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Easy and Fast Evaluation of Grasp Stability by using Ellipsoidal Approximation of Friction Cone

*Tokuo TSUJI, Kensuke HARADA, and Kenji KANEKO

Abstract— **This paper presents an easy and fast method of testing force closure when a multi-fingered hand grasps an object. Different from previous methods, we consider approximating the friction cone by using a few ellipsoids. By using this method, the total force/moment set applied to the grasped object can be obtained by a common set of multiple ellipsoids. This method is effective since we can test the force closure by simply calculating the inequalities of quadratic form. Moreover, by using the ellipsoidal approximation, we propose an easy method of evaluating the grasp stability. We show that the grasp stability can be calculated by using simple equations. The effectiveness of the proposed method is verified by several numerical examples where we show that the proposed method is fairly accurate and can evaluate the grasp stability faster than conventional methods.**

I. INTRODUCTION

Multi-fingered hand has potential ability to grasp various objects with different shape, weight or surface friction. A robot with multi-fingered hands is expected to be used for personal service assistants. In such situation, after the robot measures an object position and orientation, the grasp planning should be performed in realtime. However, the grasp planning is complex since we have to search for the contact point satisfying the following conditions; 1) the multi-fingered hand grasps the object without dropping it out from the hand, 2) each finger avoids unexpected collision with the environment, and 3) each finger joint keeps its movable range.

Among the above conditions, we focus on the first one and propose a fast and easy method of evaluating the grasp stability. The multi-fingered hand can keep on grasping the object if each finger can exert the contact forces onto the object and the resultant of contact forces balances any direction of external force/moment. This criteria is known as the force closure. Based on this concept, grasp stability can be evaluated. In most of the previous researches, The friction cone at each contact point has been approximated by using the polyhedral convex cone. In this case, the force closure is tested by solving the linear programming problem. The grasp planning has been a time consuming calculation since we have to search for the grasping posture satisfying the force closure by solving the linear programming problem for each iteration.

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The authors are with Humanoid Research Group, Intelligent Systems Research Institute, National Institute of Advanced Industrial Science and Technology (AIST), Japan tokuo.tsuji@aist.go.jp

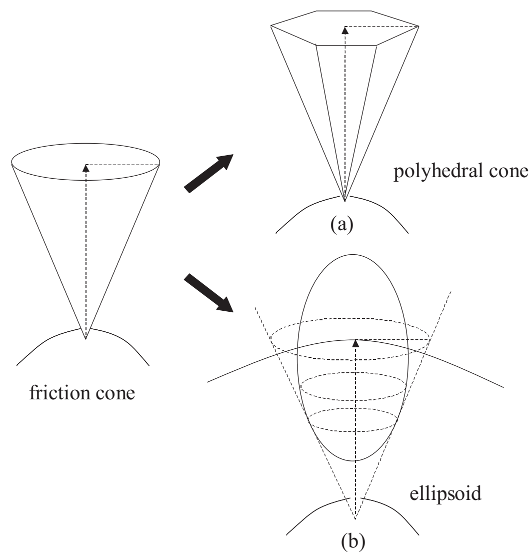

Fig. 1. Approximation of friction cone

In this case, as the number of span vectors of the polyhedral convex cone increases, we can approximate the friction cone accurately. However, at the same time, the calculation time of the force closure will increase. On the other hand, if we want to save the calculation time, we have to approximate the friction cone roughly by using smaller number of span vectors.

Knowing the above problems of force closure where the friction cone is approximated by using the convex polyhedral cone, this paper proposes a novel method of testing the force closure where the friction cone is approximated by using a few ellipsoids. Different from the conventional methods, we can test the force closure by calculating just 2^n inequalities of quadratic form for n-fingered grasping if the friction cone is approximated by using two ellipsoids. This paper also proposes a simple method of evaluating the grasp stability by using the ellipsoidal approximation of friction cone.

The rest part of this paper is organized as follows. In chapter II, we describe a general method of force closure testing. In section III.a, each friction cones is approximated by an ellipsoid, a calculation method for resultant force is formalized and the relationship between the formalization and Minkowski sum is analyzed. In section III.b, the method of product set of all ellipsoids for accurate approximation on each contact point is described. In chapter III.c, fast calculation method of grasp stability is described [1]. In chapter IV, the effectiveness of the proposed method is verified by 2D and 3D simulation.

II. Force closure

When a multi-fingered hand grasps an object, an appropriate force/moment has to be generated onto the grasped object in order to balance any direction of the external force/moment. This condition is satisfied if the set of resultant force/moment applied by each finger includes the origin. This condition is known as the force closure. The set of resultant force/moment applied to the object can be obtained by calculating the sum of contact force applied by each finger where the contact force is limited inside fiction cone.

Let us consider the contact force $\boldsymbol{f}_i$ $(i = 1, \cdots, m)$ applied at the i-th contact point position $\boldsymbol{p}_i$. The wrench $\boldsymbol{w}_i$ generated by $\boldsymbol{f}_i$ can be calculated as

$$\boldsymbol{w}_i = \begin{bmatrix} \boldsymbol{f}_i \\ \gamma \boldsymbol{p}_i \times \boldsymbol{f}_i \end{bmatrix}. \quad (1)$$

where γ is torque magnitude which is scaled with respect to the force magnitude. The space spanned by all contact forces is called the grasp wrench space (GWS). Ferrari and Canny[1] proposed two methods of generating GWS. One method generates GWS (W_{L_∞}) by calculating the sum of all combination of contact forces. It is the Minkowski sum of the wrench applied by each finger. The other method generates convex hull of contact wrenches as GWS (W_{L_1}). The set of GWS, W_{L_∞} and W_{L_1} are expressed as,

$$W_{L_\infty} = \{\oplus_{i=1}^{m} \boldsymbol{w}_i | \boldsymbol{w}_i \in W_i\} \quad (2)$$

$$W_{L_1} = ConvexHull(\{\cup_{i=1}^{m} \boldsymbol{w}_i | \boldsymbol{w}_i \in W_i\}) \quad (3)$$

where $\oplus$ is Minkowski sum, W_i is a set of the effect wrench applied at i-th each contact point. When GWS contains the origin of wrench space, we say that the grasp is force closure. If a set of vectors, $\boldsymbol{w}_i \in W_i$ positively spans $\Re^6$, then both W_{L_∞} and W_{L_1} will contain the origin. For testing whether or not, GWS contains the origin, W_{L_1} is useful because of its fast calculation. However, for the purpose of dynamic motion planning of grasped object and more accurate grasp stability evaluation, W_{L_∞} is needed. Since W_{L_∞} space shows the range which can counter external wrench or inertial wrench in any direction.

For constructing W_{L_∞}, the friction cone has been approximated by using a polyhedral convex cone. In this case, if we consider accurately approximating the friction cone by using a larger number of the faces, it results in the dramatic increase of the calculation cost. Let m and n be the number of finger and the number of face of the polyhedral cone, respectively. Its calculation cost is represented as $O(n^m)$. Reduction of the calculation cost of W_{L_∞} is important issue for grasp planning.

As for the research on force closure, Reuleaux[2] discussed force closure used in classical mechanics. Ohwovoriole[3], Salisbury and Roth[4] introduced it into the research field of robotics. Mishra, Schwartz, and Sharir[5], Nguyen[6], and other researchers[7]-[9] investigated the construction of force closure grasp by a robotic hand. Kerr and Roth[10], Nakamura, Nagai, and Yoshikawa[11], and Ferrari and Canny[1] discussed the optimal grasp under force closure.

Linear matrix inequality (LMI) method[12][13][14] is a fast method of force closure testing. LMI method can find a solution which satisfying the friction constraint quickly. Some methods are derived from LMI method, such as ray tracing method[15][16], and heuristics approach[17]. They are fast method for testing force closure and find optimal solution. However, the methods based on LMI don't generate the whole W_{L_∞}.

Borst[18] proposed a method of incremental expansion of subspace of in the weakest direction of W_{L_∞}. It is faster than conventional methods for grasp stablity evaluation of W_{L_∞}. However, the method doesn't generate the whole W_{L_∞}, since the method do not expand the subspace of W_{L_∞} in the strong direction. A much faster method for generating the whole W_{L_∞} is desirable for many applications[19]-[28].

This paper proposes a fast method of grasp stability calculation. Our method generates a good approximation of the whole W_{L_∞}. We show that our calculation is much faster than the other methods. However, more distinctive merit of our approach is that we can very easily calculate the grasp stability. Our method simply checks inequalities for force closure judgment and does not need to construct convex hull or to solve linear programming problem. If we approximate the friction cone by using a single ellipsoid, the force closure can be confirmed by checking only one inequality. Also, the grasp quality of W_{L_∞} can be calculated by using one or two simple equations.

III. Ellipsoidal approximation of friction cone

A. The resultant force set

As shown in Fig.1 (b), we approximate friction cone by ellipsoid. Considering the ith contact point $\boldsymbol{p}_i$ on an object with contact force $\boldsymbol{f}_i$ $(i = 1, \cdots, m)$, the set of the contact force can be expressed by the ellipsoid as

$$(\boldsymbol{f}_i - f_c \boldsymbol{n}_i)^t \boldsymbol{U}_i \boldsymbol{S} \boldsymbol{U}_i^t (\boldsymbol{f}_i - f_c \boldsymbol{n}_i) \leq 1 \quad (4)$$

where $\boldsymbol{S} = \mathrm{diag}[1/a^2 \;\; 1/b^2 \;\; 1/c^2]$ and a, b and c are the axis length of the ellipsoid, $\boldsymbol{U}_i$ is a 3×3 matrix composed of the unit normal vector and the unit tangent vector of the contact surface. $\boldsymbol{n}_i$ is a unit normal vector and f_c is defined as a distance between the ellipsoid center and the contact point. Here, we assume that the arbitrary contact force can be applied within the limit of Eq.(4). It means that we assume that each finger has at least 3 DOF and that only the fingertip contacts the object.

Summation of this equation for all contact points can be calculated as

$$\sum_{i=1}^{m} (\boldsymbol{f}_i - f_c \boldsymbol{n}_i)^t \boldsymbol{U}_i \boldsymbol{S} \boldsymbol{U}_i^t (\boldsymbol{f}_i - f_c \boldsymbol{n}_i) \quad (5)$$

$$= (\boldsymbol{F} - f_c \boldsymbol{N})^T \boldsymbol{T} (\boldsymbol{F} - f_c \boldsymbol{N}) \leq m \quad (6)$$

where

$$\begin{aligned} \boldsymbol{F} &= [\boldsymbol{f}_1^t \ \cdots \ \boldsymbol{f}_m^t]^t, \\ \boldsymbol{N} &= [\boldsymbol{n}_1^t \ \cdots \ \boldsymbol{n}_m^t]^t, \\ \boldsymbol{T} &= \text{block diag}[\boldsymbol{U}_1\boldsymbol{S}\boldsymbol{U}_1^t \ \cdots \ \boldsymbol{U}_m\boldsymbol{S}\boldsymbol{U}_m^t]. \end{aligned}$$

An equation w.r.t. the resultant force/moment can be derived from this equation. The resultant force/moment $\boldsymbol{w}$ can be defined by

$$\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N} = \boldsymbol{G}(\boldsymbol{F} - f_{\mathrm{c}}\boldsymbol{N}) \tag{7}$$

where

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{I} & \cdots & \boldsymbol{I} \\ \gamma\boldsymbol{p}_1\times & \cdots & \gamma\boldsymbol{p}_m\times \end{bmatrix}.$$

Note that even if Eq.(6) is satisfied, Eq.(4) is not always satisfied. Therefore, we define $\boldsymbol{G}^{\#}$ which is the weighted pseudo-inverse matrix of $\boldsymbol{G}$, so that all forces satisfy Eq.(4). Let $\boldsymbol{G}^{\#}$ be defined as

$$\boldsymbol{G}^{\#} = \boldsymbol{T}^{-1}\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}. \tag{8}$$

The $\boldsymbol{G}^{\#}$ give the least norm solution for $\boldsymbol{F}^T\boldsymbol{T}\boldsymbol{F}$. Then, by minimizing $\boldsymbol{F}^T\boldsymbol{T}\boldsymbol{F}$, the all forces are likely to be inside of ellipsoid of Eq.(4). Since $\boldsymbol{F}$ is weighted dependent on each ellipse radius. By using $\boldsymbol{G}^{\#}$, Eq.(7) can be solved w.r.t. $\boldsymbol{F} - f_{\mathrm{c}}\boldsymbol{N}$;

$$\boldsymbol{F} - f_{\mathrm{c}}\boldsymbol{N} = \boldsymbol{G}^{\#}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N}) + (\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})k. \tag{9}$$

From Eq.(6) and Eq.(9), we have

$$\begin{aligned} &(\boldsymbol{F} - f_{\mathrm{c}}\boldsymbol{N})^T\boldsymbol{T}(\boldsymbol{F} - f_{\mathrm{c}}\boldsymbol{N}) \\ &\quad = \{(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N})^T(\boldsymbol{G}^{\#})^T + k^T(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})^T\}\boldsymbol{T} \\ &\qquad\qquad \{\boldsymbol{G}^{\#}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N})(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})\boldsymbol{k}\} \\ &\quad = (\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N})(\boldsymbol{G}^{\#})^T\boldsymbol{T}\boldsymbol{G}^{\#}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N}) \\ &\qquad + 2\boldsymbol{k}^T(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})^T\boldsymbol{T}\boldsymbol{G}^{\#}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N}) \\ &\qquad + \boldsymbol{k}^T(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})^T\boldsymbol{T}(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})\boldsymbol{k}. \end{aligned} \tag{10}$$

A right-hand side of the Eq.(10) can be simplified as follows; First, the first term of the right-hand side of Eq.(10) can be simplified as

$$\begin{aligned} &(\boldsymbol{G}^{\#})^T\boldsymbol{T}\boldsymbol{G}^{\#} \\ &\quad = (\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{T}\boldsymbol{T}^{-1}\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\quad = (\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}. \end{aligned} \tag{11}$$

Then, the second term of the right-hand side of Eq.(10) is equal to zero since

$$\begin{aligned} &(\boldsymbol{I} - \boldsymbol{G}^{\#}\boldsymbol{G})^T\boldsymbol{T}\boldsymbol{G}^{\#} = \boldsymbol{T}\boldsymbol{G}^{\#} - \boldsymbol{G}^T(\boldsymbol{G}^{\#})^T\boldsymbol{T}\boldsymbol{G}^{\#} \\ &\quad = \boldsymbol{T}\boldsymbol{T}^{-1}\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\qquad -\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{T}\boldsymbol{T}^{-1}\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\quad = \boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\qquad -\boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\quad = \boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} - \boldsymbol{G}^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1} \\ &\quad = \boldsymbol{O} \end{aligned} \tag{12}$$

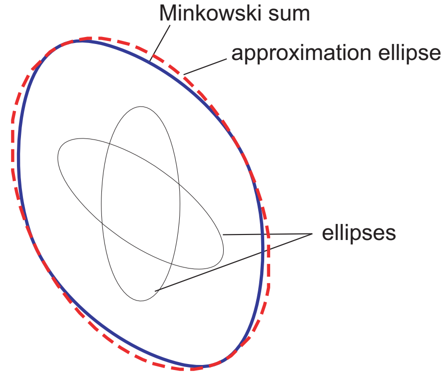

Fig. 2. Minkowski sum of two ellipses and its approximation

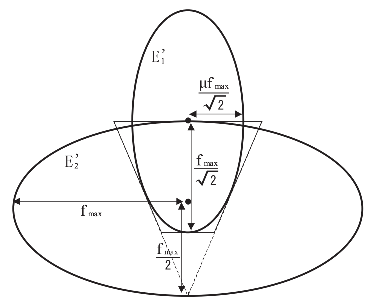

Fig. 3. Ellipsoid and friction cone

Finally, the third term of the right-hand side of Eq.(10) is always greater than $\mathbf{0}$ since T is positive definite. Therefore, by substituting Eq.(11) and Eq.(12) to Eq.(10), we obtain,

$$e(\boldsymbol{w}) = (\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N})^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N}) \le m. \tag{13}$$

Thus, we can obtain the inequality about the resultant force/moment $\boldsymbol{w}$. Therefore, we can easily confirm stable grasp condition by linear calculation.

The set of GWS W' calculated by ellipsoidal approximation is defined as,

$$W' = \{\boldsymbol{w} | (\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N})^T(\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}(\boldsymbol{w} - f_{\mathrm{c}}\boldsymbol{G}\boldsymbol{N}) \le m\}. \tag{14}$$

Here, we note that Eq.(13) is an equation of ellipsoid. However, even if we approximate the friction cone at each contact point by an ellipsoid, the Minkowski sum of the wrench does not always be an ellipsoid. Eq. (13) shows inequality of ellipsoid approximating the Minkowski sum by an ellipsoid. In Fig.2, the blue closed curve represents the Minkowski sum of two ellipses, and the red line represents an ellipsoid calculated by Eq.(13). As a result, Eq. (13) generates accurate approximation of Minkowski sum of ellipses.

B. Common set of ellipsoid

It is difficult to approximate friction cone accurately by only one ellipsoid. Thus consider extending the proposed method by defining more than two ellipsoids. In this case, the friction cone is approximated by the common set of multiple ellipsoids. First, we show the method of approximating the friction cone by using two ellipsoid where one is tangent to the side face and the other ellipsoid is tangent to the bottom face. The equation of each ellipsoid is defined as,

$$(\boldsymbol{f}_i - f_{\max}\boldsymbol{n}_i)^t \boldsymbol{U}_i \boldsymbol{S}_1 \boldsymbol{U}_i^t (\boldsymbol{f}_i - f_{\max}\boldsymbol{n}_i) \le 1 \quad (15)$$
$$(\boldsymbol{f}_i - f_{\max}\boldsymbol{n}_i/2)^t \boldsymbol{U}_i \boldsymbol{S}_2 \boldsymbol{U}_i^t (\boldsymbol{f}_i - f_{\max}\boldsymbol{n}_i/2) \le 1 \quad (16)$$

where $f_{\max}$ is maximum contact force, μ is friction coefficient, and

$$\boldsymbol{S}_1 = \mathrm{diag}\left[\left(\frac{\sqrt{2}}{\mu f_{\max}}\right)^2 \left(\frac{\sqrt{2}}{\mu f_{\max}}\right)^2 \left(\frac{\sqrt{2}}{f_{\max}}\right)^2\right]$$
$$\boldsymbol{S}_2 = \mathrm{diag}\left[\left(\frac{1}{f_{\max}}\right)^2 \left(\frac{1}{f_{\max}}\right)^2 \left(\frac{2}{f_{\max}}\right)^2\right].$$

Fig.3 approximates the friction cone by using the cross section of two ellipsoids. Here, if we define the minimum and the maximum normal component of the force, the common region defined by $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ give a good approximation of friction cone.

$\boldsymbol{T}$ in Eq.(13) is calculated by using all combination of $\boldsymbol{S}_1$ or $\boldsymbol{S}_2$ on each finger. $\boldsymbol{w}$ is obtained by the common region of Eq.(13) for all possible $\boldsymbol{T}$.

For example, in the case of two-points contact, the following four combinations can be considered for $\boldsymbol{T}$ in Eq.(13)

$$\boldsymbol{T}_1 = \text{block diag}[\boldsymbol{U}_1\boldsymbol{S}_1\boldsymbol{U}_1^t \quad \boldsymbol{U}_2\boldsymbol{S}_1\boldsymbol{U}_2^t] \quad (17)$$
$$\boldsymbol{T}_2 = \text{block diag}[\boldsymbol{U}_1\boldsymbol{S}_1\boldsymbol{U}_1^t \quad \boldsymbol{U}_2\boldsymbol{S}_2\boldsymbol{U}_2^t] \quad (18)$$
$$\boldsymbol{T}_3 = \text{block diag}[\boldsymbol{U}_1\boldsymbol{S}_2\boldsymbol{U}_1^t \quad \boldsymbol{U}_2\boldsymbol{S}_1\boldsymbol{U}_2^t] \quad (19)$$
$$\boldsymbol{T}_4 = \text{block diag}[\boldsymbol{U}_1\boldsymbol{S}_2\boldsymbol{U}_1^t \quad \boldsymbol{U}_2\boldsymbol{S}_2\boldsymbol{U}_2^t]. \quad (20)$$

Then a wrench set W'_j using T_j is defined as,

$$W'_j = \{\boldsymbol{w} | (\boldsymbol{w} - f_c\boldsymbol{G}\boldsymbol{N})^T (\boldsymbol{G}\boldsymbol{T}_j^{-1}\boldsymbol{G}^T)^{-1} (\boldsymbol{w} - f_c\boldsymbol{G}\boldsymbol{N}) \le m\}. \quad (21)$$

GWS W is defined as a common set of W'_j

$$W = \cap_j W'_j \quad (22)$$

where $j = 1, 2, 3, 4$. In the case of n contact points, 2^n inequality are judged.

Fig.4 shows an example of 2D simulation of common sets. 4 ellipses of the resultant force and moment are calculated by using combination of the ellipses at each contact point. The set of the resultant force and moment is a region which is included by all ellipses.

In the previous section, we approximated the friction cone by using two ellipsoids. Here, we assumed the maximum and the minimum normal component of the contact force. However, with this approximation, the force closure cannot be satisfied if we want the fingers to grasp the object with extremely small grasping force. We can overcome this problem by increasing the number of ellipsoids used for the approximation. Fig.5 shows the case where we approximate the friction cone by using three ellipsoids. We can see that, by simply increasing the number of ellipsoids, we can make the approximate to be more accurate one.

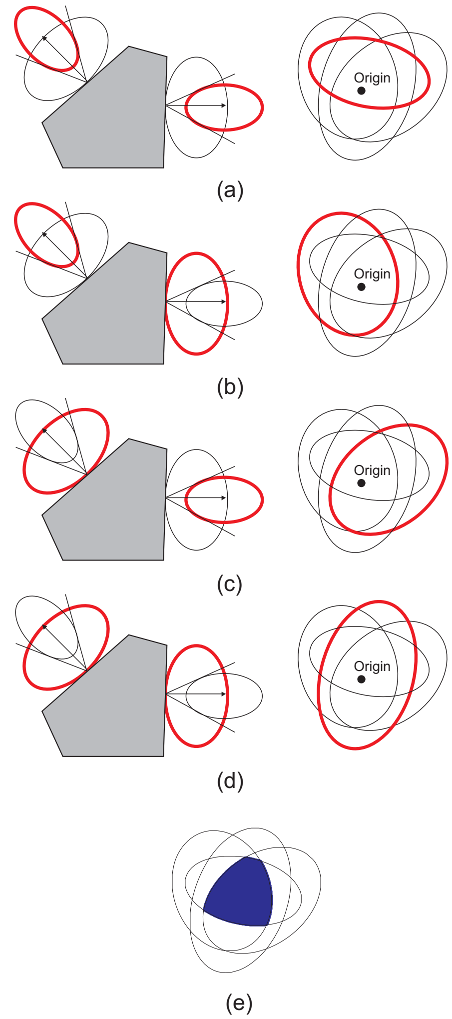

Fig. 4. Common set of ellipse

C. Criteria of grasp stability

Ferrari and Canny[1] proposed the numerical evaluation of grasp stability using a radius of the largest inscribing ball in the GWS around the origin. The radius means a distance between the origin and boundary of GWS. In our method, the distance between GWS boundary and the origin is evaluated for the same purpose. Since the equation which calculates the accurate distance between the origin and a ellipsoid cannot be solved algebraically, the approximation method is used.

We propose two different approaches for calculating approximation distance(Fig. 6). One is homothetic ellipsoid E_1 sharing the same center and passing through the origin (Fig. 6(b)) and the other is an ellipsoid E_2 sharing the same foci and passing through the origin(Fig. 6(c)).

The GWS boundary ellipsoid is expressed as,

$$(\boldsymbol{w} - \boldsymbol{c})^T A (\boldsymbol{w} - \boldsymbol{c}) = m \quad (23)$$

where $\boldsymbol{c} = f_c\boldsymbol{G}\boldsymbol{N}$ and $A = (\boldsymbol{G}\boldsymbol{T}^{-1}\boldsymbol{G}^T)^{-1}$. Let us consider the homothetic ellipsoid smaller than the GWS boundary ellipsoid having the same center with the GWS boundary ellipsoid. Let us also assume that this ellipsoid passes through

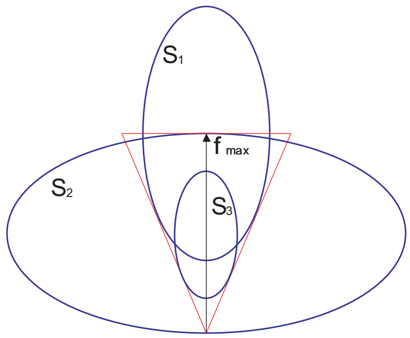

Fig. 5. Three ellipsoids

the origin $\mathbf{o}$. This ellipsoid E_1 can be expressed as

$$(\boldsymbol{w}-\boldsymbol{c})^T A((\boldsymbol{w}-\boldsymbol{c}) = e_o \tag{24}$$

where $e_o = e(\mathbf{0}) = \boldsymbol{c}^T A\boldsymbol{c}$. The unit normal vector $\boldsymbol{n}_1'$ on the homothetic ellipsoid at the origin is expressed as,

$$\boldsymbol{n}_1' = A\boldsymbol{c} \tag{25}$$

The distance d_1 between the origin and the ellipsoid along the vector is expressed as,

$$(d_1\boldsymbol{n}_1' - \boldsymbol{c})^T A(d_1\boldsymbol{n}_1' - \boldsymbol{c}) = m \tag{26}$$

This equation is expressed as a quadratic equation:

$$({\boldsymbol{n}_1'}^T A\boldsymbol{n}_1')d_1^2 - 2(\boldsymbol{c}^T A\boldsymbol{n}_1')d_1 + (\boldsymbol{c}^T A\boldsymbol{c} - m) = 0. \tag{27}$$

Note that $\boldsymbol{c}^T A\boldsymbol{n}_1' = {\boldsymbol{n}_1'}^T A\boldsymbol{c}$, since $A = A^T$. The distance is smaller one of two solutions of this equation.

Let us consider the other ellipsoid (Fig. 6(c)) smaller than the GWS boundary ellipsoid having the same foci with the GWS boundary ellipsoid. Let us also assume that this ellipsoid passes through the origin $\mathbf{o}$. Eigenvalue of A is defined as $\lambda_1, \lambda_2, \cdots, \lambda_6$. A can be expressed as,

$$A = V\Lambda V^T \tag{28}$$

where $\Lambda = diag(\lambda_1\lambda_2\cdots\lambda_6)$, and V is composed of eigenvectors of A. The ellipsoid which has the same foci of the original ellipsoids is expressed as using parameter s,

$$(\boldsymbol{w}-\boldsymbol{c})^T V\Lambda' V^T(\boldsymbol{w}-\boldsymbol{c}) = m \tag{29}$$

where

$$\Lambda' = diag\left(\frac{1}{\frac{1}{\lambda_1}-s}\ \frac{1}{\frac{1}{\lambda_2}-s}\ \cdots\ \frac{1}{\frac{1}{\lambda_6}-s}\right).$$

The ellipsoid which passes through the origin is expressed as,

$$\boldsymbol{c}^T V\Lambda' V^T\boldsymbol{c} = m. \tag{30}$$

This equation cannot be solved algebraically since it is a 6-dimensional equation. Then this equation is approximated by a 3-dimensional equation. When eigenvalue is sorted as $\lambda_1 > \lambda_2 > \cdots > \lambda_6$, the approximated matrix Λ'' is expressed as,

$$\Lambda'' = diag\left(\frac{1}{\frac{1}{\lambda_1}-s}\ \frac{1}{\frac{1}{\lambda_2}-s}\ \frac{1}{\frac{1}{\lambda_3}-s}\ \frac{1}{\frac{1}{\lambda_4}}\ \frac{1}{\frac{1}{\lambda_5}}\ \frac{1}{\frac{1}{\lambda_6}}\right) \tag{31}$$

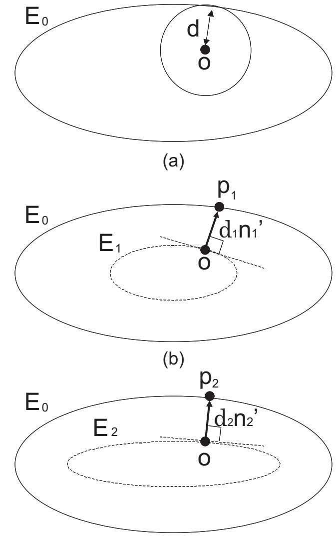

Fig. 6. Approximation distance

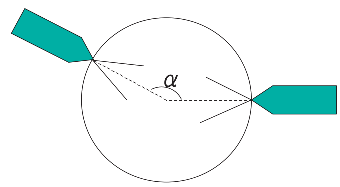

Fig. 7. The condition of simulation

Then the smallest solution of this equation is used for the ellipsoid. By using the same approach of homothetic ellipsoid, the normal vector on the ellipsoid E_2 at the origin is calculated, and distance d_2 is calculate as the distance between the origin and the E_2 boundary along the normal.

d_1 and d_2 accuracy are dependent on the ratio between the minimum length of the ellipse axis and the maximum one. If the ratio is small, d_2 has good accuracy. If the ratio is close to 1, d_1 has good accuracy. Therefore, the minimum one of d_1 and d_2 is used. Then the evaluation value d_{min} is expressed as,

$$d_{min} = \min_j \min(d_{1j}, d_{2j}) \tag{32}$$

where d_{1j} and d_{2j} are d_1 and d_2 of j-th the resultant ellipsoid, respectively. The calculation can be done algebraically and quickly.

IV. Simulation and Experiment

A. 2D simulation

In order to compare accuracies of the approximation with a conventional method using polyhedral cone and the proposed method, the resultant force in 2D was simulated. Fig.7 shows condition of the simulation of grasping circular form using two fingers. α is the angle between the directions of forces applied by two fingers. For simplicity, we considered only the resultant force, and the resultant moment was ignored. In this simulation, friction coefficient was $1/\sqrt{3}$, and the maximum

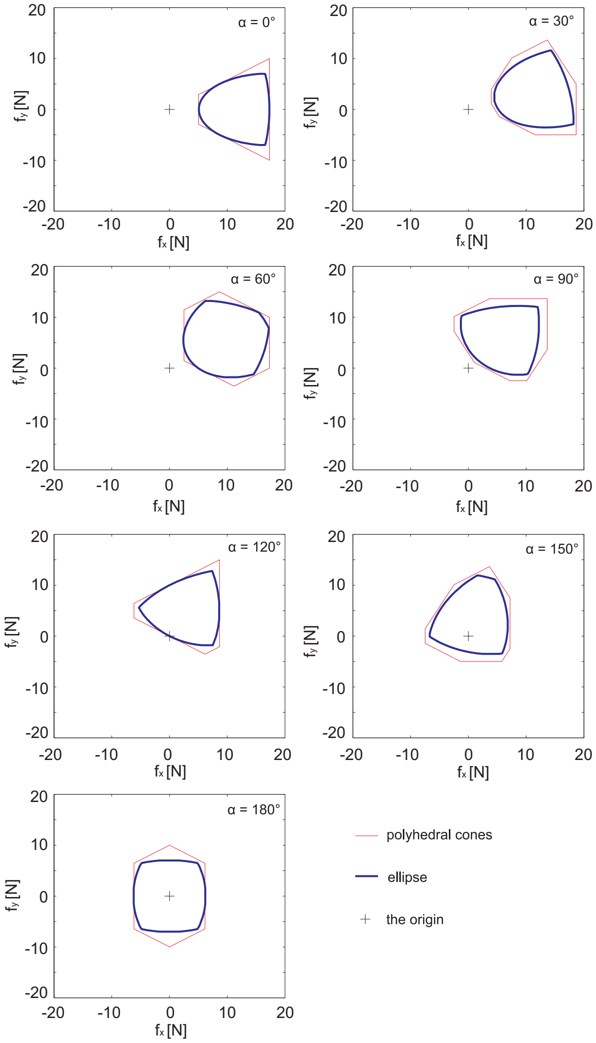

Fig. 8. Result of 2D simulation

normal force was $5\sqrt{3}\ N$. In the conventional method, the GWS was calculated by using polyhedral cone. We define that each normal force is larger than $5/\sqrt{2}\ N$.

Fig.8 shows the results when α is changed. The result of proposed method is confirmed to be smaller than the resultant force space of friction cone. The origin is not included in GWS from $\alpha = 0°$ to $\alpha = 120°$. The origin is on GWS border at $\alpha = 120°$. The origin is included in GWS from $\alpha = 120°$ to $\alpha = 180°$. We can confirm these methods had the same results whether the both GWS include the origin or not. The result of numerical evaluation of grasp stability is almost the same. The effectiveness of the proposed method is confirmed by these results of the comparison.

B. 3D simulation

3D simulation is performed with a PC (CPU: Intel Xeon 2.32 GHz, Memory: 3.2 GB). A model, bunny is grasped without considering the hand and finger kinematics. 4 facets are randomly selected and the position and normal of them are inputted to GWS generation function. We set friction coefficient to be 0.5 and the maximum normal force to

Fig. 9. Bunny model of Stanford Universiy

TABLE I
CALCULATION TIME

the number of contact	3	4
polyhedral cone	0.1948 s	2.0704 s
ellipsoid	0.0020 s	0.0046 s

be $5.0\ N$. Our method is compared with a conventional method[32] of polyhedral cone using qhull [31]. The conventional method used 6-sided polyhedral cone.

We show the result of calculation time (Table I). The average time of three contact points with polyhedral cone method was 0.1948 s. On the other hand, the calculation time with the proposed method was 0.0020 s. The time of four contact points with polyhedral cone method was 2.0704 s. On the other hand, the calculation time with the proposed method was 0.0046 s. The calculation time with the proposed method was extremely faster than that with the polyhedral cone method.

Our method is also compared with Borst method[18]. The calculation time is simulated by using the same number of wrench which is totally used in their method for expansion of the weakest direction of W_{L_∞}. The calculation time of four contact points was about 0.0500 s in our implementation.

When the wrench origin is obviously outside of GWS, the proposed method can judge instantly by testing one ellipsoid. In such case, the average calculation time is only 0.43 ms. The time of Table I is the average time when all ellipsoid are checked .

An example of GWS is shown in Fig.10. GWS generated by using polyhedral cones, GWS_p is displayed as translucent surface. GWS generated by using ellipsoids, GWS_e is displayed as opaque surface. In force space, GWS_e is almost covered by GWS_p. In torque space, GWS_e run off GWS_p. However, GWS_e is confirmed to be close to GWS_p.

We compare force closure judgment and evaluation values of grasp stability between ellipsoids and polyhedral cones(Fig. 11). The judgments of inside or outside of GWS coincide at 90% . When the judgments are different, the grasp stability value is small and the grasp is unstable. Grasp stability criteria of them are close to each other and correlation coefficient is 0.96.

C. Experiment of grasping using a HDH hand

By using a hand-arm system as shown in Fig.12, object grasp is performed. The hand is HDH hand[29] composed by 4 fingers attached to HRP-3P[30] with 7 degrees arm. This is 4 fingered hand and has the thumb with 5 joints and the index, middle and third fingers with 4 joints. The distal

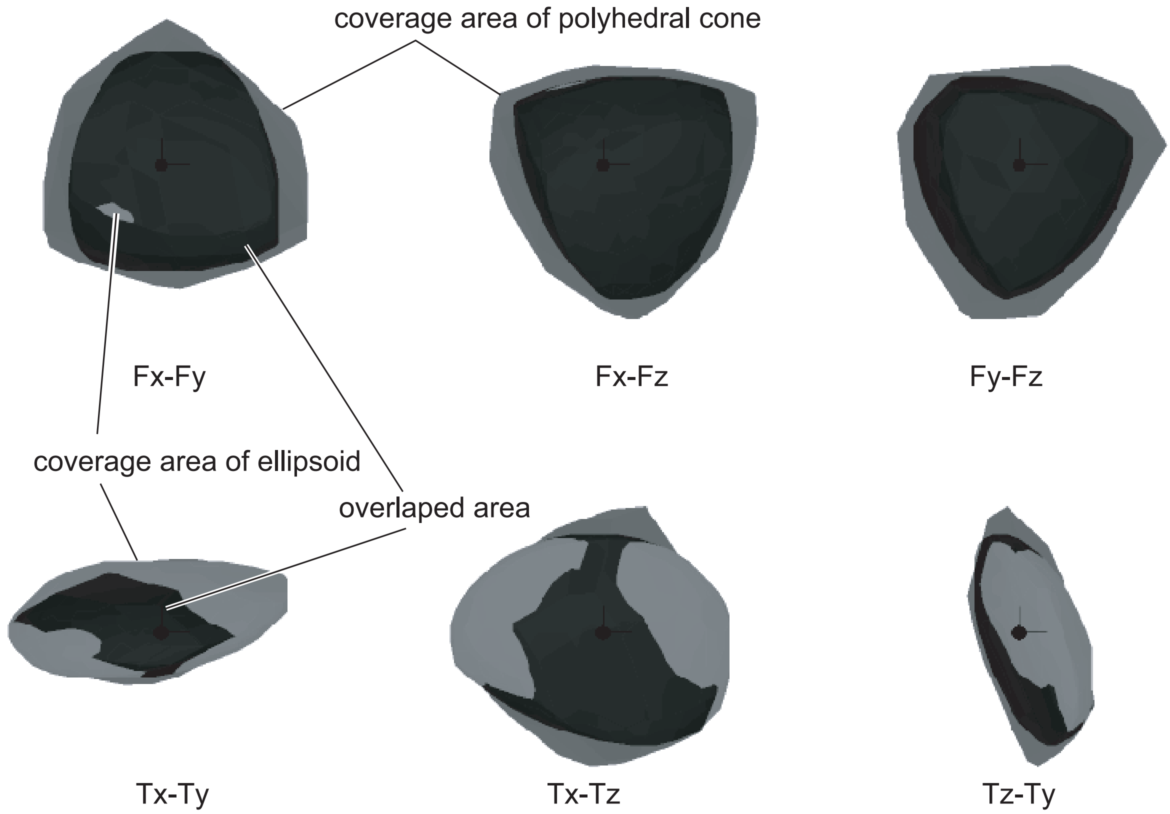

Fig. 10. Example of 3D GWS

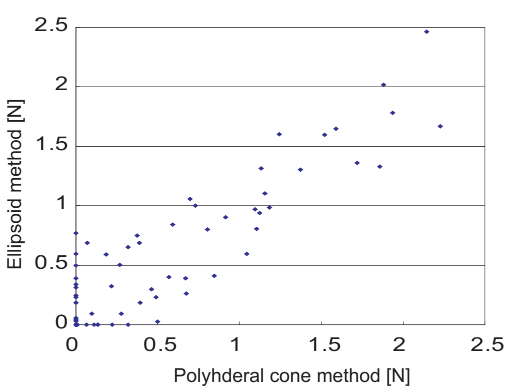

Fig. 11. Evaluation value of grasp stability

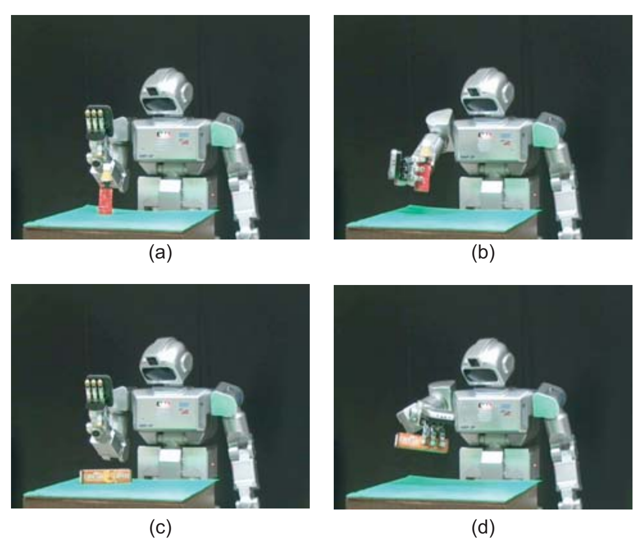

Fig. 12. Experiment of grasp

joint of each finger is not directly actuated and move along with the next joint.

At planning phase, we set that friction coefficient was 0.5, the maximum normal force is $5.0\ N$. Grasp planning is performed at (a) and (c), the grasp is executed as (b) and (d), respectively. Total time of grasp planning is less than 1 s with our method[32]. The proposed method is effective for real-time application.

V. Conclusion

Fast method of testing the force closure for multi-fingered hand was proposed in this paper. In our method, we use ellipsoids to approximate the set of contact force. If we use two ellipsoids to approximate a contact force set, the one is used to approximate the friction cone and the other is used for limiting the maximum normal component of contact force. By using this approximation method, we propose a method of testing the force closure and a method of evaluating the grasp stability. We show that, by using both methods, the force closure or the grasp stability can be evaluated very easily. The 2D numerical example show that, by using the proposed approximation method, the set of total force is close to the original set. Also, the 3D example showed that our method can test the force closure faster than the previous methods.

As a future work, we consider approximating the friction cone more accurately, and conservative calculation of torque space of W_{L_∞}. Also, in pragmatic grasping task, not only GWS but also manipulability force of fingers has to be considered. An algorithm to satisfy the both conditions has to be investigated.

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