The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Mobile Robot Behavior Coordination Using Supervisory Control of Fuzzy Discrete Event Systems

Awantha Jayasiri, George K. I. Mann, and Raymond G. Gosine

Abstract— **This paper presents a novel approach to behavior based control of mobile robots using supervisory control of Fuzzy Discrete Event Systems (FDES). Fuzzy events are triggered by the sensor readings and the inference occurs through a fuzzy rule base system. The supervisor can activate and control fuzzy controllable events simultaneously with fuzzy uncontrollable events to achieve the planned objectives. The fuzzy observability concept is incorporated to represent sensor uncertainties. Fuzzy state based controllability and observability measures are also discussed. The proposed theoretical development is then extended to discuss an application with behavior based control of mobile robots.**

Index Terms: Fuzzy discrete event systems, supervisory control, mobile robotics, behavior coordination

I. INTRODUCTION

Although fuzzy logic (FL) based systems has the capacity to handle uncertainties in sensor readings, it inherits the drawback of composing large rule bases or "curse of dimensionality" when dealing with complex control problems. This problem can be some what minimized while using modular FL systems. Behavior coordination of mobile robots using FL is best described in [1] where authors introduced the concept of context dependent blending of behaviors as a solution to the curse of dimensionality that exist in FL. This modular FL concept is well exploited in [2] to blend the motor schemas for mobile robot navigation. The lack of closed-form solutions in the fuzzy context dependent blending, makes it difficult to analyze the system stability, observability and controllability aspects, that would make the system with guaranteed stable navigation. As an alternative, discrete event systems (DES) [3] allows supervisory control and has formal methods available to investigate the above mentioned control theoretic concepts. Also, DES has the capability to handle modular and decentralized control issues of a system formally. The crisp DES and finite automata theories have been used successfully in behavior based robotics [4]. Due to the discrete nature in the crisp DES, the method can be used only for behavior arbitration. The general drawbacks of behavior arbitration involves frequent switching of behaviors at the hard boundaries and also behavior starvation that arise due to looping in a single behavior. An alternative approach with adding extra nodes to the automaton for reducing the chattering effects in DES is discussed in [5]. However, the method suffers optimum control and also failure in the presence of unmodeled obstacles [6].

When robots are driven in an unstructured environment with other moving and stationary obstacles, the occurrence of events are generally asynchronous and also highly discrete and this will make DES as the most appropriate methodology for providing high level control for mobile robots. However, DES lacks the handling of uncertainities associated within sensor readings and decision vagueness. Fuzzy discrete event systems (FDES) is defined as an extension to the formal crisp DES theory that allows us to accommodate uncertainties in the events and states [7]. Recently some applications which have been modeled using FDES such as AIDS treatment, drug delivery and fault diagnosis in complex systems [8], [9] have shown its success in decision making. While accounting these difficulties in general DES and also in fuzzy context dependent blending, the pioneering work of [6] exploited the FDES approach to successfully navigate mobile robots in unstructured environment. Motivated by this work, this paper provides the formal analysis of FDES using supervisory control.

This paper proposes a behavior coordination scheme using supervisory control of FDES. Such a supervisory control scheme has several advantages. First, it can activate both fuzzy controllable and uncontrollable events simultaneously and weight fuzzy states that represent behaviors accordingly for command fusion. Secondly, fuzzy observability is associated to represent the sensor uncertainties. For a systematic analysis fuzzy state based observability and controllability are also introduced. Coordination of behaviors of a mobile robot moving in an unmodeled environment with dead ends using proposed approach, is discussed as a practical application.

The paper is organized as follows. In section II, the newly defined supervisory control of FDES, fuzzy observability and partially observation supervisory control of FDES are presented. Coordination of behaviors of a mobile robot navigating in complex environments with dead ends, using the proposed methodology is discussed in section III. Section IV derives the conclusion.

II. SUPERVISORY CONTROL OF FDES

A. Fuzzy controllable and uncontrollable events

Here we discuss how fuzzy controllable and fuzzy uncontrollable events are combined in the formation of the fuzzy supervisor.

The fuzzy finite automaton is defined by the quadruple as $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}, \tilde{q}_0)$ where $\tilde{Q}$ is set of fuzzy states, $\tilde{\Sigma}$ represents the set of fuzzy events, $\tilde{\Sigma} = \tilde{\Sigma}_c \cup \tilde{\Sigma}_{uc}$ where $\tilde{\Sigma}_c$ and $\tilde{\Sigma}_{uc}$ are defined as set of fuzzy controllable events and set of fuzzy

---

Authors are with the Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St. John's, NL, A1B 3X5, Canada. `awantha, gmann, rgosine@engr.mun.ca`

978-1-4244-3804-4/09/$25.00 ©2009 IEEE

uncontrollable events respectively, $\tilde{\Sigma}_c \cap \tilde{\Sigma}_{uc} = \emptyset$. Then $\tilde{\delta}$ represents the fuzzy transition mapping and $\tilde{q}_0$ represents the initial fuzzy states of the system.

We define the fuzzy subset given by $\tilde{\Sigma}_{uc}$ as follows:

$$\tilde{\Sigma}_{uc}(\tilde{\sigma}) = \begin{cases} \mu_{\tilde{\sigma}} \in [0,1], & \text{if } \tilde{\sigma} \in \tilde{\Sigma}_{uc} \\ 0, & \text{if } \tilde{\sigma} \in \tilde{\Sigma}^* \setminus \tilde{\Sigma}_{uc} \end{cases} \tag{1}$$

Here $\tilde{\Sigma}_{uc}(\tilde{\sigma})$ is defined as the possibility of fuzzy event $\tilde{\sigma}$ being a member of $\tilde{\Sigma}_{uc}$ and $\mu_{\tilde{\sigma}}$ is evaluated by using the membership value of a fuzzy rule base with associated sensory information.

Note that hereafter we assume the intersection between the possibilities of the fuzzy events and strings can be modeled by by either product or the minimum. Also we define the fuzzy subset given by fuzzy controllable events, $\tilde{\Sigma}_c$ as:

$$\tilde{\Sigma}_c(\tilde{\sigma}) = \begin{cases} \left(\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}) \tilde{\cap} \bar{\tilde{k}}'(\tilde{s}\tilde{\sigma})\right), & \text{if } \tilde{\sigma} \in \tilde{\Sigma}_{\mathrm{c}} \\ 0, & \text{if } \tilde{\sigma} \in \tilde{\Sigma}^* \setminus \tilde{\Sigma}_{\mathrm{c}} \end{cases} \tag{2}$$

Here $\tilde{\Sigma}_c(\tilde{\sigma})$ is defined as possibility of fuzzy event $\tilde{\sigma}$ being a member of fuzzy controllable event set. This is evaluated after fuzzy string $\tilde{s}$ has been occurred in the system and final value is obtained using a fuzzy rule base. ($\varepsilon \leq \tilde{s}$ where ε is the null-event). $\tilde{L}_{\tilde{G}}$ is the fuzzy language generated by the system $\tilde{G}$ and $\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$ represents the degree of fuzzy string $\tilde{s}\tilde{\sigma}$ is physically possible. $\bar{\tilde{k}}'$ represents the prefix closure of the fuzzy language of marked states with the prior knowledge about the environment. $\bar{\tilde{k}}'(\tilde{s}\tilde{\sigma})$ is the possibility of fuzzy string $\tilde{s}\tilde{\sigma}$ being enabled by the supervisor with this prior knowledge and $\tilde{\cap}$ is modeled by fuzzy-AND operation (taking minimum or product of them). Now we define the possibility of a fuzzy event $\tilde{\sigma}$ being enabled by the supervisor $\tilde{S}$, after fuzzy string $\tilde{s}$ has been occurred, $\tilde{S}_{\tilde{s}}(\tilde{\sigma})$ as follows (Note that $\tilde{S}_{\tilde{s}}$ is the fuzzy subset of events the supervisor enables after occurrence of fuzzy string $\tilde{s}$.):

$$\tilde{S}_{\tilde{s}}(\tilde{\sigma}) = \begin{cases} \tilde{\Sigma}_{uc}(\tilde{\sigma}) \tilde{\cap} \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}), & \forall\tilde{\sigma}, \text{ if } \tilde{\sigma} \in \tilde{\Sigma}_{\mathrm{uc}} \\ \tilde{\Sigma}_c(\tilde{\sigma}) \tilde{\cap} \tilde{T}_{\tilde{s}}(\tilde{\sigma}), & \forall\tilde{\sigma}, \text{ if } \tilde{\sigma} \in \tilde{\Sigma}_{\mathrm{c}} \end{cases} \tag{3}$$

$\forall \tilde{\sigma} \in \tilde{\Sigma}_c$,

1) $\tilde{T}_{\tilde{s}}(\tilde{\sigma}) = \tilde{\bigcap}_{i=1}^{|\tilde{\Sigma}_{uc}|} \tilde{\delta}_i(\tilde{\sigma}), \qquad \forall \tilde{\delta}_i \in \tilde{\Sigma}_{uc}$
2) $\bar{\tilde{k}}''(\tilde{s}\tilde{\sigma}) = \bar{\tilde{k}}'(\tilde{s}\tilde{\sigma}) \tilde{\cap} \tilde{T}_{\tilde{s}}(\tilde{\sigma}) \tilde{\cap} \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma})$

$\tilde{T}_{\tilde{s}}(\tilde{\sigma})$ is evaluated by considering the conformity of the fuzzy controllable event $\tilde{\sigma}$, with all available fuzzy uncontrollable events $\tilde{\delta}_i$. $\bar{\tilde{k}}''$ represents the prefix closure of the fuzzy language of marked states with the post knowledge about the environment and $\bar{\tilde{k}}''(\tilde{s}\tilde{\sigma})$ represents the possibility of a fuzzy controllable event $\tilde{\sigma}$ being activated given string of fuzzy events $\tilde{s}$, have been occurred without rendering the system to a hazardous situation. By this definition the fuzzy supervisor is able to trigger both fuzzy uncontrollable events that are feasible in $\tilde{G}$ and fuzzy controllable events which are compliant with above uncontrollable events, simultaneously with different weighting factors.

Fuzzy controllability of FDES has been discussed previously by Qiu and Liu [10], [11]. In [10] the fuzzy controllability describes as a direct extension to crisp controllability theory presented in [3]. Later [11] discusses the fuzzy controllability under the effect of the observable projection. In order to keep the fuzzy supervisor in generating desired language for the system in operation the following theorem is defined. Note that hereafter we replace $\bar{\tilde{k}}''$ with $\bar{\tilde{k}}$.

A fuzzy language $\tilde{k}$ is fuzzy controllable if and only if:

$$\tilde{k}(\tilde{s}) \tilde{\cap} \tilde{\Sigma}_{uc}(\tilde{\sigma}) \tilde{\cap} \tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}) = \tilde{k}(\tilde{s}\tilde{\sigma}) \tag{4}$$

Here $\tilde{k} \subseteq \tilde{L}_{\tilde{G}}$ and, $\tilde{\Sigma}_{uc}(\tilde{\sigma})$ is evaluated using (1). This definition helps to obtain a final value for the possibility of fuzzy string which extend into the controllable fuzzy language.

B. The Fuzzy Observability

This has been defined as an extension to crisp observability in formal DES theory [8], [7], [11]. In [7] fuzzy event with partial observability and partial unobservability is described using $\tilde{\Sigma}_o(\tilde{\sigma}) + \tilde{\Sigma}_{uo}(\tilde{\sigma}) = 1$, where $\tilde{\Sigma}_o(\tilde{\sigma})$ and $\tilde{\Sigma}_{uo}(\tilde{\sigma})$ are the degrees of partial observability and partial unobservability of fuzzy event $\tilde{\sigma}$ respectively.

The natural projection of a fuzzy event $\tilde{\sigma}$ is defined as follows:

$$\tilde{P}(\tilde{\sigma}) = \left[\tilde{\Sigma}_{uo}(\tilde{\sigma}) \cdot \epsilon + \tilde{\Sigma}_o(\tilde{\sigma}) \cdot \tilde{\sigma}\right] \text{ for } 0 \leq \tilde{\Sigma}_{\mathrm{o}}(\tilde{\sigma}) \leq 1 \tag{5}$$

Where ϵ represents the null event. Alternative definition can be seen in [11] where authors define the natural projection of a fuzzy event without taking the degree of observability into account. The inclusion of degree of observability into events makes following partially observable fuzzy event matrix $\underline{\tilde{\sigma}}$.

$\underline{\tilde{\sigma}_{i,j}} = \tilde{\Sigma}_{uo}(\tilde{\sigma}) \times I_{i,j} + \tilde{\Sigma}_o(\tilde{\sigma}) \times \tilde{\sigma}_{i,j}$

Where $\underline{\tilde{\sigma}_{i,j}}$ is the respective element value of the resulting event matrix $\underline{\tilde{\sigma}}$ and I is the unit matrix of size $\tilde{\sigma}$.

Partial observability and fuzzy observability of FDES have been studied in [12] and [11] respectively. In [10] the fuzzy observability is defined with the assumption that effect of observable projection $P(\tilde{s}) = P(\tilde{t})$. Using the same assumption the partial observability has been defined in [12] of a fuzzy language. However the degree of this equality does not occur under sensor uncertainities and therefore in this paper we consider this relation as fuzzy. The original crisp observability theorem [3] in formal DES theory can be extended to fuzzy observability considering the possibilities of two fuzzy strings to produce the same natural projection as follows.

$\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)$ is a fuzzy subset which describes the possibility of a fuzzy string to generate its natural projection same as the natural projection of $\tilde{s}$ $\left(\text{i.e } \tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)(\tilde{s}) = 1\right)$. The fuzzy observability which is defined as in (6), can be described as for any fuzzy event string $\tilde{x}$ cannot be extend with the fuzzy event $\underline{\tilde{\sigma}}$ in the prefix closure of the fuzzy language $\tilde{k}$ (given by $\bar{\tilde{k}}$), if all following conditions hold:

1. For any fuzzy event string $\tilde{s}$ which is a member of $\bar{\tilde{k}}$.
2. For any fuzzy event $\tilde{\sigma}$ which is a member of $\tilde{\Sigma}_c$.

$$\left(\forall\tilde{s},\,\tilde{s}\in\bar{\bar{\tilde{k}}}\right)\text{ and }\left(\forall\tilde{\sigma},\,\tilde{\sigma}\in\tilde{\Sigma}_c\right)\text{ and }\left(\tilde{s}\tilde{\sigma}\in\tilde{L}_{\tilde{G}}\right)\text{ and }\left(\tilde{s}\tilde{\sigma}\notin\bar{\bar{\tilde{k}}}\right)\Rightarrow\left(\forall\tilde{x},\,\tilde{x}\in\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)\right)\text{ and }\left(\tilde{x}\tilde{\sigma}\notin\bar{\bar{\tilde{k}}}\right)\tag{6}$$

---

3. The fuzzy event string $\tilde{s}\tilde{\sigma}$ is physically possible.
4. The fuzzy event string $\tilde{s}\tilde{\sigma}$ is not a member of $\bar{\bar{\tilde{k}}}$.
5. The fuzzy event string $\tilde{x}$ is a member of $\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)$.

Assume the fuzzy string $\tilde{s}$ is consisting of n number of fuzzy events, $\tilde{s} = \tilde{\sigma}_1 \otimes \tilde{\sigma}_2 \otimes \tilde{\sigma}_3 \otimes ...\tilde{\sigma}_n$ where here $\tilde{\sigma}_i,\quad i = 1, 2, 3, ..n$ represents the i^{th} fuzzy event and "$\otimes$" represents the standard composition of fuzzy relations. It may be Max-Min or Max-Product. Let $\tilde{P}(\tilde{s}) = \tilde{l}$ is the natural projection of fuzzy string $\tilde{s}$. This yields to (7) which is obtained by the standard composition of fuzzy relations in fuzzy events considering the natural projection of each fuzzy event individually.

$$\begin{aligned}&\tilde{P}(\tilde{s}) = \tilde{l} = \tilde{P}\left(\tilde{\sigma}_1 \otimes \tilde{\sigma}_2 \otimes \tilde{\sigma}_3 \otimes \ldots \tilde{\sigma}_n\right)\\&\Rightarrow \tilde{P}\left(\tilde{\sigma}_1\right) \otimes \tilde{P}\left(\tilde{\sigma}_2\right) \otimes \tilde{P}\left(\tilde{\sigma}_3\right) \otimes \ldots \tilde{P}\left(\tilde{\sigma}_n\right)\end{aligned}\tag{7}$$

C. Fuzzy Partially Observation Supervision

This has already been discussed in [11] where authors discuss the existence of a nonblocking fuzzy supervisor. This is a direct extension of DES nonblocking partially observable supervisor discussed in [3] to FDES. With the fuzzy controllability and fuzzy observability discussed in this paper, a fuzzy partially observation supervisor $\tilde{S}^{\tilde{P}}$ is discussed below for mobile robot navigation.

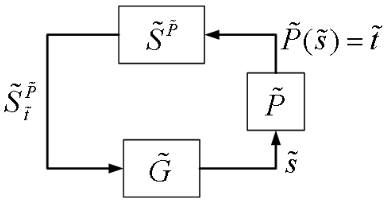


Fig. 1. The feedback loop of fuzzy supervisory control with partial observability

The supervisor $\tilde{S}^{\tilde{P}}$, of the system shown in figure 1, executes feasible fuzzy uncontrollable and fuzzy controllable events after partial observation of $\tilde{s}$ where $\tilde{P}(\tilde{s}) = \tilde{t}$.

Following is the definition we can derive for possibility of enabling fuzzy event $\tilde{\sigma}$ by fuzzy partially observation supervisor.

$$\tilde{S}^{\tilde{P}}_{\tilde{t}}(\tilde{\sigma}) = \begin{cases}\tilde{\Sigma}_{uc}(\tilde{\sigma})\tilde{\cap}\tilde{L}_{\tilde{G}}(\tilde{s}\tilde{\sigma}), & \forall\tilde{\sigma},\ \text{if } \tilde{\sigma}\in\tilde{\Sigma}_{\text{uc}}\\ \tilde{\Sigma}_c(\tilde{\sigma})\tilde{\cap}\tilde{T}_{\tilde{s}'}(\tilde{\sigma}), & \forall\tilde{\sigma},\ \text{if } \tilde{\sigma}\in\tilde{\Sigma}_{\text{c}},\\ & \tilde{s}'\in\tilde{P}^{-1}\left(\tilde{P}(\tilde{s})\right)\end{cases}\tag{8}$$

III. MOBILE ROBOT'S BEHAVIOR COORDINATION USING PROPOSED METHOD

Let $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}, \tilde{q})$ be a fuzzy automaton which it's fuzzy states represent the activation levels of the n behaviors of a complex system. Composite fuzzy event matrix $\tilde{\gamma}_t$ at time t is constructed by adding several corresponding fuzzy controllable and uncontrollable event matrices using fuzzy-OR operator (taking maximum).

For any given time $t \geq 0$,
$(\tilde{q}_{1,t}, ..., \tilde{q}_{n,t}) \in \tilde{Q}_t$,
$\tilde{Q}_t \otimes \tilde{\gamma}_{t+1} \to \tilde{Q}_{t+1}$,
$\forall \tilde{q}_{i,t} \in \tilde{Q}_t, \sum_{i=1}^{n} \tilde{q}_{i,t} = 1$,

Here $\tilde{q}_{i,t}$ represents the activation level of i^{th} behavior at time t. With these conditions all possible fuzzy uncontrollable events are included into the fuzzy language marked for the desirable behavior of the system. This satisfies our fuzzy controllability condition discussed in (4). So the existence of a fuzzy supervisor is guaranteed.

The fuzzy states in the fuzzy automaton which represent the behaviors of the system, change their values when corresponding fuzzy events triggered. The activation level of the reactive behaviors are controlled through fuzzy uncontrollable events where the activation level of deliberative behaviors are controlled through fuzzy controllable events.

Consider a fuzzy automaton $\tilde{G}_1 = (\tilde{Q}_1, \tilde{\Sigma}_1, \tilde{\delta}_1, \tilde{q}_1)$ represents a behavior based robotic navigation system consists of five behaviors, namely Route Follow, Go to Target, Avoid Obstacle, Wall Follow and Avoid Dead Ends. This can be depicted as in figure 2. With this fuzzy automaton followings are hold.

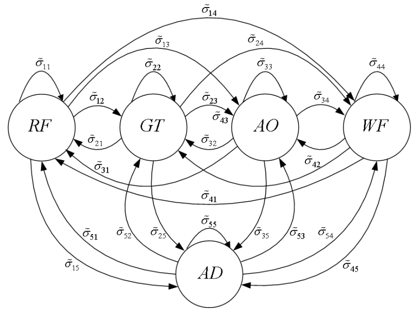


Fig. 2. The four behaviors represented by a fuzzy automaton

$\tilde{Q}_1 = (RF, GT, AO, WF, AD)$
$\tilde{\Sigma}_{1c} = (\tilde{\sigma}_{11}, \tilde{\sigma}_{21}, \tilde{\sigma}_{31}, \tilde{\sigma}_{41}, \tilde{\sigma}_{51}, \tilde{\sigma}_{12}, \tilde{\sigma}_{22}, \tilde{\sigma}_{32}, \tilde{\sigma}_{42}, \tilde{\sigma}_{52})$
$\tilde{\Sigma}_{1uc} = (\tilde{\sigma}_{13}, \tilde{\sigma}_{23}, \tilde{\sigma}_{33}, \tilde{\sigma}_{43}, \tilde{\sigma}_{53}, \tilde{\sigma}_{14}, \tilde{\sigma}_{24}, \tilde{\sigma}_{34}, \tilde{\sigma}_{44}, \tilde{\sigma}_{54},$
$\tilde{\sigma}_{15}, \tilde{\sigma}_{25}, \tilde{\sigma}_{35}, \tilde{\sigma}_{45}, \tilde{\sigma}_{55})$

$\tilde{q}_1$ is the initial fuzzy states representation of this fuzzy automaton. $\tilde{\delta}_1$ is shown in figure.

A. Modeling of Avoid Obstacle Behavior

This is a reactive behavior controlled by fuzzy uncontrollable events. This behavior slides the robot to a direction which is perpendicular to the line connecting both robot and obstacle, when distance from robot to the obstacle exceeds it's limit. Otherwise it directs to the nearest way point. Triangular membership functions are used to realize all fuzzy rule bases in modeling behaviors and the MIN-MAX-CENTROID defuzzification technique is employed to finally obtain a crisp value for the fuzzy events.

B. Modeling of Wall Follow Behavior

This is also a reactive behavior controlled by fuzzy uncontrollable events. This behavior forces robot to keep a minimum distance with the obstacles. The direction of this behavior is opposite to the direction of the nearest obstacle. If this distance is higher than it's minimum, the direction of this behavior is same as the direction suggested by Avoid Obstacle behavior.

C. Modeling of Avoid Dead Ends Behavior

This reactive behavior is designed to carefully avoid the dead end situations. When a dead end is identified on robots path, a memory flag is made "High" (i.e. = 1). Then a virtual object is placed for robot to follow until getting out from the dead end as in [13]. When it is avoided the flag is made "Low" (i.e. = 0). The direction of this behavior is towards the wall direction.

D. Modeling of Route Follow Behavior

This is a deliberative behavior used to navigate the robot through way points. This behavior is controlled by fuzzy controllable events.

E. Modeling of Go to Target Behavior

This is also a deliberative behavior controlled by fuzzy controllable events and used for path optimization. This aims to the next near most way point to the current robot orientation.

F. Fuzzy Supervisor Synthesis

The specification of the supervisor in (3) can be achieved using a fuzzy rule base which describes the conformity of controllable and uncontrollable fuzzy events. A final defuzzified weight is used to partially activate or deactivate the fuzzy controllable events.

For example assume at time t the possibility of fuzzy controllable event controlling Route Follow behavior $\Sigma_c(\tilde{\gamma}_{1c})$ is given by 0.8 and final defuzzified value obtained for $\tilde{T}_{\tilde{s}}(\tilde{\gamma}_{1c})$ is 0.3. Also assume the intersection between two possibilities is modeled by the product of them. Then $\tilde{S}_{\tilde{s}}(\tilde{\gamma}_{1c})$ would be;

$\tilde{S}_{\tilde{s}}(\tilde{\gamma}_{1c}) = \tilde{T}_{\tilde{s}}(\tilde{\gamma}_{1c}) \tilde{\cap} \tilde{\Sigma}_c(\tilde{\gamma}_{1c}) = 0.3 \times 0.8 = 0.24$

Other possibilities also can be calculated same way. The final fuzzy event for the time step $t+1$ considering all fuzzy controllable and uncontrollable events would be,

$$\underline{\tilde{\gamma}_{t+1}} = \begin{bmatrix} \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{RF}) & \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{GT}) & \underline{\mu_{AO}} & \underline{\mu_{WF}} & \underline{\mu_{AD}} \\ \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{RF}) & \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{GT}) & \underline{\mu_{AO}} & \underline{\mu_{WF}} & \underline{\mu_{AD}} \\ \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{RF}) & \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{GT}) & \underline{\mu_{AO}} & \underline{\mu_{WF}} & \underline{\mu_{AD}} \\ \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{RF}) & \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{GT}) & \underline{\mu_{AO}} & \underline{\mu_{WF}} & \underline{\mu_{AD}} \\ \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{RF}) & \tilde{S}_{\tilde{s}}(\tilde{\gamma}_{GT}) & \underline{\mu_{AO}} & \underline{\mu_{WF}} & \underline{\mu_{AD}} \end{bmatrix}$$

Here $\underline{\mu_{AO}}, \underline{\mu_{WF}}$ and $\underline{\mu_{AD}}$ are the final values obtained from the defuzzification steps of avoid obstacle, wall follow and avoid dead ends behaviors respectively. The fuzzy automaton $\tilde{Q}_{t+1}$ which gives the activation levels of behaviors can be calculated as mentioned. Assume $\underline{\vec{A}_{t+1}}$ as the final coordinated action of all different behaviors at time $t+1$;

$\underline{\vec{A}_{t+1}} = \sum_{i=1}^{n} \tilde{q}_{i,t+1} \times \vec{\mathbf{a}}_{i,t+1}$

Where $\vec{\mathbf{a}}_{i,t+1}$ is the unit vector representing i^{th} behavior and $\tilde{q}_{i,t+1} \in \tilde{Q}_{t+1}$.

G. Measure of Fuzzy State Based Controllability, C_t

Fuzzy supervisor synthesis is to mitigate inconsistencies between fuzzy states. In between behaviors following properties can be identified.

1. Route Follow and Go to Target behaviors are consistent with each other with higher degree.

2. Avoid Obstacle and Wall Follow behaviors are consistent with each other with moderate degree.

3. Avoid Dead ends behavior is highly inconsistent with deliberative behaviors and also with Wall Follow behavior.

4. Deliberative and other reactive behaviors mentioned above are consistent with each other with lesser degree as these together represent safe operation of the robot until the final goal but the direction suggested by these may be fairly contradictory.

Based on this and adopting from [7] we can construct a consistency matrix W, with above knowledge. Here the element $w_{i,j}$ represents the measure of inconsistency between fuzzy states i and j.

$$W = \begin{bmatrix} 0.0 & 0.5 & 0.9 & 0.9 & 1.0 \\ 0.5 & 0.0 & 0.8 & 0.8 & 1.0 \\ 0.9 & 0.8 & 0.0 & 0.3 & 0.7 \\ 0.9 & 0.8 & 0.3 & 0.0 & 1.0 \\ 1.0 & 1.0 & 0.7 & 1.0 & 0.0 \end{bmatrix}$$

Note $C_t = (1 - \tilde{q}_t \cdot W \cdot \tilde{q}_t^T)$ represents degree of inconsistency between fuzzy states [7], which is further identified as fuzzy state based controllability of the system at time t.

H. Measure of Fuzzy State Based Observability, O_t

The inconsistency of the current fuzzy states with the worse case scenarios (only one behavior controlling the mobile robot alone) with respect to the consistency matrix W gives an indication about the incompatibility of the decisions made by the supervisor as defined below. Note that being less represents more observability than being more.

$O_t = \sum_{i=1}^{|\tilde{Q}|} q_{max,i} \cdot W \cdot \tilde{q}_t^T$

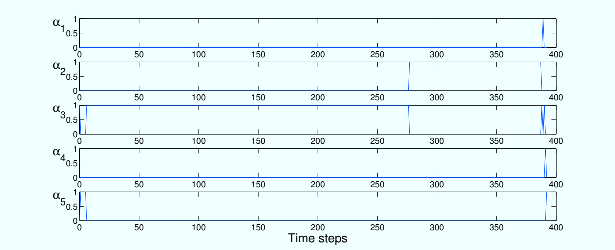
Here $q_{max,i}$ is a state matrix where its i^{th} element is 1 and all others are 0, which represents the worse case scenario.

I. Simulation Results with Dead Ends

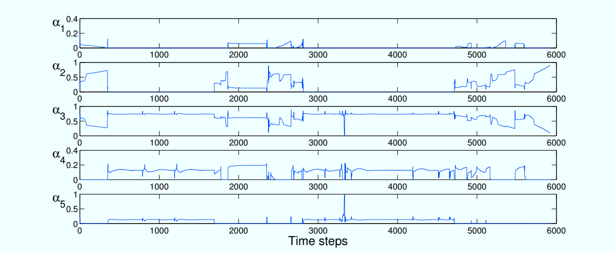
Simulations are carried out using MobileSim Version 0.4.0 provided by ActivMedia robots with Pioneer 3 DX robot. A 10m × 12m simulated environment space was used and start and end points are identified. The way points are given manually and a particle filter [14] is used to localize the robot in 10m intervals. In other points robot localization is performed only using odometry data. Modeled and unmodeled obstacles with dead ends are used to examine the performance. Robot's laser range finder is used for localization module where distance to the obstacles are obtained by using the embedded sonar ring. Time steps which represent the decision cycles of the robot coordination, consist of a rotation and a translation command for each. Rotation is used for angular correction and translation used for move the robot to the final decided direction. Robot translation and rotation speeds are fixed with 50mm per second and translation cycle is 50ms.

Figure 3 shows the navigation scenario with two dead ends and unmodeled obstacles in the environment. We assumed complete observability for DES based navigation $(\Sigma_o(\sigma) = 1)$. For FDES based navigation partial observability of events is assumed $(\tilde{\Sigma}_o(\tilde{\sigma}) = 0.8$ representing 80% accuracy of associated sensors). The proposed FDES scheme shows collision free navigation to the end point while avoiding dead ends even with partial observability associated with fuzzy events.

In figure 4 $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4$ and $\tilde{\alpha}_5$ depict the evolution of “route follow”, “go to target”, “avoid obstacles”, “wall follow” and “avoid dead ends” behaviors with DES and proposed FDES based navigation in this environment respectively. Figure 5(a) shows the angles suggested by each behavior for proposed FDES based robot navigation approach and Figure 5(b) depicts fuzzy state based controllability and observability of this scheme. Figure 6 shows the mobile robot navigation in more complex dead end environments using proposed method.

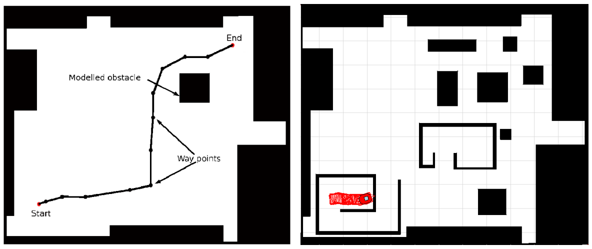


(a) Activation levels of behaviors with DES

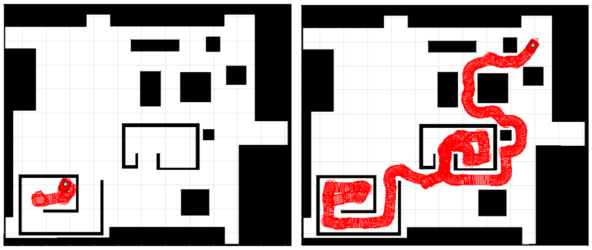


(b) Activation levels of behaviors with proposed FDES based approach

Fig. 4. Activation levels of different behavior coordination schemes



(a) Original map (b) Unmodulated coordination



(c) DES based coordination with $\Sigma_o(\sigma) = 1.0$ (d) Proposed FDES based coordination with $\tilde{\Sigma}_o(\tilde{\sigma}) = 0.8$

Fig. 3. Various behavior coordination schemes for a robot moving in unknown environment with dead ends

J. Real time Implementation with Dead Ends

This method was implemented real time in a physical robot (Pioneer 3 AT platform) with 0.8 partial observability associated with each fuzzy event. Figure 7 depicts robot’s collision free navigation from start to end while avoiding dead ends.

IV. CONCLUSION

This presents a framework for supervisory control of FDES for robust robot navigation tasks. The proposed approach eases the design of behavior based architecture with the higher modularity it offers associating fuzzy discrete event systems. Also it is able to cope with sensor imprecisions and ambiguous situations by introducing fuzzy partially observation supervisory control. The proposed approach has a better analyzing capability than the work shown in [1] by providing formal methods of system analysis such as state based controllability and observability.

Also the proposed approach is readily scalable. More behaviors can be incorporated by adding fuzzy states to the automaton. Introducing fuzzy logics to events and states makes the system more robust to sensor failures and the resulting command fusion mechanism ensures better navigation than using behavior arbitration as in crisp DES mentioned in [4].

V. ACKNOWLEDGMENTS

The authors thankful to the Memorial University of Newfoundland and Natural Sciences and Engineering Research Council of Canada (NSERC) for their financial supports.

REFERENCES

[1] A. Saffiotti, E. H. Ruspini, and K. Konolige, “Blending reactivity and goal-directedness in a fuzzy controller,” *In Proc. of the 2nd IEEE Int. Conf. on Fuzzy Systems*, pp. 134–139, 1993.

[2] R. Huq, G. K. I. Mann, and R. G. Gosine, “Mobile robot navigation using motor schema and fuzzy context dependent behavior modulation,” *Appl. Soft Comput.*, vol. 8, no. 1, pp. 422–436, 2008.

[3] C. G. Cassandras and S. Lafortune, *Introduction to Descrete Event Systems*. Boston, MA: Kluwer, 1999.

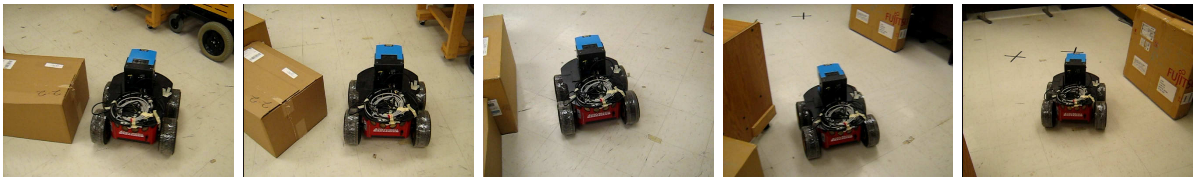
[4] J. Kosecka and R. Bajcsy, “Discrete event systems for autonomous mobile agents,” *Proceedings of Intelligent Robotic Systems*, pp. 21–31, July 1993.

[5] M. Egerstedt and X. Hu, “A hybrid control approach to action coordination for mobile robots,” *Automatica*, vol. 38, no. 1, pp. 125 – 130, 2002.

[6] R. Hug, G. Mann, and R. Gosine, “Behavior modulation technique in mobile robotics using fuzzy discrete event system,” *IEEE Transactions on Robotics*, vol. 22, no. 5, pp. pp. 903–916, 2006.

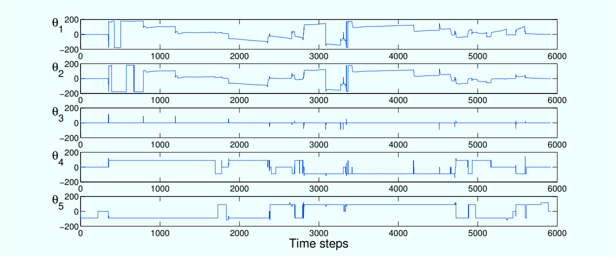
[7] F. Lin and H. Ying, “Modeling and control of fuzzy discrete event systems,” *IEEE Trans. Syst., Man, Cybern., Part B*, vol. 32, pp. pp. 408–415, Aug. 2002.

(a) Robot approaching to a sub goal (b) Turning at the dead end (c) Completing the turn (d) Following the wall of the dead end (e) Turning at the end of dead end

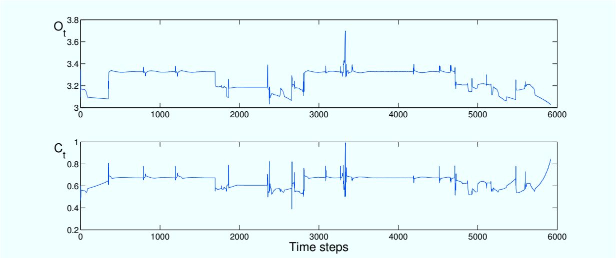


(f) Following the wall (g) Completing the turn (h) Following the wall (i) Avoiding the obstacle at left (j) Reaching to final goal

Fig. 7. Performance in real world with a dead end using proposed FDES based approach



(a) Angles suggested by each behavior with proposed approach



(b) Fuzzy state based observability and controllability for navigation using proposed approach with 80% partial observability

Fig. 5. Angels suggested by each behavior and evolution of fuzzy observability and controllability measures in proposed FDES based approach

[8] ——, "Fuzzy discrete event systems and their observability," *Proc. Joint Int. Conf. 9th Int. Fuzzy Systems Assoc. World Congr. 20th North Amer. Fuzzy Inform. Process. Soci*, pp. 1271–1276, July 2001.

[9] E. Kilic, C. Karasu, and K. Leblebicioglu, "Fault diagnosis with dynamic fuzzy discrete event systems approach," *F. A. Savaci (Ed.): TAINN 2005, LNAI 3949, Springer- Verlag Berlin Heidelberg*, pp. 117–124, 2006.

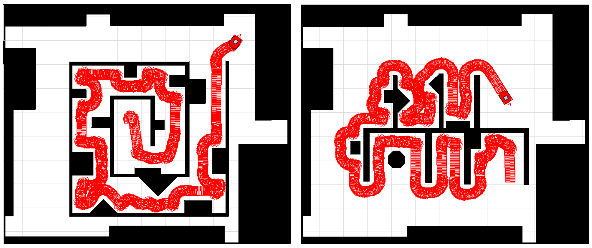
[10] D. Qiu, "Supervisory control of fuzzy discrete event systems: a formal approach," *IEEE Transaction of Systems, Man, and Cybenetics, Part B: Cybenetics*, vol. 35, no. 1, pp. pp. 72–88, February 2005.

[11] D. Qiu and F. Liu, "Fuzzy discrete event systems under fuzzy observability and a test-algorithm," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 3, June 2009.

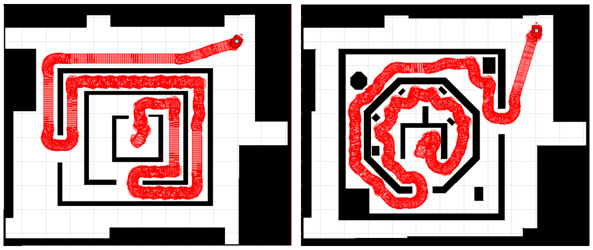
[12] Y. Cao and M. Ying, "Observability and decentralized control of fuzzy discrete-event systems," *Fuzzy Systems, IEEE Transactions on*, vol. 14, no. 2, pp. 202–216, April 2006.

[13] A. Zhu and S. X. Yang, "Neurofuzzy-based approach to mobile robot navigation in unknown environments," *IEEE Trans. Syst., Man, and Cybern., Part C*, vol. 37, no. 4, pp. 610–621, July 2007.

[14] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*. The MIT press, Cambridge, Massachusetts, 2005.



(a) In a spiral like environment with obstacles (b) In a wall like environment



(c) In a maze like environment (d) In a compound complex environment

Fig. 6. Robot navigation in complex environments using proposed approach