The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Using Robust Regressions and Residual Analysis to Verify the Reliability of LS Estimation: Application in Robotics

A. Janot(1), P.O Vandanjon(2) and M. Gautier(3)

(1) HAPTION S.A. and CEA, LIST, Service de Robotique Interactive
Atelier Relais de Soulgé Route de Laval, 53210 Soulgé sur Ouette, France
(2) Laboratoire Central des Ponts et Chaussées
Route de Bouaye BP 4129, 44341Bouguenais Cedex, France
(3) Université de Nantes 1, rue de la Noë - BP 92 101 - 44321 Nantes Cedex 03, France

***Abstract*— Usually, the identification of the dynamic parameters of robot makes use of the inverse dynamic model which is linear with respect to the parameters. This model is sampled while the robot is tracking exciting trajectories. This allows using linear least squares (LS) techniques to estimate the parameters. The efficiency of this method has been proved through experimental identifications of a lot of prototypes and industrial robots. However, it is known that LS estimators are sensitive to outliers and leverage points. Thus, it may be helpful to verify their reliability. This is possible by using robust regressions and residual analysis. Then, we compare the results with those obtained with classical LS regression. This paper deals with this issue and introduces the experimental identification and residual analysis of an one degree of freedom (DOF) haptic interface using the Huber's estimator. To verify the pertinence of our analyses, this comparison is also performed on a medical interface consisting of a complex mechanical structure.**

I. Introduction

THE usual identification method based on the inverse model and LS technique has been successfully applied to identify inertial and friction parameters of a lot prototypes and industrial robots [1]-[13] among others. The obtained results were interesting and consistent. At any case, a derivative pass band data filtering is required to calculate the joint velocities and accelerations. In addition, LS estimators are sensitive to outliers and leverage points [14]-[19].

To avoid the calculation of joint velocities and accelerations, we can use the Simple Refined Instrumental Variable (SRIV) Method [21]-[23] or the Direct and Inverse Dynamic Identification Model (DIDIM) method [24]. As these techniques are consistent at any case and practically unbiased, the control law applied to the robot must be known to use them correctly.

In 1972, Huber purposes a category of estimators generalizing the maximum likelihood estimators called the M-estimators. These estimators decrease the influence of outliers and they are close to LS estimators when the error distribution is Gaussian [15]. Holland and Welsch have proposed an algorithm based on iteratively re-weighted least squares (IRLS) which is numerically stable [17]. Hence, one can verify the reliability of LS regressions by comparing their results with those given by the Huber's estimator. An analysis of residuals is also performed.

This paper introduces the identification of a single DOF interface with LS regression and the Huber's estimator. These results are supported by those obtained on a 3 DOF medical interface.

The paper is organized as follows: second section reviews the usual method in robotics; the third section presents the Huber's estimator and the algorithm of Holland and Welsch; the experimental validation performed on a 1 DOF robot is presented in section 4.

II. Inverse Dynamic Identification Model Method

The inverse dynamic model (ID) of a rigid robot composed of n moving links calculates the motor torque vector $\mathbf{\Gamma}$ (the control input) as a function of the generalized coordinates (the state vector and it is derivative). It can be written as the following relation which explicitly depends on the joint acceleration:

$$\Gamma = \mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \qquad (1)$$

Where $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are respectively the (nx1) vectors of generalized joint positions, velocities and accelerations, $\mathbf{M}(\mathbf{q})$ is the (nxn) robot inertia matrix and $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}})$ is the (nx1) vector of centrifugal, Coriolis, gravitational and friction torques.

The choice of modified Denavit and Hartenberg frames attached to each link allows to obtain a dynamic model linear in relation to a set of standard dynamic parameters χ_S [9]-[10] :

$$\Gamma = \mathbf{D}_S(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\chi_S \qquad (2)$$

Where $\mathbf{D}_S(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is the regressor and χ_S is the vector of the standard parameters which are the coefficients XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j of the inertia tensor of link j denoted ${}^j\mathbf{J}_j$, the mass of the link j called m_j, the first moments vector of link j around the origin of frame j denoted ${}^j\mathbf{M}_j$ = [MX_j MY_j

Contacts: alexandre.janot@haption.com

$MZ_j]^T$, the friction coefficients f_{Vj}, f_{Cj} and the actuator inertia called Iaj and the offset of current measurement denoted offset.

The base parameters are the minimum number of mechanical parameters from which the dynamic model can be calculated. They are calculated from the standard inertial parameters. The minimal inverse dynamic model can be written as:

$$\Gamma = \mathbf{D}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\chi \tag{3}$$

Where $\mathbf{D}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is the minimal regressor and χ is the vector of the base parameters.

The inverse dynamic model (2) is sampled while the robot is tracking a trajectory to gat an over-determined linear such that [9]:

$$\mathbf{Y}(\Gamma) = \mathbf{W}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\chi + \boldsymbol{\rho} \tag{4}$$

With, $\mathbf{Y}(\Gamma)$ being the measurements vector, $\mathbf{W}$ the observation matrix and $\boldsymbol{\rho}$ the vector of errors.

The L.S. solution $\hat{\chi}$ minimizes the 2-norm of the vector of errors $\boldsymbol{\rho}$. $\mathbf{W}$ is a r×b full rank and well conditioned matrix where $r=N_e xn$, N_e being the number of samples, obtained by tracking "exciting" trajectories and by considering the base parameters. The LS solution $\hat{\chi}$ is given by:

$$\hat{\chi} = \left(\left(\mathbf{W}^T\mathbf{W}\right)^{-1}\mathbf{W}^T\right)\mathbf{Y} = \mathbf{W}^+\mathbf{Y} \tag{5}$$

It is calculated using the QR factorization of $\mathbf{W}$. Standard deviations $\sigma_{\hat{\chi}_i}$ are estimated using classical and simple results from statistics. The matrix $\mathbf{W}$ is supposed deterministic, and $\boldsymbol{\rho}$, a zero-mean additive independent noise, with a standard deviation such as:

$$\mathbf{C}_{\rho\rho}=E(\rho\rho^T)=\sigma_\rho^2\,\mathbf{I}_r \tag{6}$$

where E is the expectation operator and $\mathbf{I}_r$, the r×r identity matrix. An unbiased estimation of σ_ρ is:

$$\hat{\sigma}_\rho^2 = \|\mathbf{Y}-\mathbf{W}\hat{\chi}\|/(r-b) \tag{7}$$

The covariance matrix of the standard deviation is calculated as follows:

$$\mathbf{C}_{\hat{\chi}\hat{\chi}} = E[(\chi-\hat{\chi})(\chi-\hat{\chi})^T] = \sigma_\rho^2(\mathbf{W}^T\mathbf{W})^{-1} \tag{8}$$

$\sigma_{\hat{\chi}i}^2 = C_{\hat{\chi}\hat{\chi}ii}$ is the i^th diagonal coefficient of $\mathbf{C}_{\hat{\chi}\hat{\chi}}$. The relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is given by:

$$\%\sigma_{\hat{\chi}_{ri}} = 100\sigma_{\hat{\chi}_i}/\hat{\chi}_i \tag{9}$$

However, in practice, $\mathbf{W}$ is not deterministic. This problem can be solved by filtering the measurement matrix $\mathbf{Y}$ and the columns of the observation matrix $\mathbf{W}$ as described in [9],[10] and [12].

The use of LS is particularly interesting because no integration of the differential equations is required and there is no need of initial conditions. However, the calculation of the velocities and accelerations are required using well tuned band pass filtering of the joint position [9] and [12].

III. Huber's Estimator

The Huber's estimator of $\mathbf{X}$ is given by the following equation:

$$\underset{\mathbf{X}}{\mathrm{Min}} \sum_{i=1}^{r} f\left(y_i - \ell_i\mathbf{X}\right) \tag{10}$$

Where f is a convex continuous and derivable function, y_i is the i^th sample of $\mathbf{Y}$ and ℓ_i being the i^th line of $\mathbf{W}$ (that is $\mathbf{W}_{i,:}$).

In 1964, Huber defined a function keeping the same properties of the LS estimators for small errors and those of the L_1 (least absolute value) estimators for large errors [14]. ψ(.) being the derivative function of f(.), called also score function, H being a scalar fixed by the user, the Huber's function is defined by:

$$\psi_H(t) = \mathrm{Min}(H,\mathrm{Max}(-H,t)) \tag{11}$$

Generally, H varies from 1 up to 1,8. The most common choice consists in fixing H at 1,345 [14].

However, these functions do not respect the invariance property when the errors are multiplied with a scalar α. Hence, Huber modifies (10) by introducing a dispersion measurement of errors denoted d. So, (10) is substituted by (12):

$$\mathrm{Min}\ Q(\mathbf{X},d) = \sum_{i=1}^{r} \rho_H\left([y_i - \ell_i\mathbf{X}]/d\right) \tag{12}$$

As d is unknown, d and $\mathbf{X}$ are estimated simultaneously. They are the solutions of the following problem:

$$\frac{\partial Q(\mathbf{X},d)}{\partial \mathbf{X}} = \sum_{i=1}^{r} \psi_H[(y_i - \ell_i\mathbf{X})/d]\ell_i^T = \mathbf{0} \tag{13.a}$$

$$\frac{\partial Q(\mathbf{X},d)}{\partial d} = \left(-\frac{1}{d^2}\right)\sum_{i=1}^{r}(y_i - \ell_i\mathbf{X})\psi_H[(y_i - \ell_i\mathbf{X})/d] = \mathbf{0} \tag{13.b}$$

However, this algorithm is not easy to solve and can exhibit some problems of convergence [16].

An interesting algorithm was purposed by Holland and Welsch which is based on the weighted least squares (WLS). Minimize a WLS criterion turns to solve:

$$\mathrm{Min}\ Q(\mathbf{X},d) = \sum_{i=1}^{r} w_i(.)(y_i - \ell_i\mathbf{X})^2 \tag{14}$$

Where $w_i(.)$ is a weight function defined by $\psi(\rho)/\rho$. If an estimation of d noted $\hat{d}$ is known, it is sufficient to solve:

$$\sum_{i=1}^{r} w_i\left(y_i, \mathbf{X}, \hat{d}\right)\left(y_i - \ell_i \mathbf{X}\right)\ell_i^T = 0 \qquad (15)$$

The weight function $w_i(.)$ gives a weight w_i close to zero for outliers and gives a weight w_i close to one for inliers. If we choose the function $w_i(.)$ given by (16) then (15) verifies (17). Hence, we retrieve the Huber's criteria given by (13.a)

$$w_i\left(y_i, \mathbf{X}, \hat{d}\right) = \frac{\psi_H\left(\left(y_i - \ell_i \mathbf{X}\right)/\hat{d}\right)}{\left(y_i - \ell_i \mathbf{X}\right)/\hat{d}} \qquad (16)$$

$$\sum_{i=1}^{r} \psi_H\left[\left(y_i - \ell_i \mathbf{X}\right)/\hat{d}\right]\ell_i^T = \mathbf{0} \qquad (17)$$

This reasoning is based on the fact that we have an estimation of d. The most common robust estimator of dispersions used is defined by (18):

$$\hat{d} = \frac{\underset{i}{\mathrm{Med}}\left|\left(y_i - \ell_i \mathbf{X}\right) - \underset{i}{\mathrm{Med}}\left(y_i - \ell_i \mathbf{X}\right)\right|}{0{,}6745} \qquad (18)$$

Holland and Welsch show that we can estimate d and **X** simultaneously. We start with an initial estimation of **X** (obtained with the LS estimator for example), then d is estimated with (18), next we compute the w_i with (16) and **X** is estimated once again with (15) until the convergence [17].

Finally, the covariance matrix of the solution **X** solution of (17) will be:

$$\left.\begin{aligned} \mathrm{Var}\left(\hat{\mathbf{X}}\right) &= \sigma_w^2 \left(\sum_{i=1}^{r} w_i(.)\ell_i^T \ell_i\right)^{-1} \\ \sigma_w^2 &= \frac{1}{r-b}\sum_{i=1}^{r} w_i(.)\left(y_i - \ell_i \mathbf{X}\right)^2 \end{aligned}\right\} \qquad (19)$$

This algorithm is the iteratively re-weighted least squares technique (IRLS). It decreases the influence of outliers automatically, is easy to implement, has good numerical stability and does not exhibit some problems of convergence compared with (13.a) and (13.b).

IV. APPLICATIONS

A. Modeling of the one DOF system

In this section, the dynamic model of the one DOF haptic device is briefly recalled. More details about its modeling can be found in [20].

The interface to be identified is presented Fig. 1. It consists of synchronous machine and a handle actuated by means of a cable transmission. The modeling and the identification are made under the rigid model assumption. Hence, the inverse dynamic model is given by (20):

$$\Gamma = ZZ_1\ddot{q} - MX_1 g\cos(q) + MY_1 g\sin(q) + f_{v1}\dot{q} + f_{c1}\mathrm{sign}(\dot{q}) \qquad (20)$$

Where q, $\dot{q}$ and $\ddot{q}$ are respectively the joint position velocity and acceleration. The torque Γ is calculated through the current measurement, that is, $\Gamma = NK_T I$, where N is the gear ratio, K_T the torque constant and I the measured current.

The dynamic model can be thus written as a sampled linear form:

$$\mathbf{\Gamma} = \mathbf{W}\,\mathbf{X} + \boldsymbol{\rho} \qquad (21)$$

$$\mathbf{W} = \begin{bmatrix} \ddot{q}_1 & -g\cos(q_1) & g\sin(q_1) & \dot{q}_1 & \mathrm{sign}(\dot{q}_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \ddot{q}_r & -g\cos(q_r) & g\sin(q_r) & \dot{q}_r & \mathrm{sign}(\dot{q}_r) \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_r \end{bmatrix}$$

and $\mathbf{X} = \left[ZZ_1 \quad MX_1 \quad MY_1 \quad f_{v1} \quad f_{c1}\right]^T$.

Hence, we have:

$\ell_i = \left[\ddot{q}_i \quad -g\cos(q_i) \quad g\sin(q_i) \quad \dot{q}_i \quad \mathrm{sign}(\dot{q}_i)\right]$, $y_i = \Gamma_i$.

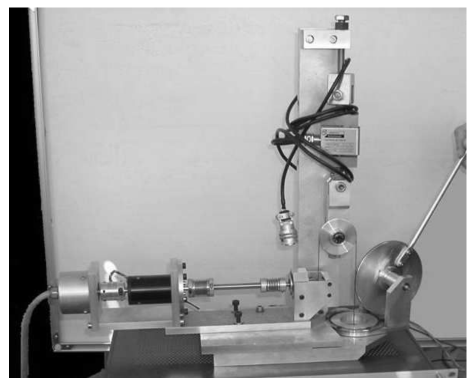

Fig. 1. Haptic device to be identified

B. Data acquisition and LS estimation

For all experimental identifications, current **I** and joint position **q** are measured, with a sampling period of 240µs. Vectors $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are derived from the position vector **q**, and all data are filtered. The cut-off frequencies of the lowpass Butterworth filter and of the decimate filter are close to 20Hz. Because of the friction model, the velocities close to zero are eliminated.

The values identified with the LS estimator are given Table 1. They are close to their nominal values.

TABLE 1: NOMINAL VALUES AND IDENTIFIED VALUES WITH THE LS ESTIMATOR

Mechanical parameters	Nominal values	Estimated values	Relative deviation
ZZ_1	$1.45\ 10^{-3}$ Kgm^2	$1.45\ 10^{-3}$ Kgm^2	0.5 %
MX_1	$2.40\ 10^{-3}$Kgm	$2.40\ 10^{-3}$Kgm	5.0 %
MY_1	0.0 Kgm	1.0 Kgm	8.7 %
f_{v1}	$1.50\ 10^{-3}$Nm/rad/s	$1.45\ 10^{-3}$Nm/rad/s	2.6 %
f_{c1}	$67.00\ 10^{-3}$ Nm	$66.00\ 10^{-3}$ Nm	0.8 %
	$\sigma_\rho = 0.01$ Nm		

In Fig. 2, the histogram of the residual vector and its estimated Gaussian are plotted. This shows clearly that the statistical assumptions made on $\boldsymbol{\rho}$ are practically not violated. Indeed, the residuals distribution is very close to a Gaussian distribution. In addition, they are few outliers, that is, points being outside of the confidence interval [$-3\sigma_\rho$ $3\sigma_\rho$].

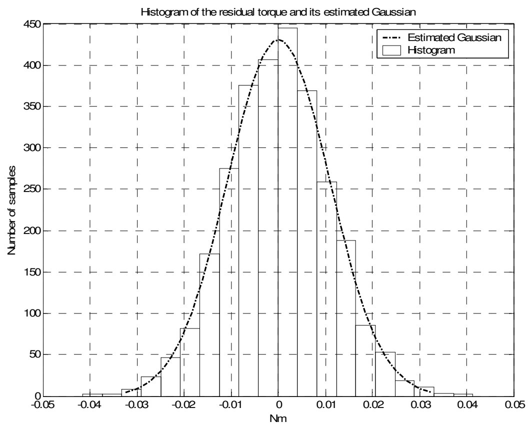

Fig. 2. Histogram of the residual torque when the data filtering is adapted

In this case, the identified values are close to their nominal values and the distribution of residuals tends to a Gaussian one. So, we can suppose that the LS estimator is consistent. Now, we use robust regression to confirm that.

C. Experimental identification of the one system with the Huber's estimator

At present the Huber's estimator with the algorithm of Holland and Welsch is used. The initial values are those identified with the LS estimator. The results are summed up in Table 2.

TABLE 2: IDENTIFIED VALUES WITH THE ALGORITHM OF HOLLAND AND WELSCH

Mechanical parameters	Estimated values	Relative deviation
ZZ_1	$1.45\ 10^{-3}$ Kgm^2	0.3 %
MX_1	$2.40\ 10^{-3}$Kgm	1.9 %
MY_1	$1.00\ 10^{-3}$Kgm	6.8 %
f_{v1}	$1.60\ 10^{-3}$Nm/rad/s	3.4 %
f_{c1}	$66.00\ 10^{-3}$ Nm	0.6 %
	$\sigma_w = 0\ .01$ Nm	

The identified values are very close to their nominal values. They equal the values identified with the LS regression. The algorithm converges after only 3 steps.

The Huber's weight function is illustrated Fig. 3. For small errors, the estimator acts as the LS estimator (weights equal 1), and for large errors, it acts like the L_1 estimator (weights decreasing in $1/\rho$).

A residuals analysis was also performed. The histogram of residuals was plotted and we retrieve exactly the same exposed in Fig. 2. In addition, we have $\sigma_\rho = \sigma_w$.

In this case, the distribution of residuals is close to a Gaussian one and the identified values are very close to those identified with the LS estimator while the Huber's estimator is reputed to be more robust than classical LS estimator. That simply means that, when everything is alright, then the LS estimator gives consistent results.

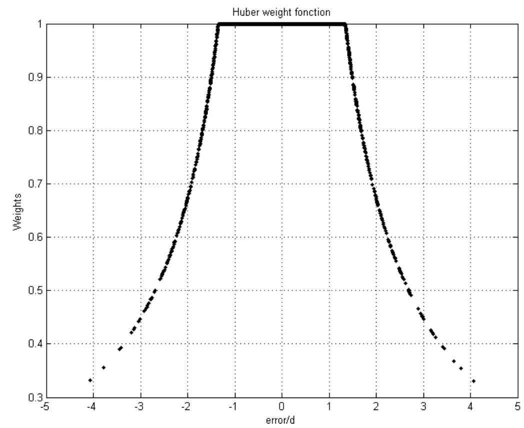

Fig. 3. Huber's weight function

We have tested the Huber's estimator with different values of H. When H tends to infinity, the Huber's estimator tends to the classical LS estimator. Hence, we retrieve the values and their relative deviations exposed Table 2. When, H tends to zero, the Huber's estimator tends to the L_1 estimator. The identified values are close to their initial values. The variations observed are less than 2%.

D. Robustness to data filtering

In this section, we analyze the robustness of the LS estimator and Huber's estimator to a bad data filtering. The data filtering is a key point of the identification process because it takes place when the robot is controlled by feedback. Thus, $\mathbf{W}$ is correlated with $\boldsymbol{\rho}$ and the LS estimator could be biased. A well designed data filtering can remove this problem. However, as shown in [22], when the data is not adapted the LS estimator is not consistent. It may be helpful to detect this trouble by using robust regressions and residuals analysis.

For all experiments, the cut-off frequency of the filters is tuned at 200Hz. In this case, the data filtering is not adapted. The LS identified values are summed up in Table 3. One notes that they are far from their nominal values.

TABLE 3: IDENTIFIED VALUES WITH THE LS ALGORITHM WHILE THE CUT OFF FREQUENCY OF FILTERS IS TUNED AT 200HZ

Mechanical parameters	Estimated values	Relative deviation
ZZ_1	$0.86\ 10^{-3}$ Kgm^2	0.10 %
MX_1	$6.80\ 10^{-3}$Kgm	0.45 %
MY_1	$-5.00\ 10^{-3}$Kgm	2.50 %
f_{v1}	$1.56\ 10^{-3}$Nm/rad/s	3.50 %
f_{c1}	$6.56\ 10^{-2}$ Nm	0.40 %
	$\sigma_\rho = 0.039$ Nm	

In Fig. 4, the histogram of residuals and its estimated Gaussian are plotted. In this case, the estimation does not match the "measurement". Furthermore, outliers appear and their quantity tends to be not negligible. This is shown with the evolution of error plotted in Fig. 5. In this case, we can suspect that the LS estimator is biased.

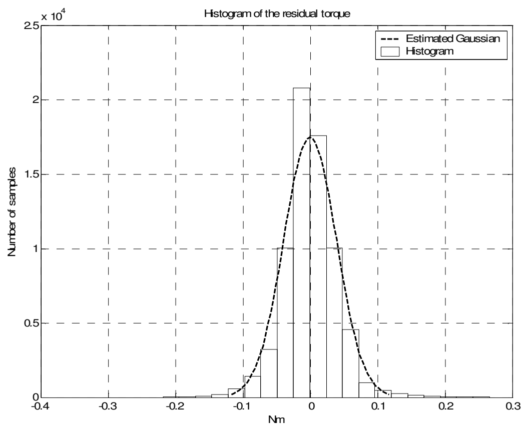

Fig. 4. Histogram of the residual torque when the data filtering is not adapted

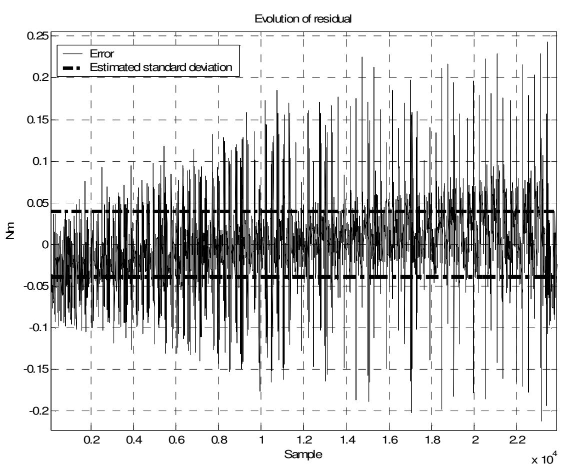

Fig. 5. Evolution of residual and estimated deviation

The Huber's estimator is initialized with the values identified with the LS estimator summed up in Table 3. These values are not compatible with their nominal values. So, the Huber's estimator is badly initialized.

The optimization converges after 5 iterations and the results are given Table 4. Although these values are quite far from their nominal values, the Huber's estimator can eliminate some outliers. However, because of the high corruption of the noise modeling resulting from a bad data filtering, it tends to be very difficult to distinguish outliers from inliers.

Hence, we try to identify the mechanical parameters by changing H. Indeed, if we decrease H, then one supposes that noise modeling is increasingly corrupted.

As an example, the results, obtained when H equals 0.2, are summed up in Table 5.

TABLE 4: IDENTIFIED VALUES WITH THE ALGORITHM OF HOLLAND AND WELSCH WHILE THE CUT OFF FREQUENCY OF FILTERS IS TUNED AT 200HZ

Mechanical parameters	Estimated values	Relative deviation
ZZ_1	$1.07\ 10^{-3}$ Kgm^2	0.15 %
MX_1	$5.29\ 10^{-3}$Kgm	0.50 %
MY_1	$-3.00\ 10^{-3}$Kgm	1.43 %
f_{v1}	$1.72\ 10^{-3}$Nm/rad/s	1.15 %
f_{c1}	$6.56\ 10^{-2}$ Nm	0.25 %
	$\sigma_w = 0.029$ Nm	

TABLE 5: IDENTIFIED VALUES WITH THE ALGORITHM OF HOLLAND AND WELSCH, THE CUT OFF FREQUENCY OF FILTERS IS TUNED AT 200HZ AND H = 0,20.

Mechanical parameters	Estimated values	Relative deviation
ZZ_1	$1.07\ 10^{-3}$ Kgm^2	0.10 %
MX_1	$5.29\ 10^{-3}$Kgm	0.30 %
MY_1	$-3.00\ 10^{-3}$Kgm	1.13 %
f_{v1}	$1.72\ 10^{-3}$Nm/rad/s	0.65 %
f_{c1}	$6.56\ 10^{-2}$ Nm	0.15 %
	$\sigma_w = 0.029$ Nm	

Once again, the identified values are not really compatible with their nominal values. This illustrates one important result from robust statistics: the M-estimators can eliminate outliers from the measurement vector but not outliers from the observation matrix. They can not eliminate the bias. When data filtering is not adapted, the observation is strongly noisy because of the joint accelerations calculation.

However, we can use robust regressions to check the reliability of the LS estimators by calculating relative variations between LS and Huber identified values. These relative variations are given by (22):

$$\%e(X^j) = 100\left|\frac{X^j_{LS} - X^j_{Huber}}{X^j_{LS}}\right| \qquad (22)$$

TABLE 6: RELATIVE VARIATIONS

	Relative variations
$\%e(ZZ_1)$	25%
$\%e(MX_1)$	23%
$\%e(MY_1)$	38%
$\%e(f_{v1})$	11%
$\%e(f_{c1})$	< 1%
$\%e(\sigma)$	34%

Compared with the previous experimental results, strong and unacceptable variations occur. This means that the LS identified values do not match with those identified with the Huber's estimator. Hence, we can suspect that the LS estimator is biased.

E. Extension to multi DOF systems

The CEA LIST has recently developed a 6DOF high fidelity haptic device for telesurgery. As serial robots are quite complex to actuate while fully parallel robots exhibit a limited workspace, this device makes use of a redundant hybrid architecture composed of two 3 DOFs branches connected via a platform supporting a motorized handle, having thus a total of 7 motors. Each branch is composed of a shoulder (link 1), an arm (link 2) and a forearm (link 3) actuated by a parallelogram loop (link 5 and 6). To provide a constant orientation between the support of the handle (link 4) and the shoulder, a double parallelogram loop is used (Fig. 6). The complete modeling of the branches can be found in [13].

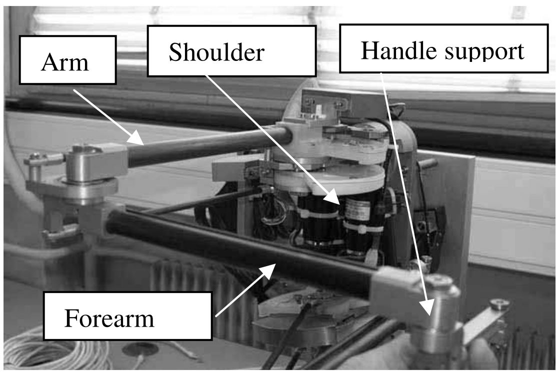

Fig. 6. CEA LIST high fidelity haptic interface. Description of the upper branch

The analysis described in the previous subsection was also applied on the medical interface described above. We retrieve the same results obtained with the one DOF system. When the data filtering is well designed, the LS estimator gives reliable results because no relative variations with the Huber's estimator are observed. Otherwise, strong variations occur. This checking may be helpful in practice.

V. Conclusion

In this paper, the Huber's estimator with the algorithm of Holland and Welsh was applied to identify mechanical parameters of robots. A residual analysis was also designed and applied. Adding both, one can check the reliability of the LS estimator.

The results obtained on the one DOF interface were detailed for a better clarity but they were supported by those obtained on a 3 DOF medical interface.

When the identification protocol is well designed, the LS estimation is consistent. Indeed, experiments have showed that the residuals tend to a Gaussian distribution and no variations are observed with the Huber's estimation. However, if something goes wrong, such as the data filtering, then strong and unacceptable variations occur and the residuals are not Gaussian. So, robust regressions and residuals analysis can be used to verify the reliability of the LS regressions. In addition, these robust regressions and analyses are easy to use and implement. Unfortunately, these robust regressions cannot eliminate the bias, we can only detect it.

References

[1] Atkeson C.G., An C.H. & Hollerbach J.M., "Estimation of Inertial Parameters of Manipulator Loads and Links", *Int. J. of Robotics Research*, vol. 5(3), 1986, pp. 101-119

[2] Ha I.J., Ko M.S. & Kwon S.K., "An Efficient Estimation Algorithm for the Model Parameters of Robotic Manipulators", *IEEE Trans. On Robotics and Automation*, vol. 5(6), 1989, pp. 386-394

[3] Kawasaki H. & Nishimura K., "Terminal-Link Parameter Estimation and Trajectory Control", *IEEE Trans. On Robotics and Automation*, vol. 4(5), 1988, pp. 485-490

[4] Khosla P.K. & Kanade T., "Parameter Identification of Robot Dynamics", *Proc. 24th IEEE Conf. on Decision Control*, Fort-Lauderdale, December 1985

[5] Kozlowski K., "Modelling and Identification in Robotics", *Springer Verlag London Limited*, Great Britain, 1998.

[6] Prüfer M., Schmidt C. & Wahl F., "Identification of Robot Dynamics with Differential and Integral Models": A Comparison, *Proc. 1994 IEEE Int. Conf. on Robotics and Automation*, San Diego, California, USA, May 1994, pp. 340-345

[7] Swevers J., Ganseman C., Tückel D.B., de Schutter J.D. & Van Brussel H., "Optimal Robot excitation and Identification", *IEEE Trans. On Robotics and Automation*, vol. 13(5), 1997, pp. 730-740

[8] Gautier M., Khalil W. and Restrepo P. P., "Identification of the dynamic parameters of a closed loop robot", *Proc. IEEE on Int. Conf. on Robotics and Automation*, Nagoya, may 1995, p. 3045-3050.

[9] Gautier M., "Dynamic identification of robots with power model", *Proc. IEEE Int. Conf. on Robotics and Automation*, Albuquerque, 1997, p. 1922-1927

[10] Gautier M. and Poignet PH. "Extented Kalman filtering and weighted least squares dynamic identification of robot", Control Engineering Practice, 2001

[11] Khalil W., Gautier M. and Lemoine P., "Identification of the payload inertial parameters of industrial manipulators", *IEEE Int. Conf. On Robotics and Automation*, Roma Italia, April 2007, pp. 4943-4948

[12] M.T. Pham, M. Gautier and P. Poignet "Identification of joint stiffness with band pass filtering", *Proc. IEEE on Int. Conf. on Robotics and Automation*, Seoul, may 2001, p. 2867-2872

[13] Janot A., Bidard C., Gosselin F., Gautier M., Keller D. and Perrot Y., "Modeling of a three DOF haptic interface", *Proc. IEEE Int. Conf. on Robotics and Automation*, Roma, April 2007, pp. 4949-4955

[14] Huber, P.J., "Robust estimation of a location parameter", *Ann. Math. Stat.*, 37, 73-101, 1964

[15] Huber, P.J., "Robust statistics: A review", *The annals of statistics*, 1041-1065, 1972

[16] Huber, P.J., "Robust statistics", Wiley, New-York, 1981

[17] Holland F.W., and Welsch R.E., "Robust regression using iteratively re-weighted least squares", *Communications in statistical A6*, 813-826

[18] Hampel, F.R. (1974). "A general qualitative definition of robustness", *Annals of Mathematical Statistics*, 42, 1971, pp. 1887-1896

[19] E. Walter et L. Pronzato, "Identification de modèles paramétriques à partir de données expérimentales", *Masson Paris*, 1994

[20] Khatounian F. et al., "Parameter identification of a single degree of freedom haptic interface", *14th IFAC Symp. on System Identification, SYSID 2006*, Newcastle, Australia, 2006, p. 249-254

[21] Young P.C., "An instrumental variable approach to ARMA model identification and estimation", *14th IFAC Symposium on System Identification, SYSID 2006*, Newcastle Australia, March 2006, pp. 410-415

[22] Vandanjon P.O., Janot A., Gautier M. et Khatounian F., "Comparison of two identification of two identification techniques: theory and application", *ICINCO 2007, 4th International Conference on Informatics in Control, Automation and Robotics; SS on Fractional Order Systems*, Angers, Mai 2007, pp. 341-347

[23] Janot A., Vandanjon P.O. and Gautier M. "Identification of Robot Dynamics with the Instrumental Variable Method", *Proc. IEEE Int. Conf. on Robotics and Automation*,, Kobe, May 2009

[24] Gautier, M., Janot, A. and Vandanjon, P.O., "A new identification method for mechatronic systems in closed-loop from only control data", *Proc. IFAC World Congress 2008*, Seoul, July 2008