Generation of Energy Saving Motion for Biped Walking Robot through Resonance-based Control Method

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Abstract—In this paper, we apply an energy saving control method to a simulation model that includes some dynamics of biped walking robots. This control method was proposed by the authors, and is based on resonance of multi-joint robots. An advantage of the control method is to work well without using exact parameter values of the controlled systems nor huge numerical calculations. This paper discusses how to apply the proposed control method to walking motions as a first step to realize energy saving biped robots. For this purpose, we consider some dynamics of the biped robots. Simulation results showed that the proposed control method can generate energy saving walking motions.

Index Terms—Resonance, Biped Walking Robot, Stiffness Optimization, Adaptive Control, Delayed Feedback Control

I. INTRODUCTION

A. Biped Walking

To clarify how to generate biped walking motion is one of important issues for not only engineering fields but also scientific fields due to interest of human motion analyses [1], [2], [3], [4]. In the robotic field, a typical approach to exert enough torque for the walking motions is to use actuators with high reduction gears. However, the high reduction gears increase friction of the actuator systems. As a result, large friction consumes much energy, and it is known that some walking robots consume much energy than human beings [2].

B. Periodic Motion and Passive Element

On the other hand, it is well-known that passive elements can effectively generate periodic motions. For example, passive pendulums oscillate for long periods of time by the gravitational effect. Therefore, we may reduce energy consumption to generate walking motions by utilizing passive elements, because walking motions are also periodic.

C. Passive Walking and Level Ground Walking

In fact, passive walking robots walk down slopes without using actuator torque [2], [3]. Therefore, the passive walking phenomena have been getting attentions of researchers to generate energy saving motions [4]. However, walking on the level ground or walking up slopes require energy supply from actuators, and can not be treated by the concept of the passive walk directly. Therefore, we have to simultaneously utilize passive elements and actuators to generate such motions.

D. Conventional Resonance

The concept of resonance is a traditional and theoretical framework that can enables optimal simultaneous use of passive elements and actuators. In the resonant conditions, potential energy of passive elements is optimally used to generate periodic motions. Then, actuator torque is minimally required. Therefore, if we want to generate certain periodic motions, resonance minimizes actuator torque. Conversely, if we use certain actuator torque, resonance maximizes velocity and amplitude of periodic motions. Thus, utilizing resonance brings about energy saving or high motion performance. However, conventional resonance can be directly applied to only linear systems, which have only one degree-of-freedom.

E. Resonance-based Motion Control Method

To effectively use passive elements and actuators for multi-joint robots, we have proposed resonance-based motion control methods [5], [6], [7], [8], [9], [10]. The proposed control methods generate periodic motions while adjusting stiffness of elastic elements installed in each joint of the robots. In the cases of some proposed controllers [5], [6], [8], desired motions are specified and fixed. Then, the stiffness is adaptively adjusted to reduce actuator torque as much as possible. An advantage of the proposed controller is to works well without using exact parameter values of the robotic systems nor huge numerical calculations.

We also proposed another type of a control method that adjusts not only stiffness but also motion patterns [7]. Simulation results of multi-joint robots with no friction showed that the proposed controller generated periodic motions, which require almost no actuator torque [7]. In this case, we could define optimal periodic motions as the motions that require no actuator torque, because the robot did not have friction. However, if the robots consume energy by friction or impact phenomena of walking motions, we should consider how to generate energy saving motions by using actuator torque.

F. Extended Resonance and Application to Biped Robots

To make clear how to generate energy saving motions of multi-joint robots by using passive elements and actuators, we have formulated resonance for the multi-joint robots [9], [10]. In this formulation, we considered a torque minimization problem, and analytically showed that the optimal actuator torque is described as linear state feedback of angular velocity. This property is the same as conventional resonance, and can simplify controller design.
Therefore, if we can apply extended resonance to biped walking robots, we can realize energy saving biped robots. However, these control methods have not been designed for walking robots, and it is unknown whether the proposed control methods can generate energy saving walking motions or not.

G. This Paper

This paper tries to apply the control method based on extended resonance to a simulation model that includes important dynamics of biped walking robots as shown in Fig.1. For this purpose, we discuss some dynamics of walking robots, and construct a 4-DOF numerical simulation. We tuned some parameters of the numerical simulation to obtain energy saving motions.

II. DYNAMICS AND OBJECTIVE

This section describes dynamics of the controlled systems and objectives in this study. In our previous papers, we treated only dynamics of multi-joint structures. In this paper, we additionally consider some dynamics of walking robots as a first step. Therefore, this paper considers impact dynamics, a condition of ankle position and falling phenomena of the sagittal plane.

A. Dynamics of Multi-Joint Structure

Dynamics of multi-joint robots without considering impact phenomena nor falling phenomena is described by

\[
R(q(t))\ddot{q}(t) + \left\{ \frac{1}{2} \dot{R}(q(t)) + S(q(t), \dot{q}(t)) + D \right\} \dot{q}(t) + g(q(t)) = -K(t)(q(t) - q_a) + \tau(t),
\]

where \( R(q) \) is a positive definite inertia matrix, \( n \in \mathbb{R} \) is the number of the joints of the robot, \( S(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is a skew symmetric matrix, \( D = \text{diag}(d_1 \cdots d_n) \) is a viscosity matrix, \( d_1 \cdots d_n \in \mathbb{R} \) are coefficients of the viscosity, \( g(q) \in \mathbb{R}^n \) is a vector of gravitational torque, \( K = \text{diag}(k_1 \cdots k_n) \) is a stiffness matrix, \( k_1 \cdots k_n \in \mathbb{R} \) are adjustable stiffness of the elastic elements installed in each joint, \( q = (q_1 \cdots q_n) \) is a vector of joint angles, \( q_a = (q_{a1} \cdots q_{an}) \) is a vector of equilibrium angles of the elastic elements, \( \tau = (\tau_1 \cdots \tau_2) \) is a vector of actuator torque, and \( t \) is time.

The stiffness \( k_1 \cdots k_n \) are assumed to be adjustable in real-time. This kind of adjustable elastic elements has been developed by many researchers [11, 12, and light and small devices are becoming available.

B. Impact Dynamics

When the swing leg contacts the ground, an impact phenomenon will occur. To make a model of the impact phenomena, we adopt the following assumptions:

- Assumption 1: The impact occurs instantly.
- Assumption 2: The impact is completely inelastic collision.
- Assumption 3: At the impact, the stance leg and the swing leg are switched each other instantly.
- Assumption 4: The actuators can not exert impulsive torque, and the actuator torque does not affect the impact dynamics.

At the impact, impulsive force is applied to the tip of the swing leg, and brings about angular momentum \( \tau_{\text{impulse}} \). Then, the angular velocity instantly changes at the impact from the conservation of the momentum.

\[
R(q^+)\dot{q}^+ - R(q^-)\dot{q}^- = \tau_{\text{impulse}}.
\]

where \( \dot{q}^+ \in \mathbb{R}^n \) is an angular velocity vector of the robot just after the impact, and \( \dot{q}^- \in \mathbb{R}^n \) is an angular velocity vector of the robot just before the impact.

We use the equation (2) for impact dynamics.

C. Condition of Ankle Position

During the walking, the ankle can not be lower to the ground. This condition is also different from our previous study.

D. Falling Phenomena

Different from robotic manipulators, walking robots can fall down, because toes and heels of the walking robots are not fixed to the ground. Therefore, we make a model to consider the falling phenomena.

To make the model, we firstly consider dynamics of the ankle of the stance leg around the ankle joint in the sagittal plane as shown in Fig.2.

\[
I_a \ddot{q}_a = -\tau_{\text{alt}} - f_l l_t + f_h l_h,
\]

where \( I_a \in \mathbb{R} \) is inertia moment of the ankle around the ankle joint, \( q_a \in \mathbb{R} \) is angle of the ankle from the ground, \( \tau_{\text{alt}} = \tau_1 - d_1 \dot{q}_1 - k_1 q_1 \), \( f_l, f_h \in \mathbb{R} \) are reaction force from the ground to the toe and the heel respectively, and \( l_t, l_h \in \mathbb{R} \) are length from the ankle joint to the toe and the heel respectively.

Since the ground can not drag the toe or the heel, the ground reaction force \( f_l, f_l \) can not be negative. Conversely, when \( f_l, f_h \) are positive, the toe and the heel do not float from the ground. Therefore, the positiveness of \( f_l, f_l \) is
a sufficient condition that we can use the equation (1) as dynamics of the robots.

To clarify the condition of positiveness of $f_i, f_1$, we consider dynamics of the vertical direction of the ankle.

$$m_a \ddot{y}_a = f_t + f_h - f_a,$$

where $m_a \in \mathbb{R}$ is mass of the ankle, $y_a \in \mathbb{R}$ is vertical displacement of the ankle, $f_a \in \mathbb{R}$ is force applied from the upper body to the ankle.

While the toe and the heel are contacting to the ground, the ankle does not move, and $\dot{q}_a$ and $\ddot{y}_a$ will be 0. From the the equation (3) and (4), we can obtain the following condition that the ground reaction force $f_t, f_h$ will be positive and the toe and the heel will not float from the ground.

$$-f_a l_h < \tau_{iall} < f_a l_h $$

The force $f_a$ can be calculated from vertical dynamics of the walking robots. In the vertical direction, only the ground reaction force is applied to the walking robots, and dynamics of mass center is given by

$$m_{body} \ddot{y}_{com} = -m_{body} g + f_a$$

where $m_{body} \in \mathbb{R}$ is total mass of the robots, $y_{com} \in \mathbb{R}$ is vertical displacement of the mass center, $g \in \mathbb{R}$ is acceleration of gravity.

E. Control Objective

Control objectives of this study are to generate walking motions and to reduce actuator torque as much as possible.

III. EXTENDED RESONANCE

This section describes extended resonance [9], [10]. Based on extended resonance, we can generate energy saving motions of multi-joint robots, and simplify controller design.

A. Cost Function

A cost function is defined as the following $L_2$ norm of actuator torque.

$$J = \frac{1}{b} \int_{iT}^{(i+1)T} \tau^T A^{-1} \tau dt,$$

where $T \in \mathbb{R}$ is a period of a cycle, $i \in \mathbb{N}$ is the number of the cycle, $b \in \mathbb{R}$ is a positive constant, $A = \text{diag}(a_1, a_2, \cdots, a_n) \in \mathbb{R}^{n \times n}$, and the elements $a_1, a_2, \cdots, a_n \in \mathbb{R}$ are positive constants.

As stated in the section II-E, one of the control objectives is to minimize the cost function $J$.

B. Boundary Condition

Initial conditions are $q(0) = q_{\text{start}}, \dot{q}(0) = v_{\text{start}}$ when $t = 0$, and terminal conditions are $q(T) = q_{\text{end}}, \dot{q}(T) = v_{\text{end}}$ when $t = T$.

In the case of walking motions, we can set the initial conditions $q_{\text{start}}, v_{\text{start}}$ to those at just after toe off, and the terminal conditions $q_{\text{end}}, v_{\text{end}}$ to those at just after heel strike.

C. Optimal Actuator Torque

We can analytically derive that the optimal actuator torque $\tau_{\text{opt}}$ can be described by the following linear state feedback form [9], [10].

$$\tau_{\text{opt}} = b A \dot{q}$$

the equation (8) is derived by an energy analysis or a Hamilton-Jacobi-Bellman Equation.

D. Discussion

Physical meaning of the above analysis is that minimum actuator torque is required to generate motions without exerting torque, which is not proportional to the angular velocity.

E. Comparison with Conventional Resonance

The analogies between conventional resonance and the proposed formulation are that actuators torque is linear state feedback of velocity, and actuators generate periodic motions by using minimum actuator torque. The differences are linearity of dynamics (conventional: linear, proposed: nonlinear), and degree-of-freedom (conventional: single, proposed: multiple). Therefore, our proposed formulation can be regarded as a kind of extension of resonance to the periodic motions of the multi-joint robots.

IV. RESONANCE-BASED MOTION CONTROL METHOD

This section describes a control method based on extended resonance, which adaptively adjust joint stiffness and motion patterns [9], [10].

A. Controller

We proposed the following controller.

$$\tau = -K_p \Delta q - K_v \dot{q} + b A \dot{q}$$

$$k = \Gamma_k Q \dot{q}$$

$$\dot{b} = -\gamma_b q^T \dot{q}$$

$$q_d(t) = (1 - \alpha) q_d(t - T_i) - \alpha q(t - T_i)$$

$$\dot{q}_d(t) = (1 - \alpha) \dot{q}_d(t - T_i) - \alpha \dot{q}(t - T_i)$$

where $K_p = \text{diag}(k_{v1}, \cdots, k_{vn})$ is a matrix of a velocity error feedback gains, $\Delta \dot{q} = \dot{q} - \dot{q}_d; q_d \in \mathbb{R}^n$ is a desired motion, $b \in \mathbb{R}$ is an estimated value of $b$ of the equation (8), $k = (k_1, k_2, \cdots, k_n)^T$ , $\Gamma_k \in \mathbb{R}^{n \times n}$ is a positive definite gain matrix of stiffness adaptation, $Q = \text{diag}(q_1 - q_{en}, \cdots, q_n - q_{en})$, $\gamma_b \in \mathbb{R}$ is a gain of the adaptive parameter tuning, $Q = \text{diag}(q_1, \cdots, q_n), \ T_i \in \mathbb{R}$ is a cycle time of the last cycle (i'th cycle), and $\alpha \in \mathbb{R}$ is a scalar constant, which is set from 0 to 1.

We should specify the desired motion in the first cycle $q_d(t) (0 \leq t < T)$. After the first cycle $T_i \leq t$, the desired motion is adjusted by the equation (12), (13).
B. Effect of Controller

The stiffness is adjusted by the equation (10) to make the natural frequency of the robots be frequency of desired motions [5], [6], [8]. The desired motions are adjusted by the equation (12), (13) of delayed feedback control structure [13]. Delayed feedback control has an ability that motions will converge to unknown periodic motions, and the unknown periodic motions require no input from the delayed feedback control. Therefore, we can expect that the terms \(-K_p\Delta q - K_v\Delta \dot{q}\) of the equation (9) will be 0 and the actuator torque will be in the optimal condition of the equation (8).

V. APPLICATION OF RESONANCE-BASED CONTROL METHOD TO WALKING ROBOT

The control methods of the previous section could generate energy saving motions of a multi-joint robot [7], [9], [10]. In addition, we can set the initial conditions of the section III-B \(q_{\text{start}}, v_{\text{start}}\) to the angle and the angular velocity at just after toe off of walking motions, and the terminal conditions \(q_{\text{end}}, v_{\text{end}}\) to those at just before heel strike. Then, our proposed formulation can treat a certain part of walking motions.

However, the proposed control method still did not consider some important conditions of walking dynamics.

A. Condition of Ankle Trajectory

One of the important conditions is that the ankle must be upper to the ground during the walking. Since the proposed controller adjusts motions online, this condition may not be satisfied. However, we confirmed that converged motions were not so different from initial desired motions in numerical simulations, and we can specify the initial desired motions. It seems to be very difficult to theoretically prove whether the proposed method can treat the condition or not. Therefore, this paper tries to generate walking motions by using the controller in numerical simulations.

B. Condition of Falling Phenomenon

The other important condition is the condition of the falling phenomena as stated in the section II-D. In dynamics of the robotic manipulators, the actuators of the all joints are assumed to exert torque with no limitations [8], [9]. To the contrary, in dynamics of biped robots, we should limit the ankle joint torque to prevent the falling phenomena. It also seems to be difficult to treat theoretically. However, our controller uses negative error feedback, and seems to generate stable periodic motions even if we limit the torque of the ankle joint. This paper tries to confirm this problem by using the numerical simulation.

C. Motion Planning and Dynamic Balancing

Other important problems of walking control are motion planning and dynamic balancing. These problems are not considered in the proposed controller explicitly. To improve the proposed controller by considering the motion planning and the dynamic balancing is our future works.

VI. SIMULATION

To investigate whether the proposed controller can generate energy saving walking motions or not, we constructed a numerical simulation. We used the 4-joint model that walks in the sagittal plane on the planar ground as shown in Fig.3.

A. Dynamics

We used the equation (1) as dynamics of the multi-joint structure, the equation (2) as the impact dynamics to calculate angular velocity vector \(\dot{q}\) just after the impact. To take into account the falling phenomena, we limited the ankle torque to satisfy the equation (5). \(m_1 = 2.0[\text{kg}], m_2 = 2.0[\text{kg}], m_3 = 2.0[\text{kg}], m_4 = 2.0[\text{kg}], l_1 = 0.5[\text{m}], l_2 = 0.5[\text{m}], l_3 = 0.5[\text{m}], l_4 = 0.5[\text{m}], l_{g1} = 0.25[\text{m}], l_{g2} = 0.25[\text{m}], l_{g3} = 0.25[\text{m}], l_{g4} = 0.25[\text{m}], I_1 = 0.04[\text{Nms}^2/\text{rad}], I_2 = 0.04[\text{Nms}^2/\text{rad}], I_3 = 0.04[\text{Nms}^2/\text{rad}], I_4 = 0.04[\text{Nms}^2/\text{rad}], d_1 = 2.0[\text{Nms}^2/\text{rad}], d_2 = 2.0[\text{Nms}^2/\text{rad}], d_3 = 2.0[\text{Nms}^2/\text{rad}], d_4 = 2.0[\text{Nms}^2/\text{rad}], l_1 = 0.15[\text{m}] \text{ and } l_h = 0.15[\text{m}], \) where \(m\) is a weight of the robot link, \(l\) is a length of the link, \(l_h\) is a length from a joint to a mass center of the link, \(I\) is an inertia moment around the mass center of the link, and the number of the suffix \(j\) represents \(j\)th link.

B. Controller

The equations from (9) to (13) were adopted as a controller.

C. Initial Desired Motion

We set the initial angular velocity \(\dot{q}(0)\) to 0. We set the initial desired motion \(q_d(t)|0 < t < T\) to approximate CGA data [14] by using trigonometric functions. Then, the initial desired motions during the period \(0 < t < T\) were \(q_{d1} = q_{c1} - \frac{\pi}{30} \cos\left(\frac{5}{3}\pi t\right)[\text{rad}], q_{d2} = q_{c2} - \frac{\pi}{30} \cos\left(\frac{5}{3}\pi t\right)[\text{rad}], \) \(q_{d3} = q_{c3} + \frac{\pi}{30} \cos\left(\frac{5}{3}\pi t\right)[\text{rad}], q_{d4} = q_{c4} - \frac{\pi}{30} \cos\left(\frac{5}{3}\pi t\right)[\text{rad}].\) After the period, we set the desired motion to \(q_d(T) = q_d(T).\)

This part seems to be improved by using appropriate motion planning methods.

D. Parameter Tuning

We tuned some parameters to obtain the energy saving walking motions. The most difficult problem was to satisfy the condition of the ankle position as stated in the section II-C. Badly tuned the equilibrium angle of the elastic elements \(q_e\) resulted in stumbles, because we adjusted the motions by using delayed feedback control. In such cases, the ankle contacted to the ground before the ankle went in front of
the stance leg. To satisfy the condition of ankle position, we tuned $q_e$ mainly. Thus, we set the equilibrium angle to $q_{e1} = 1.75\pi [\text{rad}]$, $q_{e2} = 6.2\pi [\text{rad}]$, $q_{e3} = 3.0[\text{rad}]$ and $q_{e4} = 0.87[\text{rad}]$.

We also tuned the feedback gains of the proposed controller. The structure of the equation (9), (10), (11) is almost the same as usual adaptive controllers, and we already reported an appropriate gain setting of the adaptive controller with the delayed feedback structure [7]. Therefore, the gain tuning process was not difficult problem. Thus, we set the gains to $k_{p1} = 40[\text{Nm/rad}]$, $k_{p2} = 20[\text{Nm/rad}]$, $k_{p3} = 50[\text{Nm/rad}]$, $k_{p4} = 10[\text{Nm/rad}]$, $k_{v1} = 20[\text{Nms/rad}]$, $k_{v2} = 15[\text{Nms/rad}]$, $k_{v3} = 25[\text{Nms/rad}]$, $k_{v4} = 3[\text{Nms/rad}]$, $a_1 = 0.1[\text{Nms/rad}]$, $a_2 = 0.1[\text{Nms/rad}]$, $a_3 = 1.0[\text{Nms/rad}]$, $a_4 = 0.5[\text{Nms/rad}]$, $\gamma_{k1} = 25$, $\gamma_{k2} = 10$, $\gamma_{k3} = 5$, $\gamma_{k4} = 3$, $\gamma_b = 0.2$.

We set the initial desired motion not so carefully as stated above, bad initial angle $q(0)$ resulted in the falling. Therefore, we tuned the initial angle $q(0)$ not to cause the falling phenomena. We could found suitable initial angle easily $q_1(0) = q_{e1} - 0.3[\text{rad}]$, $q_2(0) = q_{e2}[\text{rad}]$, $q_3(0) = q_{e3}[\text{rad}]$, $q_4(0) = q_{e4}[\text{rad}]$. The initial angular velocity $\dot{q}(0)$ was set to 0.

Totally, the parameter tuning processes required a period of time. However, we also confirmed there were some certain regions of the parameter combinations, which can generate stable walking motions. Additionally, to use appropriate motion planning methods and dynamic balance controllers seems to improve the proposed controller, and we will combine such methods with the proposed method in the future.
**E. PD Controller**

To confirm the energy saving effect, we conducted a simulation using only PD control \( u = -K_p \Delta q - K_v \Delta \dot{q} \), and compared the amount of the actuator torque. We set the same desired motion and the same feedback gains \( K_p, K_v \) as our proposed controller.

**F. Result**

Simulation results of tuned parameters in the case of proposed controller are shown in Fig.4. Results in the case of PD controller are shown in Fig.7. The motion was adjusted by the proposed controller as shown in Fig.4(a), (b), (c), (d), and the motion frequency was not so changed compared with PD controller as shown in Fig.7(a), (b), (c), (d). The ankle trajectories in 5 seconds (395 < \( t < 400 \)) of the proposed controller, 35 < \( t < 40 \) of the PD controller) are shown in Fig.5 and Fig.8, and these simulation results satisfied the condition of the ankle position. The parameter \( k \) and stiffness converged to almost constants as shown in Fig.4(e), (f). The actuator torque \( \tau_1, \tau_2, \tau_3 \) of the proposed controller became smaller than that of the PD controller as shown in Fig.4(g), (h), (i), Fig.7(e), (f), (g). The amount of the actuator torque \( \tau_1 \) was not so changed by the proposed controller as shown in Fig.4(j) and Fig.7(h). As show in Fig.6(a), (b), (c), (d), the actuator torque \( \tau \) and the angular velocity \( \dot{q} \) had almost linear relationship. Therefore, the simulation results satisfied the condition the extended resonanc of the equation (8), and we could confirm that the proposed controller can generate energy saving walking motions. In fact, the norm of the actuator torque in 5 seconds \( \int_{395}^{400} \tau^T \tau dt \) in the case of the proposed controller was more than 90% smaller than that in the case of PD controller \( \int_{395}^{403} \tau^T \tau dt \).

**VII. CONCLUSION**

This paper have applied an energy saving control method to robotic dynamics that includes some important dynamics of biped robots as a first step to realize energy saving biped robots. This control method was proposed by the authors [7], [9], [10], and is based on extended resonance. We discussed relations between the proposed control method and dynamics of biped robots, such as impact dynamics and a condition of falling down. After some parameter tuning processes, we could generate energy saving motions by using a simulation. The generated motion almost satisfied an optimal condition of extended resonance, and could reduce actuator torque more than 90% compared with an usual PD controller. At the same time, we discussed some problem to be improved for more robust walking. In the future, we will improve the control method by using some motion planning methods and some dynamic balancing methods. To consider more realistic conditions of biped dynamics is also our important future work.

**REFERENCES**


