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Fuzzy Logic Based Adjustment Control of a Cable-driven Auto-leveling Parallel Robot

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Abstract - **To solve the level-adjusting and force-tuning problems of high accurate and costly payloads when loading and unloading, a cable-driven auto-leveling parallel robot is developed. A hierarchical fuzzy controller, which has the ability to deal with the rule explosion problem, is proposed in this paper. After a brief introduction of the architecture of the closed-loop control system for the cable-driven auto-leveling parallel robot, the construction of the hierarchical fuzzy controller is set up, in which the force offsets of the four cables and the angle deviations of the two diagonal inclinations are chosen as input variables, and the output variables are the position changes of the four linear motion units. The hierarchical fuzzy controller contains two layers – the low level layer which generates two outputs for leveling adjustment and force tuning, and the high level layer which is used to coordinate the two outputs from the low level layer. Experimental results have demonstrated that the hierarchical fuzzy controller can achieve the control objectives with high regulation accuracy and short adjusting time, and can be easily applied to practical systems.**

I. INTRODUCTION

Nowadays, different kinds of crane systems are widely used for assembling of heavy equipments, movement of materials, and loading/unloading of fragile loads in many industrial areas [1]. Control methods, such as position regulation [2] and anti-swing control [3], etc, almost concentrate on the general crane models and are restricted in the control of hoisting and moving behaviors, under the assumption that the payload is only a particle. So they seldom consider the payload's dimension and its leveling adjustment. Actually, many payloads, such as aircrafts, satellites, and ship hulls, etc, which are heavy, valuable, fragile, and unfortunately eccentric, often need to be loaded and unloaded. Such payloads can't endure point-to-point and point-to-surface touch with the ground or the assembly platform. Besides, their centers of mass usually deviate from their geometric centers. This would cause inclination and lateral forces when loading and unloading, which usually leads to payloads' distortion, damage, or even complete destruction.

To solve this problem, various methods and mechanisms, such as link parallel platforms [4], cable parallel platforms [5], hybrid parallel platforms (combinations of link structures and cable structures) [6], and weight compensation mechanisms [7], etc, have been studied and applied. Theoretically, these methods all could be used for leveling adjustment, but have different defects [8]. Compared with other methods, cable based techniques own many merits, such as large workspaces, strong pulling forces, and fast regulating speed. Manual regulating devices using four cables are universally adopted in practical applications due to these merits. However, cable based research activities are mainly focused on the analysis of workspaces [9], the central position and pose of payloads [10], kinematics characteristics [11], and dynamic characteristics [12]. No studies have yet been reported on the leveling control of payloads' junction surface and the equilibrium problem of cables' pulling forces. Meanwhile, such manual devices have many obvious disadvantages, such as great labor intensity, low efficiency, low precision, and underlying trouble in safety.

Recently, based on the analysis of merits and drawbacks of the above techniques and mechanisms, and combining with the widely used manual regulating devices, we have designed a cable-driven auto-leveling parallel robot, whose prototype is shown in Fig. 1. It mainly consists of a rectangular worktable with four linear motion units fixed in each diagonal end, four cables seperately passing through the axial line of one linear motion unit, two angle sensors

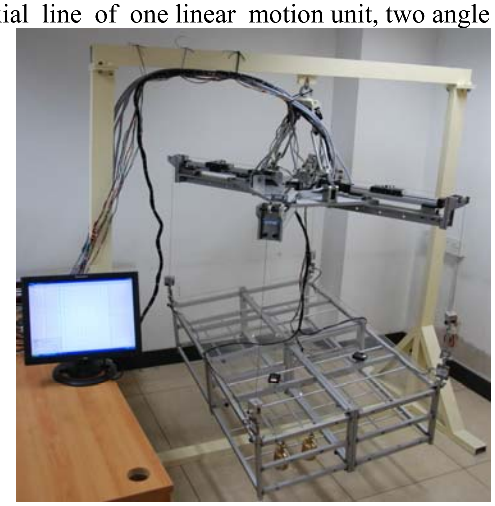

Fig. 1. The cable-driven auto-leveling parallel robot

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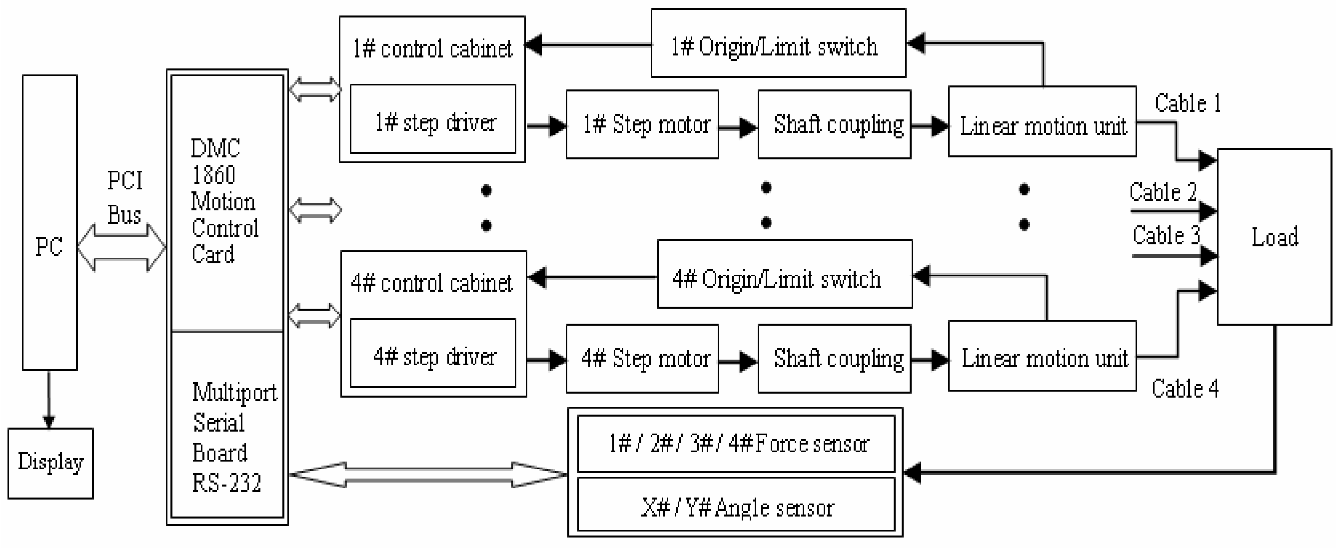

Fig. 2. Architecture of the closed-loop control system

fixed in two diagonals of the payload, four force sensors seperately connected with one cable, a rectangular payload, centroid position of which can be changed through adding or taking off heavy weights, and a computer control platform.

A 2D model for the robot is established in [13], but for the three dimensional payload, the accurate 3D model is hard to obtain for the reasons given in section II. Designing conventional controllers for the cable-driven auto-leveling parallel robot is a complex and arduous job. Fuzzy logic control [14,15], which has been proved to be a practical alternative for a variety of challenging control applications that are difficult to be solved by classical methods, can sufficiently incorporate human knowledge or experience into system design. Therefore, it may be a good choice to use fuzzy logic controllers to regulate the payloads.

However, the conventional ways of designing fuzzy logic controllers face the difficulties to transform human experience into the rule base, especially when the numbers of input variables are large. For example, for this cable-driven auto-leveling parallel robot, there are 6 inputs. If we put all the input items into the antecedent part of each fuzzy rule, a huge rule base is needed. To deal with the rule explosion problem, the most widely used method is the hierarchical scheme which has a nice property that the total number of rules increases linearly rather than exponentially [16-21].

Therefore, in this paper, a hierarchical fuzzy controller is designed for the leveling adjustment and force tuning of the cable-driven self-leveling parallel robot. Two layers are contained in the hierarchical fuzzy controller. One is the low level layer used to generate outputs for force tuning of cables and leveling adjustment of the robot's bottom surface, the other is the high level layer used to accomplish the function of coordinating the two outputs from the low level layer. In our experiment, we implement this hierarchical fuzzy controller in the cable-driven auto-leveling parallel robot. From experimental results, we can see that the hierarchical fuzzy controller can achieve the leveling adjustment of the cable-driven auto-leveling parallel robot.

The paper is organized as follows: Section II presents the detailed architecture of the closed-loop control system for the cable-driven auto-leveling parallel robot. In Section III, the hierarchical controller for the cable-driven auto-leveling parallel robot, which contains a low level layer and a high level layer, is developed. Experimental results that show the performance of the proposed hierarchical controller on the cable-driven self-leveling parallel robot are described in Section IV. Concluding remarks are given in Section V.

II. ARCHITECTURE OF THE CLOSED-LOOP CONTROL SYSTEM

The architecture of the closed-loop control system for the cable-driven parallel robot is shown in Fig. 2 in detail. It mainly contains: 1) four control cabinets(1#/2#/3#/4#) as the actuators, each of which contains a set of motor, motor driver, shaft coupling, linear motion unit, original switch, and two limit switches; 2) four cables, which are used to the connect linear motion units and corresponding force sensors; 3) four force sensors(1#/2#/3#/4#) which measure the pulling forces of the four cables (denoted as F_1, F_2, F_3 and F_4), and two angle sensors(X# / Y#) which measure the dihedral angle of the payload's bottom (or upper) surface (denoted as θ_x and θ_y); 4) a computer control platform using the scheme of "PC + DMC1860 motion control card", which processes and analyses real-time data acquired from the four force sensors and the two angle sensors through multi-port serial board (RS232), and then sends out commands to the four control cabinets to realize movement control.

The control objectives are as follows: 1) leveling of the bottom surface: the dihedral angle of payload's bottom surface (denoted as θ) should be smaller than 0.2°; 2) force tuning: for safety purpose the pulling force of each cable should almost be balanced, i.e., the deviation of each cable's pulling force should be controlled between 70% and 130% of their average; 3) setting time: the whole adjusting time is less than 60 seconds.

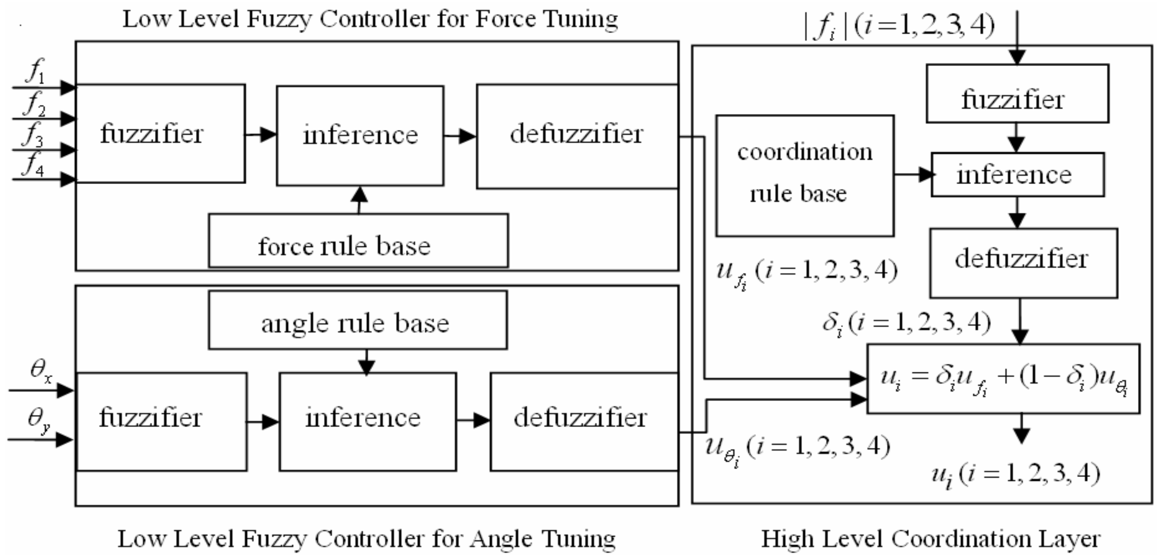

Fig.3. Structure of the hierarchical fuzzy controller

Zhang et al [13] have given the two dimensional model for the cable-driven auto-leveling parallel robot. But for three dimensional payloads, the accurate mathematical model is difficult to get as the payload mass is assumed to be unknown, and the centroid position is uncertain e. Moreover, high levels of uncertainties resulting from inevitable measurement errors and the resolution limits of the sensors, and vibration and sloshing of payloads, exist in this practical application. Therefore, designing conventional controllers for the robot becomes hard. Fuzzy logic [14,15] is an effective method for many engineering problems that can't be solved by classical methods, because it can effectively incorporate human knowledge or experience into system design. Therefore, using fuzzy logic controllers to adjust the payload may be a good choice.

As a whole, the fuzzy logic controller for the cable-driven auto-leveling parallel robot should have six inputs and four outputs. Here, we define $F_{ave}=\sum_{i=1}^{4}F_i/4$, and the force offsets of the four cables $f_i=(F_i-F_{ave})/F_{ave}(i=\ 1,2,3,4)$. The input variables are the force offsets f_1,f_2,f_3,f_4, and the angle deviations θ_x,θ_y of the two diagonal inclinations. The output variables are the length changes of the u_1,u_2,u_3,u_4 generated by the four linear motion units. In the following, we will design a fuzzy logic based controller to achieve our control goal – leveling of the payload bottom surface and force tuning of the four cables.

III. Hierarchical Fuzzy Controller for the Cable-driven Parallel Robot

We have known that the fuzzy logic based controller has 6 inputs – the force offsets $f_1,\ldots,f_4$, and the angle deviations θ_x,θ_y. Assume that 3 fuzzy sets are defined for each force offset f_i (i=1, 2, 3, 4) and 5 fuzzy sets are assigned for the angle deviations θ_x, θ_y. Using the conventional fuzzy logic control scheme that puts all the input items into the antecedent part of each fuzzy rule, we need to determine $3^4\times5^2=1525$ rules for the rule base, which is difficult to design. Meanwhile, θ_x,θ_y and its pulling force will be changed consequently and similarly when each cable's length is elongated (or shortened).

To deal with the rule explosion problem and considering the symmetry, in this study, we design a hierarchical fuzzy logic controller for the cable-driven parallel robot, which has a nice property that the total number of rules increases linearly rather than exponentially. The structure of the hierarchical fuzzy controller for the cable-driven auto-leveling parallel robot is shown in Fig. 3 in detail. From this figure, we can see that the hierarchical fuzzy controller contains two layers – the low level layer and the high level layer. In the low level layer, we design two fuzzy logic sub-controllers, which are used to generate two outputs for the force tuning of the four cables and the leveling of the bottom surface. The high level layer accomplishes the function of coordinating the two outputs from the low level layer according to the force offset of the ith (i=1,2,3,4) cable. And, four outputs u_i (i=1,2,3,4) which have the similar structures, are included in the hierarchical fuzzy controller. In the following, we will discuss how to develop the low level controllers and the high level coordinator in detail.

A. Low Level Fuzzy Sub-controller for Force Tuning

This sub-controller is used to get the ideal length changes u_{f_i} (i=1,2,3,4) needed for the force tuning to make the forces of the four cables change in a reasonable bound, without considering leveling of the payload bottom surface. This sub-controller has four inputs $f_1,\ldots,f_4$ and each of its input items is assigned with three fuzzy sets, the membership functions of which are depicted in Fig.4. (a). The output of this sub-controller for the *i*th (i=1,2,3,4) cable is u_{f_i} (i=1,2,3,4), and it is assigned with five fuzzy sets, the

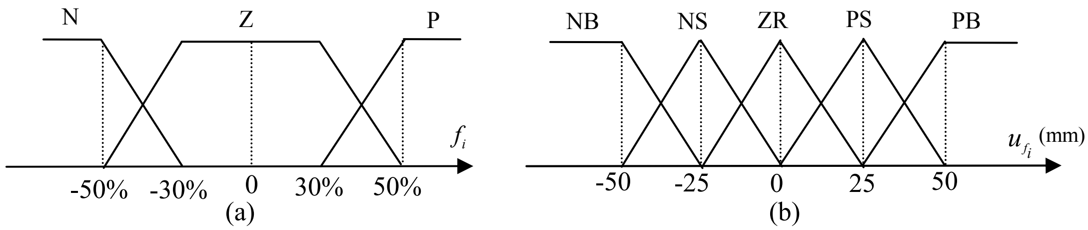

Fig.4. (a) membership functions of f_i (i=1,2,3,4) in the force rule base, (b) membership functions of u_{f_i} (i=1,2,3,4)

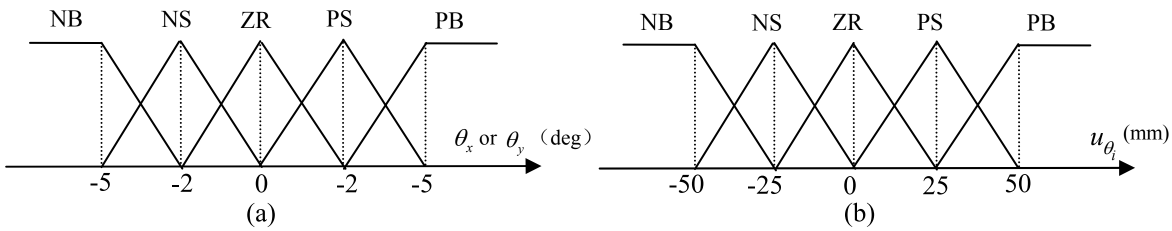

Fig. 5 (a) membership functions of θ_x and θ_y in the angle rule base, (b) membership functions of u_{θ_i} (i=1,2,3,4)

membership functions of which are shown in Fig.4. (b). Theoretically, we should determine 3^4 =81 rules in the force rule base for this low level sub-controller. But, considering the fact that $f_1+...+f_4=0$, less rules will be needed. From our experience obtained in many experiments, we have determined 59 rules for the force rule base. Taking the force rule base for the first cable f_1 for example, some of the rules are listed as follows:

If f_1 is P, f_2 is P, f_3 is P, f_4 is N, then u_{f_1} is NB,
If f_1 is P, f_2 is P, f_3 is N, f_4 is P, then u_{f_1} is NS,
If f_1 is P, f_2 is Z, f_3 is N, f_4 is Z, then u_{f_1} is ZR,
If f_1 is P, f_2 is P, f_3 is P, f_4 is N, then u_{f_1} is NB,
If f_1 is P, f_2 is P, f_3 is N, f_4 is P, then u_{f_1} is NS,
. . . . . .
If f_1 is P, f_2 is Z, f_3 is N, f_4 is Z, then u_{f_1} is ZR,
If f_1 is N, f_2 is P, f_3 is N, f_4 is P, then u_{f_1} is PB,
If f_1 is N, f_2 is N, f_3 is P, f_4 is P, then u_{f_1} is PS,
If f_1 is N, f_2 is P, f_3 is P, f_4 is P, then u_{f_1} is PB.

Then, using the singleton fuzzifier, the product inference engine, and the weighted-average defuzzifier, the output of this low level sub-controller can be expressed by

$$u_{f_1} = \frac{\sum_{k=1}^{59} w^k h^k}{\sum_{k=1}^{59} h^k},$$

where w^k (k=1,…,59) is the center of the consequent fuzzy set in the *k*th (k=1,…,59) rule, and h^k (k=1,…,59) is the firing strength of the antecedent part in the *k*th (k=1,…,59) rule, and can be expressed as

$$h^k = \mu_{\tilde{A}_1^k}(f_1) * \mu_{\tilde{A}_2^k}(f_2) * \mu_{\tilde{A}_3^k}(f_3) * \mu_{\tilde{A}_4^k}(f_4),$$

in which, $\tilde{A}_i^k$ is one of the three fuzzy sets of the *i*th (i=1,2,3,4) input item f_i (i=1,2,3,4).

B. *Low Level Fuzzy Sub-controller for Angle Tuning*

This sub-controller is used to obtain the ideal length changes u_{θ_i} (i=1,2,3,4) needed for the payload leveling adjustment, without considering the force tuning of the four cables. This sub-controller has two inputs, angle deviations θ_x , θ_y , and each is assigned with five fuzzy sets, the membership functions of which are depicted in Fig. 5 (a). The output of this sub-controller for the *i*th (i=1,2,3,4) cable is u_{θ_i} (i=1,2,3,4), and it is assigned with five fuzzy sets too, the membership functions of which are shown in Fig. 5 (b). Here, we adopt 25 fuzzy rules for this sub-controller. Taking the force rule base for the first cable for example, the rule table for u_{θ_1} is shown in Table 1 and u_{θ_i} (i=2,3,4) has the similar rule tables.

TABLE 1 RULE TABLE FOR u_{θ_1}

u_{θ_1}		θ_x				
		NB	NS	ZR	PS	PB
θ_y	NB	ZR	NS	NS	PS	PB
	NS	NS	ZR	NS	PS	PB
	ZR	NS	NS	ZR	PS	PS
	PS	NB	NS	PS	ZR	PS
	PB	NB	NS	PS	PS	ZR

Also, using the singleton fuzzifier, the product inference engine, and the weighted-average defuzzifier, the output of this low level sub-controller can be expressed by

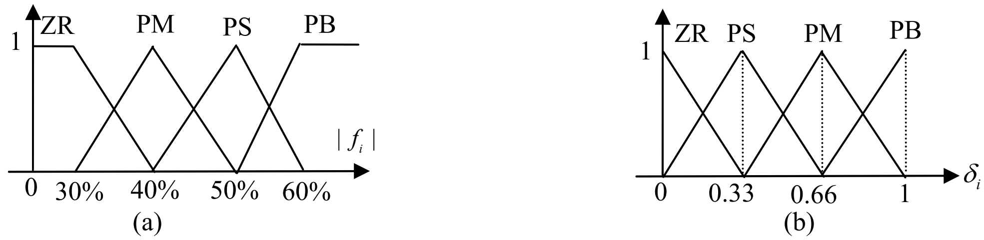

Fig. 6(a) membership functions of $| f_i |$ in the high level layer, (b) membership functions of the coordination operation δ_i (i=1,2,3,4)

$$u_{\theta_1} = \frac{\sum_{k=1}^{25} \omega^k \eta^k}{\sum_{k=1}^{25} \eta^k},$$

where ω^k is the center of the consequent fuzzy set in the *k*th (k=1,…,25) rule, and η^k is the firing strength of the antecedent part in the *k*th (k=1,…,25) rule, and can be expressed as

$$\eta^k = \mu_{\tilde{A}^k_{\theta_x}}(\theta_x) * \mu_{\tilde{A}^k_{\theta_y}}(\theta_y),$$

in which, $\tilde{A}^k_{\theta_x}, \tilde{A}^k_{\theta_y}$ is one of the five fuzzy sets of the input item θ_x, θ_y, respectively.

C. High Level Coordination Layer

The high level layer is used to accomplish the function of coordinating the two outputs from the low level sub-controllers according to the absolute force offset of the *i*th (i=1,2,3,4) cable. To achieve this goal, a coordination factor δ_i (i=1,2,3,4) is used, which can be deduced by the following fuzzy inference. And, with coordination operator, the output of the hierarchical fuzzy controller can be obtained as

$$u_i = \delta_i u_{f_i} + (1-\delta_i) u_{\theta_i} \quad (1)$$

To obtain better performance, here, the domain of the force offset of the *i*th (i=1,2,3,4) cable will be covered with four fuzzy sets shown in Fig. 6 (a). The coordination operator δ_i (i=1,2,3,4) is also assigned with four fuzzy sets shown in Fig. 6 (b). And the rules in this high level coordination layer are:

If $| f_i |$ is ZR, then δ_i is ZR;
If $| f_i |$ is PS, then δ_i is PS;
If $| f_i |$ is PM, then δ_i is PM;
If $| f_i |$ is PB, then δ_i is PB.

Once more, using the singleton fuzzifier, the product inference engine, and the weighted-average defuzzifier, the coordination operator δ_i (i=1,2,3,4) can be computed as

$$\delta_i = \frac{\mu_{ZR}(| f_i |) w_{ZR} + \mu_{PS}(| f_i |) w_{PS} + \mu_{PM}(| f_i |) w_{PM} + \mu_{PB}(| f_i |) w_{PB}}{\mu_{ZR}(| f_i |) + \mu_{PS}(| f_i |) + \mu_{PM}(| f_i |) + \mu_{PB}(| f_i |)},$$

where $w_{ZR}, w_{PS}, w_{PM}, w_{PB}$ are the centers of the fuzzy sets ZR, PS, PM, PB shown in Fig. 6 (b). Then, from (1), the output of the designed hierarchical fuzzy controller can be computed to control the lengths of the four cables to realize the control objectives.

From the design process above, we can see that, for the hierarchical fuzzy controller, only 59+25+4=81 rules are needed for each cable. The number of the rules in the hierarchical fuzzy rule base is much less than that in the conventional fuzzy control scheme.

IV. Experimental Results

This section is devoted to verify experimentally the performance of the hierarchical fuzzy controller for the cable-driven parallel robot. After adding some heavy weights, initial states of the payload are as follows: θ_x, θ_y, F_1, F_2, F_3, F_4, $1.3F_{ave}$ and $0.7F_{ave}$ are about 4.47°, 5.71°,

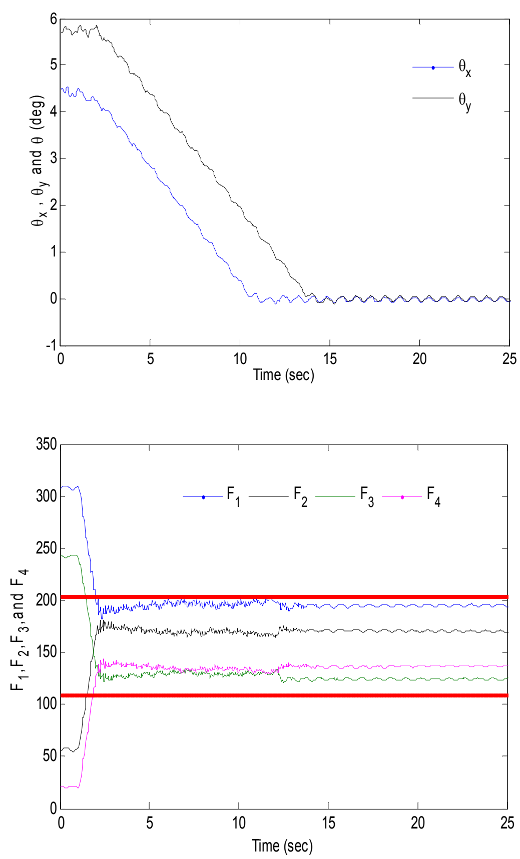

Fig. 7. Adjustment curves of leveling adjustment and force tuning.

306.9N, 56.2N, 243N, 21.3N, 204N and 110N respectively. We can see that θ_x and θ_y are much larger than 0.2º, meanwhile, F_1, F_3 are much larger than $1.3F_{ave}$, and F_2, F_4 are much smaller than $0.7F_{ave}$, i.e., the pulling forces of the four cables are extremely unbalanced. Fig. 7 gives control results of the hierarchical fuzzy controller, from which we can see the corresponding change trends of the angles θ_x, θ_y of the two diagonal inclinations and the pulling forces F_1, F_2, F_3, F_4. The hierarchical fuzzy controller started at about 1 second, and ended at about 15 second. Fig. 7 shows that the adjusting time is much less than 60 seconds, and the precision of junction surface angle θ can approach to 0° when θ_x and θ_y are gradually adjusted to be very small. Meanwhile, F_1, F_2, F_3 and F_4 are regulated and kept between $1.3F_{ave}$ and $0.7F_{ave}$, that is, the hierarchical fuzzy controller can achieve the control objectives-leveling adjustment of the bottom surface and force tuning synchronously and effectively in a very short time.

V. CONCLUSION

In order to realize the leveling adjustment of the cable-driven parallel robot's bottom surface and force tuning of its four pulling cables, and to deal with the rule explosion problem, a hierarchical fuzzy controller which has a nice property that the total number of rules increases linearly rather than exponentially, is proposed in this paper. To achieve the control objectives, four such similar controllers are needed. In the construction of each hierarchical fuzzy controller, the force offsets f_1, f_2, f_3, f_4 of the four cables and the angles θ_x, θ_y of the two diagonal inclinations are chosen as input variables. And the output variables are the length changes of the u_1, u_2, u_3, u_4 generated by the four linear motion units.. The hierarchical fuzzy controller contains two layers: the low level layer and the high level layer. Two fuzzy logic sub-controllers, which are used to generate two outputs for the force tuning of the four cables and the leveling of the bottom surface, are designed in the low level layer. The high level layer accomplishes the function of coordinating the two outputs from the low level sub-controllers according to the force offset of the *i*th (i=1,2,3,4) cable. Experimental results have demonstrated that the hierarchical fuzzy controller can achieve the control objectives with high accuracy of regulation and short adjusting time, and can be easily applied to practical device.

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