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Adaptive Dynamic Coupling Control of Human-Symbiotic Wheeled Mobile Manipulators with Hybrid Joints

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Abstract—**In this paper, adaptive dynamic coupling control is considered for hybrid joint, which could be switched to either active (actuated) or passive (under-actuated) mode, for human-symbiotic wheeled mobile manipulators. Based on Lyapunov synthesis, adaptive coupling control using physical properties of wheeled mobile manipulators proposed for passive hybrid joints ensures that the system outputs track the given bounded reference signals within a small neighborhood of zero, and guarantees semi-global uniform boundedness of all closed loop signals. The effectiveness of the proposed controls is verified through extensive simulations.**

I. INTRODUCTION

Today, robots are expected to provide various services directly to humans in environments, this situation has led to the idea of teams consisting of humans and robots working cooperatively on the same task [1]. Various names for this type of human-robot cooperation system have emerged including human-friendly robots, personal robots, assistant robots and symbiotic robots. These robots will continue to be employed also in the 21st century to cope with the increase in the elder and handicapped, the decrease in the birth rate and working population and will be introduced into non-industrial areas such as homes and offices to make a rich and comfortable life. The robots, therefore, must be with the capability of human-robot coexistence. They can be called "social robots".

Working and moving among humans requires special concerns on the mechanical compliance. A social robot should weigh not significantly more than a human, but mechanical compliance of the surface and joints is also a necessity. The most past passive compliance methods were based on the robot's structural compliance using special mechanical devices such as springs and dampers. By the passive compliance methods, the robot hardware could achieve more reliable compliance compared with the active compliance approaches. Therefore, in our previous work [4], a novelty compliant passive mechanism – the hybrid joint was proposed for mobile manipulators, which is different from the traditional spring-damper system. The hybrid joint has one clutch, when the clutch is released, the link is free, and the passive link is directly controlled by the coupling characteristics of the manipulator dynamics, as shown in Fig.1. In [4], the switching logic of the hybrid joints capable of compliantly adapting to human's motion and force was realized by switching the hybrid joints to the active mode or passive mode as needed. The operational modes of the hybrid joints need to be changed depending on the requirement of a given task. For the human-robot cooperation, i. e. , [12], a robot helps human to carry a big or long object, which is demanded in home, office and welfare site, etc., as well as factory. However, the internal force of the carried object is inevitably produced, which would damage the human collaborator. If the hybrid joint, especially in the passive mode, is introduced, which would definitely decrease the internal force and secure the human safety. On the other hand, the hybrid joint in the actuated mode could make full use of the advantage of full-actuated robots.

The hybrid joint in the under-actuated mode, being released with the actuators [4], is a typical example of the second-order nonholonomic system as [3], [10], which can rotate freely and can be indirectly driven by the effect of the dynamic coupling between the active and passive joints. The coordination of multi-manipulators using passive joints was proposed in [5] to decrease the undesired internal forces. Since the coupling between the actuated and the passive joints depends on the dynamic parameters, and is subject to errors if there are uncertainties on the values of these parameters, as in [3], [5], it is seldom found how to handle the situations in the presence of the unmodelled dynamics and external unknown disturbance. Based on the previous works [4], [2], in this paper, the dynamics uncertainty has been considered, by developing adaptive motion control for two-wheeled driven mobile manipulator with one hybrid joint, we attempt to utilize the dynamic coupling to control the passive hybrid joint with unknown modeling errors and external disturbances.

II. SYSTEM DESCRIPTION

Lemma 2.1: [6] Let $e = H(s)r$ with $H(s)$ representing an $(n \times m)$-dimensional strictly proper exponentially stable transfer function, r and e denoting its input and output, respectively. Then $r \in L_2^m \bigcap L_\infty^m$ implies that $e, \dot{e} \in L_2^n \bigcap L_\infty^n$, e is continuous, and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$, then $\dot{e} \to 0$.

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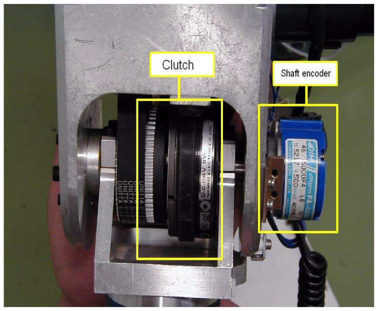

Fig. 1. The hybrid joint

A. Dynamics of Wheeled Mobile Manipulators with Hybrid Joints

Consider an n DOF fixed manipulator mounted on a two-wheeled driven mobile platform, the dynamics can be described as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \quad (1)$$

where $q = [q_v^T, q_a^T, q_h^T]^T \in R^n$ with $q_v = [x, y, \theta]^T \in R^{n_v}$ denoting the generalized coordinates for the mobile platform and $q_a \in R^{n_a}$ denoting the coordinates of the active joints, and $q_h \in R^{n_p}$ denoting the coordinates of the hybrid joints, in this paper, we focus on $n_h = 1$, and $n = n_v + n_a + n_h$. The symmetric positive definite inertia matrix $M(q) \in R^{n\times n}$, the Centripetal and Coriolis torques $V(\dot{q}, q) \in R^{n\times n}$, the gravitational torque vector $G(q) \in R^n$, the external disturbances $d(t) \in R^n$, the known input transformation matrix $B(q) \in R^{n\times m}$, the control inputs $\tau \in R^m$ and the generalized constraint forces $f \in R^n$ could be represented as, respectively $M(q) = \begin{bmatrix} M_v & M_{va} & M_{vh} \\ M_{av} & M_a & M_{ah} \\ M_{hv} & M_{ha} & M_h \end{bmatrix}$, $V(q,\dot{q}) = \begin{bmatrix} V_v & V_{va} & V_{vh} \\ V_{av} & V_a & V_{ah} \\ V_{hv} & V_{ha} & V_h \end{bmatrix}$, $f = \begin{bmatrix} J_v^T\lambda_n \\ 0 \\ 0 \end{bmatrix}$, $G(q) = \begin{bmatrix} G_v \\ G_a \\ G_h \end{bmatrix}$, $d(t) = \begin{bmatrix} d_v \\ d_a \\ d_h \end{bmatrix}$, $B(q)\tau = \begin{bmatrix} \tau_v \\ \tau_a \\ \tau_h \end{bmatrix}$, $J_v \in R^{l\times n_v}$ is the kinematic constraint matrix related to nonholonomic constraints; $\lambda_n \in R^l$ is the associated Lagrangian multipliers with the generalized nonholonomic constraints. We assume that the mobile manipulator is subject to known nonholonomic constraints. In actual implementation, we can adopt the methods of producing enough friction between the wheels of the mobile platform and the ground such that this assumption holds.

B. Reduced System

The vehicle subject to nonholonomic constraints can be expressed as

$$J_v\dot{q}_v = 0 \quad (2)$$

The effect of the constraints can be viewed as a restriction of the dynamics on the manifold Ω_n as $\Omega_n = \{(q_v, \dot{q}_v) | J_v\dot{q}_v = 0\}$.

Assume that the annihilator of the co-distribution spanned by the covector fields $J_{v_1}^T(q_v), \ \ldots, \ J_{v_l}^T(q_v)$ is an $(n_v - l)$-dimensional smooth nonsingular distribution Δ on R^{n_v}. This distribution Δ is spanned by a set of $(n_v - l)$ smooth and linearly independent vector fields $H_1(q_v), \ \ldots, \ H_{n_v-l}(q_v)$, i.e., $\Delta = \text{span}\{H_1(q_v), \ \ldots, \ H_{n_v-l}(q_v)\}$, which satisfy, in local coordinates, the following relation [7] $H^T(q_v)J_v^T(q_v) = 0$, where $H(q_v) = [H_1(q_v), \ \ldots, \ H_{n_v-l}(q_v)] \in R^{n_v\times(n_v-l)}$. Note that H^TH is of full rank. The constraint (2) implies the existence of vector $\dot{\eta} \in R^{n_v-l}$, such that

$$\dot{q}_v = H(q_v)\dot{\eta} \quad (3)$$

Considering (3) and its derivative, the dynamics of mobile manipulator can be expressed as

$$\mathcal{M}(\zeta)\ddot{\zeta} + \mathcal{V}(\zeta,\dot{\zeta})\dot{\zeta} + \mathcal{G}(\zeta) + \mathcal{D}(t) = \mathcal{U} \quad (4)$$

where $\mathcal{M}(\zeta) = \begin{bmatrix} H^TM_vH & H^TM_{va} & H^TM_{vh} \\ M_{av}H & M_a & M_{ah} \\ M_{hv}H & M_{ha} & M_h \end{bmatrix}$, $\zeta = \begin{bmatrix} \eta \\ q_a \\ q_h \end{bmatrix}$, $\mathcal{G}(\zeta) = \begin{bmatrix} H^TG_v \\ G_a \\ G_h \end{bmatrix}$, $\mathcal{D}(t) = \begin{bmatrix} H^Td_v \\ d_a \\ d_h \end{bmatrix}$, $\mathcal{V}(\zeta,\dot{\zeta}) = \begin{bmatrix} H^TM_v\dot{H} + H^TV_vH & H^TV_{va} & H^TV_{vh} \\ M_{av}\dot{H} + V_{av}H & V_a & V_{ah} \\ M_{hv}\dot{H} + V_{hv}H & V_{ha} & V_h \end{bmatrix}$, $\mathcal{U} = \begin{bmatrix} \tau_v^TH & \tau_a^T & \tau_h \end{bmatrix}^T$.

Remark 2.1: In this paper, we choose $\dot{\zeta} = [\omega, v, \dot{q}_a^T, \dot{q}_h]^T$, and $\dot{\eta} = [\omega, v]^T$, where v is the forward velocity of the mobile platform; and ω is the rotation velocity of the mobile platform.

Considering the property of the above mechanical system, we list the following properties [8] for the active hybrid joints:

Property 2.1: The inertia matrix $\mathcal{M}(\zeta)$ is symmetric and positive definite.

Property 2.2: The matrix $\dot{\mathcal{M}} - 2\mathcal{V}$ is skew-symmetric.

C. Physical Properties

When the hybrid joints are switched to the active mode, we partition ζ into $\dot{\zeta}_1 = \omega \in R$, $\dot{\zeta}_2 = [v, \dot{q}_a^T]^T$ and $\zeta_3 = q_h \in R$, according to the above partitions, corresponding to the definition of (4), we can rewrite the structure of the dynamics of mobile actuated manipulators as:

$$\begin{aligned} \mathcal{M}(\zeta) &= \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \\ \mathcal{V}(\zeta,\dot{\zeta})\dot{\zeta} &= \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_{11}\dot{\zeta}_1 + V_{12}\dot{\zeta}_2 + V_{13}\dot{\zeta}_3 \\ V_{21}\dot{\zeta}_1 + V_{22}\dot{\zeta}_2 + V_{23}\dot{\zeta}_3 \\ V_{31}\dot{\zeta}_1 + V_{32}\dot{\zeta}_2 + V_{33}\dot{\zeta}_3 \end{bmatrix} \\ \mathcal{G} &= \begin{bmatrix} G_1 & G_2^T & G_3 \end{bmatrix}^T \\ \mathcal{D} &= \begin{bmatrix} d_1 & d_2^T & d_3 \end{bmatrix}^T \\ \mathcal{U} &= \begin{bmatrix} u_1 & u_2^T & u_3 \end{bmatrix}^T \end{aligned} \quad (5)$$

Following [10], for the control design of mobile manipulators with hybrid joint, where $u_3 = 0$, it is easy to obtain $n_v + n_a - l > n_h$. It is apparent that even if $n_a = 0$, the above is also achieved. In order to make ζ_3 controllable, especially in the passive mode, we assume that matrices M_{13} and M_{31} are not equal to zero and M_{11}^{-1} exists. However, if M_{13} and M_{31} are equal to zero, while M_{12} and M_{21} are not equal to zero, which means that ζ_3 will be coupled with one vector of ζ_2, we only need to exchange ζ_1 with the vector of ζ_2. In this paper, we focus on $M_{13} = M_{31} \neq 0$. After some simple manipulations, we can obtain three dynamics as

$$M_{11}\ddot{\zeta}_1 = u_1 - V_1 - G_1 - d_1 - M_{12}\ddot{\zeta}_2 - M_{13}\ddot{\zeta}_3 \quad (6)$$

$$(M_{22} - M_{21}M_{11}^{-1}M_{12})\ddot{\zeta}_2 + (M_{23} - M_{21}M_{11}^{-1}M_{13})\ddot{\zeta}_3 + V_2 + G_2 + d_2 - M_{21}M_{11}^{-1}V_1 - M_{21}M_{11}^{-1}G_1 - M_{21}M_{11}^{-1}d_1 = u_2 - M_{21}M_{11}^{-1}u_1 \quad (7)$$

$$(M_{32} - M_{31}M_{11}^{-1}M_{12})\ddot{\zeta}_2 + (M_{33} - M_{31}M_{11}^{-1}M_{13})\ddot{\zeta}_3 + V_3 + G_3 + d_3 - M_{31}M_{11}^{-1}V_1 - M_{31}M_{11}^{-1}G_1 - M_{31}M_{11}^{-1}d_1 = -M_{31}M_{11}^{-1}u_1 \quad (8)$$

Let $\mathcal{A} = M_{22} - M_{21}M_{11}^{-1}M_{12}$, $\mathcal{B} = M_{23} - M_{21}M_{11}^{-1}M_{13}$, $\mathcal{C} = M_{32} - M_{31}M_{11}^{-1}M_{12}$, $\mathcal{D} = M_{33} - M_{31}M_{11}^{-1}M_{13}$, $\mathcal{E} = (V_{22} - M_{21}M_{11}^{-1}V_{12})\dot{\zeta}_2 + (V_{23} - M_{21}M_{11}^{-1}V_{13})\dot{\zeta}_3$, $\mathcal{F} = (V_{32} - M_{31}M_{11}^{-1}V_{12})\dot{\zeta}_2 + (V_{33} - M_{31}M_{11}^{-1}V_{13})\dot{\zeta}_3$, $\mathcal{H} = (V_{21} - M_{21}M_{11}^{-1}V_{11})\dot{\zeta}_1 + G_2 + d_2 - M_{21}M_{11}^{-1}G_1 - M_{21}M_{11}^{-1}d_1$, $\mathcal{K} = (V_{31} - M_{31}M_{11}^{-1}V_{11})\dot{\zeta}_1 + G_3 + d_3 - M_{31}M_{11}^{-1}G_1 - M_{31}M_{11}^{-1}d_1$. Then, we can rewrite (6), (7) and (8) as

$$M_{11}\ddot{\zeta}_1 = u_1 - V_1 - G_1 - d_1 - M_{12}\ddot{\zeta}_2 - M_{13}\ddot{\zeta}_3 \quad (9)$$

$$\mathcal{A}\ddot{\zeta}_2 + \mathcal{B}\ddot{\zeta}_3 + \mathcal{E} + \mathcal{H} = -M_{21}M_{11}^{-1}u_1 + u_2 \quad (10)$$

$$\mathcal{C}\ddot{\zeta}_2 + \mathcal{D}\ddot{\zeta}_3 + \mathcal{F} + \mathcal{K} = -M_{31}M_{11}^{-1}u_1 \quad (11)$$

Let $\xi = [\zeta_3^T, \zeta_2^T]^T$, considering (4) and (5), the equations (10) and (11) become

$$\mathcal{M}_1(\zeta)\ddot{\xi} + \mathcal{V}_1(\zeta,\dot{\zeta})\dot{\xi} + \mathcal{D}_1 = \mathcal{B}_1\mathcal{U}_1 \quad (12)$$

where $\mathcal{M}_1(\zeta) = \begin{bmatrix} \mathcal{D} & \mathcal{C} \\ \mathcal{B} & \mathcal{A} \end{bmatrix}$, $\mathcal{D}_1 = \begin{bmatrix} \mathcal{K} \\ \mathcal{H} \end{bmatrix}$, $\mathcal{B}_1 = \begin{bmatrix} M_{31}M_{11}^{-1} & 0 \\ M_{21}M_{11}^{-1} & I \end{bmatrix}$, $\mathcal{U}_1 = \begin{bmatrix} -u_1 \\ u_2 \end{bmatrix}$, $\mathcal{V}_1(\zeta,\dot{\zeta}) = \begin{bmatrix} V_{33} - M_{31}M_{11}^{-1}V_{13} & V_{32} - M_{31}M_{11}^{-1}V_{12} \\ V_{23} - M_{21}M_{11}^{-1}V_{13} & V_{22} - M_{21}M_{11}^{-1}V_{12} \end{bmatrix}$.

Decompose $\mathcal{V}_1(\zeta,\dot{\zeta}) = \hat{\mathcal{V}}_1 + \tilde{\mathcal{V}}_1$ such that

$$\dot{\mathcal{M}}_1 - 2\tilde{\mathcal{V}}_1 = 0 \quad (13)$$

Property 2.3: The inertia matrix $\mathcal{M}_1$ is symmetric and positive definite.

Remark 2.2: Since $v, \omega, \dot{q}_h \in R$, $M_{11}, M_{31} \in R$.

Property 2.4: The eigenvalues of the inertia matrix $\mathcal{B}_1$ are positive.

Remark 2.3: There exist the minimum and maximum eigenvalues $\lambda_{min}(\mathcal{B}_1)$ and $\lambda_{max}(\mathcal{B}_1)$, such that $\forall x \in R^{n-n_h}$, $x^T\lambda_{min}(\mathcal{B}_1)Ix \leq x^T\mathcal{B}_1x \leq x^T\lambda_{max}(\mathcal{B}_1)Ix$, and the known positive parameter b satisfying $b \leq \lambda_{min}(\mathcal{B}_1)$.

For the hybrid joints, we give the following assumptions for the actuated and passive modes, respectively,

Assumption 2.1: (Actuated Hybrid Joints)[9] The desired trajectories $\zeta_{1d}(t)$, $\zeta_{2d}(t)$, $\zeta_{3d}(t)$ and their time derivatives up to the 3rd order are continuously differentiable and bounded for all $t \geq 0$.

Remark 2.4: Since a lot of works have been done for the full-actuated mobile manipulators, such as [2], therefore, in this paper, we focus on the mobile manipulators with passive hybrid joints. Moreover, we give the following assumption.

Assumption 2.2: For the hybrid joints in the actuated mode, we could adopt the controllers , such as [2], that ensure the tracking errors for the variables ζ_1, ζ_2, ζ_3 from any $(\zeta_j(0), \dot{\zeta}_j(0)) \in \Omega$, where $j = 1, 2, 3$, $\zeta_j, \dot{\zeta}_j$ converge to a manifold Ω_{ad} specified as

$$\Omega_{ad} = \{(\zeta_j, \dot{\zeta}_j)|\ |\zeta_j - \zeta_{jd}| \leq \epsilon_{j1}, |\dot{\zeta}_j - \dot{\zeta}_{jd}| \leq \epsilon_{j2}\} \quad (14)$$

where $\epsilon_{ji} > 0$, $i = 1, 2$. Ideally, ϵ_{ji} should be the threshold of measurable noise. At the same time, all the closed loop signals are to be kept bounded.

Assumption 2.3: (Passive Hybrid Joints)[9] The desired trajectories $\zeta_{2d}(t)$, $\zeta_{3d}(t)$ and their time derivatives up to the 3rd order are continuously differentiable and bounded for all $t \geq 0$.

The control objective for the motion of the system with the unactuated hybrid joint is to design, if possible, controllers that ensure the tracking errors for the variables ζ_2, ζ_3 from any $(\zeta_j(0), \dot{\zeta}_j(0)) \in \Omega$, where $j = 2, 3$, $\zeta_j, \dot{\zeta}_j$ converge to a manifold Ω_{ud} specified as Ω where

$$\Omega_{ud} = \{(\zeta_j, \dot{\zeta}_j)|\ |\zeta_j - \zeta_{jd}| \leq \epsilon_{j1}, |\dot{\zeta}_j - \dot{\zeta}_{jd}| \leq \epsilon_{j2}\} \quad (15)$$

where $\epsilon_{ji} > 0$, $i = 1, 2$, $j = 2, 3$. Ideally, ϵ_{ji} should be the threshold of measurable noise. At the same time, all the closed loop signals are to be kept bounded.

For clarity, define the tracking errors and the filtered tracking errors as $e_j = \zeta_j - \zeta_{jd}$, and $r_j = \dot{e}_j + \Lambda_j e_j$ where Λ_j is positive definite, $j = 2, 3$. Then, based on Lemma 2.1, to study the stability of e_j and $\dot{e}_j$, we only need to study the properties of r_j. In addition, the following computable signals are defined: $\dot{\zeta}_{jr} = \dot{\zeta}_{jd} - \Lambda_j e_j$, $\ddot{\zeta}_{jr} = \ddot{\zeta}_{jd} - \Lambda_j \dot{e}_j$.

III. Adaptive Dynamic Coupling Control

A. ζ_2 and ζ_3-subsystems

Since $\dot{\xi} = \dot{\xi}_r + r$, $\ddot{\xi} = \ddot{\xi}_r + \dot{r}$,the equation (12) become

$$\mathcal{M}_1\dot{r} + \tilde{\mathcal{V}}_1 r = -\mathcal{M}_1\ddot{\xi}_r - \hat{\mathcal{V}}_1 r - \mathcal{V}_1\dot{\xi}_r - \mathcal{D}_1 + \mathcal{B}_1\mathcal{U}_1 \quad (16)$$

where $r = [r_3^T, r_2^T]^T$, $\ddot{\xi}_r = [\ddot{\zeta}_{3r}^T, \ddot{\zeta}_{2r}^T]^T$. Since $\mathcal{V}_1$ is the matrix function of $\dot{\zeta}, \zeta$, and $\mathcal{M}_1$ is the matrix consisting of the submatrices of $\mathcal{M}$, therefore, the following assumption is listed as:

Assumption 3.1: The nominal $\mathcal{B}_{10}$ for $\mathcal{B}_1$ is a known positive definite matrix, satisfying $\mathcal{B}_1 = \mathcal{B}_{10} + \Delta\mathcal{B}$ with an unknown matrix $\Delta\mathcal{B}$.

Assumption 3.2: There exist some finite positive constants $c_i > 0$ $(1 \leq i \leq 8)$ such that $\forall \zeta \in R^{2+n_a+1}$, $\forall \dot{\zeta} \in R^{2+n_a+1}$, $\|\mathcal{M}_1\| \leq c_1$, $\|\mathcal{V}_1\| \leq c_2 + c_3\|\dot{\zeta}\|$, $\|\hat{\mathcal{V}}_1\| \leq c_4 + c_5\|\dot{\zeta}\|$, $\|\mathcal{D}_1\| \leq c_6 + c_7\|\dot{\zeta}\|$, $\|\Delta\mathcal{B}\| \leq c_8$.

Assumption 3.3: There is time varying positive function ϖ which converges to zero as $t \rightarrow \infty$ and satisfies $\lim_{t\rightarrow\infty}\int_0^t \varpi(s)ds = \rho < \infty$ with finite constant ρ.

Consider the following control laws and the adaptive law as

$$\mathcal{U}_1 = -\mathcal{B}_{10}^{-1}K_P r - \frac{1}{b}\sum_{i=1}^{8}\frac{r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i} \tag{17}$$

$$\dot{\hat{c}}_i = -\sigma_i\hat{c}_i + \frac{\gamma_i\Psi_i^2\|r\|^2}{\|r\|\Psi_i+\delta_i} \tag{18}$$

where K_P is positive definite, $\gamma_i > 0$ and $\delta > 0$ and $\sigma_i > 0$, $1 \le i \le 8$, satisfying Assumption 3.3: $\int_0^\infty \delta_i(s)ds = \rho_{i\delta} < \infty$, $\int_0^\infty \sigma_i(s)ds = \rho_{i\sigma} < \infty$ with the constants $\rho_{i\delta}$ and $\rho_{i\sigma}$. Let $\hat{C} = [\hat{c}_1, \ \ldots, \ \hat{c}_8]^T$ and $\Psi = [\|\ddot{\xi}_r\|, \|\dot{\xi}_r\|, \|\dot{\zeta}\|\|\dot{\xi}_r\|, \|r\|, \|r\|\|\dot{\zeta}\|, 1, \|\dot{\zeta}\|, \|\mathcal{B}_{10}^{-1}K_P r\|]^T$, and $\Phi = C^T\Psi$.

To analyze closed loop stability for the ζ_2 and ζ_3-subsystem, consider the following Lyapunov function candidate

$$\mathbb{V}_1 = \frac{1}{2}r^T\mathcal{M}_1 r + \frac{1}{2}\tilde{C}^T\Gamma^{-1}\tilde{C} \tag{19}$$

where $\Gamma = \mathrm{diag}[\gamma_1, \ \ldots, \gamma_8]$, and $\tilde{C} = C - \hat{C}$. Its time derivative is given by

$$\dot{\mathbb{V}}_1 = r^T(\frac{1}{2}\dot{\mathcal{M}}_1 r + \mathcal{M}_1\dot{r}) + \tilde{C}^T\Gamma^{-1}\dot{\tilde{C}} \tag{20}$$

Considering (13)and Property 2.4, and substituting (16) into (20), and integrating (17), we have

$$\begin{aligned}
\dot{\mathbb{V}}_1 &= r^T(\mathcal{B}_1\mathcal{U} - \mathcal{M}_1\ddot{\xi}_r - \hat{\mathcal{V}}_1 r - \mathcal{V}_1\dot{\xi}_r - \mathcal{D}_1) + \tilde{C}^T\Gamma^{-1}\dot{\tilde{C}} \\
&= r^T[(\mathcal{B}_1 - \mathcal{B}_{10})(-\mathcal{B}_{10}^{-1}K_P r) - \frac{1}{b}\mathcal{B}_1\sum_{i=1}^{8}\frac{r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i} \\
&\quad +\mathcal{B}_{10}\mathcal{B}_{10}^{-1}(-K_P r) - \mathcal{M}_1\ddot{\xi}_r - \hat{\mathcal{V}}_1 r - \mathcal{V}_1\dot{\xi}_r - \mathcal{D}_1] \\
&\quad +\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}} \\
&\le -r^T K_P r - \sum_{i=1}^{8}\frac{r^T r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i} + \|r\|\|\mathcal{B}_1 \\
&\quad -\mathcal{B}_{10}\|\|\mathcal{B}_{10}^{-1}K_P r\| + \|r\|\|\mathcal{M}_1\|\|\ddot{\xi}_r\| \\
&\quad +\|r\|\|\mathcal{V}_1\|\|\dot{\xi}_r\| + \|\hat{\mathcal{V}}_1\|\|r\|^2 + \|r\|\|\mathcal{D}_1\| \\
&\quad +\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}} \\
&\le -r^T K_P r + \|r\|\Phi - \sum_{i=1}^{8}\frac{\hat{c}_i\Psi_i^2\|r\|^2}{\|r\|\Psi_i+\delta_i} \\
&\quad +\hat{C}^T\Sigma\Gamma^{-1}\tilde{C} - \sum_{i=1}^{8}\frac{\|r\|^2\tilde{c}_i\Psi_i^2}{\|r\|\Psi_i+\delta_i} \\
&\le -r^T K_P r + C^T\Delta + \hat{C}^T\Sigma\Gamma^{-1}(C - \hat{C}) \\
&\le -r^T K_P r + C^T\Delta + \frac{1}{4}C^T\Sigma\Gamma^{-1}C
\end{aligned} \tag{21}$$

with $\Sigma = \mathrm{diag}[\sigma_1, \ \ldots, \sigma_8]$, $\Delta = [\delta_1, \ \ldots, \delta_8]^T$. Therefore, $\dot{\mathbb{V}}_1 \le -\lambda_{min}(K_P)\|r\|^2 + C^T\Delta + \frac{1}{4}C^T\Sigma\Gamma^{-1}C$. Since $C^T\Delta + \frac{1}{4}C^T\Sigma\Gamma^{-1}C$ is bounded, there exists $t > t_1$, $C^T\Delta + \frac{1}{4}C^T\Sigma\Gamma^{-1}C \le \rho_1$ with the finite constant ρ_1, when $\|r\| \ge \sqrt{\frac{\rho_1}{\lambda_{min}(K_P)}}$, then $\dot{\mathbb{V}}_1 \le 0$, from above all, r converges to a small set Ω_1 containing the origin as $t \to \infty$,

$$\Omega_1 : \|r\| \le \sqrt{\frac{\rho_1}{\lambda_{min}(K_P)}} \tag{22}$$

Integrating both sides of the above equation gives

$$\begin{aligned}
\mathbb{V}_1(t) - \mathbb{V}_1(0) \le& -\int_0^t \lambda_{min}(K_P)\|r\|^2 ds \\
&+ \int_0^t (C^T\Delta + \frac{1}{4}C^T\Sigma\Gamma^{-1}C)ds
\end{aligned} \tag{23}$$

Since C and Γ are constant, $\int_0^\infty \Delta ds = \rho_\delta = [\rho_{1\delta}, \ \ldots, \ \rho_{8\delta}]^T$, $\int_0^\infty \Sigma ds = \rho_\sigma = \mathrm{diag}[\rho_{1\sigma}, \ \ldots, \rho_{8\sigma}]$, we can rewrite (23) as $\mathbb{V}_1(t) - \mathbb{V}_1(0) \le -\int_0^t \lambda_{min}(K_P)\|r\|^2 ds + C^T\rho_\delta + \frac{1}{4}C^T\rho_\sigma\Gamma^{-1}C < \infty$. Thus $\mathbb{V}_1$ is bounded, which implies that $r \in L_\infty$. From (23), we have $\int_0^t \lambda_{min}(K_P)\|r\|^2 ds \le \mathbb{V}_1(0) - \mathbb{V}_1(t) + C^T\rho_\delta + \frac{1}{4}C^T\rho_\sigma\Gamma^{-1}C$, which leads to $r \in L_2$. From $r_j = \dot{e}_j + \Lambda_j e_j$, it can be obtained that $e_j, \dot{e}_j \in L_\infty$. As we have established $e_j, \dot{e}_j \in L_\infty$, from Assumption 2.3, we conclude that $\zeta_j, \dot{\zeta}_j, \dot{\xi}_r, \ddot{\xi}_r \in L_\infty$. Therefore, all the signals on the right hand side of (16) are bounded, and we can conclude that $\dot{r}$ and therefore $\ddot{\zeta}_j$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_j \to 0, \dot{e}_j \to 0$ as $t \to \infty$. Since r, ζ_j, $\dot{\zeta}_j$, ζ_{jr}, $\dot{\zeta}_{jr}$, $\ddot{\zeta}_{jr}$ are all bounded it is easy to conclude that $\mathcal{U}$ is bounded from (17).

All the signals on the left hand side of (16) are bounded, therefore, $\mathcal{H}$ and $\mathcal{K}$ are also bounded, since $\mathcal{H}$ and $\mathcal{K}$ contain $\dot{\zeta}_1$, we can obtain $\dot{\zeta}_1$ is bounded.

B. ζ_1-subsystem

Finally, for system (6)–(8) under control laws (17), apparently, the ζ_1-subsystem (6) can be rewritten as

$$\dot{\varphi} = f(\nu, \varphi, \mathcal{U}) \tag{24}$$

where $\varphi = [\zeta_1^T, \dot{\zeta}_1^T]^T$, $\nu = [r^T, \dot{r}^T]^T$, $\mathcal{U} = [u_1, u_2^T]^T$. From ζ_2 and ζ_3 subsystem and their stability, the zero dynamics of ζ_1-subsystem can be addressed as [11]

$$\dot{\varphi} = f(0, \varphi, \mathcal{U}^*(0, \varphi)) \tag{25}$$

where $\mathcal{U}^*$ is the input vector at $\nu = 0$.

Assumption 3.4: [11] System (6), (7) and (8) is hyperbolically minimum-phase, i.e. zero dynamics (25) is exponentially stable. In addition, assume that the control input $\mathcal{U}$ is designed as a function of the states (ξ, φ) and the reference signal satisfying Assumption 2.3, and the function $f(\nu, \varphi, \mathcal{U})$ is Lipschitz in ν, i.e., there exists Lipschitz constants L_ν and L_f for $f(\nu, \varphi, \mathcal{U})$ such that

$$\|f(\nu, \varphi, \mathcal{U}) - f(0, \varphi, \mathcal{U}_\varphi)\| \le L_\nu\|\nu\| + L_f \tag{26}$$

where $\mathcal{U}_\varphi = \mathcal{U}^*(0, \varphi)$.

Lemma 3.1: [11] For the internal dynamics $\dot{\varphi} = f(\nu, \varphi, \mathcal{U})$ of the system, if Assumptions 2.3 and 3.2 are satisfied, then there exist positive constants L_φ and T_0, such that

$$\|\varphi(t)\| \le L_\varphi, \quad \forall t > T_0 \tag{27}$$

Theorem 3.1: Consider the system (6–8) with Assumptions 2.3 and 3.2, under the action of control laws (17) and adaptation laws (18). For compact set Ω_1, where $(\zeta(0)$, $\dot{\zeta}(0)$, $\hat{C}(0))$ $\in \Omega$, the tracking error r converges to the compact set Ω_1

defined by (22), and all the signals in the closed loop system are bounded.

Proof: From the results (23), it is clear that the tracking errors r_j converges to the compact set Ω_1 defined by (22). In addition, the signal $\tilde{C}$ is bounded. From Lemma 2.1, we can know e_2, $\dot{e}_2$, e_3, $\dot{e}_3$ are also bounded. From the boundedness of ζ_{2d}, ζ_{3d} in Assumption 2.3, we know that ζ_2, ζ_3 are bounded. Since $\dot{\zeta}_{2d}, \dot{\zeta}_{3d}$ are also bounded, it follows that $\dot{\zeta}_2, \dot{\zeta}_3$ are bounded. With C constant, we know that $\hat{C}$ is also bounded. From Lemma 3.1, we know that the ζ_1-subsystem (6) is stable, and $\zeta_1, \dot{\zeta}_1$ are bounded. This completes the proof.

C. Switching Stability

For the system switching stability between the actuated and passive mode, the following theorem is given as

Theorem 3.2: Consider the system (4) with the actuated mode (5) and the under-actuated mode (6–8), if the system is both stable before and after the switching phase using the Assumption 2.2 and (17), and assume that there exists no external impact during the switching, the system is also stable during the switching phase.

Proof: : Since $\mathbb{V}_1$ is bounded from (23), and from Theorem 3.1, we know the system is stable if the hybrid joint is passive. From Assumption 2.2, we know the system is stable for the active hybrid joints. Let $\mathbb{V}^-$ and $\mathbb{V}^+$ denote the Lyapunov function before and after the switching, and $\dot{\zeta}^+$ and $\dot{\zeta}^-$ represent the post- and pre-switch velocities, respectively. The Lyapunov function change during the switching can be simplified as $\Delta\mathbb{V} = \mathbb{V}^+ - \mathbb{V}^- = \frac{1}{2}(\dot{\zeta}^+ - \dot{\zeta})\mathcal{M}(\dot{\zeta}^+ - \dot{\zeta}) - \frac{1}{2}(\dot{\zeta}^- - \dot{\zeta})\mathcal{M}(\dot{\zeta}^- - \dot{\zeta})$. Since there is no external impact during the switching, which means that there is no extra energy injected into the system. Since the inertia of the hybrid joint and link exists, during the hybrid joint switching, if the hybrid joint is switched from the active mode to the passive mode, if without considering the friction, the motion of the link should be continuous, that is, $\dot{\zeta}^+ = \dot{\zeta}^- = \dot{\zeta}$, therefore, during the switching, the Lyapunov function is non-increasing, if considering the friction, the Lyapunov function is decreasing, that is, $\Delta\mathbb{V} \leq 0$, the motion is stable during the switching. Similarly, if the hybrid joint is switched from the passive mode to the active mode, although the joint torque is added, since the motion of the system is continuous because of the inertia, that is $\Delta\mathbb{V} \leq 0$, the motion of the system is also stable. ■

IV. Simulation

The following variables have been chosen to describe the vehicle (see also Fig. 2): τ_l, τ_r: the torques of two wheels; τ_1: the torques of the under-actuated joint, that is, $\tau_1 = 0$; θ_l, θ_r: the rotation angle of the left wheel and the right wheel of the mobile platform; v: the forward velocity of the mobile platform; θ: the direction angle of the mobile platform; ω: the rotation velocity of the mobile platform, and $\omega = \dot{\theta}$; θ_1: the joint angle of the under-actuated link; m_1, m_2: the mass of links of the manipulator; I_{z1}, I_{z2}: the inertia moment of the link 1 and the link 2; l_1, l_2: the link length of the manipulator; r: the radius of the wheels; $2l$: the distance of the wheels; d:

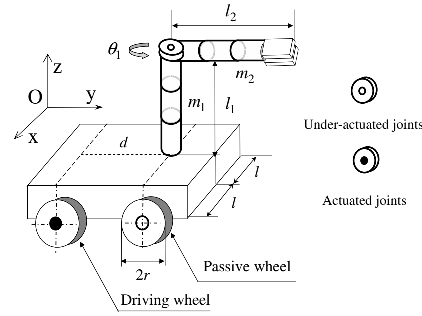

Fig. 2. The mobile under-actuated manipulator in the simulation

the distance between the manipulator and the driving center of the mobile base; m_p: the mass of the mobile platform; I_p: the inertia moment of the mobile platform; I_w: the inertia moment of each wheel; m_w: the mass of each wheel; g: gravity acceleration.

The mobile under-actuated manipulator is subjected to the following constraint: $\dot{x}\cos\theta - \dot{y}\sin\theta = 0$. Using Lagrangian approach, we can obtain the standard form with $q = [\theta_l, \theta_r, \theta_1]^T$, $\dot{\zeta} = [\dot{\zeta}_1, \dot{\zeta}_2, \dot{\zeta}_3]^T = [\dot{\theta}, v, \dot{\theta}_1]^T$, for the page limit, we omit the details of (1). Let $p_0 = \frac{1}{4}(m_p + m_1 + m_2 + 2m_w)r^2 + \frac{1}{4}(I_p + 2Iw + m_1 d^2 + m_2 d^2 + 2m_w l^2)r^2 + (I_{z1} + I_{z2})r^2/4$, $p_1 = m_2 l_2 d r^2/2$, $p_2 = m_2 l_2^2 r^2/4$, $p_3 = m_2 l_2 r^2/2$, $q_0 = (m_p + m_1 + m_2 + 2m_w)r^2/4 - \frac{1}{4}(I_p + 2I_w + m_1 d^2 + m_2 d^2 + 2m_w l^2)r^2 - (I_{z1} + I_{z2})r^2/4$, $q_1 = m_2 l_2 r/2$, $q_2 = m_2 l_2 d r/2$, $q_3 = m_2 l_2 r^2/2$, $q_4 = (I_{z1} + I_{z2})r/2$. In the simulation, we assume the parameters $p_0 = 6.0kg \cdot m^2$, $p_1 = 1.0kg \cdot m^2$, $p_2 = 0.5kg \cdot m^2$, $p_3 = 1.0kg \cdot m^2$, $p_4 = 2.0kg \cdot m^2$, $q_0 = 4.0kg \cdot m^2$, $q_1 = 1.0kg \cdot m^2$, $q_2 = 1.0kg \cdot m^2$, $q_3 = 1.0kg \cdot m^2$, $q_4 = 0.5kg \cdot m^2$, $d = 1.0m$, $r = 0.5m$, $\zeta(0) = [\pi/90, 0.2, 0.0]^T$, $\dot{\zeta}(0) = [0.5, 0.0, -0.5]^T$, The disturbances from environments on the system are introduced as $0.1\sin(t)$, $0.1\sin(t)$ and $0.1\sin(t)$ to the simulation model. The control gains are selected as $K_P = 1.0$, $\hat{C}_(0) = [10.0, \ \ldots, 10.0]^T$, $\delta_i = \sigma_i = 1/(1+t)^2$, $\gamma_{2i} = 2.0$. The desired trajectories are chosen as $\zeta_d = 0.3t$ m, $v_d = 0.3$ m/s, $\theta_{1d} = 0$ rad. The ζ_2 and ζ_3 positions tracking are shown in Figs. 3 and 4, and the corresponding velocities have shown in Fig. 6. From these figures, we know that the ζ_2 and ζ_3 tracking positions have converged to the desired trajectories, and the input torques are shown in Fig. 5, the bounded angular velocity ω is shown in Fig. 7.

V. Conclusion

In this paper, adaptive dynamic coupling control designs are carried out for dynamic balance and tracking of desired trajectories of social mobile robot with passive hybrid joints in the presence of unmodelled dynamics, or parametric/functional uncertainties. Simulation results demonstrate that the system is able to track reference signals satisfactorily, with all closed loop signals uniformly bounded.

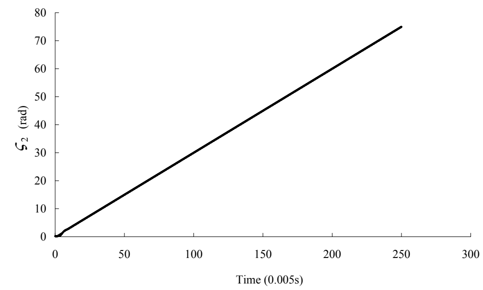

Fig. 3. Tracking the desired position ζ_2

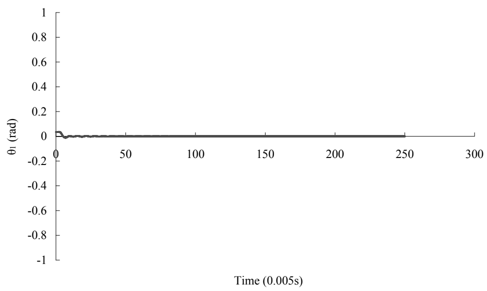

Fig. 4. Tracking the desired position of θ_1

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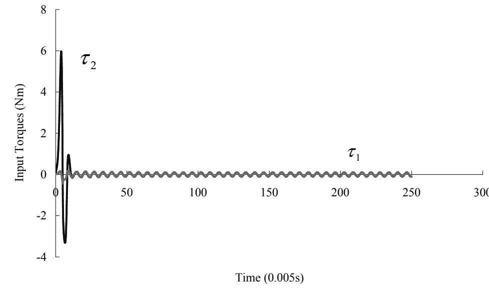

Fig. 5. Input torques

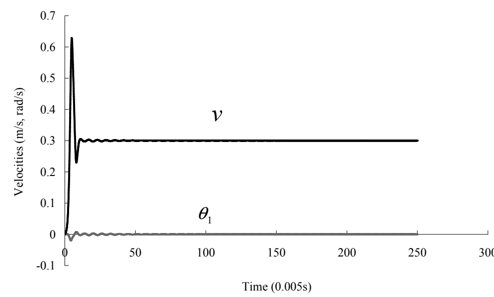

Fig. 6. The produced velocities of v and θ_1

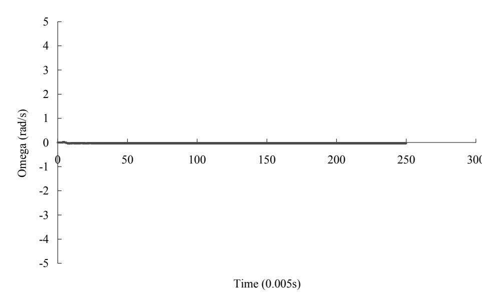

Fig. 7. The bounded ω