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A New Practical Strategy to Localize a 3D Object without Sensors

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Abstract— **Sensorless localization of 3D objects has been a significant research topic for many years. Researchers have focused on this problem from both theoretical and practical perspective where the goal is to reduce uncertainties in the orientation of a 3D object. However, to the best of our knowledge, no effective practical methods have been proposed so far to localize a polyhedron from any initial orientation to a unique orientation without sensors.**

In our previous work [1], two broad classes of 3D objects have been introduced, which can be localized from an arbitrary state to a unique state on a flat plane (the surface resting on the flat plane is established) without sensors. In this paper, a much broader class of polyhedra is introduced, which can be localized to a unique state without sensors. The main contributions of this paper are given as follows:

- **It is found that a polyhedron with an arbitrary initial state on the flat plane can be rotated to a fixed orientation (the orientation of the surface resting on the flat plane is fixed), provided that the polygon corresponding to each surface of the polyhedron can be oriented to a unique orientation in a 2D space. The method of rotating the polyhedron to a fixed orientation is given.**
- **Base on the above result, both conditions and the strategy are given for a polyhedron to be localized to a unique state.**
- **An example is given to show the validity of the strategy.**

I. INTRODUCTION

In manufacture, three-dimensional objects often need to be located and oriented to a fixed pose before assembly, grasping and other manipulation. For example, in the motor car manufacture, localizing a piston to a unique pose is necessary to subsequent assembly process; in the automatic case packing system, goods with different shapes should be turned to a proper orientation; in military applications, some high-precision manipulation also demands exact orientations. In the foremost work on localizing objects, the orientation of objects and other required information were often retrieved by high-precision and high-speed sensors, which makes the object localization system expensive and fragile. In some situation, sensors may not be suitable for use. For example, a vision system will not be effective in a dark environment; it is also rarely used in food manufacture for sanitary purpose. Therefore, it is necessary to investigate how to localize an object without sensors.

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Orienting a planar polygonal object to a unique orientation has been investigated for many years. Goldberg [2] summarized some previous work on orienting polygons and successfully designed a strategy to orient polygonal parts without sensors. Chen et al. [3] proved Goldberg's conjecture that a polygon can be oriented in $O(n)$ steps manipulation and used the information of distance between the two parallel jaws upon closure to distinguish a finite set of parts. In addition, according to Goldberg's planing algorithms [4], 2D objects with a planar projection being a piecewise algebraic convex hull can be oriented with two parallel jaws.

Sensorless localization of 2D polygons has also been investigated from the aspect of application, including sensorless part sorting [5], stable alignment pushing [6], nonprehensile manipulating [7], and feeder devices designing such as curved fences [8], conveyor with fences [9] and one joint robot [10]. Localization of 2D objects has also been considered using attractive regions of the configuration space [11]. Akella et al. [12] considered shape uncertainties of polygonal parts and proposed a sensor-less orienting strategy for a variety of classes of part shapes given a nominal part shape and tolerance bounds.

In practical applications, a 3D object on a flat plane will come to rest in one stable state [13]. For a 3D object in a stable state, there are two kinds of uncertainties: the surface resting on the flat plane and the orientation of the contact surface are both unknown. Due to the interaction of these two kinds of uncertainties, localizing a 3D polyhedral object to a unique state cannot be simply decomposed into 2D object localizations. Berretty et al. [14] have conducted a theoretical study on localizing 3D polyhedra and proved that one action exists which orients a polyhedron from any two states to one state. Thus, a polyhedron with n stable states can be oriented to a known state with $O(n)$ manipulation operations. They also provided a method of adjusting the state of the polyhedron by pushing the polyhedron from different directions.

Some previous work focused on uncertainty reduction for 3D objects orientation in some special cases, which does not require the object to be localized to a unique pose or which restricts the initial state of the object in a subset of all the states. For example, Erdmann et al. [15] utilized the method of tilting the table to successfully reduce the uncertainty in the 3D orientation of parts and proposed a directed-graph constructing planner that determines a sequence of tilting operations designed to minimize the uncertainty in the 3D orientation of parts. Zhang et al. [16] proposed to move the polyhedron forward on step devices. Thus, the polyhedron will slip to keep in the primary state or roll to an new state by setting a different speed. For some polyhedral parts, using the algorithm proposed in [16] a part can be oriented from its most initial 3D orientations to a unique 3D orientation. Manipulating 3D objects by rolling is an important and effective approach to orient 3D objects. Ceccarelli et al. [17] adopted a minimally complex robotic dexterous mechanism to rotate a polyhedron about edges belonging to a fixed surface from a given configuration to another reachable one. These methods do not guarantee that a polyhedron can be localized from any initial 3D orientation to the unique final 3D orientation.

On the other hand, many authors are concerned with the design of devices which can be used to localize certain class of 3D objects. Moll et al. [18] presented a component for the automated design of parts-orienting devices and proposed a method of orienting parts by minimizing the entropy of the pose distribution. Recently, Goemans et al. [19] proposed a new primitive which can feed a broad class of three dimensional parts by reorienting and rejecting all but those in the desired orientation. They also gave a practical algorithm for its design.

In this paper, we study conditions and the practical strategy for a polyhedron to be localized from an arbitrary initial state to a unique state without sensors. An example is also given to show the validity of the proposed strategy.

The rest of this paper is organized as follows. In Second II, conditions are presented for a polyhedron to be localized to a unique state: the first one is the condition for a polyhedron to be rotated to a fixed orientation on a flat plane, and the second one is the condition for a polyhedron to be localized from an arbitrary state to a unique state on a flat plane. In Section III, an example is given to show the validity of the proposed strategy. Some conclusions are given in Section IV.

II. Localize a Polyhedron from an Arbitrary Initial State to a Unique State

Usually, a polyhedron on a flat plane has two kinds of uncertainties:

1) State Uncertainty: the bottom surface, which rests on the flat plane, is unknown;
2) Orientation Uncertainty: the orientation of the polyhedron (denoted by the orientation of the bottom surface) is unknown.

In this section, we will give a new practical strategy by which a polyhedron can be rotated from an arbitrary state to a unique state where the bottom surface is fixed. This strategy will be implemented in two steps:

1) Rotate a polyhedron with an arbitrary surface resting on the flat plane to a fixed orientation;
2) Rotate a polyhedron from an arbitrary state to a unique state.

A. Orient a polyhedron with an arbitrary surface resting on the flat plane to a fixed orientation

In this subsection, the conditions for a polyhedron to be rotated to a fixed orientation on a flat plane are investigated (Lemma) and the corresponding strategy is given.

Firstly, several notations and definitions, which will be used in the Lemma, are defined and explained as follows.

(1) Symbols E_{ij}, $E_{b(ij)}$, $\min_j\{\phi_{cij}\}$ and $\min_j\{\phi_{aij}\}$. Assume that a polyhedron has n surfaces and its i^{th} surface rests on a flat plane (Fig. 1). Denote by E_{ij} the common edge of the surfaces i and j and by $E_{b(ij)}$ the j^{th} edge of the surface i. Denote by O_i the projection of the center of gravity cg of the polyhedron on the i^{th} surface and by V_{ij} the j^{th} vertex of the i^{th} surface, where V_{ij} and $V_{i(j+1)}$ are conjunctive. Let us denote by ϕ_{cij} the clockwise angle from O_iV_{ij} to $V_{ij}V_{i(j+1)}(E_{b(ij)})$ and by ϕ_{aij} the counter-clockwise angle from O_iV_{ij} to $V_{ij}V_{i(j-1)}(E_{b(i(j-1))})$. Let $\min_j\{\phi_{cij}\}$ be the minimal ϕ_{cij} of the i^{th} surface and let $\min_j\{\phi_{aij}\}$ be the minimal ϕ_{aij} of the i^{th} surface.

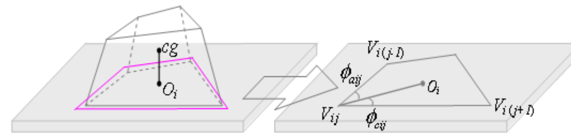
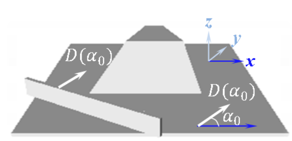
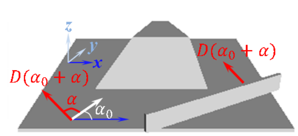


Fig. 1. A polyhedron on a flat plane and its i^{th} surface with ϕ_{cij} and ϕ_{aij}

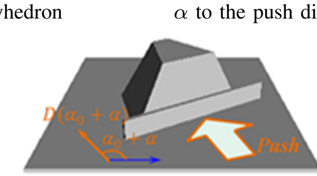
(2) Push action $P[D(\alpha_0), \alpha]$ denotes a flat-jaw normal push to a polyhedron on a flat plane along the direction, which has an angle α with $D(\alpha_0)$. $D(\alpha_0)$ is the initial normal direction of the push jaw, which has an angle α_0 with the reference direction (such as x-axis of the world frame). The flat jaw moves with a fixed orientation along the direction $D(\alpha_0+\alpha)$ for a distance. The distance is specified to be long enough to make one edge in the bottom surface of the polyhedron aligned with the flat jaw (Fig.2).



(a) Initial relative position of push jaw and the polyhedron



(b) Rotate the push jaw at an angle α to the push direction



(c) Push along the direction $D(\alpha_0+\alpha)$ to make the polyhedron aligned with the push jaw

Fig. 2. Denotation of $P(D(\alpha_0), \alpha)$

(3) Relative-Orientation Transference Mapping. A polyhedral object on the flat plane is pushed by $P(D(\alpha_0), 0)$

to the orientation in which one edge $E_{b(ij)}$ of the bottom surface is aligned with the push jaw. Then the polyhedron can be reoriented by $P(D(\alpha_0), \alpha)$ from the current orientation (denoted by $E_{b(ij)}$) to another orientation (denoted by $E_{b(ix)}$), which can be expressed as

$$f_r : (E_{b(ij)}, P(D(\alpha_0), \alpha)) \to E_{b(ix)} \quad (1)$$

The mapping f_r in Eq. (1) is defined as the **Relative-Orientation Transference Mapping**. We have that

a) if $-(\pi/2 - \varphi_{cij}) < \alpha < \pi/2 - \varphi_{ai(j+1)}$, then $E_{b(ix)} = E_{b(ij)}$;
b) if $\pi/2 - \varphi_{ai(j+1)} < \alpha < 3\pi/2 - \varphi_{ai(j+1)} - \varphi_{ci(j+1)} - \varphi_{ai(j+2)}$, then $E_{b(ix)} = E_{b(i(j+1))}$;
c) if $-(3\pi/2 - \varphi_{cij} - \varphi_{aij} - \varphi_{ci(j-1)}) < \alpha < -(\pi/2 - \varphi_{cij})$, then $E_{b(ix)} = E_{b(i(j-1))}$;
d) if $\alpha > 3\pi/2 - \varphi_{ai(j+1)} - \varphi_{ci(j+1)} - \varphi_{ai(j+2)}$, then $E_{b(ix)} = E_{b(i(j+l_+))}$ $(l_+ > 1)$; and
e) if $\alpha < -(3\pi/2 - \varphi_{cij} - \varphi_{aij} - \varphi_{ci(j-1)})$, then $E_{b(ix)} = E_{b(i(j+l_-))}$ $(l_- < -1)$.

Lemma. *For a n-surface polyhedron with an arbitrary surface $S_{b(k)}$ resting on the flat plane, if satisfying:*

a) $\min\limits_{j=1,\cdots,m_i}\{\phi_{cij}\}$ *is unique for* $i = 1, \cdots, n$, *where* m_i *is the vertex number of the surface* $S_{b(i)}$, *and* $\min\limits_{j=1,\cdots,m_i}\{\phi_{cij}\} > \varphi_{cij} + \varphi_{aij} + \varphi_{ci(j-1)} - \pi$, *or*
b) $\min\limits_{j=1,\cdots,m_i}\{\phi_{aij}\}$ *is unique for* $i = 1, \cdots, n$ *and* $\min\limits_{j=1,\cdots,m_i}\{\phi_{aij}\} > \varphi_{ai(j)} + \varphi_{ci(j)} + \varphi_{ai(j+1)} - \pi$,

the polyhedron can be oriented with the push $P(D(\alpha_0), \alpha)$ to a fixed orientation of the push jaw, i.e. the edge $E_{b(kj_k)}$ aligned with the push jaw is established for each $S_{b(k)}$.

Proof. Since the proofs for the lemma with the condition a) and b) satisfied are similar, we just give the proof for the lemma with the condition a) satisfied as follows.

Firstly, we give a strategy by which the polyhedron can be rotated on the flat plane from an arbitrary initial orientation to a fixed orientation. In this strategy, α_0, ϕ_{cij}, n, m_i are the inputs.

(i) Set $i = 1$, and push the polyhedron with $P(D(\alpha_0), 0)$, where α_0 is an arbitrary angle. Without loss of generality, we set $\alpha_0 = 0$.
(ii) Set $\alpha = -(\frac{\pi}{2} - \min\limits_j\{\phi_{cij}\}) + \varepsilon$, $l = 1$.
(iii) Push the polyhedron with $P(D(\alpha_0), \alpha)$.
(iv) $\alpha_0 = \alpha_0 + \alpha$. $l = l + 1$. Go to (iii) until $l = m_i$.
(v) $i = i + 1$. Go to (ii) until $i = n$.

Assume that the n surfaces of the polyhedron are sorted by $\min\limits_j\{\phi_{cij}\}$:

$$\min_j\{\phi_{cij}\} < \min_j\{\phi_{c(i+1)j}\}$$

We also assume that the polyhedron initially rests on the k^{th} surface $S_{b(k)}$. k is unknown and the orientation of the k^{th} surface is also unknown.

In the case with $k = 1$,

- when $i = 1$, the rotational angle satisfies: $|\alpha| = \frac{\pi}{2} - \min\limits_j\{\phi_{c1j}\} - \varepsilon$, so the polyhedron will rotate to a fixed relative orientation of the push jaw(This has been proved in [6], and the proof is briefly given in the Appendix).
- when $i > 1$, the rotational angle $\alpha = -(\frac{\pi}{2} - \min\limits_j\{\phi_{cij}\}) + \varepsilon$, and satisfies: $0 > \alpha > -(\frac{\pi}{2} - \min\limits_j\{\phi_{c1j}\})$. According to the Relative-Orientation Transference Mapping
$$f_r : (E_{b(ij)}, P(D(\alpha_0), \alpha)) \to E_{b(ij)},$$
the relative orientation between the polyhedron and the push jaw will not change.

In the case with $k \geq 2$,

- when $i < k$, the rotational angle α satisfies: $-(3\pi/2 - \varphi_{ckj} - \varphi_{akj} - \varphi_{ck(j-1)}) < \alpha < -(\frac{\pi}{2} - \min\limits_j\{\phi_{ckj}\})$. According to the Relative-Orientation Transference Mapping
$$f_r : (E_{b(ij)}, P(D(\alpha_0), \alpha)) \to E_{b(ij-1)},$$
each push($P(D(\alpha_0), \alpha)$) will change the polyhedron from one relative orientation of the push jaw to another.
- when $i \geq k$, the orientation change of the polyhedron is the same as that in the case with $k = 1, i \geq k$.

Therefore, the final relative orientation between the polyhedron and the push jaw is fixed. □

The strategy in the proof of Lemma is named as **Orientation-fixed Strategy**. After the strategy,

- the final push direction is known and marked as $D(\alpha_f)$;
- for each $S_{b(k)}$, the edge $E_{b(kj_k)}$ aligned with the push jaw is determined and defined as **Orientation-fixed Edge**.

In the next subsection, we will investigate how to eliminate the uncertainty of $S_{b(k)}$.

B. A broad class of polyhedra which can be rotated to a unique state without sensors

In this subsection, we give conditions and a strategy for a polyhedron to be rotated from an arbitrary state to a unique state. To this end, several terms are needed which are defined and explained as follows.

(1) Transitional Angle. The polyhedron rotates around an edge from resting on one surface to resting on another. A rotational angle which is equal to or a little greater than θ_{ij} can cause the polyhedron to over-rotate from surface $S_{b(i)}$ to surface $S_{b(j)}$. The angle θ_{ij} is defined as the "Transitional Angle". This definition is the same as the definition of the transition angle in Ref. [15].

(2) Stable-state Transference Mapping. Assume that the polyhedron will only be transferred from its current stable state to another stable state by a rotation around one edge of the bottom surface. The new stable state (denoted by $S_{b(x)}$) after the rotation is determined by the current stable state of the polyhedron (denoted by $S_{b(i)}$), the pivot edge (denoted by E_{ij}) and the rotating angle θ and can be expressed as

$$f : (S_{b(i)}, E_{ij}, \theta) \to S_{b(x)} \quad (2)$$

The mapping f in Eq. (2) is defined as the **Stable-State-Transference Mapping**. We have that

- if $0 < \theta < \theta_{ij}$ then $x = i, S_{b(x)} = S_{b(i)}$; and
- if $\theta_{ij} \leq \theta < \theta_{ij} + \delta$ then $x = j,\ S_{b(x)} = S_{b(j)}$, where δ is a small positive number.

(3) Stable-state Transference Graph. We first define a set of edges:

$$\begin{aligned} S_E &= \{E_{b(ij_{Ti})}| \\ &\quad f_r : (E_{b(ij_i)}, P(D(\alpha_f), \alpha_T)) \to E_{b(ij_{Ti})}, \\ &\quad E_{b(ij_i)} \text{ is the Orientation-fixed Edge of } S_{b(i)}, \\ &\quad i = 1, 2 \cdots n, 1 \leq j_i, j_{Ti} \leq m_i, \alpha_T \in [0, 2\pi)\} \\ &= \{E_{ik}|f_r : (E_{ij}, P(D(\alpha_f), \alpha_T)) \to E_{ik}, \\ &\quad E_{ij} \text{ is the Orientation-fixed Edge of } S_{b(i)}, \\ &\quad i = 1, 2 \cdots n, 1 \leq j, k \leq \sum_{i=1}^{n} m_i/2, \alpha_T \in [0, 2\pi)\}. \end{aligned}$$

Stable-state Transference Graph is a directed graph and is defined as

$$\begin{aligned} G_S(\alpha_T) &= (V_S, A_S) \\ &= \{(S_{b(i)}, \langle S_{b(i)}, S_{b(k)}\rangle)|E_{ik} \in S_E\}, \end{aligned}$$

where $\langle S_{b(i)}, S_{b(k)}\rangle$ denotes a directed arc from $S_{b(i)}$ to $S_{b(k)}$.

Remark: According to Relative-Orientation Transference Mapping a)−e), we can obtain that $j_{T_i} = j_i + l$ $(l = -(j_i - 1), \cdots, 0, \cdots m_i - j_i)$. In the case with $E_{b(ij_i)}$ and α_f fixed, for any $\alpha_T \in [0, 2\pi)$, l has a unique value, which determines a unique edge $E_{b(ij_{Ti})}$ of the surface $S_{b(i)}$. Thus, $E_{b(ij_{Ti})}(E_{ik})$ is the ground-touching edge of the surface $S_{b(i)}$, around which the polyhedron rotates to the next stable surface $S_{b(k)}$, and $S_{b(k)}$ is uniquely determined by E_{ik}.

Theorem. *A n-surface polyhedron on a flat plane can be rotated without sensors from any initial stable state to a unique stable state if the polyhedron satisfies:*

a) *the conditions of the Lemma, and*
b) *there exists $\alpha_T \in [0, 2\pi)$ such that $G_S(\alpha_T)$ has only one loop $G_s = (V_s, A_s)$ ($G_s \subseteq G_S$).*
c) *there is only one maximum in $\{\theta_{ij}|\langle S_{b(i)}, S_{b(j)}\rangle \in A_s\}$.*

Proof. Assume that all the surfaces of the polyhedron are numbered differently with $i = 1, 2 \cdots n$. It is also assumed that the initial state of the polyhedron is the state where $S_{b(K_1)}$ lies on the flat plane. K_1 and the orientation of surface $S_{b(K_1)}$ are unknown. Define two sets Θ, Θ_s as follows:

$$\begin{aligned} \Theta &= \{\theta_{ij}|\langle S_{b(i)}, S_{b(j)}\rangle \in A_S\}, \\ \Theta_s &= \{\theta_{ij}|\langle S_{b(i)}, S_{b(j)}\rangle \in A_s\}. \end{aligned}$$

The following strategy will localize the polyhedron from resting on surface K_1 to a fixed state. In the strategy, α_T, Θ, Θ_s, m and n are the inputs.

(i) Set $k = 1$, $\theta = \max\{\Theta\} + \varepsilon$.
(ii) Execute Orientation-fixed Strategy and $P(D(\alpha_f), \alpha_T)$, where $D(\alpha_f)$ is the push direction after Orientation-fixed Strategy is executed.
(iii) Rotate the polyhedron at an angle θ around the edge aligned with the push jaw.
(iv) $k = k + 1$. If $k \geq n$, $\theta = \max\{\Theta_s\} - \varepsilon$. Go to (ii) until $k = n + m - 2$, where m is the number of the nodes V_s in the graph G_s.

In the above strategy, after (iii), the polyhedron will be rotated to a new state or stay in the same state. This can be classified into three cases:

Case 1, $k <= n - 1$. The rotational angle θ:

$$\theta = \max\{\Theta\} + \varepsilon > \theta_{ij}.$$

According to the Stable-state Transference Mapping

$$f : (S_{b(i)}, E_{ij}, \theta) \to S_{b(x)},$$

the polyhedron will be rotated to the new state, where K_{k+1} lies on the flat plane.

Case 2, $n \leq k \leq n + h_1 - 1$ $(1 \leq h_1 \leq m - 1)$. The rotational angle θ:

$$\theta = \max\{\Theta_s\} - \varepsilon > \theta_{ij}(\theta_{ij} \in \Theta_s \backslash \max\{\Theta_s\}).$$

The polyhedron will be rotated to the new state where K_{k+1} lies on the flat plane.

Case 3, $n + h_1 \leq k \leq n + h_2 - 1$ $(h_1 < h_2 \leq m - 1)$. The rotational angle θ:

$$\theta = \max\{\Theta_s\} - \varepsilon < \max\{\Theta_s\}.$$

The polyhedron will stay the same.

Therefore, no matter what K_1 is at the beginning and which orientation the surface K_1 has, there exists a sequence of surfaces $S_{b(K_1)}, S_{b(K_2)} \cdots S_{b(K_l)}$ $(1 \leq l \leq n + m - 1, S_{b(K_k)} \in V_S, 1 \leq k \leq n + m - 2)$. It can be proved that $S_{b(K_n)} \in V_s$ and K_l is unique, which is given as follows.

i) The Proof for $S_{b(K_n)} \in V_s$: assume that $S_{b(K_n)} \overline{\in} V_s$. Define a set S_N.

$$\begin{aligned} S_N = \quad &\{S_{b(L_v)}|S_{b(L_v)} \in V_S, v = 1, 2 \cdots, \\ &f : (S_{b(L_v)}, E_{L_v L_{v+1}}, \max\{\Theta\}) \to S_{b(L_{v+1})}\}. \end{aligned}$$

G_s is a loop subgraph of G_S, so we can obtain:

$$S_{b(L_v)} \in V_s \Rightarrow S_{b(L_{v+1})} \in V_s \quad (3)$$

$$S_{b(L_{v+m})} = S_{b(L_v)} \quad (4)$$

Thus, we can define a subset of S_N:

$$\begin{aligned} S_M = \quad &\{S_{b(L_v)}|S_{b(L_v)} \in V_s, v = 1, 2 \cdots, \quad (5) \\ &f : (S_{b(L_v)}, E_{L_v L_{v+1}}, \max\{\Theta\}) \to S_{b(L_{v+1})}\} \end{aligned}$$

According to Eq. (3), (5) and Case 1, we can obtain:

$$\begin{aligned} &S_{b(K_k)} \in S_M \Rightarrow S_{b(K_{k+1})} \in S_M \Rightarrow \cdots \Rightarrow \\ &\quad S_{b(K_n)} \in S_M (K_k \neq K_{k+1}, 1 \leq k < n) \quad (6) \end{aligned}$$

According to the assumption that $S_{b(K_n)} \overline{\in} V_s$ and Eq. (6), we can obtain:

$$\begin{aligned} &S_{b(K_n)} \overline{\in} S_M \Rightarrow S_{b(K_{n-1})} K_{n-1} \overline{\in} S_M \Rightarrow S_{b(K_{n-2})} \overline{\in} S_M \\ &\cdots \Rightarrow S_{b(K_1)} \overline{\in} S_M (K_i \neq K_j, i, j \in \{1, \cdots n\}, i \neq j). \end{aligned}$$

There are at least n different surfaces in $V_S \backslash V_s$, i.e. $N\{V_S \backslash V_s\} \geq n$. There are n and m surfaces in V_S and V_s respectively, so there are $n - m$ surfaces in $V_S \backslash V_s$, i.e.

TABLE I

THE EDGES OF ALL THE SURFACES OF THE HEXAHEDRON

Surfaces&Edges		Edges	Length	Surfaces&Edges		Edges	Length
S_1	$V_{11}V_{12}$	v_1v_2	10.00	S_4	$V_{41}V_{42}$	v_3v_4	7.07
	$V_{12}V_{13}$	v_2v_3	4.12		$V_{42}V_{43}$	v_4v_8	4.68
	$V_{13}V_{14}$	v_3v_4	7.07		$V_{43}V_{44}$	v_8v_7	3.24
	$V_{14}V_{11}$	v_4v_1	5.39		$V_{44}V_{41}$	v_7v_3	4.35
S_2	$V_{21}V_{22}$	v_1v_2	10.00	S_5	$V_{51}V_{52}$	v_4v_1	5.39
	$V_{22}V_{23}$	v_2v_6	4.58		$V_{52}V_{53}$	v_1v_5	5.10
	$V_{23}V_{24}$	v_6v_5	5.00		$V_{53}V_{54}$	v_5v_8	3.13
	$V_{24}V_{21}$	v_5v_1	5.10		$V_{54}V_{51}$	v_8v_4	4.68
S_3	$V_{31}V_{32}$	v_2v_3	4.12	S_6	$V_{61}V_{62}$	v_5v_6	5.00
	$V_{32}V_{33}$	v_3v_7	4.35		$V_{62}V_{63}$	v_6v_7	2.58
	$V_{33}V_{34}$	v_7v_6	2.58		$V_{63}V_{64}$	v_7v_8	3.24
	$V_{34}V_{31}$	v_6v_2	4.58		$V_{64}V_{61}$	v_8v_5	3.13

TABLE II

ϕ_{cij} AND $\min\{\phi_{ci}\}$

Surfaces	ϕ_{cij}	Radian of ϕ_{cij}	Surfaces	ϕ_{aij}	Radian of ϕ_{aij}
S_1	ϕ_{c11}	0.40*	S_4	ϕ_{c41}	0.63
	ϕ_{c12}	0.86		ϕ_{c42}	0.50*
	ϕ_{c13}	0.59		ϕ_{c43}	0.84
	ϕ_{c14}	1.29		ϕ_{c44}	1.16
S_2	ϕ_{c21}	0.43*	S_5	ϕ_{c51}	1.26
	ϕ_{c22}	0.62		ϕ_{c52}	0.41
	ϕ_{c23}	0.57		ϕ_{c53}	0.36*
	ϕ_{c24}	1.61		ϕ_{c54}	0.95
S_3	ϕ_{c31}	0.79	S_6	ϕ_{c61}	0.49*
	ϕ_{c32}	0.21*		ϕ_{c62}	0.86
	ϕ_{c33}	0.96		ϕ_{c63}	0.68
	ϕ_{c34}	1.41		ϕ_{c64}	1.06

$N\{V_S \backslash V_s\} \leq n-m$. However, $m > 0$, the above two results conflicts with each other. Therefore, $K_n \in M$.

ii) The proof that K_l is unique: assume that the only one state with $\max\{\Theta_s\}$ is $S_{b(L_f)}$, so $S_{b(L_f)} \in S_M$.

It can be proved that $S_{b(L_f)}$ can be achieved from S_{K_n} by at most $m-1$ rotations of the polyhedron around $E_{ij}(\langle S_i, S_j\rangle \in A_s)$, i.e. $h_1 \leq m-1$: assume that $K_n = L_w$; according to Case 2 and Eq. (4), if $w > f$, $h_1 = m-w+f < m$; if $w < f$, $h_1 = f - w < m$; so $h_1 \leq m-1$.

Also, it can be prove that $S_{b(K_l)} = S_{b(L_f)}$: according to Case 3, when $n + h_1 \leq k \leq n + h_2 - 1 (h_1 \leq h_2 \leq m-1)$, the polyhedron will stay in the same state, i.e. $S_{K_l} = S_{b_f}$. □

III. LOCALIZATION OF A POLYHEDRAL OBJECT WITH THE PROPOSED STRATEGY

In this section, as an example, a polyhedron (Fig. 3) is localized with the proposed strategy, which shows the validity of the strategy.

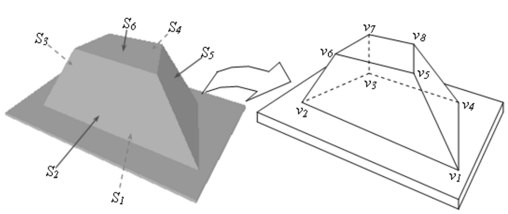


Fig. 3. The polyhedron to be localized to a unique state without sensors

The shape parameters of the polyhedron are listed in TABLE I, where $V_{ij}V_{i(j+1)}$ is the j^{th} edge of the i^{th} surface of the polyhedron.

First, ϕ_{cij} and $\min\limits_j\{\phi_{cij}\}$ are computed and listed in TABLE II.

By comparison, we find that $\min\limits_j\{\phi_{cij}\}$ in each surface is unique (Note that the condition a) of the Lemma is satisfied). The parameters $(S_{b(i)}, \phi_i, m_i, E_{b(ij_i)})$ are listed in TABLE III, where

TABLE III

$S_{b(i)}$, ϕ_i, EDGE NUMBER m_i OF EACH SURFACE AND THE ORIENTATION-FIXED EDGE $E_{b(ij_i)}$

Surfaces(i)	$\min\limits_j\{\phi_{cij}\}$	$S_{b(i)}$	ϕ_i	m_i	$E_{b(ij_i)}$
S_1	0.40	$S_{b(3)}$	1.17	4	v_1v_2
S_2	0.43	$S_{b(4)}$	1.14	4	v_1v_2
S_3	0.21	$S_{b(1)}$	1.36	4	v_3v_7
S_4	0.50	$S_{b(6)}$	1.07	4	v_4v_8
S_5	0.36	$S_{b(2)}$	1.21	4	v_5v_8
S_6	0.49	$S_{b(5)}$	1.08	4	v_5v_6

- $S_{b(i)}$ denotes the i^{th} one of all the surfaces sorted by using $\min\limits_j\{\phi_{cij}\}$ in the manner that $\min\limits_j\{\phi_{cij}\} < \min\limits_j\{\phi_{ci+1j}\}$,
- $\phi_i = \frac{\pi}{2} - \min\limits_j\{\phi_{cij}\}$ and $\phi_i > \phi_{i+1}$,
- m_i is the number of the vertices in $S_{b(i)}$, which denotes the replications of $P(D(\alpha_0), \phi_i)$, and
- $E_{b(ij_i)}$ is the Orientation-fixed Edge of $S_{b(i)}$.

With the strategy presented in the Lemma, the polyhedron, which initially rests on $S_{b(k)}$, can be oriented to a fixed orientation, which is determined by $E_{b(kj_k)}$ $(1 \leq k \leq n)$.

We now localize the polyhedron from resting on the k^{th} surface (k is unknown) to a unique state.

By a discrete search for α_T in $[0, 2\pi)$, we obtain that there exists $\alpha_T = 0$ such that $G_S(\alpha_T)$ has only one loop. By using the mapping

$$f_r : (E_{b(kj_k)}, P(D(\alpha_0), 0)) \rightarrow E_{b(kj_{Tk})},$$

we obtain that

$$E_{b(kj_{Tk})} = E_{b(kj_k)}).$$

Note that $S_{b(k)}$, $E_{b(kj_k)}$, $S_{b(l)}$ and θ_{kl} are given in TABLE IV, where

- $S_{b(k)}$ is the surface resting on the flat plane,
- $E_{b(kj_k)}$ is the edge of $S_{b(k)}$ to determine the pivot edge around which the polyhedron rotates out of its k^{th} state,

TABLE IV

$S_{b(k)}$, $E_{b(kj_k)}$, $S_{b(l)}$ AND θ_{kl}

$S_{b(k)}$	$E_{b(kj_k)}$	$S_{b(l)}$	θ_{kl}
$S_{b(1)}$	v_3v_7	$S_{b(6)}$	0.25
$S_{b(2)}$	v_5v_8	$S_{b(5)}$	0.30
$S_{b(3)}$	v_1v_2	$S_{b(4)}$	0.86
$S_{b(4)}$	v_1v_2	$S_{b(3)}$	0.96
$S_{b(5)}$	v_5v_6	$S_{b(4)}$	0.58
$S_{b(6)}$	v_4v_8	$S_{b(2)}$	0.88

- $S_{b(l)}$ is the conjunct surface of $S_{b(k)}$ at $E_{b(kj_k)}$ (i.e. E_{kl}), and
- θ_{kl} is the Transitional Angle from $S_{b(k)}$ to $S_{b(l)}$.

By using the mapping

$$f : (S_{b(i)}, E_{ij}, max\{\theta_{ij}\}) \to S_{b(j)},$$

we can get the following $S_{b(i)} \Rightarrow S_{b(x)}$ state-transference sequences:

- $S_{b(1)} \Rightarrow S_{b(6)} \Rightarrow S_{b(2)} \Rightarrow S_{b(5)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)}$,
- $S_{b(2)} \Rightarrow S_{b(5)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)}$,
- $S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)}$,
- $S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(3)}$,
- $S_{b(5)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)}$,
- $S_{b(6)} \Rightarrow S_{b(2)} \Rightarrow S_{b(5)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)} \Rightarrow S_{b(4)} \Rightarrow S_{b(3)}$.

The loop subgraph of G_S can be obtained:

$$G_s = \{(S_{b(3)}, \langle S_{b(3)}, S_{b(4)} \rangle), (S_{b(4)}, \langle S_{b(4)}, S_{b(3)} \rangle)\}.$$

Thus, $\Theta_s = \{\theta_{34}, \theta_{43}\}$ and $\theta_{34} < \theta_{43}$ (max$\{\Theta_s\}$ is unique). Therefore, **the polyhedron will finally be localized to the unique state, where $S_{b(4)}$ rests on the flat plane.**

IV. CONCLUSIONS

In this paper, a new practical strategy was proposed to localize a polyhedron on the flat plane from an arbitrary state to a unique state without sensors. The strategy can be implemented in two steps: eliminating the uncertainty of polyhedral bottom-surface orientations and eliminating the uncertainty of polyhedral states.

Firstly, based on the analysis of the 2D-polygon orienting methods, it is found that an arbitrary one of multiple polygons can be oriented to a fixed orientation provided that each of these polygons can be oriented to a unique orientation in a 2D space. Based on this result, conditions have been given for a polyhedron to be oriented to a fixed orientation on the flat plane, and an approach has been presented to eliminate the orientation uncertainty of the polyhedron.

Secondly, from the fixed orientation established above, a polyhedron on the flat plane can be rotated to other desired orientations. Thus, for each state of the polyhedron, different transitional directions from this state to others can be achieved by the rotation of the polyhedron on the flat plane. By the study of the transference of polyhedral states along different transitional directions, we obtained the conditions for a polyhedron to be localized to a unique state without sensors. A strategy was further proposed to localize the polyhedron.

Finally, an example was given to show the validity of the proposed strategy.

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