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# Sliding Angle Reconstruction and Robust Lateral Control of Autonomous Vehicles in Presence of Lateral Disturbance

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***Abstract*— In this paper the problem of path following control of autonomous vehicles subject to sliding is addressed. First a kinematic model is built which takes sliding effects into account by introducing two additional tire sliding angles. Since the tire sliding angles cannot be directly measured by sensors, an adaptive robust Luenberger observer is designed. With this observer, the tire cornering stiffness instead of the sliding angles is identified in presence of time-varying lateral disturbance. The Lyapunov stability theory guarantees that the estimated cornering stiffness would converge to a neighborhood of the real value when control inputs excitated the system persistently. But due to the existence of the lateral disturbance which causes loss of accuracy of the sliding angle reconstruction, the previously designed anti-sliding controller whose effectiveness completely depends on the estimation of the sliding angles cannot yield satisfactory results. To overcome this problem a tire-oriented kinematic model is built in which the inaccuracy of the sliding angle reconstruction is modeled in form of additive disturbances to the kinematic model. By transforming the tire-oriented kinematic model into a perturbed chained system, a sliding mode controller, which is robust to both the sliding effects and the negative effects of the lateral disturbance is designed with the help of the natural algebraic structure of the chained systems. Simulation results show that the proposed methods can provide accurate estimation of the sliding angles and guarantee high anti-sliding control accuracy even in presence of time-varying lateral disturbance.**

***Index Terms*— Lateral control, sliding control, autonomous vehicles**

## I. Introduction

Automatic steering control has been studied actively, but most of them focus their interests on control law design under the pure rolling condition which is never true for actual applications. Sometimes stability and controllability of the autonomous systems may be violated because of unexpected sliding effects.

Until now there are very few papers dealing with sliding. [1] prevents cars from skidding by robust decoupling of car steering dynamics which is achieved by feedback of the integrated yaw rate into front wheel steering. [2] copes with the control of WMR (Wheeled Mobile Robot) not satisfying the ideal kinematic constraints by using slow manifold methods, but the parameters characterizing the sliding effects are assumed to be exactly known. In [3] a controller is designed based on the averaged model allowing the tracking errors to converge to a limit cycle near the origin. In [4] a general singular perturbation formulation is developed which leads to robust results for linearizing feedback laws ensuring trajectory tracking. But the schemes of [3][4] only take into account sufficiently small sliding effects and they are too complicated for real-time practical implementation. In [5] [6] Variable Structure Control (VSC) is used to eliminate the harmful sliding effects when the bounds of the sliding effects have been known. The trajectory tracking problem of mobile robots in the presence of sliding is solved in [7] by using discrete-time sliding mode control. But the controllers [5]-[7] counteract sliding effects **only** relying on high-gain controllers which is not realistic because of limited bandwidth and low level delay introduced by steering systems. Moreover a robust adaptive controller is designed in [8] which compensates sliding by parameter adaptation and VSC. But the adaptive laws make the controller too complicated to be realized. In the latest research papers [9][10], kinematic-based observers are designed with the concept of classical feedback control theories to estimate the sliding effects. But because only one GPS is available, a numerical derivation is necessary for state vector estimation. In [11] an adaptive observer is designed to estimate the sideslip angle. But all the front/rear-side acceleration and the yaw rate have to be measured.

In this paper body side-acceleration and yaw rate of vehicles are measured which provides necessary measurements for sliding estimation. From these data the cornering stiffness which is only relevant to tire-ground characteristics is identified in presence of lateral disturbances. Because it is rather difficult to identify a time-varying variable especially when it cannot be modeled precisely, the advantage of the proposed scheme is that instead of the time-varying sliding effects, the cornering stiffness which is nearly time-invariant or varies slowly may be identified with high accuracy by using an adaptive robust Luenberger observer.

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Furthermore a sliding mode controller is to be designed based on chained system theory to cope with inaccuracy of sliding angle estimation, leading to satisfactory sliding control results. This paper is organized as follows, in section II the path following problem is described and a kinematic model considering sliding with two sliding angles is constructed. In section III the sliding angles are reconstructed by using adaptive robust observer. In section IV an anti-sliding controller is designed based on chained system theories. In section V some comparative control results are presented to validate the proposed control laws.

## II. KINEMATIC MODEL FOR PATH FOLLOWING CONTROL

In this paper the vehicle is simplified into a bicycle model. The kinematic model is expressed with respect to the path in frame $(M, \eta_t, \eta_n)$, variables necessary in the kinematic model are denoted as follows: (see Figure 1)

- $C$ is the path to be followed.
- $O$ is the center of the vehicle virtual rear wheel.
- $M$ is the orthogonal projection of $O$ on path $C$.
- $\eta_t$ is the tangent vector to the path at $M$.
- $\eta_n$ is the normal vector at $M$.
- $y$ is the lateral deviation between $O$ and $M$
- $s$ is the curvilinear coordinates (arc-length) of point $M$ along the path from an initial position.
- $c(s)$ is the curvature of the path at point $M$.
- $\theta_d(s)$ is the orientation of the tangent to the path at point $M$ with respect to the inertia frame.
- $\theta$ is the orientation of the vehicle centerline with respect to the inertia frame.
- $\tilde{\theta} = \theta - \theta_d$ is the orientation error.
- $l$ is the vehicle wheelbase.
- $v$ is the vehicle longitudinal linear velocity.
- $\delta$ is the steering angle of the virtual front wheel

So the vehicle movement can be described by $(y, s, \tilde{\theta})$. In this paper an anti-sliding control law

$$\delta = K(s, y, \tilde{\theta}, v) \tag{1}$$

will be designed to guarantee

$$\lim_{t \to \infty} y = 0 \tag{2}$$

and $\tilde{\theta}$ is bounded in presence of sliding.

When autonomous vehicles move without sliding, the ideal kinematic model of the vehicles is (see [12] for details).

$$\begin{cases} \dot{s} = \dfrac{v \cos \tilde{\theta}}{1 - c(s)y} \\ \dot{y} = v \sin \tilde{\theta} \\ \dot{\tilde{\theta}} = v\Big(\dfrac{\tan \delta}{l} - \dfrac{c(s) \cos \tilde{\theta}}{1 - c(s)y}\Big) \end{cases} \tag{3}$$

But when autonomous vehicles move on a steep slope or the ground is slippery, tire sliding would occur inevitably, (3) is no longer valid. The violation of the pure rolling constraints is described by introducing the rear sliding angle $\alpha_r$ and the front sliding angle (also called Steering Angle Bias) $\delta_b$ (Figure 2).

$$\delta_b = \delta - \frac{l_f \gamma + v \tan \beta}{v} \tag{4}$$

$$\alpha_r = \frac{-l_r \gamma + v \tan \beta}{v} \tag{5}$$

where $\beta$ is the body sideslip angle of the vehicle and $\gamma$ is the yaw rate at the mass center, $l_f$ $(l_r)$ is the distance between the front (rear) wheel and the mass center.

Similar developing methods lead to a tire-oriented kinematic model with sliding [9]

$$\begin{cases} \dot{s} = \dfrac{v \cos(\tilde{\theta} + \alpha_r)}{1 - c(s)y} \\ \dot{y} = v \sin(\tilde{\theta} + \alpha_r) \\ \dot{\tilde{\theta}} = v\Big[\cos \alpha_r \dfrac{\tan(\delta + \delta_b) - \tan \alpha_r}{l} - \dfrac{c(s) \cos(\tilde{\theta} + \alpha_r)}{1 - c(s)y}\Big] \end{cases} \tag{6}$$

## III. SLIDING ANGLE RECONSTRUCTION

### A. 2DOF lateral vehicle dynamics

The proposed robust adaptive observer is deduced from a 2DOF dynamic model of a vehicle which is simplified into a bicycle. Since it is possible to decouple the longitudinal and lateral dynamics, the following linear model of the lateral vehicle dynamics is used for observer design [13][14].

$$\dot{x} = Ax + Bu + \Psi \xi \tag{7}$$

where

$$x = \begin{bmatrix} v_y & \gamma \end{bmatrix}^T \tag{8}$$

$$A = \begin{bmatrix} -\dfrac{p_1}{mv} & -v + \dfrac{p_2}{mv} \\ \dfrac{p_2}{I_z v} & -\dfrac{p_3}{I_z v} \end{bmatrix} \tag{9}$$

$$B = \begin{bmatrix} \dfrac{k_f}{m} & \dfrac{k_f l_f}{I_z} \end{bmatrix}^T \tag{10}$$

$$\Psi = \begin{bmatrix} \dfrac{1}{m} & -\dfrac{l_d}{I_z} \end{bmatrix}^T \tag{11}$$

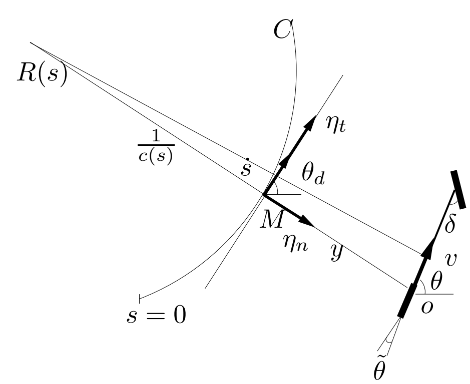


Fig. 1. Notation and path frame description

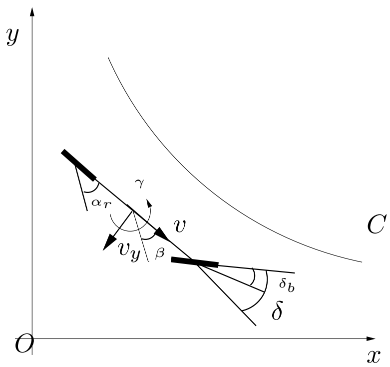


Fig. 2. Notations of sliding effects

$$I_z = m l_r l_f \tag{12}$$
$$p_1 = k_f + k_r \tag{13}$$
$$p_2 = k_r l_r - k_f l_f \tag{14}$$
$$p_3 = k_f l_f^2 + k_r l_r^2 \tag{15}$$

$u$ is the vector of the steering law $\delta$. The state vectors $[v_y, \gamma]$ are the lateral velocity and the yaw rate at the mass center. $\xi$ is the time-varying lateral disturbance, $l_d$ is the distance between the disturbance location to the mass center. $k_f$ ($k_r$) represents the cornering stiffness of the front (rear) tire which is approximately constant or varies slowly.

Therefore by assuming the vehicle moves with a constant velocity, differentiating (7) may yield

$$\dot{X} = AX + BU + \Psi\dot{\xi} \tag{16}$$

where

$$X = \dot{x} = \begin{bmatrix} \dot{v}_y \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} a_g - v\gamma \\ \dot{\gamma} \end{bmatrix} \tag{17}$$
$$U = \dot{u} \tag{18}$$

where $a_g$ is the body side-acceleration.

## B. State Equation of Sideslip Angle

Remark that $a_g$ can be written as

$$a_g = \begin{bmatrix} v & 0 & v \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \\ \dot{\theta} \end{bmatrix} = T\dot{\tilde{x}} \tag{19}$$

Furthermore the following equation describing the sliding dynamic relationship between the sideslip angle $\beta$ and the yaw rate $\gamma$ is considered

$$\dot{\tilde{x}} = A_1\tilde{x} + B_1 u + \Psi_1 \xi \tag{20}$$

where

$$\tilde{x} = \begin{bmatrix} \beta & \gamma & \theta \end{bmatrix}^T \tag{21}$$

$$A_1 = \begin{bmatrix} -\frac{p_1}{mv} & -1 + \frac{p_2}{mv^2} & 0 \\ \frac{p_2}{I_z} & -\frac{p_3}{I_z v} & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{22}$$

$$B_1 = \begin{bmatrix} \frac{k_f}{mv} & \frac{k_f l_f}{I_z} & 0 \end{bmatrix}^T \tag{23}$$

$$\Psi_1 = \begin{bmatrix} \frac{1}{mv} & -\frac{l_d}{I_z} & 0 \end{bmatrix}^T \tag{24}$$

Substitute (20) into (19), it can be obtained that

$$a_g = TA_1\tilde{x} + TB_1 u + T\Psi_1\xi \tag{25}$$

Hence the expression of the sideslip angle $\beta$ can be obtained by solving the above equation.

$$\begin{aligned}\beta &= \frac{k_f}{k_f + k_r}u - \frac{k_f l_f - k_r l_r}{v(k_f + k_r)}\gamma - \frac{m}{k_f + k_r}a_g + \frac{1}{k_f + k_r}\xi \\ &= \hat{\beta} + \frac{1}{k_f + k_r}\xi\end{aligned} \tag{26}$$

Remark that the side-acceleration $a_g$ is measurable and $\gamma$ can be obtained by a yaw gyroscope, so the unmeasurable sideslip angle $\beta$ can be reconstructed by (26) with troublesome estimation errors as long as the cornering stiffness $k_f, k_r$ has been identified. Therefore the identification of $k_f, k_r$ becomes a key issue for estimating the sideslip angle as well as the two tire sliding angles by considering (4)(5).

## C. Identifying cornering stiffness by using adaptive robust observer

Due to the existence of the derivative of the lateral disturbance $\dot{\xi}$ in (16), the following linear robust Luenberger observer is applied to identify $k_f, k_r$ [15]

$$\begin{cases} \dot{X} = AX + BU + \Psi\dot{\xi} \\ \dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}U + K(X - \hat{X}) + \Lambda sign(X - \hat{X}) \end{cases} \tag{27}$$

where $\hat{A}, \hat{B}$ are the matrices containing the estimation of the unknown cornering stiffness $\hat{k}_f, \hat{k}_r$.

$$\hat{A} = \begin{bmatrix} -\frac{\hat{k}_f + \hat{k}_r}{mv} & -v + \frac{\hat{k}_r l_r - \hat{k}_f l_f}{mv} \\ \frac{\hat{k}_r l_r - \hat{k}_f l_f}{I_z v} & -\frac{\hat{k}_f l_f^2 + \hat{k}_r l_r^2}{I_z v} \end{bmatrix} \tag{28}$$

$$\hat{B} = \begin{bmatrix} \frac{\hat{k}_f}{m} & \frac{\hat{k}_f l_f}{I_z} \end{bmatrix}^T \tag{29}$$

$K = diag(k_1, k_2)$ is a matrix such that $(A - K)$ is Hurwitz. $\hat{X}$ is the estimated value. $\Lambda = diag(\lambda_1, \lambda_2)$ is the gain of the signum function.

Define the error of the state estimation $e = \hat{X} - X$, the following equation holds by considering (27)

$$\dot{e} = A_e e + \widetilde{W}\phi - \Psi\dot{\xi} - \Lambda sign(e) \tag{30}$$

where

$$A_e = A - K \tag{31}$$
$$\widetilde{W} = \begin{bmatrix} W_1\hat{X} + W_2\dot{u} & W_3\hat{X} \end{bmatrix} \tag{32}$$
$$\phi = \begin{bmatrix} \hat{k}_f - k_f \\ \hat{k}_r - k_r \end{bmatrix} \tag{33}$$

And in $\widetilde{W}$

$$W_1 = \begin{bmatrix} -\frac{1}{mv} & -\frac{l_f}{mv} \\ -\frac{l_f}{I_z v} & -\frac{{l_f}^2}{I_z v} \end{bmatrix} \tag{34}$$

$$W_2 = \begin{bmatrix} \frac{1}{m} & \frac{l_f}{I_z} \end{bmatrix}^T \tag{35}$$

$$W_3 = \begin{bmatrix} -\frac{1}{mv} & \frac{l_r}{mv} \\ \frac{l_r}{I_z v} & -\frac{{l_r}^2}{I_z v} \end{bmatrix} \tag{36}$$

The adaptive learning rules for $\hat{k}_f, \hat{k}_r$ may be obtained by using Lyapunov stability theory. The Lyapunov function is defined as

$$V = e^T P_0 e + \phi^T Q_0 \phi \tag{37}$$

where $P_0 = diag(p_a, p_\gamma)$ and $Q_0$ are positive symmetric definite matrices and $P_0$ satisfies ${A_e}^T P_0 + P_0 A_e = -Q_1 < 0$, then the time derivative of $V$ is

$$\begin{aligned} \dot{V} < &-e^T Q_1 e + 2\phi^T (\widetilde{W}^T P_0 e + Q_0 \dot{\phi}) \\ &+2|e|^T (|P_0 \Psi \dot{\xi}| - P_0 \Lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}) \end{aligned} \tag{38}$$

Assume an estimation of the maximum disturbance derivative $\dot{\xi}$ is available, let

$$\dot{\phi} = \begin{bmatrix} \dot{\hat{k}}_f \\ \dot{\hat{k}}_r \end{bmatrix} = -Q_0^{-1} \widetilde{W}^T P_0 e \tag{39}$$

$$\lambda_i > max(|\Psi \dot{\xi}|_i) \tag{40}$$

It can be obtained that

$$\dot{V} < -e^T Q_1 e \tag{41}$$

which guarantees the convergence of the linear Luenberger observer.

From (41) it is concluded that $e \to 0, \dot{e} \to 0$ . When $\widetilde{W}$ is full rank, the identification errors of the cornering stiffness $\phi$ is stable and bounded by the values of the disturbance derivative. In this case, $\dot{u}$ which appears in the notation of $\widetilde{W}$ functions as the excitation instead of control inputs of the systems. It is also noted that if the lateral disturbance is constant, the estimated values of the cornering stiffness would converge to its real values.

After the cornering stiffness has been identified accurately, the vehicle sideslip angle $\beta$ may be estimated and the unknown sliding angles $\delta_b, \alpha_r$ can be reconstructed. But due to the existence of the lateral disturbance, the reconstructed $\delta_b, \alpha_r$ are not identical to the real values.

## IV. Robust Anti-sliding controller design based on Chained System Theory

To accomplish anti-sliding control relying on the estimated values $\hat{\delta}_b, \hat{\alpha}_r$ which suffer from a certain amount of reconstruction errors, a sliding mode controller is to be designed based on chained system theory.

Because the tire sliding introduces weak effects on the longitudinal motion, substituting the estimation of $\delta_b, \alpha_r$ into (6) and ignoring the negative effects of the estimation errors on the longitudinal direction, we obtain that

$$\begin{cases} \dot{s} = \dfrac{v\cos(\tilde{\theta} + \hat{\alpha}_r)}{1 - c(s)y} \\ \dot{y} = v\sin(\tilde{\theta} + \hat{\alpha}_r) + \varepsilon_1 \\ \dot{\tilde{\theta}} = v\Big[\cos\hat{\alpha}_r \dfrac{\tan(\delta + \hat{\delta}_b) - \tan\hat{\alpha}_r}{l} - \dfrac{c(s)\cos(\tilde{\theta} + \hat{\alpha}_r)}{1 - c(s)y}\Big] + \varepsilon_2 \end{cases} \tag{42}$$

where $\varepsilon_i$ is the vector indicating the negative effects caused by the estimation errors of $\delta_b, \alpha_r$ on the lateral and orientation kinematics.

Considering the kinematic model (42), via state transformation as following [16]

$$(a_1, a_2, a_3) = (s, y, (1 - c(s)y)\tan(\tilde{\theta} + \hat{\alpha}_r)) \tag{43}$$

a perturbed chained system (44) can be obtained

$$\text{derivation w.r.t } \boldsymbol{t} \begin{cases} \dot{a}_1 = \dfrac{v\cos(\tilde{\theta} + \hat{\alpha}_r)}{1 - yc(s)} = m_1 \\ \dot{a}_2 = v\sin(\tilde{\theta} + \hat{\alpha}_r) + \varepsilon_1 \\ \quad = a_3 m_1 + \varepsilon_1 \\ \dot{a}_3 = \dfrac{d}{dt}((1 - yc(s))\tan(\tilde{\theta} + \hat{\alpha}_r)) \\ \quad = m_2 + \eta \end{cases} \tag{44}$$

where

$$\begin{aligned} m_2 = &-vc(s)\sin(\tilde{\theta} + \hat{\alpha}_r)\tan(\tilde{\theta} + \hat{\alpha}_r) \\ &- v\frac{dc(s)}{ds}\frac{\cos(\tilde{\theta} + \hat{\alpha}_r)}{1 - yc(s)}\tan(\tilde{\theta} + \hat{\alpha}_r)y \\ &+ v\frac{1 - yc(s)}{\cos^2(\tilde{\theta} + \hat{\alpha}_r)}\Bigg(\cos(\hat{\alpha}_r)\frac{\tan(\delta + \hat{\delta}_b) - \tan\hat{\alpha}_r}{l} \\ &- c(s)\frac{\cos(\tilde{\theta} + \hat{\alpha}_r)}{1 - yc(s)}\Bigg) + \frac{1 - yc(s)}{\cos^2(\tilde{\theta} + \hat{\alpha}_r)}\frac{d}{dt}\hat{\alpha}_r \end{aligned} \tag{45}$$

$$\eta = \frac{(1 - yc(s))\varepsilon_2}{\cos^2(\tilde{\theta} + \hat{\alpha}_r)} - c(s)\varepsilon_1\tan(\tilde{\theta} + \hat{\alpha}_r) \tag{46}$$

Noting that in (44) $\varepsilon_1$ and $\eta$ play a role as two additive disturbances to a ideal chained system, so (44) can be converted into a perturbed single-input linear system by computing the derivative with respect to the state variable $a_1$.

$$\text{derivation w.r.t } \boldsymbol{a}_1 \begin{cases} a_1' = 1 \\ a_2' = a_3 + \dfrac{\varepsilon_2}{m_1} \\ a_3' = \dfrac{m_2}{m_1} + \dfrac{\eta}{m_1} = u + \dfrac{\eta}{m_1} \end{cases} \tag{47}$$

where $u$ is the virtual control input of the disturbed single-input system (47). Because the single-input model (47) contains uncertain bounded disturbances, sliding mode control theories are applied to design a robust controller which may guarantee the system states converge to a neighborhood near the origin.

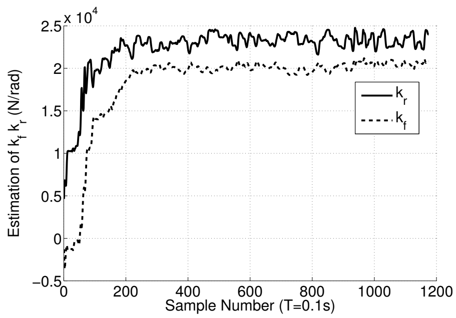


Fig. 3. The estimation of $k_f, k_r$

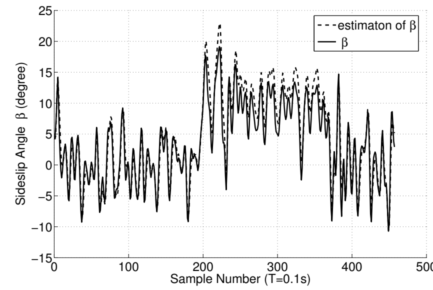


Fig. 4. Estimation of sideslip angle

**Theorem 1**: Considering the system (44) where $(a_1, a_2, a_3) = \big(s, y, (1 - c(s)y)\tan(\tilde{\theta} + \hat{\alpha}_r)\big)$, define

$$z = \Lambda a_2 + a_3 = \Lambda y + (1 - yc(s))\tan(\tilde{\theta} + \hat{\alpha}_r) \qquad (48)$$

the achievement of sliding motions on the sliding surface (48) can be guaranteed by the control law

$$u = -K_s z - \Lambda_s a_3 - \rho \text{sign}(z) \qquad (49)$$

where

$$\rho \geq |\varsigma| = \left|\frac{\Lambda_s \varepsilon_2 + \eta}{m_1}\right| \qquad (50)$$

Please refer to [6] for more details. The physical steering angle is obtained by inverse conversion of the virtual robust control law $u$.

Due to the definition of $a_2, a_3$ in (43), it is proven that the lateral deviation $y$ and the orientation error $\tilde{\theta}$ are globally uniformly ultimately bounded in presence of the sliding effects and lateral disturbances.

## V. Simulation Results

### A. Cornering stiffness identification

In order to validate the adaptive learning rules (39), a simulation is performed in which the parameters are set as m=500kg, $l_f$=1.1m, $l_r$=1.3m, $k_f$=20000N/rad, $k_r$=25000N/rad. The sine-like lateral disturbance which is the most common type of disturbances is set as $\xi = 550N$ and $l_d = 0.3m$ The vehicle velocity is set as a constant $v = 8.3km/h$. In the simulations, values of the cornering stiffness are initialized to zero, the gains are set as $p_a = 500000$, $p_\gamma = 2750000$, $k_1 = 20$, $k_2 = 3$, $\lambda_i = 10$. The larger $||k_1||, ||k_2||$ are, the faster the state estimation errors $e$ tend to zero, but the slower the estimation of $\hat{k}_f, \hat{k}_r$ converges to the real values. On the other hand if $p_a, p_\gamma$ go too large, it may cause too much vibration of the estimation results of $\hat{k}_f, \hat{k}_r$, so in actual implementations these gains should be tuned gradually to make an optimal compromise between convergence rate and accuracy.

The estimation results of $k_f, k_r$ are shown by Figure 3. From this figure it is known that although $\hat{k}_f$ and $\hat{k}_r$ have important initial errors, finally they would converge into a neighborhood of their real values which is necessary for sideslip angle reconstruction. Then the sideslip angle $\beta$ is reconstructed based on (26) without the knowledge of the lateral disturbance, the result is shown by Figure 4. In this figure the solid line indicates the sideslip angle which is obtained by solving the dynamic model (20) providing the normal lab values of the sideslip angle. The dashed line depicts the estimated value of $\hat{\beta}$. Because the cornering stiffness has been identified, the estimation of the sideslip angle may be obtained correspondingly. But a certain amount of estimation errors are also noted which is caused by the lateral disturbance.

### B. Robust anti-sliding control

In this section, some simulation results are presented. A reference path consisting of straight lines and curves is followed (see Figure 5), the sliding effects are introduced to the system and the time-varying sine-like lateral disturbances are also introduced. To simulate all the other external unexpected disturbances, noises are always added to the system through the same channel as the control inputs. To compare the control performances with the previous works, the control laws of [9] as well as [12] which didn't take sliding effects into account are applied under the same condition except that we set the controller gains as $k_p = 0.09, k_d = 0.6$.

The simulation results of the lateral deviation of the path-following controllers are shown by Figure 6 respectively. The dotted line represents the results of the controller without any sliding compensation, the dashed line presents the results of the sliding controller of [9] only with classical sliding angle observation and compensation, the solid line depicts the results of the proposed robust anti-sliding controller (49). Because the control law of [12] does not take any sliding effects into account and from theoretical point of view, the PD-type virtual control law is not robust against disturbances, it is clear that it suffers from sliding greatly. When the sliding effects appear, its lateral deviation becomes more significant than the others. Although the sliding controller [9] is effective to correct the negative sliding effects in common cases, it still cannot yield a satisfactory results either with obvious bias when the lateral disturbances is important. While the robust control law proposed in this paper provides satisfactory simulation results, it has good transient responses and is

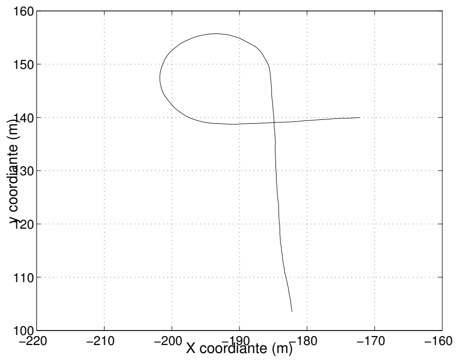


Fig. 5. Path to be followed

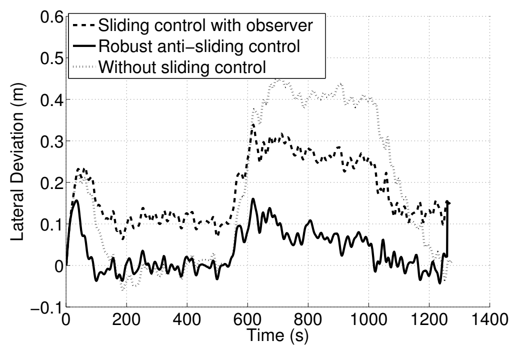


Fig. 6. Lateral deviation

robust against not only the sliding effects but also the time-varying disturbance. The simulation results show that the sliding effects affect the lateral deviation weakly, the lateral deviation is obviously less than the others without robust sliding compensation.

In the simulation, it is also found that the cornering stiffness changes slowly with the tyre characteristics and rapidly with the road condition. If the ground condition varies greatly, the convergence speed of the lateral control would become relatively slow which is a direct consequence of prolonged process of parameter estimation. But with the help of the proposed robust sliding controller, the stability of the lateral control still can be guaranteed.

## VI. CONCLUSION

Unknown sliding angles are reconstructed by using an adaptive robust Luenberger observer in presence of time-varying lateral disturbance. The stability of the observer is proven via Lyapunov analysis. It is noted that the existence of the time-varying lateral disturbance causes errors of sliding angle reconstruction. To utilize the estimated sliding angles into sliding compensation, a tire-oriented kinematic model which integrates the effects of the lateral disturbance as additive disturbances to the ideal kinematic model is used. From this model, a particular perturbed chained system is evolved. The use of the attractive structure of chained systems together with the sliding mode control leads to a robust controller which is robust to both the sliding effects and the lateral disturbances.

## REFERENCES


[1] J. Ackermann, “Robust control prevents car skidding”. *IEEE Control Systems Magazine*, pp: 23-31, June,1997.

[2] I. Motte, H. Campion, “Control of sliding mobile robots : a slow manifold approach”, MNTS, 2000.

[3] W. Leroquais, B. D’Andrea-Novel, “Vibrational control of wheeled mobile robots not satisfying ideal velocity constraints: the unicycle case”, *European Control Conference*. July 1-4, Brussels, 1997.

[4] B. D’Andrea-Novel, G. Campion and G. Bastin, “Control of wheeled mobile robots not satisfying ideal constraints: a singular perturbation approach”, *International Journal of Robust Nonlinear Control*, 5: 243-267, 1995.

[5] Y. L. Zhang, J. H. Chung, S. A. Velinsky,“ Variable structure control of a differentially steered wheeled mobile robot”, *Journal of intelligent and Robotic Systems*. 36: 301-314, 2003.

[6] H. Fang, R. Lenain, B. Thuilot, P. Martinet, “Sliding Mode Control of Automatic Guidance of Farm Vehicles in the Presence of Sliding”. *The 4th International Symposium on Robotics and Automation,* Queretaro, Mexico August 25-27. pp. 582-587, 2004.

[7] M. L. Corradini and G. Orlando, “Experimental testing of a discrete-time sliding mode controller for trajectory tracking of a wheeled mobile robot in the presence of skidding effects”. *Journal of robotic systems*, 19(4), 177-188, 2002.

[8] H. Fang, R. Lenain, B. Thuilot, P. Martinet, “Robust Adaptive Control of Automatic Guidance of Farm Vehicles in the Presence of Sliding”, *IEEE International Conference on Robotics and Automation*, Barcelona, April, 18-22, pp. 3113-3118, 2005.

[9] Lenain R. Thuilot B. Cariou C, Martinet P, “Sideslip angles observer for vehicle guidance in sliding conditions: Application to agricultural path tracking tasks”, *In Proc. of the intern. Conf. On Robotics and Automation*, Orlando, Florida, May , pp. 3183-3188, 2006.

[10] Lenain R. Thuilot B. Cariou C, Martinet P. , “Mobile robot control in presence of sliding: Application to agricultural vehicle path tracking”, *In Proc. of the 45th IEEE Conference on Decision & Control*, San Diego, CA, USA, December 13-15, pp. 6004-6009, 2006.

[11] T. Hiraoka, H. Kumamoto, O. Nishihara and K. Tenmoku , “Cooperative Steering System Based on Vehicle Sideslip Angle Estimation from Side Acceleration Data at Percussion Centers”, Proc. of IEEE International Vehicle Electronics Conference 2001(IVEC2001), pp.79-84 (2001.9)

[12] Thuilot B, Cariou C, Martinet P, and Berducat M, “Automatic guidance of a farm tractor relying on a single CP-DGPS”, *Autonomous robots* , 13(1): 87-104,2002

[13] Abe, M,“vehicle dynamics and control ” (in Japanese), Sankaidou, 1992

[14] Hiraoka, T., Kumamoto, H. and Nishihara, O., “ Sideslip angle estimation and active front steering system based on lateral acceleration data at centers of percussion with respect to front/rear wheels”, JSAE Review Vol.25, 2004, pp.37-42.

[15] Stephant J., Charara A., Meizel D , “Virtual Sensor, Application to vehicle sideslip angle and transversal forces”, *IEEE Transactions on Industrial Electronics*, Vol. 51, n.2, Avril 2004.

[16] Samson, C, Control of chained systems, “Application to path following and time-varying point stabilization of mobile robot”, *IEEE Trans,on Automatic Control*, 39(12): pp. 2411-2425, 1995.