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Measurement Noise Estimator Assisted Extended Kalman Filter for SLAM Problem

Won-Seok Choi, Jeong-Gwan Kang, and Se-Young Oh, *Member, IEEE*

Abstract—**This paper addresses the measurement noise of Extended Kalman Filter-based Simultaneous Localization And Mapping (EKF-SLAM). The Extended Kalman Filter (EKF) is based on the Gaussian noise with zero mean and should know the correct prior knowledge of control and measurement noise covariance matrices. If these conditions are not satisfied, EKF unavoidably diverges. The present paper proposes the method of a new adaptive kalman filter to be supported by Measurement Noise Estimator (MNE), which estimates the measurement noise distribution including biased noise and noise covariance, whenever the update step executes. We evaluate this method under well-known benchmark environment for SLAM problem. Simulation results show that the proposed algorithm overcomes degrading performance of the standard EKF under the condition of wrong knowledge of sensor statistics.**

I. INTRODUCTION

THE Simultaneous Localization And Mapping (SLAM) problem is one of the most significant subject among the researchers of the autonomous vehicle and mobile robot, for the last twenty years. The SLAM problem, as the name explains, consists of estimating the pose of a moving platform on the basis of the map and estimating the map including features on the basis of the pose of a moving platform [1]. Due to the mutual dependence of robot pose and the map, the noise of robot pose arise the uncertainty of map estimation and vice versa. Therefore, this problem requires a solution in a high dimensional space.

Kalman Filter (KF) based technique is one of the popular approaches to solve the SLAM problem. Several successful SLAM algorithms have been developed with various sensors [2], [3] and for various environments [4], [5], [6]. In general, Extended Kalman Filter-based SLAM (EKF-SLAM) represents the robot pose and the feature positions in the form of a state vector and the state uncertainty in the form of an error covariance matrix. The covariance matrix includes the covariance of the robot pose and the each feature position, and the cross-correlations among the robot pose and the each feature position and among feature positions. However, the standard EKF-SLAM has the one significant weakness that the computational burden increases geometrically. Several recent papers have focused on this problem [7], [8].

Particle Filter (PF) based approaches is an alternative to solve the SLAM problem. Because the uncertainty is represented as the distribution of particles, PF can be emancipated from the key assumption in EKF that the noise should is model by Gaussian distribution, and can remove a redundant uncertainty owing to linearization of nonlinear model. However, this approach requires a large number of particles for the sufficient performance. Furthermore, in case of the SLAM problem that there are many targets to be estimated, the number of particles increases geometrically. Some researchers have developed Rao-BlackWellized Particle Filter (RBPF) based approaches [9], [10]. These methods have solved the curse of dimensionality by using each advantage of KF and PF. The uncertainty of each feature is represented by Gaussian distribution in the form of KF, and the current robot pose is represented by the particles including the robot trajectory in the form of PF. The significant meaning of this combination is that the trajectory, stored in each particle, replaces cross-correlation factor among the features. Due to the property, the computation complexity increases in arithmetical progression in the respect to the number of features. However, such simplification may be unable to correct the previous features.

To return KF, the performance of EKF-SLAM depends on the accuracy of the prior knowledge of the control noise covariance matrix $\mathbf{Q}$ and the measurement noise covariance matrix $\mathbf{R}$, this is a first matter of our concern. In a practical application, it is too difficult to get the precision prior knowledge, furthermore the real noise covariance may be time-variant. Hence, originating from the innovation adaptive estimation approach [11], some approaches to estimate these matrices have been developed [12], [13]. The general strategy is how to reducing difference between the theoretical covariance of innovation sequences and the corresponding actual covariance of the innovation sequence. To put it plainly, the quantity of individual innovation is regarded as the standard deviation of the covariance of the innovation sequence, but the correlations among innovations are not considered. In this paper, the noise covariance matrix is estimated with the correlations among innovations, without the uncertainty of the robot. Therefore, the proposed approach can decrease the redundant computation burden and

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Won-Seok Choi, Jeong-Gwan Kang are with the Department of Electronic and Electrical Engineering, the POhang university of Science and TECHnology (POSTECH), Pohang, KOREA,
(onlydol:naroo1@postech.ac.kr)

Se-Young Oh is with the Department of Electronic and Electrical Engineering, the POhang university of Science and TECHnology (POSTECH), Pohang, KOREA,
(phone: +82-54-279-2214, fax:+82-54-279-5594, syoh@postech.ac.kr)

be suitable for a real-time application.

In KF, a second matter of our concern is a white noise assumption in KF that the means of control and measurement noise should be zero. In real application, noise may have a biased error owing to mismodeling, extreme non-linearity and variation of system parameters. This type of noise can result in not only degrading performance of EKF-SLAM, but also the divergence due to feature mismatching in the data association process. In [14], a neural network aided extended Kalman filter have been proposed to estimate a biased error of the control noise.

In consideration of the two matter of our concern as mentioned above, the proposed method estimates the measurement noise covariance matrix and the biased measurement noise. We evaluate this method under well-known benchmark environment for SLAM problem [17]. Simulation results show that the proposed algorithm is very effective compared with the standard EKF-SLAM under practical condition where the measurement noise is time-variant biased noise.

This paper is organized as follows. Section II presents the standard EKF-SLAM algorithm and some statistics property for the background knowledge. Section III presents the proposed algorithm to estimate the measurement noise covariance and its bias. In Section IV, the performance of the proposed method is evaluated in comparison with the standard EKF-SLAM. Conclusion is presented in Section V.

II. Background

A. The Standard EKF-SLAM

The basic formulation of the EKF-SLAM begins at the definition of model functions. The state transition (control) is modeled by a nonlinear function $\mathrm{f}(\bullet)$ and the measurement of the state is modeled by a nonlinear function $\mathrm{h}(\bullet)$

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathrm{f}\left(\mathbf{x}_k, \mathbf{u}_{k+1}\right) + \mathbf{q}_k \\ \mathbf{z}_{k+1} &= \mathrm{h}\left(\mathbf{x}_{k+1}\right) + \mathbf{r}_{k+1} \end{aligned} \tag{1}$$

where the subscript k is time index, and $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{z}$ are the state vector, the control input vector, and the measurement vector, respectively. The control noise $\mathbf{q}$ is modeled by Gaussian white distribution with the covariance matrix $\mathbf{Q}$. The measurement noise $\mathbf{r}$ is modeled by Gaussian white distribution with the covariance matrix $\mathbf{R}$.

For the SLAM problem, the state vector consists of the vehicle pose $\mathbf{x}_v$ and the feature (landmark) position $\mathbf{x}_m$, and its uncertainty is represented by the covariance matrix $\mathbf{P}$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_v^T & \mathbf{x}_m^T \end{bmatrix}^T, \qquad \mathbf{P} = \begin{bmatrix} \mathbf{P}_{vv} & \mathbf{P}_{vm} \\ \mathbf{P}_{vm}^T & \mathbf{P}_{mm} \end{bmatrix} \tag{2}$$

where $\mathbf{P}_{vv}$ is the covariance of vehicle pose, $\mathbf{P}_{mm}$ is the covariance of each feature position, and $\mathbf{P}_{vm}$ is the covariance of cross-correlation among the vehicle pose and feature.

The procedure of the standard EKF algorithm consists of the prediction step (3) and the update step (4), of which formulations are,

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k} &= \mathrm{f}\left(\hat{\mathbf{x}}_{k|k}, \hat{\mathbf{u}}_{k+1}\right) \\ \mathbf{P}_{k+1|k} &= \nabla \mathrm{f}_k \, \mathbf{P}_{k|k} \nabla \mathrm{f}_k^T + \mathbf{Q}_{k+1} \end{aligned} \tag{3}$$

$$\begin{aligned} \mathbf{y}_{k+1} &= \mathbf{z}_{k+1} - \mathrm{h}\left(\hat{\mathbf{x}}_{k+1|k}\right) \\ \mathbf{S}_{k+1} &= \nabla \mathrm{h}_{k+1} \mathbf{P}_{k+1|k} \nabla \mathrm{h}_{k+1}^T + \mathbf{R}_{k+1} \\ \mathbf{K}_{k+1} &= \mathbf{P}_{k+1|k} \nabla_{\mathbf{x}} \mathrm{h}_{k+1}^T \cdot \mathbf{S}_{k+1}^{-1} \\ \hat{\mathbf{x}}_{k+1|k+1} &= \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_{k+1} \mathbf{y}_{k+1} \\ \mathbf{P}_{k+1|k+1} &= \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1} \mathbf{S}_{k+1} \mathbf{K}_{k+1}^T \end{aligned} \tag{4}$$

where $\hat{\mathbf{x}}$ presents the estimated state, $\mathbf{y}$ is the innovation sequences, $\mathbf{S}$ is theoretical covariance of innovation sequences, and $\mathbf{K}$ is the Kalman gain. The Jacobians of the control model and the measurement model are given as;

$$\nabla \mathrm{f}_k = \left. \frac{\partial \mathrm{f}}{\partial \mathbf{x}_k} \right|_{\left(\hat{\mathbf{x}}_{k|k}, \hat{\mathbf{u}}_{k+1}\right)}, \qquad \nabla \mathrm{h}_{k+1} = \left. \frac{\partial \mathrm{h}}{\partial \mathbf{x}_k} \right|_{\left(\hat{\mathbf{x}}_{k+1|k}\right)} \tag{5}$$

B. Statistic Property

The KF-based approach requires the noise covariance which can be calculated as,

$$\begin{aligned} \boldsymbol{\mu} &= \mathrm{E}\left(\mathbf{x}\right) = \frac{1}{n} \sum_i \mathbf{x}_i \\ \boldsymbol{\Omega} &= \mathrm{E}\left(\left(\mathbf{x} - \boldsymbol{\mu}\right)\left(\mathbf{x} - \boldsymbol{\mu}\right)^T\right) = \frac{1}{n} \sum_i \mathbf{x}_i \mathbf{x}_i^T - \boldsymbol{\mu}\boldsymbol{\mu}^T \end{aligned} \tag{6}$$

where $\mathrm{E}(\bullet)$ is expectation, μ is mean, and Ω is covariance, n is the number of samples.

A whole sample set S_c can divide into two subsets S_a and S_b, of which samples present as bellow,

$$\begin{aligned} & S_c = S_a \bigcup S_b, \qquad S_a \bigcap S_b = \varnothing \\ & S_a = \left\{x_{a,1}, x_{a,2} \ldots x_{a,n_a}\right\}, \quad S_b = \left\{x_{b,1}, x_{b,2} \ldots x_{b,n_b}\right\} \end{aligned} \tag{6}$$

where n_a and n_b are the number of elements in each subset.

Assumed that the statistics of each subset are known, the mean and the covariance of superset S_c can be calculated in the form of a multivariate as;

$$\begin{aligned} \boldsymbol{\mu}_c &= \frac{1}{n_c} \sum_{S_c} \mathbf{x} = \frac{1}{n_a + n_b} \left(\sum_{S_a} \mathbf{x} + \sum_{S_b} \mathbf{x} \right) \\ &= \frac{n_a \cdot \boldsymbol{\mu}_a + n_b \cdot \boldsymbol{\mu}_b}{n_a + n_b} \end{aligned} \tag{7}$$

$$\begin{aligned} \boldsymbol{\Omega}_c &= \frac{1}{n_c} \sum_{S_c} \mathbf{x}\mathbf{x}^T - \boldsymbol{\mu}_c \boldsymbol{\mu}_c^T = \frac{1}{n_c} \left(\sum_{S_a} \mathbf{x}\mathbf{x}^T + \sum_{S_b} \mathbf{x}\mathbf{x}^T \right) - \boldsymbol{\mu}_c \boldsymbol{\mu}_c^T \\ &= \frac{1}{n_c} \left(n_a \boldsymbol{\Omega}_a + n_b \boldsymbol{\Omega}_b \right) + \frac{n_a \boldsymbol{\mu}_a \boldsymbol{\mu}_a^T + n_b \boldsymbol{\mu}_b \boldsymbol{\mu}_b^T}{n_a + n_b} - \boldsymbol{\mu}_c \boldsymbol{\mu}_c^T \\ &= \frac{n_a \boldsymbol{\Omega}_a + n_b \boldsymbol{\Omega}_b}{n_a + n_b} + \frac{n_a n_b \left(\boldsymbol{\mu}_a - \boldsymbol{\mu}_b\right)\left(\boldsymbol{\mu}_a - \boldsymbol{\mu}_b\right)^T}{\left(n_a + n_b\right)^2} \end{aligned} \tag{8}$$

where $\boldsymbol{\Omega}_c$ is a covariance matrix form. If variables are independent, equation (8) can be simplified in the form of a variance vector;

$$\boldsymbol{\Omega}_c = \frac{n_a \boldsymbol{\Omega}_a + n_b \boldsymbol{\Omega}_b}{n_a + n_b} + \frac{n_a n_b \left(\boldsymbol{\mu}_a - \boldsymbol{\mu}_b\right)^2}{\left(n_a + n_b\right)^2} \tag{9}$$

where $(\ )^2$ is the element by element operation.

In this paper, this property is employed as computing the noise character. One subset S_a is utilized in a prior knowledge, the other S_b is utilized in a current knowledge, and the super set S_c presents a corrected current knowledge

III. Measurement Noise Estimator assisted EKF-SLAM

A. The Algorithm

Most researches about adaptive Kalman filter technique have presented how to extract the reasonable information from innovations acquired for certain time interval. The existing works extract the useful information from the disparity $\Delta\mathbf{C}$ between the desired value and the prediction value: the corresponding actual covariance of innovation sequence $\mathbf{C}$ and the theoretical covariance of the innovation sequences $\mathbf{S}$ in (4).

$$\begin{aligned}\mathbf{C} &= \mathbf{y}\cdot\mathbf{y}^T \\ \Delta\mathbf{C} &= \mathbf{C}-\mathbf{S}\end{aligned} \tag{10}$$

In (10), $\mathbf{S}$ is the function of measurement noise covariance $\mathbf{R}$. The measurement noise covariance $\mathbf{R}$, which makes the disparity $\Delta\mathbf{C}$ minimum, is optimal value in existing method.

In the proposed algorithm, we use the weighted sample mean and variance of innovations at time k to obtain a current knowledge of measurement. From the statistical viewpoint, the feature's uncertainty covariance includes the meaning of the inverse form of weight of innovation sample. To put it precisely, the correlation between the vehicle and the features can separate owing to considering the vehicle pose as a static variable. At a single discrete time k, the vehicle uncertainty can be excluded from the process of calculating weighted mean and weighted variance, because the influences of the vehicle uncertainty on each innovation sample are same and can be canceled. As time goes by, the influences become a statistical zero, therefore it can be ignored.

To update a current estimate including mean and variance of the measurement noise, the proposed Measurement Noise Estimator (MNE) employs the number of innovation samples (7),(9), instead of the concept of uncertainties. Therefore, it is possible to estimate the properties of measurement noise by using the weighted sample mean and variance that we have focused on. The detail implementation of Measurement Noise Estimator (MNE) presents in Section III-B.

We explain the MNE assisted EKF-SLAM using a range-bearing sensor model. The measurement and its noise model given as

$$\mathbf{z} = \begin{bmatrix} r & \theta \end{bmatrix}^T, \quad \mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} \tag{11}$$

$$\mathbf{z}_k = \mathbf{z}_{k,true} + \mathbf{b}_k + \mathrm{N}(0,\mathbf{R}_k) \tag{12}$$

where r is range, θ is bearing, $\mathbf{z}$ is actual measurement, $\mathbf{z}_{true}$ is true measurement, $\mathbf{b}$ is a biased noise, and $\mathrm{N}(\bullet)$ is a Gaussian distribution function.

The proposed algorithm is described.

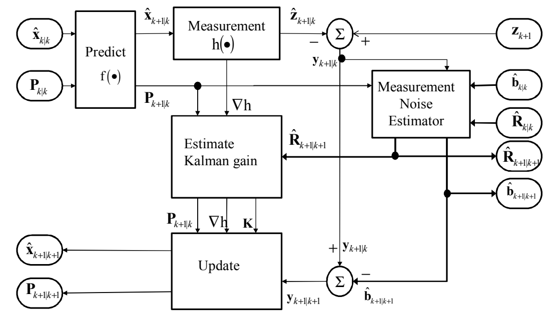


Fig. 1. The block diagram of MNE assisted EKF-SLAM.

1) Initialize

$$\begin{aligned}\hat{\mathbf{x}}_0 &= \mathbf{0}, \quad \mathbf{P}_0 = \mathbf{0} \\ \hat{\mathbf{b}}_0 &= \mathbf{0}, \quad \hat{\mathbf{R}}_0 = \lambda\cdot\mathbf{I}\end{aligned} \tag{13}$$

2) Predict $\hat{\mathbf{x}}$ and $\mathbf{P}$, using (3)

3) Obtain measurement $\mathbf{z}_{k+1}$

4) Compute augmented innovation sequence $\mathbf{y}_{k+1|k}$ for already stored features, for batch-mode.

$$\mathbf{y}_{k+1|k} = \mathbf{z}_{k+1} - \mathrm{h}\left(\hat{\mathbf{x}}_{k+1|k}\right) \tag{14}$$

where the dimension of $\mathbf{y}_{k+1|k}$ is $2N\times 1$ and N is the number of measurements corresponding with stored features.

5) Estimate the mean and variance of measurement noise, using MNE of which function representation is,

$$\left(\hat{\mathbf{b}}_{k+1|k+1}, \hat{\mathbf{R}}_{k+1|k+1}\right) = \mathrm{F}_{MNE}\left(\hat{\mathbf{b}}_{k|k}, \hat{\mathbf{R}}_{k|k}, \mathbf{y}_{k+1|k}\mathbf{P}_{k+1|k}\right) \tag{15}$$

6) Make augmented measurement noise covariance $\hat{\mathbf{R}}_{aug}$ of which dimension is $2N\times 2N$ and augmented measurement biased noise $\hat{\mathbf{b}}_{aug}$ of which dimension is $2N\times 1$, for batch-mode.

$$\begin{aligned}&\hat{\mathbf{R}}_{k+1|k+1} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}, \qquad \hat{\mathbf{b}}_{k+1|k+1} = \begin{bmatrix} b_r \\ b_\theta \end{bmatrix} \\ &\hat{\mathbf{R}}_{aug} = \mathrm{diag}\begin{bmatrix} \sigma_r^2 & \sigma_\theta^2 & \sigma_r^2 & \sigma_\theta^2 & \cdots & \sigma_r^2 & \sigma_\theta^2 \end{bmatrix} \\ &\hat{\mathbf{b}}_{aug} = \begin{bmatrix} b_r & b_\theta & b_r & b_\theta & \cdots & b_r & b_\theta \end{bmatrix}^T\end{aligned} \tag{16}$$

where $\mathrm{diag}(\bullet)$ is the function to make a diagonal matrix regarding a vector as a diagonal term.

7) Compute the Kalman gain, using (4).

8) Correct innovation sequence with augmented $\hat{\mathbf{b}}_{aug}$

$$\mathbf{y}_{k+1|k+1} = \mathbf{y}_{k+1|k} - \hat{\mathbf{b}}_{aug} \tag{17}$$

9) Update $\hat{\mathbf{x}}$ and $\mathbf{P}$, using (4).

10) Correct measurement to store a new feature.

$$\mathbf{z}_{k+1|k+1} = \mathbf{z}_{k+1|k} - \hat{\mathbf{b}}_{k+1|k+1} \tag{18}$$

11) Augment corrected measurement for new features

For $\hat{\mathbf{R}}_0$ in step 1), λ prefers to be large rather than small in the basis of the real noise model, because the larger one can prevent the wrong correction due to the premature estimation in the early stages.

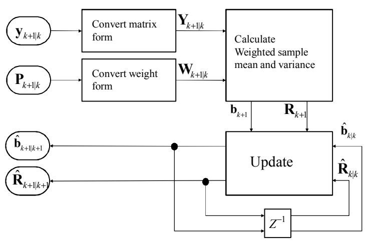


Fig. 2. The block diagram of MNE.

B. Measurement Noise Estimator

Measurement Noise Estimator (MNE) is developed as a simple feedback system with input of innovation sequence $\mathbf{y}_{k+1|k}$ and the predicted state covariance $\mathbf{P}_{k+1|k}$ in the main KF procedure, and output (internal state) of the biased measurement noise $\hat{\mathbf{b}}_{k+1|k+1}$ and the measurement noise variance $\hat{\mathbf{R}}_{k+1|k+1}$. (Fig. 2)

The procedure of MNE is described,

1) Convert the innovation sequence vector $\mathbf{y}$ into the matrix form $\mathbf{Y}$ of which dimension is $2\times N$

$$\mathbf{y}=\begin{bmatrix}\mathbf{y}_{f1}{}^T & \mathbf{y}_{f2}{}^T & \cdots & \mathbf{y}_{fN}{}^T\end{bmatrix}^T,\quad \mathbf{y}_{fi}=\begin{bmatrix}y_{fi}^r\\ y_{fi}^\theta\end{bmatrix} \tag{19}$$

$$\mathbf{Y}=\begin{bmatrix}\mathbf{y}_{f1} & \mathbf{y}_{f2} & \cdots & \mathbf{y}_{fN}\end{bmatrix} \tag{20}$$

where, N is the number of measurements corresponding with stored features, f_i presents the corresponding index with already stored features, and y_{fi}^r and y_{fi}^θ are range and bearing value of i -th innovation.

2) Calculate the innovation weight. The input is given as,

$$\mathbf{P}=\begin{bmatrix}\mathbf{P}_{vv} & \mathbf{P}_{vm}\\ \mathbf{P}_{vm}^T & \mathbf{P}_{mm}\end{bmatrix},\quad \mathbf{P}_{mm}=\begin{bmatrix}\sigma_{r,1}^2 & \cdots & \cdots & \cdots & \cdots\\ \vdots & \sigma_{\theta,1}^2 & \cdots & \cdots & \vdots\\ \vdots & \vdots & \ddots & \cdots & \vdots\\ \vdots & \vdots & \vdots & \sigma_{r,M}^2 & \vdots\\ \vdots & \cdots & \cdots & \cdots & \sigma_{\theta,M}^2\end{bmatrix} \tag{21}$$

where M is the number of total observed features. The innovation weight $\mathbf{W}$ is defined as

$$\mathbf{W}=\begin{bmatrix}\mathbf{w}_{f1} & \mathbf{w}_{f2} & \cdots & \mathbf{w}_{fN}\end{bmatrix} \tag{22}$$

where $\mathbf{w}_{fi}=\begin{bmatrix}1/\sigma_{r,i}{}^2 & 1/\sigma_{\theta,i}{}^2\end{bmatrix}^T$

3) Calculate the weighted sample mean $\mathbf{b}_{k+1}$ and variance $\mathbf{R}_{k+1}$ with respect to the current knowledge.

$$\mathbf{b}_{k+1}=\sum_i \mathbf{w}_{fi}\bullet\mathbf{y}_{fi}\Big/\sum_i \mathbf{w}_{fi} \tag{23}$$

$$\mathbf{R}_{k+1}=\sum_i \mathbf{w}_{fi}\bullet\left(\mathbf{y}_{fi}-\mathbf{b}_{k+1}\right)^2\Big/\sum_i \mathbf{w}_{fi} \tag{24}$$

where product and division are the element by element operation, and the dimension of $\mathbf{b}_{k+1}$ and $\mathbf{R}_{k+1}$ is 2×1

4) Update the biased measurement noise $\hat{\mathbf{b}}_{k+1|k+1}$ and the measurement noise variance $\hat{\mathbf{R}}_{k+1|k+1}$, using (7), (9).

In the final step for using (7), (9), the numbers of samples of two subsets are required. One number n_a about a prior knowledge corresponds with the total number of innovations throughout the navigation, and the other number n_b about a current knowledge corresponds to the number N of measurements corresponding with stored features. Naturally, the number n_a should increase with time due to accumulating the innovation samples. Thus, the adaptation speed decreases gradually and the over-fitting problem happens. To prevent this problem, n_a should be below a certain value. On the other hand, the small number n_a on the basis of n_b makes the system sensitive, so the system can be diverse. In this paper, the number n_a is regarded as a known constant, which is specified as 50 in our simulation.

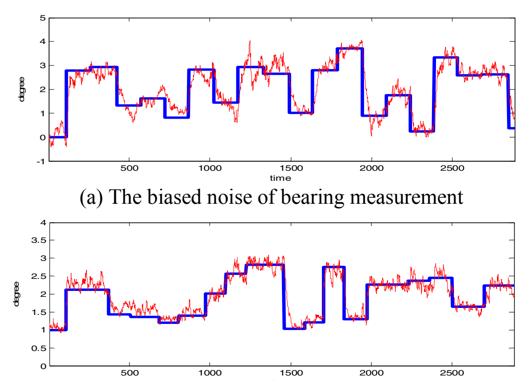
IV. Simulation Results

To evaluate the performance of the proposed method, we used well-known Matlab simulation for SLAM which is the opened software packages for SLAM [17]. We have developed the MNE assisted EKF-SLAM which is based on EKF-SLAM source in this packages.

We assumed that the control noise is white Gaussian noise in every simulation. The standard deviation (STD) of linear velocity is 0.3 m/s and the STD of steering angle is 3 degree. In simulation environment, the measurement noise is time-variant with a non-zero mean, and its initial values are specified as: (σ_r = 0.1 m, σ_θ = 1 degree, b_r = 0 m, and b_θ = 0 degree). The standard EKF-SLAM employs these initial values as the internal noise parameters ($\bar{\sigma}_\theta$, $\bar{\sigma}_r$) until the end.

For the sake of the plain evaluation without side factor, the range uncertainties (σ_r , b_r) are time-invariant, and the only bearing uncertainties (σ_θ , b_θ) frequently vary between 1-3 degree and 0-4 degree, respectively. Identifications of the corresponding feature are known. The same control and observation data is used in both simulations: The standard EKF-SLAM and the MNE assisted EKF-SLAM. The vehicle navigates three loops.

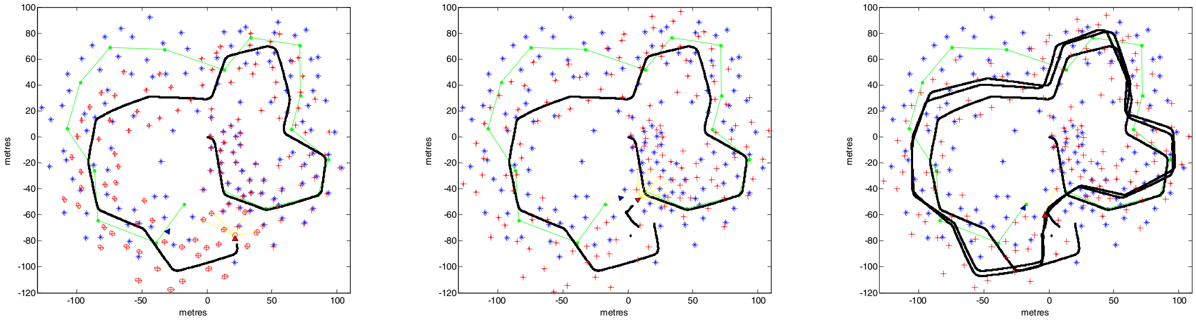
Fig. 3 shows that both $\hat{\mathbf{R}}$ and $\hat{\mathbf{b}}$ using the proposed method keep track of the true value. Thank to this tracking capacity, the performance of the proposed method can improve. Fig. 4 consists of the vehicle trajectory and features at three significant moments: just before loop closing, just



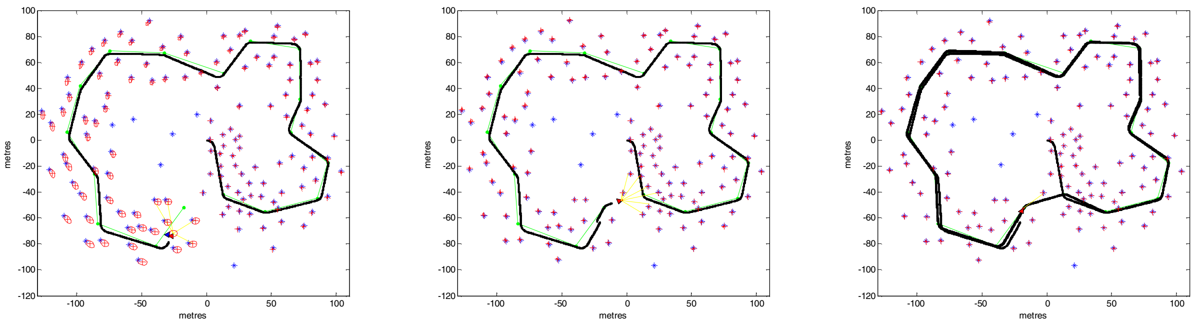
(a) The biased noise of bearing measurement

(b) The standard deviation of bearing measurement noise

Fig. 3. The estimation of measurement noise; blue lines present true noise and red lines present estimated noise



(a) Before loop closing. Time index k=944 (b) After loop closing, Time index k =1000 (c) Three loop. Time index k =2887



(d) Before loop closing. Time index k=944 (e) After loop closing, Time index k =1000 (f) Three loop. Time index k =2887

Fig. 4. Simulation results in case of a time-variant biased noise; (a),(b) and (c) are the performance of the standard EKF-SLAM, (d), (e), and (f) are the performance of the MNE assisted EKF-SLAM; blue (*) is a true feature, red (+) is an estimated feature, green line is a reference (true) trajectory, and black line is an estimated trajectory

after loop closing, and the last. Firstly, Fig. 4-(a) and Fig. 4-(d) show that the possibility of mismatching in the proposed algorithm is less than the standard EKF-SLAM's from the standpoint of the data association problem. Secondly, although the identification of feature is known, the estimation and the uncertainties of vehicle and featured are too inaccurate to be corrected reasonably in the standard EKF-SLAM after loop closing (Fig. 4-(b)), however, the proposed method works well (Fig. 4-(e)). Finally, even if the vehicle moves the same path over and over again, EKF cannot correct (Fig. 4-(c)) owing to its optimistic nature [15].

If the true state $\mathbf{x}_k$ is known, we can use the average Root Mean Square Error (RMSE) of features, and the average Normalized Estimation Error Squared (NEES) $\bar{\varepsilon}_k$ to characterize the performance of SLAM algorithm by using the Monte Carlo test.[1]

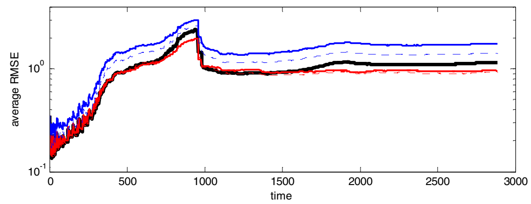
$$\begin{aligned} \varepsilon_k &= \left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)^T \mathbf{P}_{k|k}^{-1} \left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right) \\ \bar{\varepsilon}_k &= \frac{1}{N}\sum_{i=1}^{N} \varepsilon_{i,k} \end{aligned} \tag{24}$$

[1] For a consistent evaluation, a thorough examination is needed via multiple Monte Carlo runs [16]. The criterion of this test is the 95% probability concentration region for the average NEES with the 3-demensional vehicle pose and 50 runs, and is bounded by the interval 2.35-3.72 [15]. If it is higher than the upper bound, this algorithm is optimistic, and if it is below the lower bound, the algorithm is conservative. Thanks to the optimistic nature of the EKF, we can mention briefly that the lower average NEES is, the better performance is.

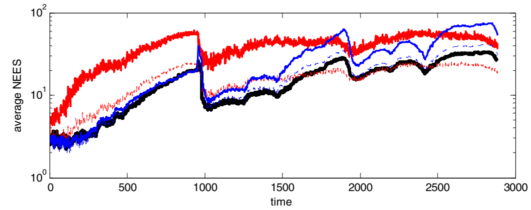
where N (=50) is the number of Monte Carlo runs. To evaluate the effects of $\hat{\mathbf{R}}$ and $\hat{\mathbf{b}}$ separately, we examine in two types of environment. In Monte Carlo test 1, we assumed that the "actual" measurement noise is a white Gaussian noise, and we used the only estimate of variance $\hat{\sigma}_\theta$. The actual σ_θ varies between 1-3 degree, and the static $\bar{\sigma}_\theta$,employed by the standard EKF-SLAM, are 0.5, 1, 3, and 6 degree. In Monte Carlo test 2, the actual b_θ varies between 0-4 degree as well as the actual σ_θ varies, and the static $\bar{\sigma}_\theta$ is 2 degree (the middle of the actual varying range). The other setting parameters are same as the single run simulation.

In Monte Carlo test 1, in case that $\bar{\sigma}_\theta$ is smaller than the actual value σ_θ (red lines), NEES of the standard EKF tends to get worse (Fig. 5-(b)), because the uncertainty $\mathbf{P}$ become smaller than the optimal owing to excessive reliance of measurement. To the contrary, in case that $\bar{\sigma}_\theta$ is larger than the actual value σ_θ (blue lines), RMSE of the standard EKF tends to get worse (Fig. 5-(a)), because the correcting quantity is small owing to distrust of the measurement. Generally speaking about the standard EKF, the larger difference between $\bar{\sigma}_\theta$ and σ_θ (solid color lines) is, the worse performance is. Consequently, Fig. 5 shows that the proposed algorithm (black line) is good in comparison with the standard EKF-SLAM from both standpoints. However, even the proposed algorithm cannot prevent the consistency from breaking, but only can reduce the breaking speed.

In Monte Carlo test 2, we also observed that the

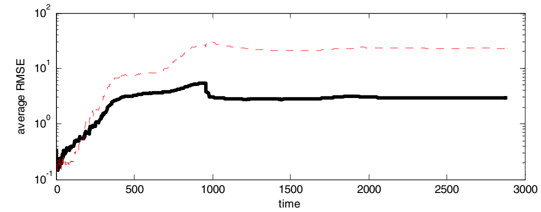


(a) Average RMSE of features.

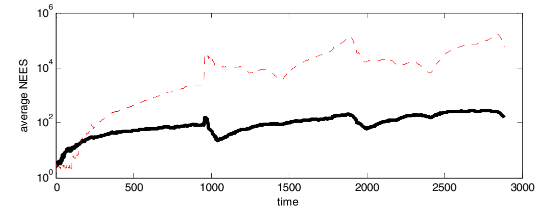


(b) Average NEES of the vehicle pose

Fig. 5. Results of Monte Carlo test 1 in case of a time-variant white noise. Black line presents the MNE-EKF, and color lines present the EKF with the static value of noise variance $\bar{\sigma}_\theta$: Red solid, red dashed, blue dashed, and blue solid present 0.5, 1, 3, and 6, respectively



(a) Average RMSE of features



(b) Average NEES of the vehicle pose

Fig. 6. Results of Monte Carlo test 2 in case of a time-variant biased noise; black line presents the MNE-EKF, red line presents EKF with $\bar{\sigma}_\theta$ =2 (the middle of the varying interval of σ_θ).

performance of the proposed algorithm (block line) is better than the standard EKF-SLAM's in the environment of a time-variant biased noise (Fig. 6). Unfortunately, the proposed algorithm rather worse than the standard EKF-SLAM until the time index 100, because of the oscillation of the estimate (Fig. 3). Therefore, if the actual noise is white Gaussian and we can know this fact, estimating the only variance $\hat{\mathbf{R}}$ is better, otherwise some technique to diminish the oscillation is recommended.

V. Conclusion

This paper presents an adaptive online approach to estimate the measurement noise distribution for the SLAM problem by using the mean and variance of innovation. The proposed method overcomes two practical problems of KF-based approaches; a time-variant noise and a color noise with non-zero mean. The performance of the proposed method is excellent in comparison with the standard EKF-SLAM in practical environment which includes a time-variant color noise. Based on this approach, we will keep researching on the subject of simultaneous estimating control and measurement noise.

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