

# NDT Scan Matching Method for High Resolution Grid Map

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**Abstract**—A new convergence calculation method of the Normal Distributions Transform (NDT) scan matching for high resolution of grid maps is proposed. NDT scan matching algorithm usually has a good effect on large grids, so it is difficult to generate the detailed map with small grids. The proposed method employs Interactive Closest Point(ICP) algorithm to find corresponding point, and it also enlarges the convergence area by modifying the eigenvalue of normal distribution so that the evaluation value is driven effectively for the pairing data. In addition, outlier elimination process is implemented to the scanning for sub-grid scale object. The scanning data from Laser Range Finder(LRF) have error but its set of detected small object can be clustered to determine the Center of Mass(CoM) and the outlier data. The outlier commonly locates behind true points and it can be eliminated when the robot observes from other point. Experimental result shows the effectiveness of the proposed convergence algorithm and outlier elimination method.

## I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is important technique to know unknown environment by the mobile robot. Normal Distributions Transform (NDT) algorithm [2] is one of the scan matching methods for the convergence calculation. In our previous study[1], we employ the grid map with NDT scan matching; The method has the features of fast convergence speed and memory saving for data administration. When we implement grid map for navigation of small robots, the grid size should be small and the detailed map is necessary according to the robot size. However, there are problems to apply the NDT to the detailed grid map. Since the grid size is small, it can capture only a few input points in one scanning, and there is only a few grid having the normal distribution value. Furthermore, a small error makes the input data be in the outside of the reference grid area.

To solve the problems, we proposed two techniques for the high resolution grid map with NDT scan matching. First, the Interactive Closest Point(ICP) algorithm is implemented to find corresponding point even if the input scan points are in the outside of the reference grid area. Secondly, the convergence area adjustment method is proposed to improve the evaluation value with all the pairing data. The normal distribution area enlarges as its eigenvalue becomes large. By using the characteristics, the phased adjustment of convergence area was proposed. The convergence area

is expanded according to the average distance between the initial input scanning points and the corresponding reference grids. After the input points come close to the reference grids, the input points are evaluated and adjusted precisely by using the original normal distribution.

This paper proposes additional scan matching techniques for detailed mapping. Since the resolution of Laser Range Finder(LRF) is usually limited, the small object is not only ignored but also recognized in the false location by the insufficient reflection. The false recognition is divided into 3 cases: edge of object, locating in front of large object, surface of wall. Each recognition is corresponding to the location from the sensor. According to the scanning resolution of LRF, the reliability of scanning area is decided to 2 cases: "reliable", "suspicious". Measurement points are judged by the false recognition, and outlier is deleted. These techniques are implemented to the grid mapping and the experimental results show the feasibility of proposed method.

This paper is organized as follows. Section II discusses related works about environmental mapping. In section III, we describe the basic technique of grid mapping with NDT algorithm. Section IV explains proposed high resolution grid mapping and the difference from last study. Section VI shows experimental results and the effectiveness of the proposed method. Finally, we conclude this paper in section VII.

## II. PREVIOUS WORK

Scan matching is necessary to prevent the accumulation of the error of input data. Iterative Closest Point (ICP) algorithm [4] is one of the scan matching algorithms. The algorithm estimates the optimal pose by minimizing the sum of squared distance between corresponding points. To find the nearest corresponding point of each input point, the calculation load takes  $O(MN)$ . Since the mapping data is managed by collecting each coordinate value, the volume of data is increasing as the number of input scan is increasing. The major calculation load of the ICP spends for the searching the nearest corresponding point. By using k-d tree space quantization algorithm, it takes  $O(N \log(M))$  [5]. Furthermore, additional calculation  $O(n \log n)$  is required for building the k-d tree structure. Elias method [6] is another scan matching algorithm to find a nearest corresponding point. The method divides the scanning area into grids and checks them sequentially so that the corresponding point is found fast. In this case, ICP needs linear time. It depends on the shape of the map data whether it is better to use an Elias method or a k-d tree method.

Biber proposed the Normal Distributions Transform (NDT) algorithm as a new scan matching method [2]. This

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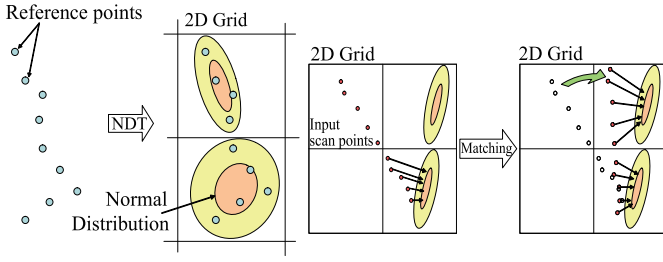


Fig. 1. NDT converts for reference scan

Fig. 2. Flow of NDT scan matching

method divides the scanning space into grids at regular intervals, and the distributions of scanning data in each grid are approximated by the normal distribution. The ICP algorithm matches the input point with the reference point, but the NDT algorithm matches the input point with the normal distributions of the reference grid. NDT algorithm do not need to seek the reference grids for the input scan, so the calculation cost is only  $O(N)$  and the matching speed is fast. Martin [8] and Takeuchi [9] apply NDT to three dimensions. Takeuchi proposed new convergence algorithm which widens the size of grid locating in long-distance to catch the large error input. Nora [10] applied a multi grid size algorithm for matching, and an iterative calculation algorithm with Lievenberg-Marquardt method for effective convergence. Martin also recently proposed new approach to appearance based place recognition exploiting NDT [11]. Furthermore, they presented a comparison of 3D scan registration algorithms: ICP, NDT[12].

The advantage of the NDT algorithm is that the calculation amount is few. However, the convergence performance depends on the grid size because the outlier from the reference grid is not used. In the previous study of NDT, the grid is only used for matching input points with reference grids; the grid size is usually larger than several tens of centimeters, so it's not useful to display map. Thus, the point data is also stored to display detailed environmental map. In our last study[1], we implemented NDT algorithm and grid mapping with small grid. Since the grid size is small, the map has enough resolution and the data volume for mapping doesn't relate to the scanning number of times; the point data is not stored. We proposed high resolution grid mapping method with NDT and showed effectiveness. In this paper, we compare the several grid sizes and the search area in our proposed method, and evaluate the accuracy of convergence. In addition, we implement outlier elimination algorithm for small object sensing.

### III. NDT GRID MAP

This section briefly describes the NDT scan matching algorithm and the grid map. We will call "NDT Grid Map" that employs grid mapping and NDT scan matching. We can expect fast convergence calculation and ease of data management. Since the grid map, robots can use directly it for navigation.

#### A. Normal Distributions Transform (NDT)

The NDT algorithm is one of the scan matching methods. In this algorithm, the scanned space is divided into cells, and the input points in each cell are converted into normal distribution which characterizes the distribution of points. The latest input scan is evaluated in each cell by the normal distribution to converge them to the feasible shape. Since the evaluation is performed only in the cell including both input scan points and normal distribution, the calculation load is  $O(N)$ .

The NDT process is as follows. First, normal distribution is calculated by using point data in each cell(Fig.1). The cell having normal distribution is defined "reference scan". Second, the input scan points are matched to the reference scan (Fig.2). For the process, it is necessary to store and update an average coordinate and a covariance matrix of reference scan in each grid.

Now, we define following variables.

- $p = (p_i)'_{i=1,2,3} = (t_x, t_y, \theta)'$ : A coordinate transformation parameter between the reference scan and the input scan, where  $(t_x, t_y)'$  describes the translation and  $\theta$  is the rotation.
- $x_i$ : A point coordinate of the input scan.
- $x'_i$ : The input scan data which were transformed by coordinate transformation parameter  $p$
- $q_i, \Sigma_i$ : The mean coordinate and the covariance matrix of the normal distribution corresponding to the point  $x'_i$
- $M_i$ : The total number of the points in the cell.

The evaluation function is defined as,

$$E(X, p) = \sum_i^{N-1} \exp \frac{-(x'_i - q_i)' \Sigma_i^{-1} (x'_i - q_i)}{2}. \quad (1)$$

The  $x'_i$  is calculated using the input data  $x_i$  and the transformation parameter  $p$  as follow:

$$x'_i = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} x \cos \theta - y \sin \theta + t_x \\ x \sin \theta + y \cos \theta + t_y \end{pmatrix}. \quad (2)$$

Since the Newton's Algorithm is a nonlinear function minimization algorithm, an optimizing function for NDT scan matching must be;

$$f(p) = -E(X, p). \quad (3)$$

Parameter  $p$  is updated by following equation;

$$\Delta p = -H^{-1} g, \quad p_{new} = p_{old} + \Delta p, \quad (4)$$

where  $g$  and  $H$  are calculated for each input points by following equations;

$$g = \sum_{i=0}^{N-1} \tilde{g}_i, \quad H = \sum_{i=0}^{N-1} \tilde{H}_i. \quad (5)$$

$\tilde{g}_i$  and  $\tilde{H}_i$  are partial differential and second partial differential of optimizing function  $f(p)$  with  $p$ . (For details, please refer to [2])

The computational amount for  $q_j$  and covariance  $\Sigma_j$  is reduced by the following equations;

$$m_j = m_{jold} + x_j, M_j = M_{jold} + N_j, S_j = S_{jold} + x_j x_j \quad (6)$$

$$q_j = \frac{m_j}{M_j}, \quad \Sigma_j = \frac{S_j - q_j m_j^t}{M_j} \quad (7)$$

where  $N_j$  is number of input points for each cell at  $j$  scan.

Each cell has  $m_j$  and  $S_j$ , and they are updated by eq. (6). When the scan matching is operated, the average  $q_j$  and the covariance matrix  $\Sigma_j$  are calculated by eq. (7).

### B. Grid Map

Grid map is one of the representation methods. In this method, space is divided into a lattice of congruent cells. The cell is minimum resolution and the measured points in the cell are destroyed. Thus, memory consumption does not increase even if a new input scan was stored. To share the data of NDT scanning match, each grid stores following elements:

- $M$  : The count of inputted data
- $m$  : A summation of the coordinate vectors
- $S$  : A matrix which becomes the covariance matrix

## IV. THE HIGH RESOLUTION GRID MAPPING WITH NDT

### A. new convergence algorithm

Grid size is a key of NDT algorithm. If the grid size is small, the map becomes high resolution, but the total computational amount increases. On the contrary, if a grid size is big, a computational amount becomes little, but the resolution becomes low.

When the grid size is small, the pair of a reference scan and an input scan will be easily changed, and the evaluation value will be unsteadily changed, even if the input error is small. Furthermore, the NDT scan matching does not make pair with the neighboring grid. Thus, the available pairs decrease and the convergence calculation becomes difficult.

To solve the problems, we propose following three operations for NDT algorithm.

#### 1) Search for neighboring grids

The NDT algorithm doesn't need to search a reference scan for an input scan. However, when the grid size is small, even if the error of input is small, it reduces the number of matching pair. The problem is solved by using Elias method searching for nearest reference scan from neighboring grids.

#### 2) Expansion of normal distribution

When the input scan is matched with the reference scan by using above process, if the distance of the two scans is too long, the degree of conformance becomes small and it does not affect convergence process. As a solution of the problem, the normal distribution expansion method is proposed; it enlarged the skirts of the normal distribution to give an effectual value to a long-distance pair. Specifically, we enlarge two eigenvalues of the covariance matrix  $\Sigma$  of the eq. (7).

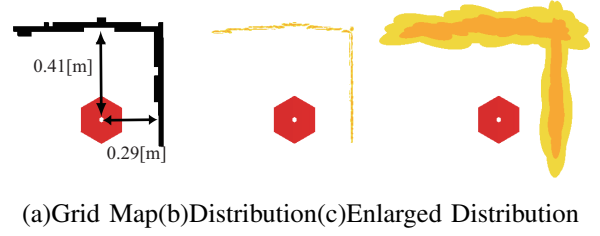


Fig. 3. Enlarged normal distributions of reference scans

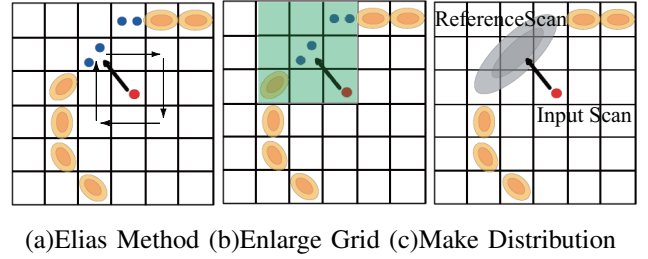


Fig. 4. Enlarged Reference Scan

Let  $\varphi_j$  be an eigenvector of an eigenvalue  $\lambda_j$ , and let  $k$  be an expansion coefficient of the eigenvalue, a new covariance matrix  $\Sigma'_j$  is given as follow;

$$\Sigma'_j = V \Lambda' V^t \quad (8)$$

$$V = [\varphi_1 \ \varphi_2], \quad \Lambda' = \begin{bmatrix} k\lambda_1 & 0 \\ 0 & k\lambda_2 \end{bmatrix}$$

#### 3) Fusion around minority reference cell

If a cell had only one or two points of data, it cannot make normal distribution. Small grid makes this problem frequently at the first scan time and far place. To solve the problem, we implement the summing up process to the neighboring around the cell which has three or less points. Fig.4 shows the summing up process. When a reference cell for an input scan has points less than three like Fig.4 (a) and it cannot make normal distribution, The neighboring cells around the cell are checked like Fig.4 (b) so that the summed up cell is made like Fig.4 (c). Extra data space is not necessary to implement the algorithm, since it is sufficient to calculate sum of eq.(6) about the neighboring cells.

### B. Evaluation of proposed method

Fig.5(a) shows experimental data; gray points indicate input scan, black points indicate reference scan. We compare several grid size and the changes of the evaluation value with the error of input scan are compared by several grid sizes and expansion coefficients. Fig.5(b)-(f) shows evaluation value  $E(p)$  of eq.(1). The input error is given  $\pm 40$  [cm] in each  $x$ ,  $y$  axis.

First, Fig.5(b), (c) show the evaluation value of the original NDT algorithm by using 2 and 20 [cm] grid. The distribution

of evaluation value in (c) is smoother rather than (b), and the area of local minimum are decreased. The original NDT algorithm, which doesn't match with neighboring grids, has good performance by using large grid size. Next, Fig.5(d), (e) show the evaluation value of the proposed NDT algorithm by using 2[cm] grid with expansion coefficient  $k=10$ . (e) employs also the fusion process around minority reference cell. The evaluation value of (d) is increased overall compared with (b) in the same 2[cm] grid, and the local minimum also decreases. The distribution of evaluation value of (e) is expanded rather than (d), so that the function make easy to converge large error inputs. Fig.5(f) shows the evaluation value using 2[cm] grid with expansion coefficient  $k=100$  and fusion algorithm. The distribution of evaluation value of (f) is also expanded rather than (e), thus the proposed algorithm can be better as the expansion coefficient increases. Based on the result, our last study[1] proposed an phased adjustment algorithm of convergence area by reducing gradually the expansion coefficient  $k$ . However, the phased adjustment takes much time, and if the input error is not so big and the expansion coefficient is fixed, the convergence result is not so different. Thus, in this report, we employ fixed expansion coefficient  $k=10$ .

### C. Experiment using NDT grid map

Fig.6(a) shows experimental environment. The multi-legged robot "ASTERISK"[13] and LRF (URG-04LX : Hokuyo Electric Co., Ltd.) are employed. In this paper, a grid size is defined 2[cm] and the size of a total grid space is defined 8[m] based on the spec of LRF. The 2[cm] grid is strict for the LRF, since it's accuracy is  $\pm 10$ [mm]. The dead reckoning has error since the calibration error of the joint and the slipping on the floor. This sensor measures a 2 dimensional plane, and the range covering  $\pm 120$  degrees. It can get 683 points' distances at one scan.

Fig.6(b)-(d) show experimental results. All maps consist of 2[cm] grids, and the black point represents scanned grids, the gray line represents trajectory of robot. (b) is made by using only dead reckoning, (c) employs original NDT algorithm: it means  $k=1$ , (d) is generated by using proposed algorithm with  $k=10$ . The error of dead reckoning is corrected by using NDT algorithm and more effective modification is conducted by using our proposed method.

## V. RECOGNITION FOR SUBGRID SCALE OBJECT

The detectable scale of LRF is limited by the resolution performance, and the small object leads false recognition. Our LRF has  $\pm 10$ [mm] resolution in radiation direction, but the angular resolution capability is 0.36[deg]. Thus, the resolution in the tangential direction decreases as the length in the radiation direction increases. If the detect point was not so long, the 2[cm] grid is strict size for the small object which smaller than 2[cm]. In this section, we propose subgrid size object recognition method.

### A. Variation of false detection for subgrid object

The small object can be detected by using latest scanning data, but it should be not stored because the false detection

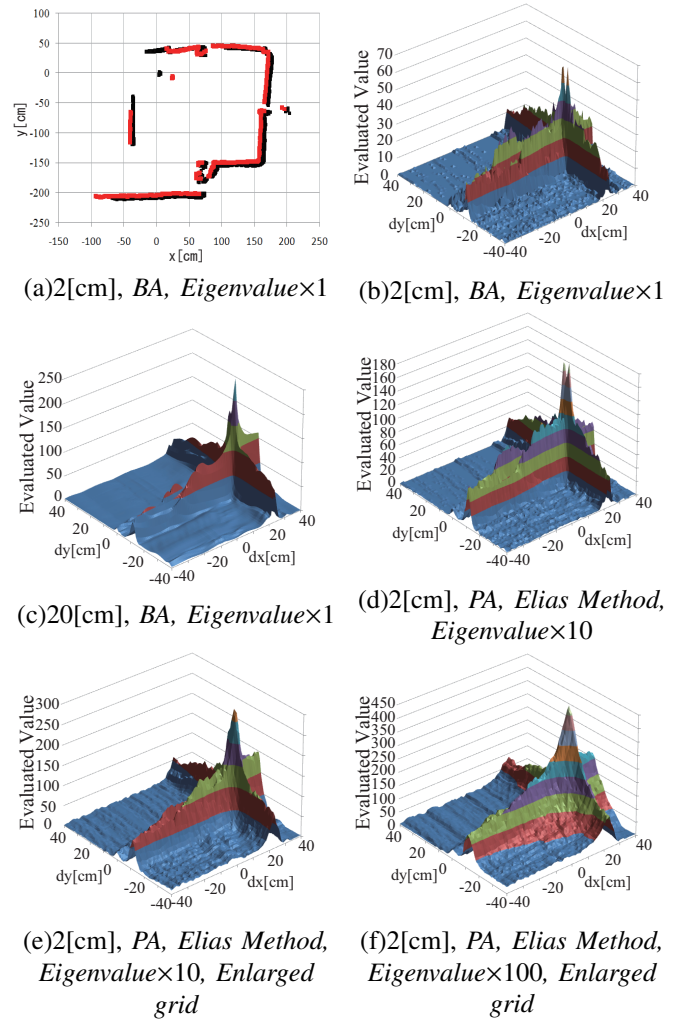


Fig. 5. The evaluation value

occurs frequently. Because of the detection principle of LRF, the detection reaction is changed by the object condition: scale, surface shape, material etc., and it is a reason of false detection. Fig.7 shows one shot scanning examples for a small object. The LRF is located  $(x,y)=(0,0)$ . The object is iron stick whose diameter is 1.3[cm]. The black circle is true position and the gray points are data from LRF. (a) is the result that only a metal stick scanned. The false detections exist at the edge of object, and the error position tends to be located behind the object. Thus, the nearest data cloud is dependable. (b) is the result which scans a metal stick in front of wall. In this case, the small object is not completely detected: all scanned points shift to backward. The dotted line is setup position for the metal stick, so the detected result has 5[cm] error. From fig.6, another current error is found in the surface of wall. Now, we categorize the error pattern into three: edge of object, small object in front of wall, surface of wall.

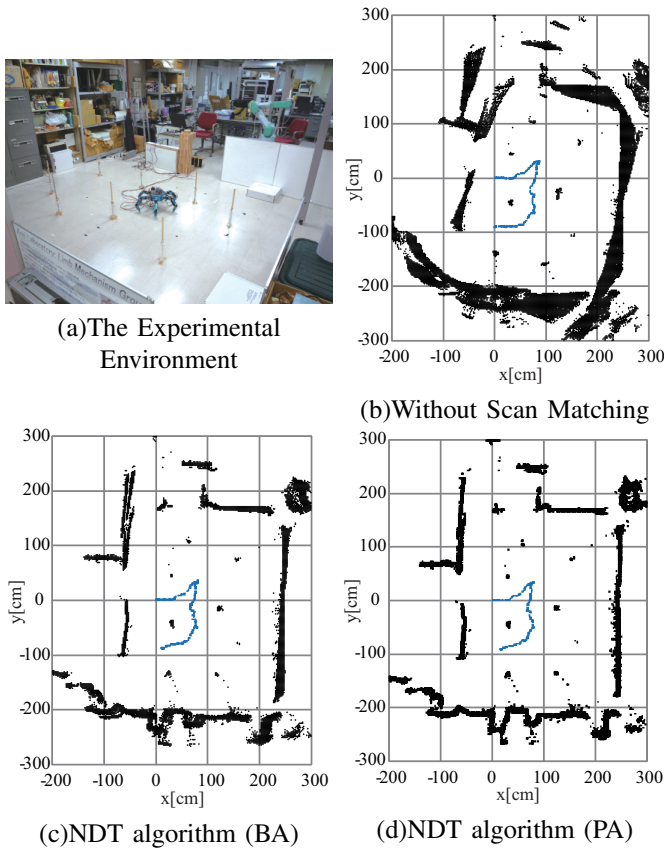


Fig. 6. The Experiment

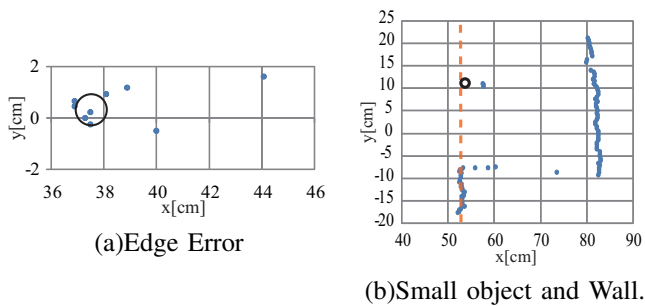


Fig. 7. Errors of LRF

### B. Elimination method for false detection

The catalog spec of LRF(URG-04LX) indicates that the smallest object detection is defined 1[deg]; 3 neighboring laser spot. It means 2[cm] object can be detected near than 120[cm]. However, our preliminary experiments says that the reliable area is within 50[cm] for 2[cm] size objects and there are a lot of false detection in the area from 50 to 120[cm]. To change the false detection algorithm, we divide the scanning area into two areas; Area 1 is defined within 50[cm], Area 2 is the other.

1) *Edge of object*: The false detection of edge can be removed before inputting grid map, since the each data set

of object can be checked by clustering method in one shot scanning data. The process is as follows;

- a-1 Apply clustering process to the new input scan data and select the cluster with small number.
- a-2 Calculate Center of Mass(CoM) position of the cluster.
- a-3 Remove the data in back of the CoM position.
- a-4 Convert the point data into grid data and store only the grid being at the forefront.

These processes are implemented in both Area 1 and 2.

2) *Small object in front of wall*: In Area 2, when the object is in front of wall/big object, the detected points move to backward. However, in Area 1, the detected points are true value in the same situation; they don't move to backward. Thus, we implement following process;

- b-1 Delete the grid not observed in the Area 1.
- b-2 If there are former grid data in the short of radiation direction, don't store the new scanned points; it might be false detection moving to backward.

3) *Surface of wall*: The false detection grids at the surface of wall/big object tends to be observed only a few times. Thus, we can check the false detection by the number of scanned times, delete them. In practical use, we define the frequency of the checking once every three times.

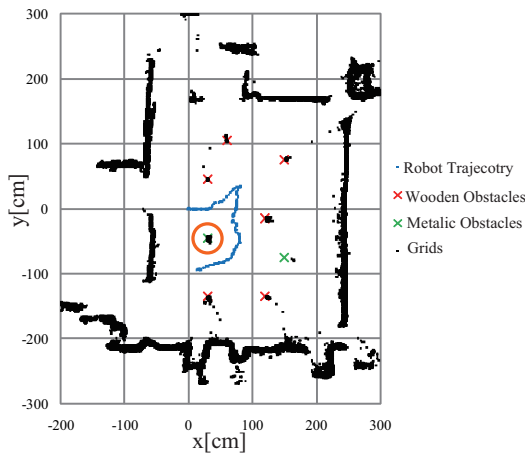
4) *Confirmation of detection result*: There are still some leavings after the above elimination process. When the scanned grid is converted to the reference scan grid, the grids are checked in the radiation direction and if there are former grid data in short, eliminate the old grids. For the detection of small object, this process is employed in Area 1 because the detection in Area 2 is not reliable. Meanwhile, since the big object is stably observed, we employ this process both in Area 1 and 2, so that the mobile object can be eliminated.

## VI. ENVIRONMENTAL MAP WITH SMALL OBJECT

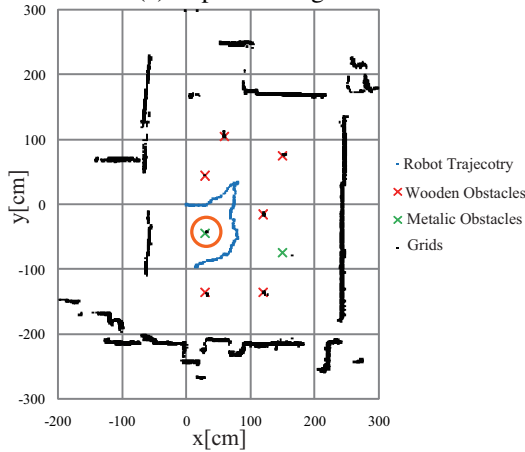
By using the proposed NDT scan matching algorithm and subgrid object error detection, the environmental map with small objects is generated. Six wooden sticks whose diameter is 1.5[cm] and two iron sticks whose diameter is 1.3[cm] are prepared in the experimental environment. The expansion coefficient of the proposed NDT algorithm is set  $k=10$ . Fig.8 shows experimental result. Fig.8(a) is made without error elimination algorithm. Fig.8(b) is using the proposed elimination algorithm. The errors around the small objects and the surface of wall are almost eliminated. The average error of detected CoM positions is decreased from 4.1[cm] to 2.3[cm]. Fig.9 is the enlarged view of the surroundings of the iron stick at the center in Fig.8. Fig.9(a) shows the first map by using section V-B but the errors still remain; the robot is moving around start position. Fig.9(b) shows the final map, it is confirmed that the errors are almost eliminated since the robot is located opposite side of the iron stick.

## VII. CONCLUSION

High resolution grid mapping with NDT scan matching and the subgrid object recognition are proposed. The convergence algorithm consists of three processes: search for neighboring cell, expansion of normal distribution, fusion



(a)Map including errors



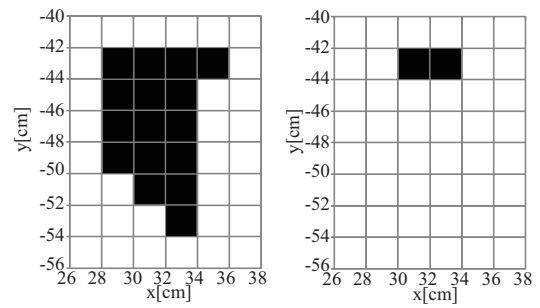
(b)Errors are eliminated by proposed method.

Fig. 8. The Experimental Results

around minority reference cell. Each process contributes to derive effective evaluation value, as a result the grid map with small cell is available to know the detailed environment. In addition, the subgrid size object, which is difficult to know precisely depending on the sensor spec, is recognized by the error rejection algorithm for the vague input. Experimental result shows the effectiveness of the detailed environmental map generated by the NDT high resolution grid mapping. The method will be used for navigation of small robot. Our next target is making of 3D environmental map.

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(a)Errors are included (b)Errors are eliminated

Fig. 9. Efficacy of the Elimination Method

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