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Stiffness Control of Multi-DOF Joint

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***Abstract*—This paper deals with mechanical stiffness control of multi-DOF joint. It fundamentally mimics the skeleto-muscular system of human articulation, in which at least two muscles handle one rotary axis under their antagonistic (counteractive) action. In the first part of the paper one introduces basic formula for controlling the multi-DOF rotary joint that is assumed to be driven by a couple of novel actuators called ANLES (Actuator with Non-Linear Elastic System). It mimics a skeletal muscle in the sense of having a non-linear elasticity. Next the paper describes the structure of the ANLES that is designed and constructed for controlling the wrist joint of an anthropomorphic robot. The experimental results using three DOF joint controlled by four ANLES reveal that the joint angle and the joint stiffness can be independently controlled by the proposed formula.**

I. Introduction

It is easily found that some dexterous motions of human articulations are due to the capability of regulating the stiffness in accordance with a task that he/she is about to do. The skeleto-muscular system of human articulations is able to regulate its stiffness mechanically rather than by efferent command from the CNS using exteroceptive force feedback. The key mechanism for regulating the stiffness is the antagonistic structure of the skeleto-muscular system; one agonist and its antagonist muscles counteractively drive one articulation. Simultaneous stretching of both muscles provides high stiffness of the articulation and both relaxing gives us the low stiffness. It is notified that the non-linear elasticity of the muscles is prerequisite for the agonist-antagonist structure for regulating the stiffness. Some amount of displacement of joint angle requires a respectively small torque at the joint under the equilibrium state of low stretching of both muscles. On the other hand the equilibrium state under high stretching requires a respectively large torque to provide the same amount of angle displacement. So the stiffness is regulated according to magnitude of stretching of both muscles. It is obvious that linear elasticity does not provide such a stiffness change. A vast amount of physiological studies have elucidated skeletal muscles have the non-linear elasticity like this [1][2][3][4].

Some studies for investigating the stiffness of human arms elucidate that the stiffness ellipse of the arm's endpoint is adjustable in its volume by stretching muscles [5], but its shape is roughly determined by the arm's posture [6].

Some studies in the field of robotics deal with the antagonistic control of joints [7][8][9][10][11][12] and pointed out the importance of the non-linear characteristics of the elastic elements to control the stiffness of the joint [9][10][11], but there have been few papers that propose the control method of stiffness in the practical point of view, although some theoretical approach for stiffness control provides valuable insights[11][13][14].

This study assumes artificial joints that are controlled by at least two actuator units having a similar elastic characteristic to human voluntary muscles. It is called the *antagonistically driven joints* (ADJ).

There have been some approaches to comprise the ADJ using linear actuators that works like muscle. The most successful approach developed so far will be those of using the McKibben type pneumatic actuator[15][16]. Although the pneumatic rubber actuator inherently has non-linear elastic characteristics, it has some drawbacks such as, the difficulty of designing the non-linear elasticity, the heat sensitivity, large volume of apparatus for supplying compressed air, etc. Koganezawa proposed to use a conical spring that is embedded between the actuator and the joint to be controlled [17]. However it also has the similar drawbacks to the rubber actuator such as the difficulty for designing the non-linear elasticity and the large volumes of the overall transmission system.

There are some recent another approaches to develop the non-linear elastic module used to control the stiffness of ADJ [18][19], which presented ingenious mechanical devices to design the non-linear elasticity, although they are prototypical.

This paper proposes the alternative mechanism used for an artificial muscle to comprise the ADJ. It is on the same line of [18][19][20] in the sense of composing the non-linear elasticity through converting the force generated by a normal linear spring on the process of its transmission[21][22][23].

In the following section, the basic formula for controlling the stiffness of the ADJ having multiple DOF is derived, in which the control of the stiffness and the control of the joint angle are decoupled by the dual equations.

In the third section, a new mechanism is proposed, which fundamentally mimics a skeletal muscle that has non-linear elasticity mentioned above. We call it as the *actuator with non-linear elastic system* (ANLES). It shows the structure of the ANLES accompanied by the design of its non-linear elasticity. It follows the experimental results to show non-linear characteristics of the ANLES sufficiently corresponding to those of the designed one.

The fourth section is devoted to show the three DOF rotary joint controlled by four ANLESes, which associates a wrist joint of an anthropomorphic robot. The validity of the proposed equations derived in the second section is clearly shown by the experimental results of the stiffness regulation under constant joint angle and the joint angle regulation under constant stiffness.

In the final section, some conclusive remarks are described.

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II. Basic formula for controlling the stiffness and rotation of multi-DOF joint

A. Kinetics of a multiple DOF joint system driven by multiple tendons

Let us consider the joint having three rotation axes (roll, yaw, pitch) that are driven by *m* tendons as shown in Fig.1. Each tendon is stretched by the individual actuator that is similar to a voluntary skeletal muscle in the sense of having non-linear elasticity. Let us call it *actuator with non-linear elastic system* (ANLES). Its tension force vector for the *j*th tendon is denoted by $\boldsymbol{\xi}_j$ of which modulus is assumed to be a non-linear function with respect to one individual variable ϕ_j,

$$\left|\xi_j\right| = \left|\xi_j\right|(\phi_j) \tag{1}$$

It is also assumed to be a mass located above the joint. Denoting $\boldsymbol{r}_{jk}$ for the moment arm vector from the *k*th joint axis to the *j*th tendon and $\boldsymbol{r}_{gk}$ for the *k*th joint axis to the gravitation force vector, we can derive a kinetically equilibrium state equation in terms of torque,

$$\begin{bmatrix} \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j1}\times\boldsymbol{\xi}_j)+\boldsymbol{r}_{g1}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_1 \\ \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j2}\times\boldsymbol{\xi}_j)+\boldsymbol{r}_{g2}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_2 \\ \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j3}\times\boldsymbol{\xi}_j)+\boldsymbol{r}_{g3}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_3 \end{bmatrix} = \boldsymbol{0} \tag{2}$$

where, $\boldsymbol{e}_k, k=1,2,3$ represents the unit vector about the *k*th joint axis. Let us consider that infinitesimal torque $\Delta\boldsymbol{\tau}$ is loaded at the joint and it gives rise to infinitesimal rotation of joints $\Delta\theta_k$ $(k=1,2,3)$. The relationship between them can be derived by differentiating (2),

$$\Delta\boldsymbol{\tau} = \begin{bmatrix} \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\boldsymbol{r}_{j1}\times\frac{\partial\xi_j}{\partial\theta_i}+\frac{\partial\boldsymbol{r}_{j1}}{\partial\theta_i}\times\xi_j)+\frac{\partial\boldsymbol{r}_{g1}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_1 \\ \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\boldsymbol{r}_{j2}\times\frac{\partial\xi_j}{\partial\theta_i}+\frac{\partial\boldsymbol{r}_{j2}}{\partial\theta_i}\times\xi_j)+\frac{\partial\boldsymbol{r}_{g2}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_2 \\ \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\boldsymbol{r}_{j3}\times\frac{\partial\xi_j}{\partial\theta_i}+\frac{\partial\boldsymbol{r}_{j3}}{\partial\theta_i}\times\xi_j)+\frac{\partial\boldsymbol{r}_{g3}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_3 \end{bmatrix} \tag{3}$$

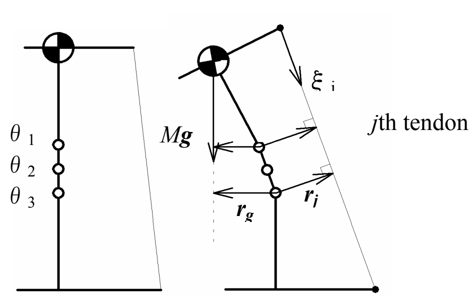


Fig.1 Model of the three D.O.F. joints driven by m tendons.

$\partial\xi_j/\partial\theta_i$ is rewritten by using the length of *j*th tendon l_j,

$$\frac{\partial\boldsymbol{\xi}_j}{\partial\theta_i} = \frac{\partial\boldsymbol{\xi}_j}{\partial l_j}\frac{\partial l_j}{\partial\theta_i} = \frac{\partial\boldsymbol{\xi}_j}{\partial l_j}\gamma_{ji} \tag{4}$$

where, $\gamma_{ji}\equiv\partial l_j/\partial\theta_i$ is a moment radius of the *j*th tendon with respect to the *i*th rotary axis. $\partial\boldsymbol{\xi}_j/\partial l_j$ can also be rewritten as follows,

$$\frac{\partial\boldsymbol{\xi}_j}{\partial l_j} = \frac{\partial\xi_j}{\partial\phi_j}\frac{\partial\phi_j}{\partial l_j} = \frac{\partial\xi_j}{\partial\phi_j}\frac{1}{\delta_j} \tag{5}$$

with $\delta_j\equiv\partial l_j/\partial\phi_j$ be a change rate of the tendon length with respect to the variable of the ANLES.

The tension force vector $\boldsymbol{\xi}_j$ is expressed by using its unit vector $\boldsymbol{\eta}_j$,

$$\boldsymbol{\xi}_j = \left|\boldsymbol{\xi}_j\right|\boldsymbol{\eta}_j \tag{6}$$

Using the notations of (4) ,(5) and (6), (3) is rewritten as,

$$\Delta\boldsymbol{\tau} = \begin{bmatrix} \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\frac{\partial|\xi_j|}{\partial\phi_j}\frac{\gamma_{ji}}{\delta_j}\boldsymbol{r}_{j1}\times\boldsymbol{\eta}_j+\frac{\partial\boldsymbol{r}_{j1}}{\partial\theta_i}\times|\xi_j|\boldsymbol{\eta}_j)+\frac{\partial\boldsymbol{r}_{g1}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_1 \\ \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\frac{\partial|\xi_j|}{\partial\phi_j}\frac{\gamma_{ji}}{\delta_j}\boldsymbol{r}_{j2}\times\boldsymbol{\eta}_j+\frac{\partial\boldsymbol{r}_{j2}}{\partial\theta_i}\times|\xi_j|\boldsymbol{\eta}_j)+\frac{\partial\boldsymbol{r}_{g2}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_2 \\ \left[\sum_{i=1}^{3}\left(\sum_{j=1}^{m}(\frac{\partial|\xi_j|}{\partial\phi_j}\frac{\gamma_{ji}}{\delta_j}\boldsymbol{r}_{j3}\times\boldsymbol{\eta}_j+\frac{\partial\boldsymbol{r}_{j3}}{\partial\theta_i}\times|\xi_j|\boldsymbol{\eta}_j)+\frac{\partial\boldsymbol{r}_{g3}}{\partial\theta_i}\times M\boldsymbol{g}\right)\Delta\theta_i\right]\cdot\boldsymbol{e}_3 \end{bmatrix}$$
$$= \boldsymbol{S}\Delta\boldsymbol{\theta} \tag{7}$$

where, $\Delta\boldsymbol{\theta}=[\Delta\theta_1\ \Delta\theta_2\ \Delta\theta_3]^T$ and $\mathbf{S}=\{s_{ij}\,|\,(i,j=1,2,3)\}\in\Re^{3\times3}$ is identified as a stiffness matrix with respect to the rotation. It is practically difficult to manipulate all elements of the stiffness matrix (6 independent elements). In order to do so, it requires more than 9 tendons [11]. The authors consider it is not practical and not necessary to control all of them. The experiments in the field of the motion physiology [5] shows that human can regulate the stiffness-ellipsoid at the end-point of the arm merely in its volume rather than in its shape. So let us consider to manipulate only the diagonal three elements of the stiffness matrix. For the general consideration dealt with off-diagonal elements of the stiffness matrix, please refer the paper [21].

The stiffness vector consisting of the diagonal elements of the stiffness matrix is calculated by the following equation,

$$\boldsymbol{s}\equiv\begin{bmatrix}s_{11}\\ s_{22}\\ s_{33}\end{bmatrix} = M\boldsymbol{\lambda}_0+\boldsymbol{\lambda}_1\begin{bmatrix}\partial|\boldsymbol{\xi}_1|/\partial\phi_1\\ \vdots\\ \partial|\boldsymbol{\xi}_m|/\partial\phi_m\end{bmatrix}+\boldsymbol{\lambda}_2\begin{bmatrix}|\boldsymbol{\xi}_1|\\ \vdots\\ |\boldsymbol{\xi}_m|\end{bmatrix} \tag{8}$$

with

$$\boldsymbol{\lambda}_0 = \begin{bmatrix}\left[\frac{\partial\boldsymbol{r}_{g1}}{\partial\theta_1}\times g\right]\cdot\boldsymbol{e}_1\\ \left[\frac{\partial\boldsymbol{r}_{g2}}{\partial\theta_2}\times g\right]\cdot\boldsymbol{e}_2\\ \left[\frac{\partial\boldsymbol{r}_{g3}}{\partial\theta_3}\times g\right]\cdot\boldsymbol{e}_3\end{bmatrix},\quad \boldsymbol{\lambda}_2 = \begin{bmatrix}\left(\frac{\partial\boldsymbol{r}_{11}}{\partial\theta_1}\times\boldsymbol{\eta}_1\right)\cdot\boldsymbol{e}_1 & \cdots & \left(\frac{\partial\boldsymbol{r}_{m1}}{\partial\theta_m}\times\boldsymbol{\eta}_m\right)\cdot\boldsymbol{e}_1\\ \left(\frac{\partial\boldsymbol{r}_{12}}{\partial\theta_2}\times\boldsymbol{\eta}_1\right)\cdot\boldsymbol{e}_2 & \cdots & \left(\frac{\partial\boldsymbol{r}_{m2}}{\partial\theta_2}\times\boldsymbol{\eta}_m\right)\cdot\boldsymbol{e}_2\\ \left(\frac{\partial\boldsymbol{r}_{13}}{\partial\theta_3}\times\boldsymbol{\eta}_1\right)\cdot\boldsymbol{e}_3 & \cdots & \left(\frac{\partial\boldsymbol{r}_{m3}}{\partial\theta_3}\times\boldsymbol{\eta}_m\right)\cdot\boldsymbol{e}_3\end{bmatrix}$$

$$\boldsymbol{\lambda}_1 = \begin{bmatrix} \frac{\gamma_{11}}{\delta_1}(\boldsymbol{r}_{11}\times\boldsymbol{\eta}_1)\cdot\boldsymbol{e}_1 & \cdots & \frac{\gamma_{m1}}{\delta_m}(\boldsymbol{r}_{m1}\times\boldsymbol{\eta}_m)\cdot\boldsymbol{e}_1 \\ \frac{\gamma_{12}}{\delta_1}(\boldsymbol{r}_{12}\times\boldsymbol{\eta}_1)\cdot\boldsymbol{e}_2 & \cdots & \frac{\gamma_{m2}}{\delta_m}(\boldsymbol{r}_{mk}\times\boldsymbol{\eta}_m)\cdot\boldsymbol{e}_2 \\ \frac{\gamma_{13}}{\delta_1}(\boldsymbol{r}_{13}\times\boldsymbol{\eta}_1)\cdot\boldsymbol{e}_3 & \cdots & \frac{\gamma_{m3}}{\delta_m}(\boldsymbol{r}_{m3}\times\boldsymbol{\eta}_m)\cdot\boldsymbol{e}_3 \end{bmatrix},$$

Let us consider the infinitesimal rotation of ANLESes that give rise to the infinitesimal variation of the stiffness vector under holding a constant joint angle. We derive the following equation from (8),

$$\Delta\boldsymbol{s} = \left[\boldsymbol{\lambda}_1 diag\left\{\frac{\partial^2|\boldsymbol{\xi}|_1}{\partial\phi_1^2},\cdots,\frac{\partial^2|\boldsymbol{\xi}|_m}{\partial\phi_m^2}\right\} + \boldsymbol{\lambda}_2 diag\left\{\frac{\partial|\boldsymbol{\xi}|_1}{\partial\phi_1},\cdots,\frac{\partial|\boldsymbol{\xi}|_m}{\partial\phi_m}\right\}\right]\Delta\boldsymbol{\phi} \equiv \boldsymbol{\Lambda}\Delta\boldsymbol{\phi} \quad (9)$$

Eq. (9) suggests that the second-order derivatives of tensions with respect to the variables of ANLESes play an important role for regulating the stiffness, which implies to require the non-linear elasticity (this fact is also suggested by the paper [13]). Next, let us consider the infinitesimal variation of the tension vector $\boldsymbol{\xi}_i$ due to the variation of ϕ_i that give rise to the infinitesimal rotation of the joints. We have the following equation from (2),

$$\begin{bmatrix} \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j1}\times\Delta\boldsymbol{\xi}_j + \Delta\boldsymbol{r}_{j1}\times\boldsymbol{\xi}_j) + \Delta\boldsymbol{r}_{g1}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_1 \\ \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j2}\times\Delta\boldsymbol{\xi}_j + \Delta\boldsymbol{r}_{j2}\times\boldsymbol{\xi}_j) + \Delta\boldsymbol{r}_{g2}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_2 \\ \left[\sum_{j=1}^{m}(\boldsymbol{r}_{j3}\times\Delta\boldsymbol{\xi}_j + \Delta\boldsymbol{r}_{j3}\times\boldsymbol{\xi}_j) + \Delta\boldsymbol{r}_{g3}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_3 \end{bmatrix} = \boldsymbol{0}$$

This can be rewritten by using (4)(5) and (6),

$$\boldsymbol{\Gamma}\Delta\boldsymbol{\theta} = -\boldsymbol{\Sigma}\Delta\boldsymbol{\phi} \quad (10)$$

where, $\Delta\boldsymbol{\phi} = [\Delta\phi_1 \cdots \Delta\phi_m]^T$,

$$\boldsymbol{\Gamma} = \left\{\left[\sum_{j=1}^{m}(\boldsymbol{r}_{jk}\times\frac{\gamma_{jl}}{\delta_j}\frac{\partial\xi_j}{\partial\phi_j}\boldsymbol{\eta}_j + \frac{\partial\boldsymbol{r}_{jk}}{\partial\theta_k}\times\xi_j) + \frac{\partial\boldsymbol{r}_{gk}}{\partial\theta_k}\times M\boldsymbol{g}\right]\cdot\boldsymbol{e}_k \;\middle|\; (k,l=1,2,3)\right\} \in \Re^{3\times3},$$

$$\boldsymbol{\Sigma} = \left\{\left[\boldsymbol{r}_{jk}\times\frac{\partial\xi_j}{\partial\phi_j}\boldsymbol{\eta}_j\right]\cdot\boldsymbol{e}_k \;\middle|\; (k=1,2,3,\ j=1,\cdots,m)\right\} \in \Re^{3\times m}$$

The general solution of (10) with respect to $\Delta\boldsymbol{\phi}$ is

$$\Delta\boldsymbol{\phi} = -\boldsymbol{\Sigma}^{\dagger}\boldsymbol{\Gamma}\Delta\boldsymbol{\theta} - \boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\Delta\boldsymbol{\zeta} \quad (11)$$

where, $\boldsymbol{\Sigma}^{\dagger}\in\Re^{m\times3}$ represents the generalized inverse of $\boldsymbol{\Sigma}$ and $\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\in\Re^{3\times3}$ represents the null projection operator that projects arbitrarily specified vector $\Delta\boldsymbol{\zeta}\in\Re^m$ on the complementary space of $\boldsymbol{\Sigma}$.

B. The formulation for the ANLES to vary the joint stiffness under keeping joint angles to be constant

Let us first consider to regulate stiffness of the joint without giving rise to displacement of the joint angles. This must be carried out by the displacement of the ANLES's variable vector $\Delta\boldsymbol{\phi}$ while holding $\Delta\boldsymbol{\theta}=\boldsymbol{0}$ in (11),

$$\Delta\boldsymbol{\phi} = -\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\Delta\boldsymbol{\zeta} \quad (12)$$

Substitution of (12) into (9) provides,

$$\Delta\boldsymbol{s} = -\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\Delta\boldsymbol{\zeta} \quad (13)$$

Solving (13) with respect to $\Delta\boldsymbol{\zeta}$ we have,

$$\Delta\boldsymbol{\zeta} = -\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\right)^{\dagger}\Delta\boldsymbol{s} \quad (14)$$

Substitution of it into (12) gives us,

$$\Delta\boldsymbol{\phi}_s = \boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\right)^{\dagger}\Delta\boldsymbol{s} \quad (15)$$

where, $\Delta\boldsymbol{\phi}_s$ is the infinitesimal variation of the ANLES's variable vector to provide the stiffness variation $\Delta\boldsymbol{s}$ with no giving rise to the rotation of the joint angles.

C. The Formulation for the ANLES to vary the joint angles under keeping the joint stiffness to be constant

Next it is capable to derive the formula for regulating angles of joints without bringing about the variation of the stiffness of the end-point. The general solution of (9) with respect to $\Delta\boldsymbol{\phi}$ will be,

$$\Delta\boldsymbol{\phi} = \boldsymbol{\Lambda}\Delta\boldsymbol{s} + \boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\Delta\boldsymbol{\psi} \quad (16)$$

where $\Delta\boldsymbol{\psi}\in\Re^m$ is a vector that is arbitrarily assignable.

Since we aim to keep the stiffness being constant, $\Delta\boldsymbol{s}$ in (16) should be zero,

$$\Delta\boldsymbol{\phi} = \boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\Delta\boldsymbol{\psi} \quad (17)$$

Substituting it into (10), we have

$$\boldsymbol{\Gamma}\Delta\boldsymbol{\theta} = -\boldsymbol{\Sigma}\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\Delta\boldsymbol{\psi}\,. \quad (18)$$

Solving it with respect to $\Delta\boldsymbol{\psi}$ and substituting into (17), we have,

$$\Delta\boldsymbol{\phi}_a = -\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\left(\boldsymbol{\Sigma}\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\right)^{\dagger}\boldsymbol{\Gamma}\Delta\boldsymbol{\theta}\,. \quad (19)$$

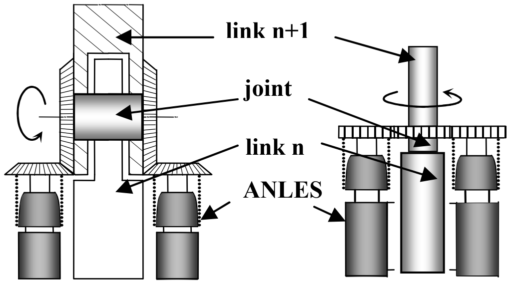
where $\Delta\boldsymbol{\phi}_a$ is the infinitesimal variation of the ANLES's variable vector to give rise to the joint rotation with no interference with the joint stiffness. $\Delta\boldsymbol{\phi}_s$ in (15) is simplified by making use of the nilpotent property of projection operator,

$$\begin{aligned}\Delta\boldsymbol{\phi}_s &= \boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\right)^{\dagger}\Delta\boldsymbol{s} \\ &= \boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\right)^T\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\right)^T\right)^{-1}\Delta\boldsymbol{s} \quad (20) \\ &= \boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\boldsymbol{\Lambda}^T\left(\boldsymbol{\Lambda}\boldsymbol{P}^{\perp}(\boldsymbol{\Sigma})\boldsymbol{\Lambda}^T\right)^{-1}\Delta\boldsymbol{s}.\end{aligned}$$

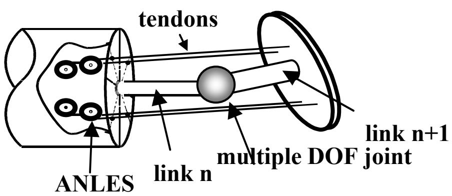
Similar procedure can be applied to (19) as follows,

$$\begin{aligned}\Delta\boldsymbol{\phi}_a &= -\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\left(\boldsymbol{\Sigma}\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\right)^{\dagger}\boldsymbol{\Gamma}\Delta\boldsymbol{\theta} \\ &= -\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\boldsymbol{\Sigma}^T\left(\boldsymbol{\Sigma}\boldsymbol{P}^{\perp}(\boldsymbol{\Lambda})\boldsymbol{\Sigma}^T\right)^{-1}\boldsymbol{\Gamma}\Delta\boldsymbol{\theta}.\end{aligned} \quad (21)$$

(20) is the basic formula for manipulating the stiffness without giving rise to the variation of joint angles. As a dual equation (21) is that of manipulating the joint angles without giving rise to the variation of stiffness. However please notice that the control of the stiffness and joint angles individually by using (20) and (21) may not succeed perfectly. It depends on the number of tendons that are individually

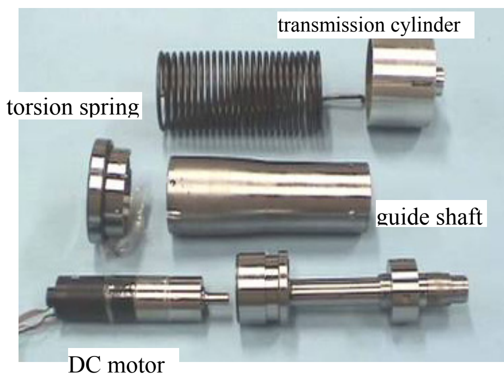


(a) Type A: The case of single joint



(b) Type B: the case of multiple DOF joint

Fig. 2 two types of ADJ



(a) Parts of the ANLES (Type A)

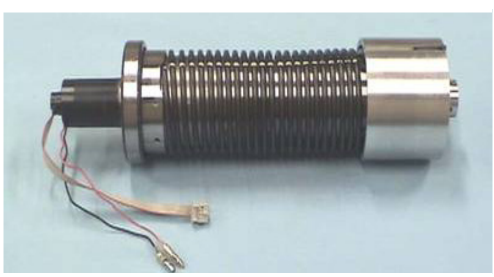

(b) Assembled view of the ANLES (Type A)

Fig.3 The fabricated ANLES type (A)

controlled by ANLES and their disposition. Please see the paper [21] for some case studies.

III. ANLES

The ADJ will be classified into two types. Type (A) in Fig.2(a) has a single axis joint that is controlled by a pair of ANLESes. It does not necessary require tendon (or wire) if the actuator is rotary type. On the other hand a joint having a couple of DOF , spherical or universal joints, will require a couple of tendons, each one of which is controlled by the individual ANLES as shown in Fig.2 (b). The actuator used for type (B) will be a linear actuator that strains the tendon. In this section two types of ANLES individually developed for controlling the type (A) ADJ and the type (B) ADJ are introduced. Firstly the structure of the ANLES of type (A) and the type (B) are introduced. Subsequently the design method of the non-linear elasticity, which is common with both types, is introduced followed by the actually developed example of type (B) ANLES. Please refer to [21] for the details of Type (A); the design of its non-linear elasticity, the ADJ controlled by a pair of type (A) ANLES and its experimental results of the stiffness control.

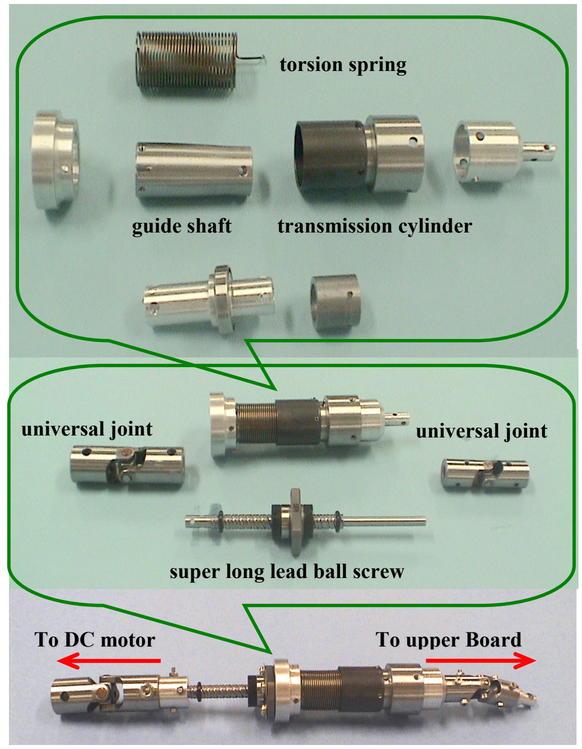


Fig.4 ANLES designed for controlling the type B joint

A. Structure and design of the ANLES

Fig.3 shows the structural parts and their assembled appearance designed for ANLES (Type (A)). It consists of DC-motor, guide-shaft, torsion spring and transmission cylinder (pulley). The torque generated by DC-motor rotates the guide shaft. The guide-shaft rotates the transmission cylinder via the torsion-spring. The transmission cylinder may be combined with a pulley that winds wire. The diameter of the guide shaft smoothly thins down along the rotation axis. When the DC-motor rotates the guide shaft, the torque is transmitted to the transmission cylinder via the torsion spring.

The ANLES designed for controlling the ADJ of type (B) (Fig.4) has almost the same structure as the ANLES (type (A)) shown in Figs.3. The method for designing the guide-shaft to obtain the non-linear elasticity combined with the torsion spring is identical. But this type of ANLES needs

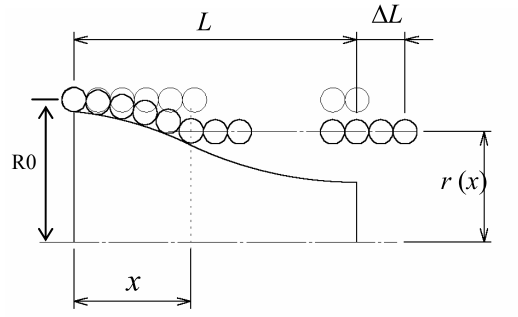


Fig.5 Model of the guide-shaft

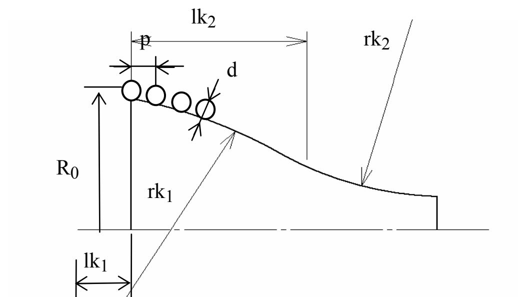


Fig.6 Parameters for designing the guide shaft

a transmitter to transform the rotational motion to the translational motion, and also its vice versa with minimum transmission loss. We therefore employ the super long lead ball screw ($\phi 6$ diameter of the rod with 6 mm lead) into the guide-shaft as shown in Fig.4 and located the DC-motor outside the guide-shaft. The radius of the guide shaft plus radius of the spring wire at the location x along the shaft axis is denoted by $r(x)$. The torsion spring covering the guide shaft coils on the surface of the guide shaft from its edge of large radius. Let us suppose the torsion spring wraps the guide shaft from the left edge to the location x as shown in Fig.5. The wire length of the torsion spring wrapping on the shaft is calculated by,

$$l_a(x) = \int_0^x \sqrt{p^2 + \left(2\pi r(x)\right)^2}\,\frac{dx}{p}. \tag{22}$$

where p is the pitch of the torsion spring. The wire length of the torsion spring covering the part of $L-x$ is calculated by,

$$\overline{l}_r(x) = \frac{L-x}{p}\sqrt{\left(2\pi r(x)\right)^2 + p^2}. \tag{23}$$

The wire length Δl protruding outside the guide-shaft (the part of ΔL along the axis in Fig.5) due to coiling is then calculated by,

$$\Delta l(x) = l - l_a(x) - \overline{l}_r(x), \tag{24}$$

where l is the total wire-length of the torsion spring. Δl obtained by Eq.(24) gives us the torsion angle,

$$\phi(x) = \Delta l(x)/r(x). \tag{25}$$

The wire length that is not yet wrapped on the guide-shaft is,

$$l_r(x) = l - l_a(x) = \overline{l}_r(x) + \Delta l(x). \tag{26}$$

where, $l_r(x)$ is the expansion length of the spring wire that actually works as a spring at location x. In this state the additional torque required to coil the spring furthermore by an infinitesimal torsion angle $\Delta\phi$ is,

$$\Delta T_g(x) = \left(EI / l_r(x)\right)\Delta\phi. \tag{27}$$

where, E is the modulus of longitudinal elasticity and I is the second moment of area of the torsion spring wire.
(27) leads the spring coefficient as a function of x,

$$K(x) = \Delta T_g(x)/\Delta\phi = EI/l_r(x). \tag{28}$$

We can obtain the relation between the torsion angle $\phi(x)$ and the torque $T_g(x)$. Hence the $T_g(x)$ and also $K(x)$ may be denoted by $T_g(\phi)$ and $K(\phi)$ respectively in lieu of using the intermediate parameter x. Now we have a free-hand to obtain the function $T_g(\phi)$ through designing $r(x)$; the radius function along the axis. We propose some configuration parameters for designing the guide-shaft as shown in Fig. 6 in which $r(x)$ consists of two curvatures having radius rk_1 and rk_2 that are smoothly connected at the location lk_2.

B. Stiffness regulation of the joint controlled by two ANLESes

Non-linear elastic characteristics of an ANLES should be designed such that an ADJ driven by two ANLESes has stiffness that is variable in the specified range and also that is able to regulate by the torsion angles of two ANLESes as linearly as possible. The torque of the joint exerted by two ANLESes is denoted by

$$T_j = k_1(\theta)T_{g1}(\hat{\phi} + \phi(\theta)) - k_2(\theta)T_{g2}(\hat{\phi} - \phi(\theta)) \tag{29}$$

under no gravitational effect. Where, T_{g1} and T_{g2} are torques generated by the respective ANLESes, which are the function with respect to the torsion angle of the ANLES ϕ that is also the function with respect to the joint angle.
$k_1(\theta)$ and $k_2(\theta)$ are the coefficients of transmitting torques to the joint which value may be varied according to the joint angle. $\hat{\phi}$ is the torsion angle of ANLESes that are initially twisted, that may be called *initial torsion angle*. The stiffness of the joint is then calculated by the following equation,

$$s(\hat{\phi}) \equiv \partial T_j / \partial\theta\,|_{\theta\to 0} \tag{30}$$

As found in the definition of the stiffness in (30), the stiffness is essentially determined by the initial torsion angle of ANLESes.

C. Design of the ANLES (type B)

In Table 1 the design parameters of the guide shaft are listed and its outcome of the non-linear elasticity of the ANLES shown in Fig.4 is illustrated in Fig.7 accompanied by the results of measuring the elasticity.

Table 1 Designed parameters of guide shaft of the ANLES (Type (B)) shown in Fig.4.

lk_1	Position of the center of the first curvature along axis	-5 mm
lk_2	Changing position of curvatures	25.2 mm
rk_1	Radius of the first curvature	450 mm
rk_2	Radius of the second curvature	400 mm
d	Diameter of the spring wire	0.8 mm
p	Pitch of the spring	1.0 mm
n	Winding number of the spring	28
R_0	Onset radius of the spring	8.0 mm

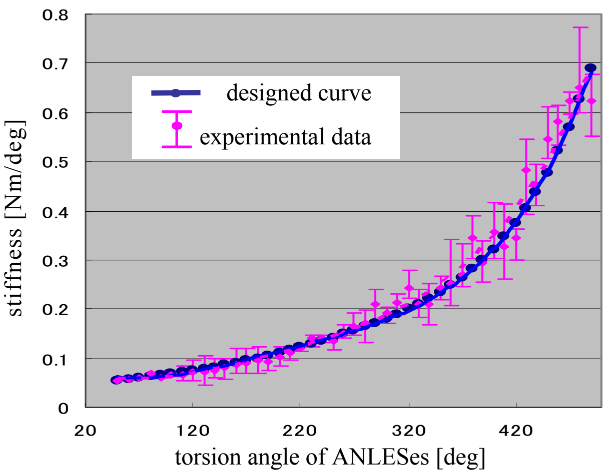


Fig.7 Stiffness of the ADJ (type(B)) shown in Fig.4 controlled by the torsion angle of the

The error bar shows the variation range of five trials. This non-linear elastic curve is used for the stiffness and angle control described in the following section.

IV. Three DOF wrist joint controlled by ANLES

A. Structure

Fig.8 shows the three DOF joint that mimics a wrist joint. It is controlled by four ANLESes (type (B)) shown in Fig.4. The DC motor for each ANLES, embedded under the lower plate, rotates the ball screw rod via the universal joint. If all of the DC motors rotate the same torsion angle with the same direction, the same amount of torque is generated and it is transformed to the same amount of traction force by the super-long-lead ball screws. These forces work as torques about the rotary joint, the same amplitude but opposite direction, hence they are *antagonistically* cancelled and the rotary joint will not rotate. In this case the DC motor purely twists the torsion spring of the ANLES, which enhances the stiffness about the rotary joint with no rotation. If the traction forces of four rods differ the joint rotates (extension/flexion and radial flexion/ ulnar flexion) to reach the equilibrium state that is determined not only by the traction forces but also by the moment arms; the normal length between the axis of the rotary joint and each rod, which is varied by the angle of the rotary joint.

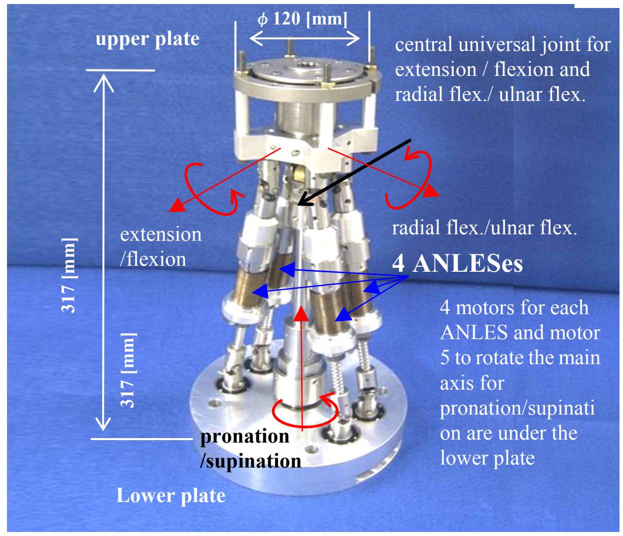


Fig.8 Three DOF wrist joint controlled by ANLESes

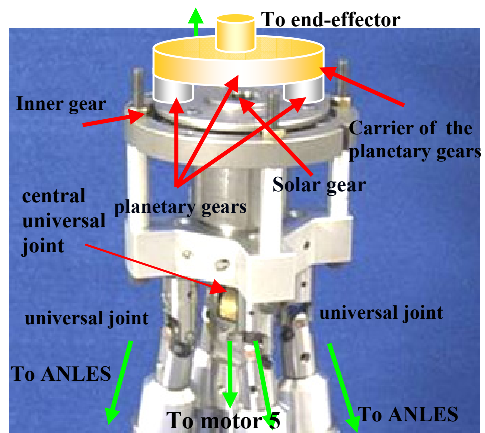


Fig.9 The structure of the upper plate

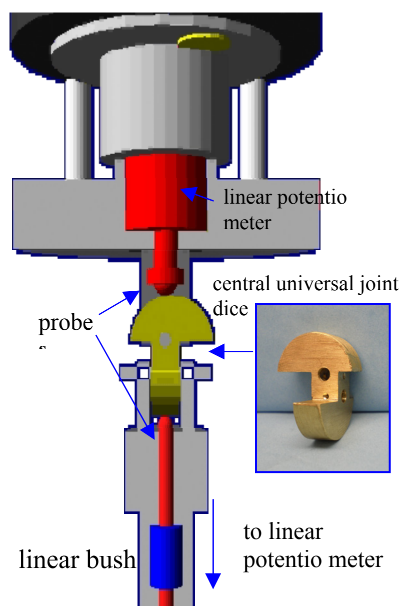


Fig.10 The measurement device of two angles: extension/flexion, radial flex./ ulnar flex.

The antagonistic control of three DOF joint fundamentally requires six ANLESes. However it is almost all impossible to equip two more ANLESes to control the stiffness and the angle of the pronation/supination due to the size limitation. Alternatively we device a mechanism in the upper plate. As shown in Fig.9, a planetary gear system is employed. The four ANLESes are connected to the inner gear via individual universal joint. The solar gear is connected to the central universal joint and gets the torque from the motor 5 that rotates the center axis. An end-effector is connecting to the carrier of the planetary gears. The active torque of the motor 5 is transmitted to the carrier and rotates the end-effector. On the other hand, if the external rolling torque is loaded on the end-effector, it rotates the inner gear to incline the ball-screw rods of the ANLESes, which brings about the corresponding twisting of the ANLESes. Therefore a preliminary twist angle of ANLESes also specifies the stiffness about the pronation/supination as well as the flexion/extension and the radial flex./ulnar flex. Hence the stiffness about the pronation/supination is not independent to those of the flexion/extension and the radial flex./ulnar flex.

The rotation angles of the ext./flex. and radial flex./ulnar flex. are measured by the originally developed central universal joint as shown in Fig.10. The dice of the central universal joint transforms the two rotations into the linear displacements for the linear potentiometers.

B. The experiment of the angle and stiffness control

We carried out the experiments to confirm the basic formula ((20) and (21)). This section shows the results of the joint angle control under holding a constant stiffness and the stiffness control under holding a constant joint angle. The 1.2 kg mass is attached at the 125 mm distance from the joint. This mass represents a hand that we plan to construct and integrate with the 3 DOF wrist joint. ANLESes are twisted individually by some torsion angles using the normal PID control of the DC motors, which gives rise to the rotation of the joint to take a equilibrium torque balance. After the joint settles at some angle, the joint angle is measured. In this state the stiffness of the joint is measured by taking some small torque about the joint and measured the corresponding rotation.

The torsion angles given to each ANLES are calculated as follows. First, the non-linear elastic curve shown in Fig.7 is fitted with a polynomial with respect to the torsion angle ϕ to obtain the tension vector $\xi_i(\phi_i),\ \ (i=1,2)$ ((1) and (6)). Using ξ_i we can calculate the (20) or (21) to obtain the torsion angle vector $\Delta\boldsymbol{\phi}_s$ with respect to the specified stiffness variation Δs or $\Delta\boldsymbol{\phi}_a$ with respect to the specified infinitesimal joint angle $\Delta\theta$.

Fig.11 shows the stiffness when four ANLESes are twisted the same amount of angle. The error bar is the variation range of 5 measurements. As shown the experimental results are well coincident with the theoretical curve. The stiffness order of the pronation/supination is less than 1/10. It is a drawback of this test machine.

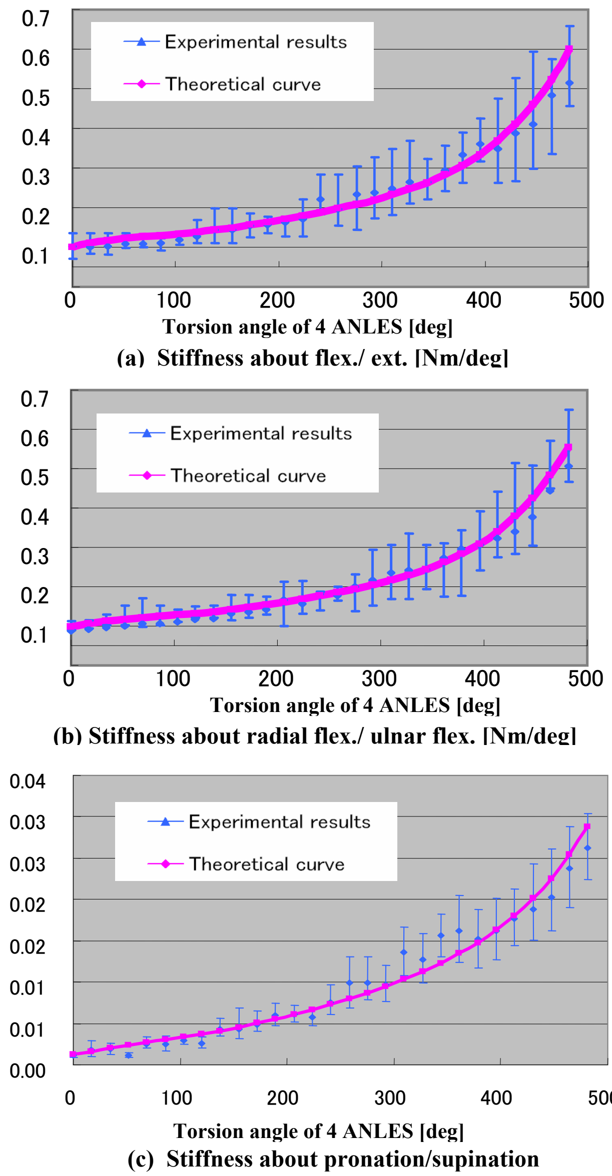


Fig.11 Stiffness change by the torsion angle of ANLESes

Fig.12 is the result of controlling the angle change (extension/flextion) while holding the constant stiffness. The twist angle of corresponding two ANLESes are changed according to the Eq.(21) under constant stiffness being 0.12 [Nm/deg]. It is the results when the twist angles of ANLESes are given in 28 steps as shown in the third figure. For the radial flex./ulnar flex. the similar results are obtained.

IV. CONCLUSIONS

This paper totally deals with the theory and the mechanism for controlling the joint stiffness. The proposed formula and the mechanism of ADJ using the ANLES followed by its design method are verified by the experiments. The experimental results shown in Fig.11 and 12 clearly prove the validity of the equations (20) and (21).

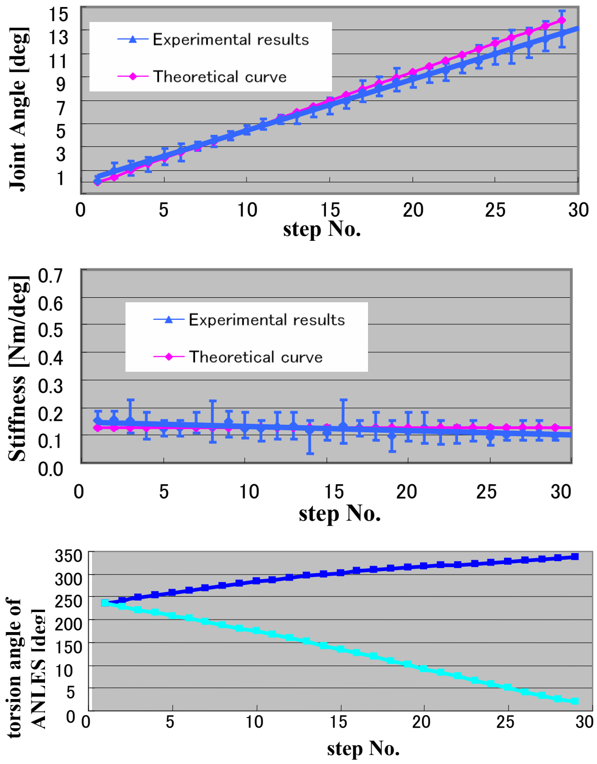


Fig.12 The angle control while holding the constant stiffness about extension/flexion

We are now developing the refined 3 DOF wrist joint that will overcome the following drawbacks depicted in the first machine.

(1) The stiffness range of the pronation/supination is too small. -> It will increase about 5 times.

(2) The size and the weight are too large. -> It will be almost half ; 140 mm tall and 1.4 kg.

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