

A rigid body attitude estimation for Bio-logging application: A quaternion-based nonlinear filter approach

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Abstract— Bio-logging is a new interdisciplinary research area at the intersection of animal behavior and bioengineering. It involves several applications such as determination of specific parameters (attitude, acceleration, and position) via a new generation of Mechatronic systems. The aim of this paper concerns the animal motion estimation problem using low-cost sensors fusion. A quaternion-based nonlinear observer for the tracking of rigid body attitude (orientation) and heading using measurements provided from low cost inertial and magnetic sensors is presented. The algorithm combines low-frequency, 3-axis accelerometer and 3-axis magnetometer, data with high frequency 3-axis gyroscope measurement. Then, to increase the performance and reduce the computational requirements, we exploit the sensor readings directly in the designed observer. Using the estimated attitude, the linear acceleration is then derived. This latter will be used in the future to evaluate the animal energy index and its mechanical work. Finally, some experimental results, using the measurements provided by an inertial sensor put on dog are given to illustrate the efficiency of the proposed algorithm.

I. INTRODUCTION

Orientation or attitude tracking of rigid body has wide range of applications in robotics, Unmanned Aerial Vehicles (U.A.V), virtual reality, biomechanics, and recently Bio-logging. Therefore it has received a lot of attention from both the industry and the scientific community.

In fact, Bio-logging is a large scientific domain which concerns animal behavior by using adapted-animal mechatronic systems. This scientific field of research aims at understanding natural systems and preserving species that are very small, sensitive, fragile, and live in extreme environments. Bio-logging was introduced during the International Symposium on Bio-logging Science [1]. In recent years, Bio-logging science has grown up quickly, and Bio-logging technology continues to advance rapidly due to the spectacular development in the electronic field and embedded systems. Now the challenge is focused to produce a new bio-logger that can provide maximum accurate information on the animal with minimum energy consumption and a great storage capacity of information.

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It is practically concerned with the determination of animal energy index, physiology, ecology and behavior derived by measurement of specific parameters such as attitude, acceleration and position using animal-attached archival units.

In this work, we focus on the attitude estimation of the animals such as the badger and the penguin. In literature, only few approaches have been applied to the animal's attitude estimation using only measurements from 3D-accelerometer and 3D-magnetometer [2], [3], [4]. The accelerometers are used to measure the gravity vector in body coordinates, and then tilt angles are calculated from three orthogonal accelerations. The magnetometers were used to measure a local magnetic field vector, and then the heading angle was calculated. Nevertheless these solutions are not ideal for dynamic motion since non-gravitational acceleration (linear acceleration) affects the gravity measurement. Also, in these works, a low-pass filter is used to extract the gravity projection from the acceleration readings, which leads to lose the information in the case of fast animal movements. As reported in other work [5], the authors assume that the gravity vector projection can be derived from the running mean over a one second interval of the total acceleration. Nevertheless, this approximation is not valid over time and depends on other parameters as the animal species and its movement type.

To circumvent this problem, we propose in this paper to add 3D-gyroscope to the sensors already used. Note that, the attitude estimation idea based on sensor fusion of accelerometers, magnetometers and gyroscopes in Bio-logging have practically the same concerns that the methods used in others fields (UAV, Robotics,...) except that our application is limited by other constraints such as lack of GPS and speed sensor (marine animals diving), given the environment nature. It also imposes fast dynamic motions (linear acceleration not neglected) for the animal in relation to the rehabilitation area [6], virtual reality domain [7], and micro air vehicles field [8], [9], [10]. Then the proposed algorithm doesn't require the knowledge of the system model or the inertia matrix and simply uses rigid body kinematic equation. Generally, the attitude estimation is based on linear complementary filter, Extended Kalman Filter (E.K.F) or nonlinear observers.

In this paper, an approach based nonlinear observer exploits the sensors readings directly to track the orientation for a rigid body contrary to the work developed in [10] that must first algebraically compute the orientation. In this case

the performance is increased and the computational requirements are reduced. This work will be applied in Bio-logging area to study the Penguins and Badgers behaviors. Using the estimated attitude, linear acceleration can be derived. This will be used in Bio-logging to evaluate the animal energy index.

This paper is organized as follows: Section II presents the problem formulation. Then, we describe the Bio-logging application, the future prototype design, which will be logged on animals, the attitude parameterization and the rigid body kinematic model. Section III details both of the structure of the nonlinear attitude observer and the choice of the observer gain. Section IV is dedicated to some experimental results. Some conclusions end the paper.

II. PROBLEM FORMULATION

A. General ideas on Bio-Logging application

The main contribution of this work is developed within NaviMeles project which concerns the attitude estimation of two animals' species; the Badger and the Penguin. Without loss of generality, this goal can be achieved using rigid body attitude estimation and can then be applied to animals [11]. The first prototype "P3D+", that is under development, will be equipped with kinematics sensing unit (3-axis accelerometers, 3-axis magnetometers, and 3-axis gyroscopes), temperature sensor, pressure sensor, data acquisition unit and power unit. This electronic device allows studying the animal motion (attitude), acceleration, position, index energy.... Once the animal returns to its starting zone (especially for the Penguins), the system is recovered and all calculations are done offline by analyzing the unit recorded measurements and applying the suggested approach. Note that the following approach as introduced before can either be applied on humans to measure their attitude and thereafter to estimate their linear accelerations and the energy consumption.

We are interested in this paper to Bio-logging application that concerns the study of animal behavior using electronic devices (logger) [12]. These systems will be logged on the animals as illustrated by Fig. 1.

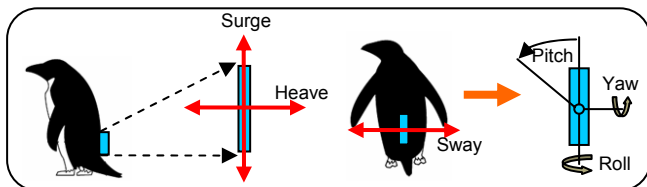


Fig. 1. Schematic diagram showing how electronic device is logged on penguin

B. Attitude representation using unit quaternion

In this study, one chooses to model with the well known quaternion representation [13]. In fact, the quaternion eliminates the singularity problem and computational

inefficiency associated with solving the transcendental equations in the Euler angles representations.

The quaternion, denoted by q is a hyper-complex number of rank 4 such that:

$$q = s + v = s + v_x i + v_y j + v_z k \quad (1)$$

where $v = v_x i + v_y j + v_z k$ represents the imaginary vector, and s is the scalar element.

Since the unit quaternion is considered here, then (1) can be written such that:

$$q = \left[\cos\left(\frac{\phi}{2}\right) \quad e_1 \sin\left(\frac{\phi}{2}\right) \quad e_2 \sin\left(\frac{\phi}{2}\right) \quad e_3 \sin\left(\frac{\phi}{2}\right) \right]^T \quad (2)$$

Let $\hat{e} = [e_1 \quad e_2 \quad e_3]$ denotes the Euler axis of rotation vector and ϕ represents the rotation angle.

We invite the reader to refer to [14], [15] for a more detailed description of quaternion algebra.

Two coordinate frames (Fig. 2) need to be defined to compute the rigid body attitude (orientation).

- $G(x_G, y_G, z_G)$: Fixed inertial coordinate system attached to the earth which is tangent on its surface (NED: North, East, Down).

- $B(x_B, y_B, z_B)$: Mobile inertial coordinate system attached to the body. This frame has its origin at the inertial/magnetic sensor and coincides with the gravity centre of the rigid body. The axes of this reference also coincide with the body axes (the animal in our case) and the sensitive axis of the sensor.

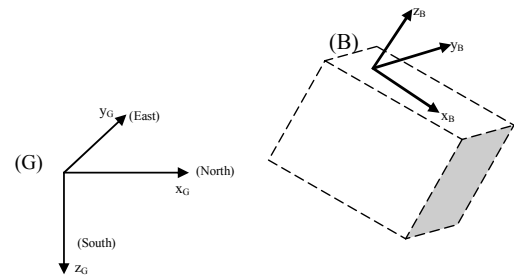


Fig. 2. Definition of the reference frames

C. Rigid body kinematic model

Denoting by $\omega = [\omega_x \quad \omega_y \quad \omega_z]^T$ the angular velocity vector of the rigid body measured in the mobile inertial coordinate system B and $w = [0 \quad \omega_x \quad \omega_y \quad \omega_z]^T$ its equivalent update quaternion. The kinematic equation, in terms of the unit quaternion, that describes the relation between the rigid body attitude variation and the angular velocity is given by [15]:

$$\dot{q} = \frac{1}{2} q \otimes w \quad (3)$$

where \otimes denotes quaternion multiplication.

For small rotation angle and assuming that ϕ changes linearly with time, the orientation expressed by $q(t)$ as a function of time becomes [16]:

$$q(t) = \left(1 \quad \frac{1}{2} \omega_x t \quad \frac{1}{2} \omega_y t \quad \frac{1}{2} \omega_z t \right)^T \quad (4)$$

The equation (3) is then developed to form the system (\mathfrak{S}):

$$\mathfrak{S}: \begin{cases} \dot{q} = \begin{pmatrix} \dot{s} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} s \\ v_x \\ v_y \\ v_z \end{pmatrix} \otimes \begin{pmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \frac{1}{2} \left(I_{3 \times 3} s + [v^\times] \right) \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \\ y = (A_x \quad A_y \quad A_z \quad H_x \quad H_y \quad H_z)^T \end{cases} \quad (5)$$

where

$q = [s \quad v_x \quad v_y \quad v_z]^T \in \mathfrak{R}^4$ is the state and $y \in \mathfrak{R}^6$ is the system output. The accelerometer and magnetometer measure respectively the specific force $A = [A_x \quad A_y \quad A_z]^T$ and the direction of the earth's magnetic field $H = [H_x \quad H_y \quad H_z]^T$ in the mobile inertial coordinate system B . Note that $I_{3 \times 3}$ is the identity matrix and

$$[v^\times] = \begin{pmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{pmatrix} \quad (6)$$

III. THE NONLINEAR OBSERVER FOR ATTITUDE ESTIMATION

A. Observer Design

This section is related to the developed algorithm which will be subsequently used to track the attitude of two species such as Penguin and Badger. Since the attitude cannot be measured directly using sensors, then it must be estimated. Note that the gyroscopes typically suffer from gyro-bias which causes the orientation estimate to drift away from the true orientation, but they lead to a good attitude estimate when integrating for short time.

Accelerometers/magnetometers are good sensors in attitude estimation only for static motion. So, the observer approach developed in this paper must take advantages from accelerometers, magnetometers and gyroscopes to estimate the most accurate attitude.

In order to estimate the attitude, the following Nonlinear Attitude Observer (N.L.A.O) is proposed (Fig. 3):

$$\dot{\hat{q}} = \begin{pmatrix} \dot{\hat{s}} \\ \dot{\hat{v}}_x \\ \dot{\hat{v}}_y \\ \dot{\hat{v}}_z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \hat{s} \\ \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{pmatrix} \otimes \begin{pmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \otimes \begin{pmatrix} 1 \\ K(y - \hat{y}) \end{pmatrix} \quad (7)$$

$\hat{q} = [\hat{s} \quad \hat{v}_x \quad \hat{v}_y \quad \hat{v}_z]^T \in \mathfrak{R}^4$ and $K \in \mathfrak{R}^{3 \times 6}$ represent respectively the estimated state and the observer gain. $\delta(\hat{q}) = (y - \hat{y})$ is the modeling error which represents the difference between the real and estimated measurements and

$$\hat{y} = (\hat{A}_x \quad \hat{A}_y \quad \hat{A}_z \quad \hat{H}_x \quad \hat{H}_y \quad \hat{H}_z)^T \quad (8)$$

where

$$\hat{A} = [0 \quad \hat{A}_x \quad \hat{A}_y \quad \hat{A}_z]^T = \hat{q}^{-1} \otimes m \otimes \hat{q} \quad (9)$$

$$\hat{H} = [0 \quad \hat{H}_x \quad \hat{H}_y \quad \hat{H}_z]^T = \hat{q}^{-1} \otimes n \otimes \hat{q} \quad (10)$$

m represents the gravity vector in the earth coordinates. It is always down and can be expressed in quaternion form as:

$$m = [0 \quad 0 \quad 0 \quad 9.8]^T \quad (11)$$

n represents the local magnetic field in the earth coordinates and can be expressed in quaternion form as:

$$n = [0 \quad 0.5 \cos(60^\circ) \quad 0 \quad 0.5 \sin(60^\circ)]^T \quad (12)$$

The error quaternion corresponding to the modeling error can be written in the following form:

$$\delta \tilde{q} = (1 \quad \delta(\hat{q}))^T \quad (13)$$

The scalar part of the error quaternion is very close to unit since the incremental quaternion corresponds to a small angle rotation.

As reported in [17], the authors used a simple addition of the error quaternion to update the state. In (7) we use rather a quaternion multiplication of this error.

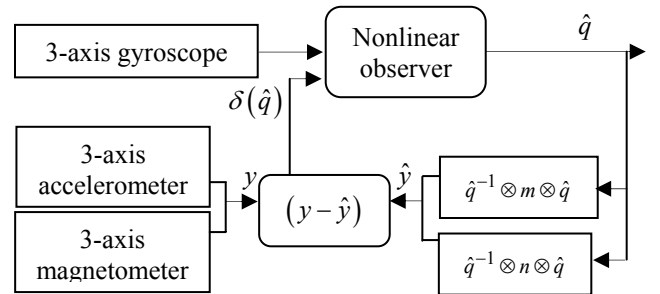


Fig. 3. Block diagram of the estimation algorithm

B. Choice of the observer gain

The observer gain $K = kK'$ in (7) is used to minimize the modeling error $\delta(\hat{q})$ using the gain K' and to tune the cross-over frequency between gyroscopes on one hand and

accelerometers/magnetometers [8], [9] using the gain k on the other hand.

1) *Choice of K'* : Since $\delta(\hat{q})$ represents the error vector, then let us find a solution that minimizes the scalar squared error criterion function [18]:

$$\xi(q) = \delta(\hat{q})^T \delta(\hat{q}) \quad (14)$$

Several approaches have been proved to be effective for many regression problems and allow to minimize a nonlinear function such as $\xi(q)$. In this paper, Levenberg Marquardt Algorithm (L.M.A) was used since it outperforms the Gauss-Newton Algorithm (G.N.A) and the method of gradient descent. The L.M.A is more robust than G.N.A, which means that in many cases it finds a solution even if it starts very far of the final minimum. Then the unique minimum can be written in the following form [19]:

$$\Delta(\hat{q}) = K' \delta(\hat{q}) = \left[X^T X + \lambda I_{3 \times 3} \right]^{-1} X^T \delta(\hat{q}) \quad (15)$$

where λ is a constant that adjusts the convergence rate of the algorithm and guarantees that the inverted term will be nonsingular.

X is the Jacobian matrix defined as:

$$X = 2 \begin{bmatrix} 0 & A_z & -A_y & 0 & H_z & -H_y \\ -A_z & 0 & A_x & -H_z & 0 & H_x \\ A_y & -A_x & 0 & H_y & -H_x & 0 \end{bmatrix}^T \quad (16)$$

2) *Choice of k* : The gain factor k is used to tune the balance between measurement noise and response time of the observer. It relates to the stability and accuracy of the observer and allows fusing low bandwidth accelerometer/magnetometers readings with high bandwidth gyros measurements [8], [9]. It is showed in several works [8] that the nonlinear filter has a better convergence if k is chosen somewhere between 0.1 and 1.

Finally, the gain K takes the following form:

$$K = k \left[X^T X + \lambda I_{3 \times 3} \right]^{-1} X^T \quad (17)$$

IV. EXPERIMENTAL RESULTS

In this section, we present some experimental results to show the efficiency of the proposed nonlinear observer. Some experiments are realized using Inertial Measurement Units (IMU) called “MTi” from Xsens Technologies [20] in order to have the three measurement types (acceleration, angular velocity and earth’s magnetic field). In addition, this unit gives the quaternion data and Euler angles.

In the first set of experiments, we use a turning table and the “MTi” was initially placed in its centre with its xyz axes aligned with north-east-down directions. After that, it is rotated 90° about the z -axis at a variable angular velocity and then rotated -90° (in the reverse direction) for two cycles.

Fig. 4 shows the performance of the proposed approach in estimating the orientation of the “MTi”. In this figure, for more clarity reasons, we choose to represent the attitude with Euler angles due to their intuitivity. It is noted that the algorithm was able to track the motion under static conditions (between 0-8s, 12.5-20s for example) and in the dynamic state moving (between 8-12.5s, 20-24s for example). Moreover, one can remark for this experiment, that in the case when the attitude is computed from only accelerometer and magnetometer, it is disturbed when high linear accelerations occur.

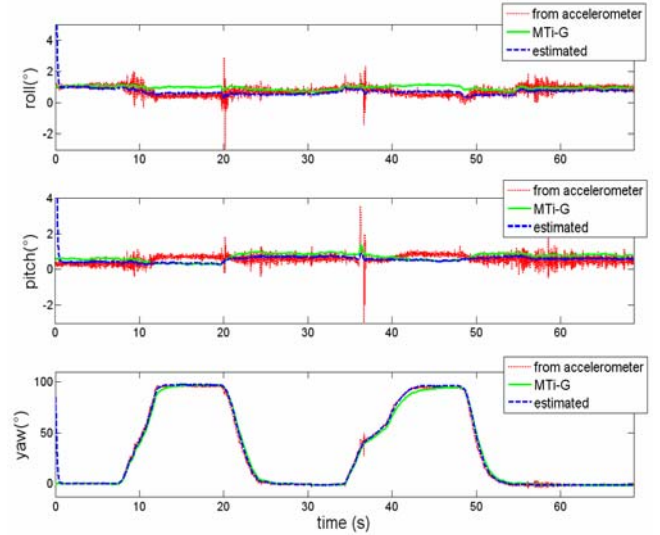


Fig. 4. Experimental and estimated Euler angles (1st experiment)

In the second set of experiments, we use an integrated GPS and an Inertial Measurement Units (IMU) called “MTi-G”. This device is attached to the back of a dog such as its xyz axes coincide with those of the animal. The “MTi-G” allows recording its 3D-acceleration, 3D-angular velocity and 3D-Earth’s magnetic field during the motion (Fig. 5). After that, all these measurements are used to generate the estimated attitude using the proposed nonlinear observer.

Fig. 6 shows the animal accelerations recorded using the “MTi-G”. For an initial period of time (between 0-18s), the dog is at rest. Then its accelerations are almost constant and represent the gravity projection. When the animal start to gallop (between 18s-140s), the acceleration values increase and the variation range is between $-20m/s^2$ and $20m/s^2$.

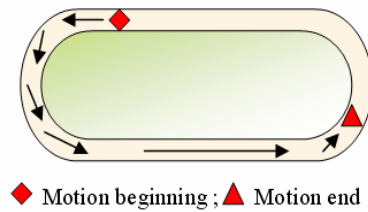


Fig. 5. Animal motion description

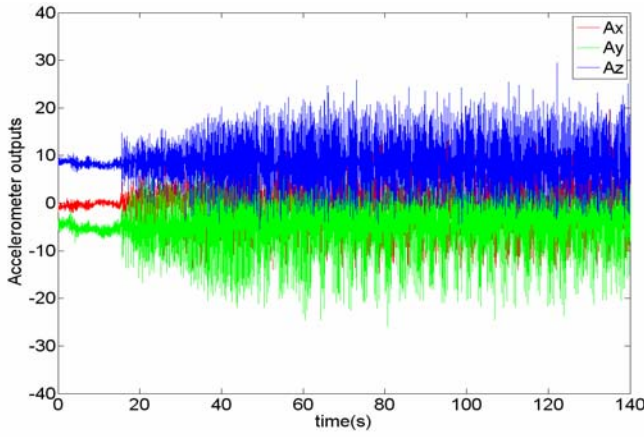


Fig. 6. Accelerometer outputs from MTi-G

Figs. 7 and 9 show the performance of the filter in estimating the orientation of the “MTi-G” that is attached to the animal. Fig. 8 plots the difference between estimated quaternion by “MTi-G” and those estimated by the proposed filter. Note that this attitude is presented respectively using the quaternion (Fig. 7) and Euler angles form (Fig. 9). The quaternion and Euler angles readings from “MTi-G” were also depicted for comparison with those obtained using the proposed approach in this paper. During the first period (between 0-18s) we can see a negligible attitude variation (quaternion or Euler angles forms) since the dog is at rest. After this period, there is a small variation in roll and pitch angles since the “MTi-G” retains a horizontal position on the animal back but the yaw angle varies widely as the animal, often, changes its direction to move and to take a turn during its movement (Fig. 5). This also corresponds to visual observations of the animal during the motion. The conclusion is that the mismatch between the orientation estimate using a nonlinear observer and the “MTi-G” is smaller than (6°), even during fast motions. Note that the animal realizes a half circle and so we will have a change in its yaw angle of 180° (Fig. 9). In (quasi) static situations (so during no or only slow movements) the mismatch is smaller than (3°). This precision is enough for our Bio-logging application. Finally, we can note that the “MTi-G” can not be considered as a reference since it has another attitude filtering algorithm and can commit small error.

Fig. 10 shows the attitude estimation in terms of Euler angles estimated directly from accelerometer measurements against the estimated values from the filter for the same motion as mentioned before. Note that the accelerometer fails to track the true attitude since it is sensitive to linear accelerations while the observer is highly successful in reconstructing the pitch, roll and yaw estimates. The linear accelerations of the dog can be measured using the estimated quaternion, depicted in Fig. 6, such as:

$$\hat{a} = \text{inv}(M(\hat{q}))f - g \quad (18)$$

where $M(\hat{q})$ is the rotation matrix expressed in terms of estimated quaternion such as [12]:

$$M(\hat{q}) = \begin{pmatrix} 2(\hat{q}_0^2 + \hat{q}_1^2) - 1 & 2(\hat{q}_1\hat{q}_2 + \hat{q}_0\hat{q}_3) & 2(\hat{q}_1\hat{q}_3 - \hat{q}_0\hat{q}_2) \\ 2(\hat{q}_1\hat{q}_2 - \hat{q}_0\hat{q}_3) & 2(\hat{q}_0^2 + \hat{q}_2^2) - 1 & 2(\hat{q}_0\hat{q}_1 + \hat{q}_2\hat{q}_3) \\ 2(\hat{q}_0\hat{q}_2 + \hat{q}_1\hat{q}_3) & 2(\hat{q}_2\hat{q}_3 - \hat{q}_0\hat{q}_1) & 2(\hat{q}_0^2 + \hat{q}_3^2) - 1 \end{pmatrix} \quad (19)$$

$g = [0 \ 0 \ 9.81]^T$ is the gravity vector and f is the accelerometer readings.

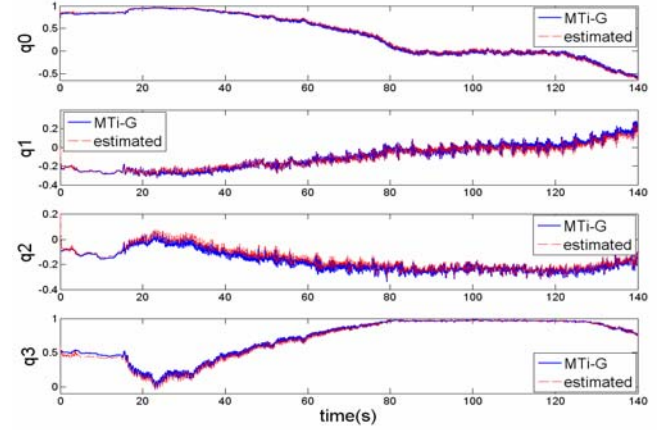


Fig. 7. Experimental and estimated quaternion (Second experiment)

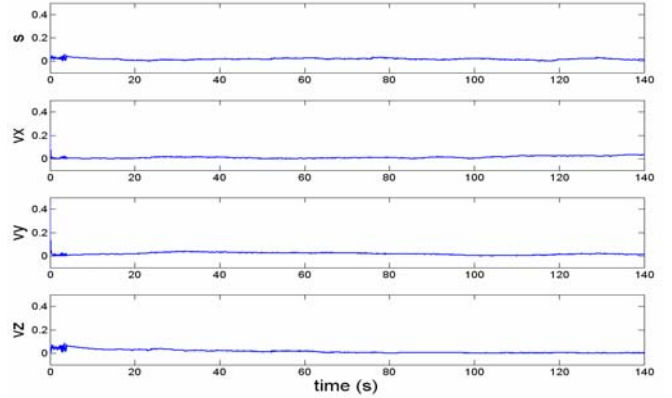


Fig. 8. Difference between estimated quaternion by “MTi-G” and those estimated by the filter

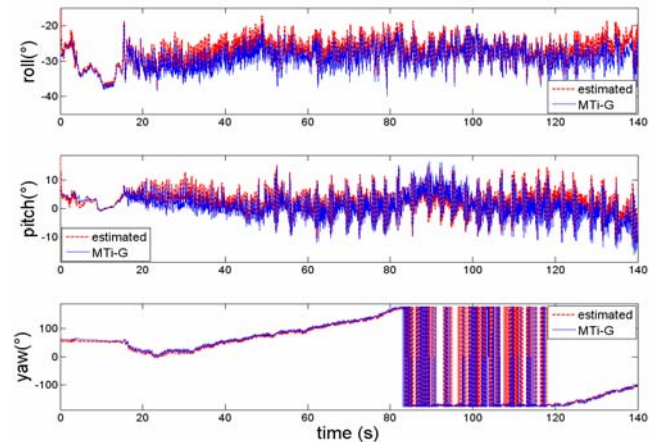


Fig. 9. Experimental and estimated Euler angles (Second experiment)

In Fig. 11, to be more clear, we plot the difference between estimated linear accelerations of the animal by “MTi-G” against those estimated by equation (18). The conclusion is that using the one accurate estimated attitude, we are able to reconstruct the animal linear acceleration with a small error which lead in future works to evaluate the energy index of the species considered in the “NaviMeles” project.

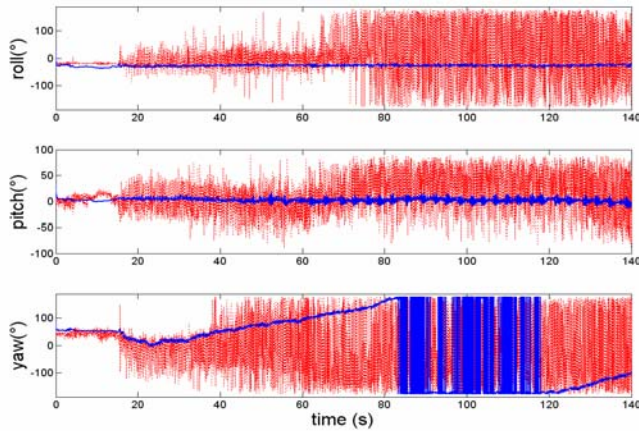


Fig. 10. Estimation results of Euler angles using observer (line) and from accelerometer (dashed line)

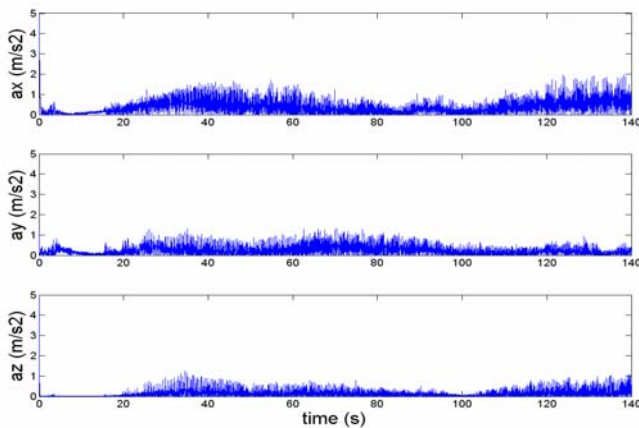


Fig. 11. Difference between estimated linear accelerations by “MTi-G” and those estimated by equation (18)

V. CONCLUSION

In this paper a nonlinear observer for rigid body attitude estimation exploiting gravity vector, earth’s magnetic field and biased angular velocity readings was derived. The goal of this work, dedicated to Bio-logging topic, is to produce an animal posture tracking over wide motion range. Since the attitude measurements, which are provided by accelerometers and magnetometers, represent the true attitude only at low frequencies and become disturbed when the accelerometers sudden noises and vibrations, we have proposed to associate these measurements with gyroscopic readings to estimate the best orientation and linear accelerations of the animal. The obtained experimental results point out that the algorithm is able to track orientation under several motions and with fast dynamics.

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