceleration Pattern

Model of Deceleration Profile

Tsuru et al.[12] and Wada et al.[11] showed that deceleration pattern of expert drivers can be characterized using risk index KdB as follows based on the results of the experiments with real cars (Fig.4).

P1) Index $K_{dB}$ is changed with the same slope $dK_{dB}/dD=dK_{dB}(t_0)/dD$ as in phase I or constant slope phase in Fig.4.

P2) Constant deceleration is held after peak deceleration until $V_r=0$. as in phase II or constant deceleration phase in Fig.4.
Deceleration Model of Constant slope phase

Expert driver’s deceleration behavior was modeled by constant slope feature (P1) and the peak hold feature (P2). The peak hold feature is difficult to be installed in the automatic braking system due to lack of robustness against situation changes. Thus, let us focus on constant slope feature for applying deceleration profile generation method.

Suppose that we deal with only approaching condition because we are considering deceleration assist system. And assume that 4×10^{-7} V_r /D^2 ≥1 is satisfied. In such case, definition of KdB of eq.(1) can be rewritten as eq.(4).

\[ K_{dB}(t) = 10 \log_{10} \left( 4 \times 10^{-7} \frac{V_r(t)}{D^2(t)} \right) \]  

(4)

Differentiating \( K_{dB} \) by gap \( D \) yields

\[ \frac{dK_{dB}}{dD} = \frac{10}{\ln 10} \frac{\frac{dV_r(t)}{dt} - \frac{3}{D(t)}}{V_r^2(t)} \]  

(5)

Then P1 can be written as eq.(6).

\[ \frac{dK_{dB}}{dD} = \frac{dK_{dB}(t_{bi})}{dD} \]  

(6)

Substituting eq. (5) into eq. (6) and solving by relative acceleration leads to deceleration profile model eq.(7).

\[ \dot{V}_r(t) = \left( \frac{3}{D(t)} - \frac{3}{D(t_{bi})} \right) V_r^2(t) \]  

(7)

By integrating eq.(7) by time yields profile of relative velocity as eq.(8).

\[ V_r(t) = \frac{V_r(t_{bi})}{D^3(t_{bi})} D^3(t) \exp \left[ \frac{3}{V_r^2(t_{bi})} \left( \frac{1}{D(t)} - \frac{3}{D(t_{bi})} \right) (D(t) - D(t_{bi})) \right] \]  

(8)

Constant relative velocity situation

For the sake of simplicity, we deal with the situation approaching to a preceding car driving with a constant velocity. In this situation, relative deceleration is zero until braking behavior of the following car’s driver. Based on the assumption, substituting \( \dot{V}_r(t_{bi}) = 0 \) into eqs.(7) and (8) leads to eqs.(9) and (10), respectively.

\[ \dot{V}_r(t) = \left( \frac{3}{D(t)} - \frac{3}{D(t_{bi})} \right) V_r^2(t) \]  

(9)

\[ V_r(t) = \frac{V_r(t_{bi})}{D^3(t_{bi})} D^3(t) \exp \left[ \frac{3}{V_r^2(t_{bi})} (D(t) - D(t_{bi})) \right] \]  

(10)

where \( d(t) = D(t)/D(t_{bi}) \). From eq.(9), \( dV_r/dt \) reaches at zero when \( V_r = 0 \) otherwise \( dV_r/dt \) always takes positive value, say, deceleration. In addition, from eq.(10), \( V_r \) reaches at zero when \( D \) goes to zero otherwise \( V_r \) always takes negative value because \( V_r(t_{bi}) < 0 \) is assumed. From these results, the derived deceleration profile results in collision with \( V_r = 0 \) under the given assumptions as long as the calculated deceleration can be generated, that is, the state is uniquely converged to its equilibrium point \( [V_r, D] = [0, 0]^T \).

Fig.5 illustrates calculated results of eqs.(9) and (10) with \( V_r(t_{bi}) = -20 \text{Km/h} \) without relative deceleration until brake initiation. Two lines in each graph show \( D(t_{bi}) = 25 \text{m} \) and \( 50 \text{m} \). Very smooth deceleration profile can be obtained with only simple calculation of eq.(9).

As the results, our model can generate very smooth and safe(collision-less) deceleration profile with simple calculation but without any complex calculation such as solving optimal problem.

C. Model of Expert Driver’s Deceleration Timing

We have analyzed expert drivers’ braking timing with experiments with real cars and it has been shown that brake judgment line eq.(11) can describe timing of expert driver based on the index KdB_c.

\[ \phi(V_r, V_p, D) = K_{dB,c}(a) - b \log_{10} D - c = 0 \]  

(11)

The coefficients \( a, b, \) and \( c \) are determined so that the approximated error of the equation is minimized in terms of
least squares. For the experimental results with test drivers, $a=0.2$, $b=-22.66$, $c=74.71$ were obtained.

It has already been shown that the judgment line of brake initiation can discriminate between normal safe driving and micro data of crashes very well. Probability that the plots for the normal driving were located in the upper area than the line is 0.00694. On the other hand, for the crash data, probability that the crash data was located lower than the line is 0 [5].

IV. DECELERATION CONTROL METHOD FOR COLLISION AVOIDANCE BASED ON PERCEPTUAL RISK

The proposed system aims that the vehicle equipped with the system starts to decelerate automatically and avoid collision if the driver does not decelerate or decelerates insufficiently even in high risk situation against the preceding vehicle by driver’s failure etc. In order to realize effective system, brake initiation timing, the target converged status, and deceleration profile play important roles. Especially, automatic brake should start as early as possible in the case of high risk situation to realize smooth and safe deceleration as long as the driver does not feel discomfort. For this problem, we propose a new control method by introducing driver’s perceptual risk.

A. Determination Method of Brake Initiation

In the previous section, driver’s brake initiation timing was modeled as brake judgment line. Here, a method is proposed to apply the model to brake initiation judgment for automatic braking system. Brake judgment model obtained in the previous section is an averaged result of driver’s brake initiation. Thus, drivers might rely on the system excessively and do not act any braking because the system starts to brake automatically just when the driver will start brake if the model is employed as brake initiation algorithm without any change and it works perfectly. In addition, there are individual differences in brake initiation timing. For example, aggressive drivers start to brake in more risky situations. Thus, it is important to take individual differences of brake initiation into account in brake initiation judgment to avoid discomfort.

Therefore, brake assist control is initiated when the state enter the dangerous status define by eq.(12) by adding an offset of the line $\Delta c$ to eq.(11) as follows:

$$\Omega_{\text{dangerous}} = \{(V_r, V_p, D) | \phi(V_r, V_p, D) \geq \Delta c\} \quad (12)$$

where $\Delta c$ is determined by taking individual difference into account.

B. Determination Method of Final Target Status

Final converged status of two vehicles after collision avoidance by deceleration assist is important because it affects driver’s recovering behavior after the assist and peripheral traffic flow especially for the system that works during relatively earlier stage. In this paper, the final target status will be determined based on driver’s perceptual risk.

Function $\phi(V_r, V_p, D)$ defined in eq.(11) can be understood as driver’s perceptual risk for collision. Eq.(11) with assumption of $-V_r + aV_p > 0$ yields eq.(13).

$$\phi(V_r, V_p, D) = 10\log_{10}(4 \times 10^7 \times \frac{-V_r + aV_p}{D^3}) - b \log_{10} D - c = 0 \quad (13)$$

Here, let us consider a method to determine the target converged status by specifying driver’s perceptual risk at the final status. Namely, the driver’s risk at the converged status is specified as eq.(14) by taking safe margin $\Delta d$ into account.

$$\Omega_{\text{conv}} = \{(V_r, V_p, D) | \phi(V_r, V_p, D) = \Delta d\} \quad (14)$$

Now, also assume that $V_r = 0$ at the converged status. Therefore, converged gap satisfying eq.(14) is calculated as eq.(15) given $V_r = 0$ and current $V_p$.

$$D = 4 \times 10^{\gamma \times \frac{c + \Delta d}{10} \times aV_p} \quad (16)$$

Namely, converged gap is determined by specifying preceding car’s velocity $V_p$. As seen from equation, $D_{\text{conv}} = 0$ at $V_p = 0$. So, eq.(16) is derived by adding gap offset $\Delta D > 0$.

$$D_{\text{conv}} = 4 \times 10^{\gamma \times \frac{c + \Delta d}{10} \times aV_p} + \Delta D \quad (16)$$

By setting target converged gap as eq.(16), converged status can be determined as $[V_r, V_p, D]_T = [0, V_p, D_{\text{conv}}]_T$, $\phi(V_r, V_p, D) \leq \Delta d$ is realized.

C. Generation Method of Deceleration Pattern

Let us consider a way to generate deceleration pattern based on the formulated expert driver’s deceleration profile $P_1$, say constant slope feature as eq.(10). Namely, velocity control based on eq.(10) is employed. It should be noted that the profile calculated by eq.(10) has an equilibrium point $[V_r, D]_T = [0, 0]^T$. Thus, we need to have a way to change the equilibrium to $[V_r, D]_T = [0, D_{\text{conv}}]_T$. Let us define variable $\delta$ as eq.(17).

$$\delta(t) = \frac{D(t) - D_{\text{conv}}}{D_{\text{conv}} - D_{\text{ini}}} \frac{D(t)}{D_{\text{ini}}} \quad (17)$$

Replacing $d(t)$ by this $\delta(t)$ yields a new desired velocity profile eq.(18).

$$V_r(t) = V_r(t_{ini}) \delta^3(t) \exp\left[3(1 - \delta(t))\right] \quad (18)$$

Let us consider characteristics of eq.(18). Differentiating eq.(18) by time with assumption that the preceding vehicle does not decelerate yields eq.(19).

$$\frac{dV_r(t)}{dt} = 3\delta^3(t) \exp\left[3(1 - \delta(t))\right] \quad (19)$$

Following car decelerates when $\delta(1 - \delta(t)) < 0$ is satisfied because $V_r(t_{ini}) < 0$. In addition, $1 - \delta(t) > 0$ means deceleration of the following car in the case of approaching
status $V_r<0$ because $\frac{d}{dt}\delta(t) = \frac{V_r}{D_{ds} - D_{cw}} < 0$ is satisfied. Consequently, the following car always decelerates in approaching condition $V_r<0$ and the equilibrium point is represented as $[V_r, \delta]^T = [0, 0]^T$.

D. Concept of Automatic Braking System

Consequently, the following control method has been obtained for automatic braking system for collision avoidance (Fig.6):

1) $\phi(V_r, V_p, D)$ is calculated in real-time from measured $V_r(t), D(t)$ and $V_p(t)$ or $V_o(t)$.

Brake control starts when the status is judged as dangerous status $\Omega_{\text{dangerous}}$ by eq. (12).

2) Desired relative velocity can be determined by the profile model eq. (18).

3) The brake control terminates if $V_r >= 0$.

Acceleration command from the given velocity profile is generated by the following simple method as an example:

$$G = -k_p \left( V_r^d (D) - V_r(t) \right), \quad (20)$$

where $k_p$ is feedback gain. Please note that in the proposed system, $D$ can be measured by a milliwave radar equipped with the car and $V_r$ can be obtained by numerically differentiating the measured $D$. Velocity $V_p$ can be obtained by $V_r + V_o$ given $V_o$.

![Fig. 6 System flow of automatic braking system](image)

E. Simulation Results of Proposed Automatic Braking System

In order to show the effectiveness of the proposed braking method, some numerical simulations are performed with the sequences given in Fig.6. Fig 7 illustrates the behavior of the proposed control method in approaching with a constant relative velocity. Suppose that the velocity of the following car is $V_o=80\text{km/h}$ for both conditions. Fig.7-(a) and (b) illustrate the results with velocity of the preceding car are $40\text{km/h}$ and $60\text{km/h}$, respectively. As seen in both figures, it is found that smooth velocity profiles are realized and they avoid the collision. As same as the results of the method proposed in literature [11], deceleration is relatively increased rapidly just after the brake initiation, and then smoother profile realizes the target velocity. In addition, as seen from the figure, the converged gap in Fig.7-(a) with larger preceding car velocity is greater than that with smaller preceding vehicle.

![Fig. 7 Simulation results](image)

V. EXPERIMENTS USING DRIVING SIMULATOR

A fixed-base driving simulator developed by Kagawa University was utilized for the experiments (Fig.8). A 100inch screen and two 80inch screens are located in front of the cockpit. Distance from driver’s eyes to the main screen is 2.3m in depth. Visual angle by the three screens is about 140deg. Vehicle motion is calculated by vehicle dynamics simulation software CarSim based on driver’s operation.

Experimental scenario was car following situation in a lane as shown in Fig.1. A following car with constant velocity approached to the preceding car that drives in a constant velocity. Following car always drives faster than the preceding car. Participants sit on the driver’s seat and take driving posture and step on the gas pedal even though the operation is not reflected to the vehicle motion and the car drives at constant velocity automatically. After that, the following car started to decelerate automatically based on the proposed control method. As the experimental conditions, the preceding car velocity was $V_p=40, 60, 80\text{km/h}$ and there are three conditions for relative velocity as $V_r=-20, -40, -60\text{km/h}$ for each $V_p$ condition. Order of the experimental conditions is randomized.

Participants were observed deceleration of the following vehicle by the proposed control method and asked to evaluate subjectively the converged status, say, the converged gap of two vehicles after termination of the deceleration control based on the five levels of 1) near, 2) slightly near, 3) good, 4) slightly far, and 5) far after each trial. Participants were five male students of Kagawa University of 21 to 24 yrs. Parameters in the control method were set as $\Delta c=\Delta d=0$. The offset for the converged gap was set as $\Delta D=5\text{m}$ based on the rehearsal experimental results.
Fig. 8 Driving Simulator

Fig. 9 shows subjective evaluation results of all participants. For almost all conditions, the participants evaluated as "good". There was some participants who evaluated "slightly near" in \( V_r = -40, -60 \text{km/h} \) conditions. This implies that deceleration profile to the converged status affects driver’s perceptual risk even for the same converged status. Investigation of this mechanism will be one of the important future studies. In addition, distance feeling of the driving simulator can be another reason of that. In addition, all participants have positive comments about the total deceleration patterns from free comments. This shows that the proposed method realize smooth deceleration control for overall.

![Graph of subjective evaluation results](image)

**VI. CONCLUSION**

A new deceleration control method as a driver assistance system to avoid rear-end collision was proposed based on driver’s perceptual risk of collision for collision avoidance system that works also in relatively low emergency level. The simulation results showed that smooth profile can be generated appropriately with the proposed method. In addition, from the experimental results, smooth deceleration was realized to avoid collision with the proposed method and almost good results were obtained about converged status by the subjective evaluation. However, it is found that some participants evaluated the same target status differently in the case of the different deceleration profiles. This implies that the deceleration profile affects the driver’s perceptual risk largely.

Further investigation for personal adaptation of the brake initiation timing and converged status is needed as a future study because drivers are sensitive to them and it leads to discomfort easily. As another future study, robustness of the method will be investigated including high emergency situation or complex traffic environment.

In addition, influence of the system on the traffic flow will be investigated because the brake assistance system affects driving behavior of following traffics. It is expected that negative effect is small because our proposed method generates very smooth and human-like deceleration profile. Furthermore, the method will be applied to other wide range of traffic environment such as curve deceleration control etc.

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