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Multiplicative Potential Energy Function for Swarm Control

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Abstract— **This paper presents a novel method for shape control of a swarm of robots based on region control concept. Multiplicative potential energy function is used to form the overall desired shape of the entire swarm. The shape formed using this method is a union of all the regions defined by corresponding inequality functions. This proposed method is a complement to our previous method where the additive potential energy is used to form the desired shape. By combining the multiplicative and additive potential energies, a variety of complicated shapes can be formed. Lyapunov-like function is presented for convergence analysis of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed method.**

I. INTRODUCTION

Cooperative control of multi-robot systems has been the subject of extensive research in recent decades. One important research problem in cooperative control of multi-robot systems is to maintain a desired formation during movements. In behavior-based formation control [1]-[5], a desired set of behaviors is implemented onto individual robots. By defining the relative importance of all the behaviors, the overall behavior of the robot is formed. In leader-following control strategy [6]-[10], the leaders are identified and the follower are defined to follow their respective leaders. In virtual structure method [11]-[14], the entire formation is considered as a single entity and desired motion is assigned to the structure.

In general, behavior-based is suitable for controlling a swarm of robots. However, it is difficult to analyze the overall system mathematically and show that the system converges to the desired formation. Both leader-following and virtual structure methods are easier to analyze but not suitable for controlling a large group of robots because the constraint relationships among robots become more complicated as the number of robots in the group increases. To alleviate the problem, Belta and Kumar [15] proposed a control method for a large group of robots to move along a specified path. However, this proposed control strategy has no control over the desired shape since the shape of the whole group is dependent on the number of the robots in the group. For large number of robots, the shape is fixed as an elliptical shape whereas for a small number of robots the shape is fixed as a rectangular shape. Some shape control methods for a group of robots are studied in [16]-[17]. Pimenta et al. [16] study the problem of controlling a large group of robots in 2D generation task. This approach is based on an analogy with fluids in electrostatic fields. In [17], interpolated implicit functions are used to control a swarm of robots to generate 2D patterns. This control method enables the robots to form various 2D shapes but it requires a lot of constraint points to specify the shape. The above mentioned methods are used only for static shape generation and the desired shape is only limited to a boundary. Cheah et al. [18] proposed a region-based shape control method for a large group of robots to form a certain shape. Robots are required to spread out inside the region specified by the desired shape which is moving along a specified path. The dynamics of the robots are also considered in the stability analysis of the system. However, only limited shapes are feasible since the desired shape is defined as the intersection of various regions specified by corresponding inequality functions.

In this paper, we propose a new shape control method for a swarm of robots. A novel way of defining the desired shape is presented with the aid of multiplicative potential energy function. The desired shape in this case is the union of all the regions defined by corresponding inequality functions. This proposed method is a complement of our previous method [18] and by combining these two approaches, a variety of shapes which is not feasible in our previous method, can be formed. In the proposed shape control method, each robot moves together in the desired shape as a group and at the same time maintains a minimum distance from each other. The robots in the group only need to communicate with their neighbors and not the entire community. The robots do not have specific identities or roles within the group. Therefore, the proposed method does not require specific orders or positions of the robots inside the region and hence different shapes can be formed even for a swarm of robots. Lyapunov theory is used to show the stability of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed controller.

II. ROBOT DYNAMICS

We consider a group of N fully actuated mobile robots whose dynamics of the i^{th} robot with n degrees of freedom can be described as [20], [21]:

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i)\dot{x}_i + g_i(x_i) = u_i \quad (1)$$

where $x_i \in R^n$ is a generalized coordinate, $M_i(x_i) \in R^{n\times n}$ is an inertia matrix, $C_i(x_i, \dot{x}_i) \in R^{n\times n}$ is a matrix of Coriolis and centripetal terms, $D_i(x_i) \in R^{n\times n}$ represents the damping force, $g_i(x_i) \in R^n$ denotes a gravitational force vector, and $u_i \in R^n$ denotes the control inputs.

Several important properties of the dynamic equation described by (1) are given as follows [20], [21]:

Property 1: The inertia matrix $M_i(x_i)$ is symmetric and

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positive definite for all $x_i \in R^n$.

Property 2: The Coriolis and centripetal matrix $C(x, \dot{x})$ is characterized by the following property $s^T[\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)]s = 0$ for all $s \in R^n$, $x_i \in R^n$.

Property 3: The damping matrix $D_i(x_i)$ is positive definite for all $x_i \in R^n$.

Property 4: The dynamic model described by (1) is linear in a set of unknown parameters $\theta_i \in R^p$ as

$$\begin{aligned} & M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i)\dot{x}_i + g_i(x_i) \\ = \; & Y_i(x_i, \dot{x}_i, \dot{x}_i, \ddot{x}_i)\theta_i \end{aligned} \tag{2}$$

where $Y_i(x_i, \dot{x}_i, \dot{x}_i, \ddot{x}_i) \in R^{n \times p}$ is a known regressor matrix.

III. Shape Control of Multi-Robot System

In this section, we present a shape control method for a swarm of robots using multiplicative potential energy function. First, an overall desired region of specific shape is defined for all the robots to stay inside. This overall desired region can be formed by many different regions. Second, a minimum distance is specified between each robot and its neighboring robots. Thus, the group of robots will be able to form a desired shape while maintaining a minimum distance among each other.

Let us define several regions by the following inequality functions:

$$\begin{aligned} R_1 &: h_1(\Delta x_{io1m}) = [f_{11}(\Delta x_{io11}), ..., f_{1M_1}(\Delta x_{io1M_1})]^T \leq 0 \\ R_2 &: h_2(\Delta x_{io2m}) = [f_{21}(\Delta x_{io21}), ..., f_{2M_2}(\Delta x_{io2M_2})]^T \leq 0 \\ &\vdots \\ R_L &: h_L(\Delta x_{ioLm}) = [f_{L1}(\Delta x_{ioL1}), ..., f_{LM_L}(\Delta x_{ioLM_L})]^T \leq 0 \end{aligned} \tag{3}$$

where, $\Delta x_{iolm} = x_i - x_{olm}$, $l = 1, 2, ..., L$, $m = 1, 2, .., M_l$, x_{olm} is a point inside f_{lm}, R_l is the closed region defined by h_l, M_l is the number of functions to form the region R_l and L is the number of desired regions. $f_{lm}(\Delta x_{iolm})$ are chosen to be continuous and twice partially differentiable that satisfy $|f_{lm}(\Delta x_{iolm})| \to \infty$ as $||\Delta x_{iolm}|| \to \infty$. $f_{lm}(\Delta x_{iolm})$ is chosen in such a way that the boundedness of $f_{lm}(\Delta x_{iolm})$ ensures the boundedness of $\frac{\partial f_{lm}(\Delta x_{iolm})}{\partial \Delta x_{iolm}}$, $\frac{\partial^2 f_{lm}(\Delta x_{iolm})}{\partial \Delta x_{iolm}^2}$. The final desired region R is formed by taking the union of all the L regions i.e.

$$R = R_1 \cup R_2 \cup ... \cup R_L \tag{4}$$

Each region shall move at the same speed that is, $\dot{x}_{o11} = ... = \dot{x}_{o1M} = \dot{x}_{o21} = ... = \dot{x}_{o2M} = ... = \dot{x}_{oL1} = ... = \dot{x}_{oLM}$, so that R can maintain its original shape during the course of movement. This implies that the points x_{olm} of all the regions are just offsets from one another i.e. $x_{o11} = x_o + c_{11}, ...x_{o1M} = x_o + c_{1M}, ..., x_{oL1} = x_o + c_{L1}, ...x_{oLM} = x_o + c_{LM}$ where x_o is a point inside the region R, c_{lm} are some constants. For simplicity of presentation, the index of x_{olm} is dropped and we will denote Δx_{iolm} by Δx_i where $\Delta x_i = x_i - x_o$. By combining different desired regions, various shape can be formed. For example, a star shape can be formed by choosing the objective functions as follows:

$$\begin{aligned} R_1 &: h_1(\Delta x_i) = f_{11}(\Delta x_i) = \frac{(x_{i_1} - x_{o11_1})^2}{a_1^2} + \frac{(x_{i_2} - x_{o11_2})^2}{b_1^2} - 1 \leq 0 \\ R_2 &: h_2(\Delta x_i) = f_{12}(\Delta x_i) = \frac{(x_{i_1} - x_{o11_1})^2}{a_2^2} + \frac{(x_{i_2} - x_{o11_2})^2}{b_2^2} - 1 \leq 0 \end{aligned} \tag{5}$$

where $x_i = [x_{i_1}, x_{i_2}]^T$, a_1, b_1 and a_2, b_2 are the semimajor and semiminor axes of the two ellipses, $(x_{o11_1}(t), x_{o11_2}(t))$ represents the common center of the two ellipses. In this case, only one inequality function is used to formed one region. To form a letter N we can define the following functions:

$$\begin{aligned} R_1 &: h_1(\Delta x_i) = \begin{cases} f_{11}(\Delta x_i) = (x_{i_1} - x_{o11_1})^2 - w^2 \leq 0 \\ f_{12}(\Delta x_i) = (x_{i_2} - x_{o11_2})^2 - d^2 \leq 0 \end{cases} \\ R_2 &: h_2(\Delta x_i) = \begin{cases} f_{21}(\Delta x_i) = (x_{i_1} + x_{i_2} - x_{o21_1} - x_{o21_2})^2 \\ \qquad -w^2 \leq 0 \\ f_{22}(\Delta x_i) = (x_{i_2} - x_{o21_2})^2 - d^2 \leq 0 \end{cases} \\ R_3 &: h_3(\Delta x_i) = \begin{cases} f_{31}(\Delta x_i) = (x_{i_1} - x_{o31_1})^2 - w^2 \leq 0 \\ f_{32}(\Delta x_i) = (x_{i_2} - x_{o31_2})^2 - d^2 \leq 0 \end{cases} \end{aligned}$$

where (x_{o11_1}, x_{o11_2}), (x_{o21_1}, x_{o21_2}) and (x_{o31_1}, x_{o31_2}) are the centers of the 3 regions, w and d are half of the width and height of the rectangles as illustrated in Fig. 1. In this case each region is formed by two inequality functions and the final overall region is the union of all the 3 rectangles.

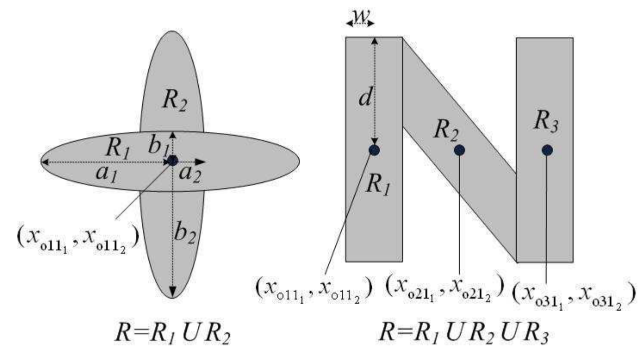

Fig. 1. Examples of desired regions. Grey regions are the desired regions.

Let P_l be the potential energy function associated with R_l, $l = 1, 2, .., L$, where:

$$P_l(\Delta x_i) = \sum_{m=1}^{M_l} \frac{1}{2} k_{lm} \left[max\left(0, f_{lm}\left(\Delta x_i\right)\right)\right]^2 \tag{6}$$

where k_{lm} are positive constants.

In our previous approach [18], we define the potential energy P of the desired region as a summation of the potential energy associated with each region i.e. $P = P_1 + P_2 + ... + P_L$. The desired region R resulted from this summation of the potential energy is the intersection of all the regions R_l that is $R = R_1 \cap R_2 \cap ... \cap R_L$. For example a desired region R which is an intersection of two ellipses specified in (5) can be formed by defining the potential energy P associated with R as $P = P_1 + P_2$. An illustration of this desired region is shown in Fig. 2.

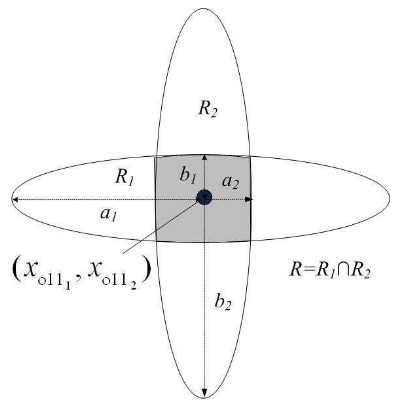

Fig. 2. Example of desired regions as an intersection of two ellipses. Grey region is the desired region.

Using this idea of summation of potential energy, various shapes can be formed. However, there are some limitations on this method. For example, it is not possible to form the star shape, which is the union of two ellipses, or the alphabet N, which is the union of three rectangles as shown in Fig. 1.

This paper presents a new method using multiplicative potential energy function. The proposed method in a way is a complement to our previous method and by combining this two approaches, a wide range of shapes can be formed. Let P_T be the potential energy associated with the desired region R and is defined by:

$$P_T = P_1 \times P_2 \times .. \times P_L \tag{7}$$

where P_l is defined in (6). It should be noted that P_l has a minimum of zero at the desired region where all the functions $f_{lm} \leq 0$. Therefore, P_T has a minimum value of zero when x_i is within any of the desired regions. That is, the overall desired region R is the union of all the regions R_1, R_2,...,R_L as defined in (3).

Using equations (6) and (7), the potential energy function of the final desired region R can be written as:

$$P_{Ti}(\Delta x_i) = \prod_{l=1}^{L} \sum_{m=1}^{M_l} \frac{1}{2} k_{lm} \left[max\left(0, f_{lm}\left(\Delta x_i\right)\right)\right]^2 \tag{8}$$

From (8) we can see that the potential energy is at the minimum value (zero) at the final desired region. This potential function will ensure that robots move toward the overall region formed by union of all the regions $R_1, R_2, ..., R_L$. An illustration of potential energy of a star shape defined in (5) is shown in Fig. 3. We can see that the potential energy is positive outside the overall desired region and zero within the desired region.

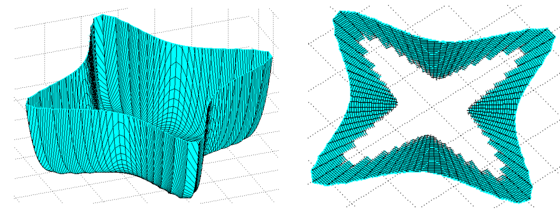

Fig. 3. Illustration of star-shape potential energy function. White region is the overall desired region.

Partial differentiating the potential energy function described by (8) with respect to Δx_i, we have:

$$\begin{aligned}
\frac{\partial P_{Ti}(\Delta x_i)}{\partial \Delta x_i} &= \\
&\sum_{m=1}^{M_1} k_{1m} max(0, f_{1m}(\Delta x_i)) \left(\frac{\partial f_{1m}(\Delta x_i)}{\partial \Delta x_i}\right)^T \prod_{l \neq 1}^{L} P_l \\
&+ \sum_{m=1}^{M_2} k_{2m} max(0, f_{2m}(\Delta x_i)) \left(\frac{\partial f_{2m}(\Delta x_i)}{\partial \Delta x_i}\right)^T \prod_{l \neq 2}^{L} P_l \\
&\vdots \\
&+ \sum_{m=1}^{M_L} k_{Lm} max(0, f_{Lm}(\Delta x_i)) \left(\frac{\partial f_{Lm}(\Delta x_i)}{\partial \Delta x_i}\right)^T \prod_{l \neq L}^{L} P_l \\
&\triangleq \quad \Delta \zeta_i
\end{aligned} \tag{9}$$

Note that when the robot i is outside the desired region, the control force $\Delta \zeta_i$ described by (9) is activated to attract the robot toward the desired region. When the robot is inside the desired region, then $\Delta \zeta_i = 0$.

Next, we define a minimum distance between robots by the following inequality:

$$g_{ij}(\Delta x_{ij}) = r^2 - ||\Delta x_{ij}||^2 \leq 0 \tag{10}$$

where $\Delta x_{ij} = x_i - x_j$ is the distance between robot i and robot j and r is a minimum distance between the two robots. For simplicity, the minimum distance between robots is chosen to be the same for all the robots. Note from the above inequality that the function $g_{ij}(\Delta x_{ij})$ is twice partially differentiable. From (10), it is clear that

$$g_{ij}(\Delta x_{ij}) = g_{ji}(\Delta x_{ji}) \tag{11}$$

and

$$\frac{\partial g_{ij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = -\frac{\partial g_{ji}(\Delta x_{ji})}{\partial \Delta x_{ji}} \tag{12}$$

A potential energy for the local objective function (10) is defined as:

$$Q_{ij}(\Delta x_{ij}) = \sum_{j \in N_i} \frac{k_{ij}}{2} [max(0, g_{ij}(\Delta x_{ij}))]^2 \tag{13}$$

where k_{ij} are positive constants and N_i is a set of neighbors around robots i. Any robot that is at a distance smaller than r_N from robot i is called neighbor of robot i. r_N is a positive number satisfy the condition $r_N > r$.
Partial differentiating (13) with respect to Δx_{ij}, we get

$$\frac{\partial Q_{ij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = \sum_{j \in N_i} k_{ij} max(0, g_{ij}(\Delta x_{ij})) \left(\frac{\partial g_{ij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T \triangleq \Delta\rho_{ij} \quad (14)$$

Note that $\Delta\rho_{ij}$ is a resultant control force acting on robot i by its neighboring robots. Similarly, when robot i maintains minimum distance r from its neighboring robots, then $\Delta\rho_{ij} = 0$. The control force $\Delta\rho_{ij}$ is activated only when the distance between robot i and any of its neighboring robots is smaller than the minimum distance r. We consider a bidirectional interactive force between each pair of neighbors. That is, if robot i keeps a distance from robot j then robot j also keeps a distance from robot i.
Next, we define a vector $\dot{x}_{ri}$ as

$$\dot{x}_{ri} = \dot{x}_o - \alpha_i \Delta\zeta_i - \gamma\Delta\rho_{ij} \quad (15)$$

where $\Delta\zeta_i$ is defined in (9), $\Delta\rho_{ij}$ is defined in (14), α_i and γ are positive constants. Let

$$\Delta\epsilon_i = \alpha_i \Delta\zeta_i + \gamma\Delta\rho_{ij}, \quad (16)$$

we have

$$\dot{x}_{ri} = \dot{x}_o - \Delta\epsilon_i \quad (17)$$

When the robot i keeps a minimum distance from all its neighboring robots inside the desired region (as illustrated in figure 4), then $\Delta\epsilon_i = 0$.

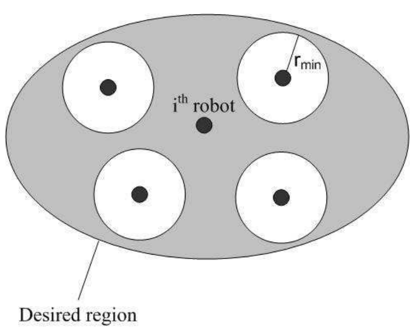

Fig. 4. Desired region seen by robot i

Differentiating (15) with respect to time we get

$$\ddot{x}_{ri} = \ddot{x}_o - \Delta\dot{\epsilon}_i \quad (18)$$

A sliding vector for robot i is then defined as:

$$s_i = \dot{x}_i - \dot{x}_{ri} = \Delta\dot{x}_i + \Delta\epsilon_i \quad (19)$$

where $\Delta\dot{x}_i = \dot{x}_i - \dot{x}_o$. Differentiating (19) with respect to time yields

$$\dot{s}_i = \ddot{x}_i - \ddot{x}_{ri} = \Delta\ddot{x}_i + \Delta\dot{\epsilon}_i \quad (20)$$

where $\Delta\ddot{x}_i = \ddot{x}_i - \ddot{x}_o$. Substituting equations (19) and (20) into (1), and using property 4 we have

$$\begin{aligned} & M_i(x_i)\dot{s}_i + C_i(x_i, \dot{x}_i)s_i + D_i(x_i)s_i \\ & + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\theta_i = u_i \end{aligned} \quad (21)$$

where $Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\theta_i = M_i(x_i)\ddot{x}_{ri} + C_i(x_i, \dot{x}_i)\dot{x}_{ri} + D_i(x_i)\dot{x}_{ri} + g_i(x_i)$. The controller for multi-robot systems is proposed as

$$u_i = -K_{si}s_i - K_p\Delta\epsilon_i + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\hat{\theta}_i \quad (22)$$

where K_{si} are positive definite matrices, $K_p = k_p I$, k_p is a positive constant and I is an identity matrix. The estimated parameters $\hat{\theta}_i$ are updated by

$$\dot{\hat{\theta}}_i = -L_i Y_i^T(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})s_i \quad (23)$$

where L_i are positive definite matrices.
The closed-loop dynamic equation is obtained by substituting (22) into (21):

$$\begin{aligned} & M_i(x_i)\dot{s}_i + C_i(x_i, \dot{x}_i)s_i + D_i(x_i)s_i + K_{si}s_i \\ & + Y_i(x, \dot{x}, \dot{x}_{ri}, \ddot{x}_{ri})\Delta\theta_i + K_p\Delta\epsilon_i = 0 \end{aligned} \quad (24)$$

where $\Delta\theta_i = \theta_i - \hat{\theta}_i$. Let us define a Lyapunov-like function for multi-robot systems as

$$\begin{aligned} V &= \sum_{i=1}^{N} \frac{1}{2} s_i^T M_i(x_i) s_i + \sum_{i=1}^{N} \frac{1}{2} \Delta\theta_i^T L_i^{-1} \Delta\theta_i \\ &+ \sum_{i=1}^{N} \frac{1}{2} \alpha_i k_p \prod_{l=1}^{L} \sum_{m=1}^{M_l} k_{lm} [max(0, f_{lm}(\Delta x_i)]^2 \\ &+ \frac{1}{2} \sum_{i=1}^{N} \frac{1}{2} \gamma k_p \sum_{j \in N_i} k_{ij} [max(0, g_{ij}(\Delta x_{ij}))]^2 \end{aligned} \quad (25)$$

Differentiating (25) with respect to time and substituting (23) and (24) into it, we can show that

$$\begin{aligned} \dot{V} = & -\sum_{i=1}^{N} s_i^T K_{si} s_i - \sum_{i=1}^{N} s_i^T D_i(x_i) s_i \\ & -\sum_{i=1}^{N} k_p \Delta\epsilon_i^T \Delta\epsilon_i \end{aligned} \quad (26)$$

We are ready to state the following theorem:

Theorem: *Consider a group of N robots with dynamic described by (1), the adaptive control law (22) and the parameter update laws (23) give rise to the convergence of $\Delta\epsilon_i \to 0$ and $s_i \to 0$ for all $i = 1, 2, ..., N$, as $t \to \infty$.*

Proof: From (26), we can conclude that s_i and $\Delta\epsilon_i \in L^2$ and $\Delta\theta_i$ is bounded. Differentiating equations (9) and (14), it can be shown that $\Delta\dot{\zeta}_i$ and $\Delta\dot{\rho}_{ij}$ are bounded and hence $\Delta\dot{\epsilon}_i$ is bounded. From (18), $\ddot{x}_{ri}$ is bounded if $\ddot{x}_o$ is bounded. From the closed-loop equation (24), we can conclude that $\dot{s}_i$ is bounded. Applying Barbalat's lemma [21], we have $\Delta\epsilon_i \to 0$ and $s_i \to 0$ as $t \to \infty$. From (19), $\Delta\dot{x}_i \to 0$. Since, $\Delta\epsilon_i = \alpha_i\Delta\zeta_i + \gamma\Delta\rho_{ij} = 0$, reasonable weightages for $\Delta\zeta_i$ and $\Delta\rho_{ij}$ can be obtained by adjusting α_i and γ.

Remark: The proposed controller can be extended to the case of dynamic region with rotation and scaling. In this case, the global objective functions can be defined as follows:

$$f_G(\Delta x_{Ri}) = [f_{G1}(\Delta x_{Ri}),\ f_{G2}(\Delta x_{Ri}),\ \ldots,\ f_{GM}(\Delta x_{Ri})]^T \leq 0 \tag{27}$$

where $\Delta x_{Ri} = x_{Ri} - x_o = RS\Delta x_i$, $R(t)$ is a time-varying rotation matrix and $S(t)$ is a time-varying scaling matrix. The details can be found in [19].

IV. SIMULATION

This section presents some simulation results to illustrate the performance of the proposed controller. In the simulation, 100 robots are used to form different shapes while moving along a path specified by $x_{o_1} = t$, $x_{o_2} = 2\sin(t)$. The mass of each robot is set to $1\ kg$. The control gains are set as $K_{si} = diag\{20, 20\}$, $k_p = 4$, $k_{ij} = 250$, $k_{lm} = 0.01$, $\gamma = 1$ and $\alpha_i = 1$ for all the simulations.

A. Desired Region as a Union of two Triangles

In this section, the group of robots forms a star shape specified by the union of two triangles. All the 100 robots are initially spread out as shown in Fig. 5. All the robots then move toward the desired shape and spread out inside the desired shape so as to maintain minimum distance among themselves. The movement of the entire group at various time instances is shown in Fig. 5.

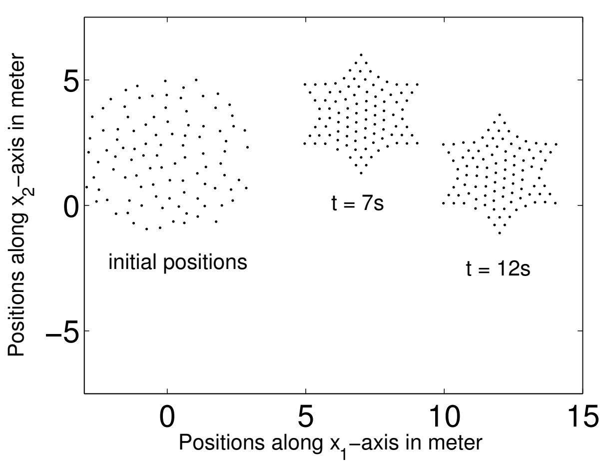

Fig. 5. A group of 100 robots forms a star shape as a union of two triangles

B. Desired Region as a Union of two Ellipses

The group of robots, in this case, forms a cross shape defined by the union of two ellipses. The simulation results showing the movement of the robots at various time instances are shown in Fig. 6.

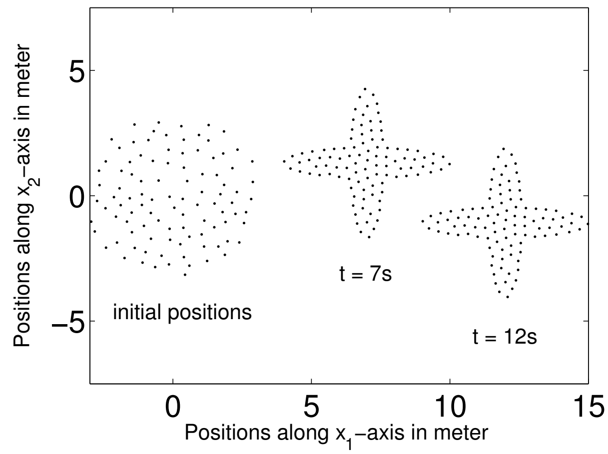

Fig. 6. A group of 100 robots forms a cross shape as a union of two ellipses

C. Rotation of a star shape

In this section, the simulation on rotation of desired region is presented. The group of robots forms a star shape defined by union of two triangles and move along a straight line while rotating at the same time. The swarm of robots rotates counter clockwise about its centroid at the speed of $45°/s$. Simulation results are presented in Fig. 7 and a square marker is added at one of the vertices of the star shape to mark the movement of the rotation.

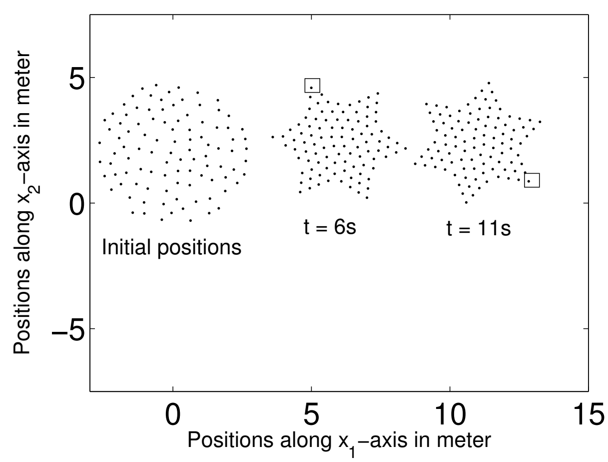

Fig. 7. A group of 100 robots forms a star shape and rotates counter clockwise

D. Static Region with Obstacles Avoidance

The group of robots moves along a corridor towards the desired region while avoiding obstacles along the pathway. The grey area represents the boundaries enclosing the obstacles and the walls. When the robot goes into the boundary area, it will try to move out to avoid hitting the obstacles or the walls. Positions of all the robots at different time instances are shown in Fig. 8.

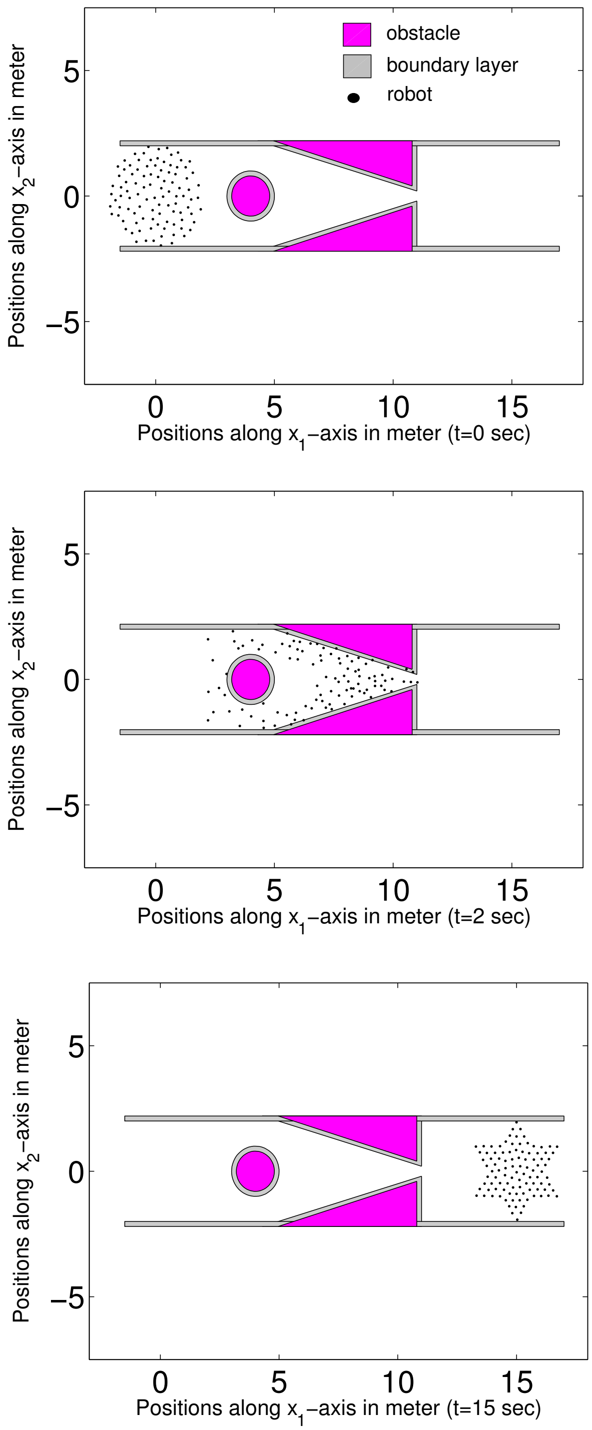

Fig. 8. A group of 100 robots moves pass the obstacles to form a star shape

V. Conclusion

In this paper, we have proposed a novel method of shape control based on region control method. Both multiplicative and additive potential energy functions have been introduced for shape control of a swarm of robots. The additive potential energy term is used to form desired region which is the intersection of different regions whereas the multiplicative term is used to form desired region as the union of various regions. With these two approaches, various complicated shapes can now be formed easily. It has been shown that all the robots are able to move as a group inside the desired shape while maintaining minimum distance from each other. Lyapunov-like function has been proposed for the stability analysis of the multi-robot systems. Simulation results have been presented to illustrate the performance of the proposed controller.

References

[1] C. Reynolds, "Flocks, herds and schools: A distributed behavioral model,". *Computer Graphics*, vol. 21, pp. 25–34,1987.

[2] D.C. Brogan and J.K. Hodgins, "Group behaviors for systems with significant dynamics," *Autonomous Robots*, vol. 4, pp. 137–53, March 1997.

[3] T. Balch and R.C. Arkin, "Behavior-based formation control for multi-robot systems," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, Dec. 1998.

[4] J.H. Reif and H. Wang, "Social potential fields: A distributed behavioral control for autonomous robots," *Robotics and Autonomous Systems,* vol. 27, pp. 171-194, 1999.

[5] J.R. Lawton,R.W. Beard, and B.J. Young, "A Decentralized Approach to Formation Maneuvers," *IEEE Transactions on Robotic and Automation*, vol. 19, no. 6, pp. 933–941,Dec. 2003.

[6] P.K.C Wang, "Navigation strategies for multiple autonomous robots moving in formation," *Journal of Robotics Systems*, vol. 8, no. 2, pp. 177–195, 1991.

[7] J.P. Desai, V. Kuma, and P. Ostrowski, "Modeling and control of formations of nonholonomic mobile robots,", *IEEE Transaction on Robotic and Automation*, vol.17, pp. 905–908, Dec. 2001.

[8] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Transaction on Robotic and Automation*, vol. 18, no. 5, pp. 813–825, 2002.

[9] P. Ogren, M. Egerstedt and X. Hu, "A control Lyapunov function approach to multi-agent coordination," *IEEE Transaction on Robotic and Automation*, vol. 18, no. 5, pp. 847–851, October 2002.

[10] J. Fredslund and M. J. Mataric, "A general algorithm for robot formations using local sensing and minimal communication," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 837–846, October 2002.

[11] M.A. Lewis and K.H. Tan, "High Precision Formation Control of Mobile Robots Using Virtual Structures," *Autonomous Robots*, vol. 4, no. 4, pp. 387–403, 1997.

[12] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *IEEE Transactions on Robotics and Automation*, vol.17, no. 6, pp. 947–951, 2001.

[13] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups," in *Proceeding of Decision and Control Conference*, pp. 2968-2973, Orlando, FL, Dec. 2001.

[14] R.W. Beard, J. Lawton, and F.Y. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Transactions on Control Systems Technology,* vol. 9, no. 6, pp. 777–790, Nov. 2001.

[15] C. Belta and V. Kumar, "Abstraction and control for groups of robots," *IEEE Transactions on Robotics*, vol. 20, no. 5, pp. 865–875, 2004.

[16] L.C.A. Pimenta, N. Michael, R.C. Mesquita and G.A.S Pereira and V. Kumar, "Control of Swarms Based on Hydrodynamic Models," *IEEE International Conference on Robotics and Automation*, pp. 1948-1953, May 19-23, 2008, Passadena, C.A, USA.

[17] M.A. Hsieh, V. Kumar and L. Chaimowicz, "Decentralized controllers for shape generation with robotic swarms," *Robotica*, vol. 26, pp. 691–701, 2008.

[18] C.C. Cheah, S.P. Hou and J.J.E Slotine "Region Following Formation Control for Multi-Robot Systems," *IEEE International Conference on Robotics and Automation*, pp. 3796-3801, May 19–23, 2008, Passadena, C.A, USA.

[19] S.P. Hou, C.C. Cheah and J.J.E Slotine "Dynamic Region Following Formation Control for a Swarm of Robots," *IEEE International Conference on Robotics and Automation*, pp. 1929–1934, May 12-17, 2009, Kobe, Japan.

[20] T.I. Fossen, *Guidance and Control of Ocean Vehicles*. Baffins Lane, Chichester: John Wiley & Sons Ltd. 1994

[21] J.J.E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, New Jersy: Prentice Hall, 1991.