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Heuristic Approach for Multiple Queries of 3D N-Finger Frictional Force Closure Grasp

Nattee Niparnan, Thanathorn Phoka and Attawith Sudsang

Abstract— **This work proposes a necessary condition for n-finger force closure grasp which considers true quadratic force cone without linearization. The condition finds its use as a heuristic for multiple queries force closure test. The heuristic works as a filtering criteria which improves the overall running time of the entire set of queries. An empirical example shows that our approach could speed up the force closure test be the factor of four. This work improves our earlier works [1], [2] to cover the case of n-finger grasp.**

I. INTRODUCTION

In grasping research, one prominent goal is to have a method that allows a robot to securely grasp an object. This motivates extensive studies in the topic of grasp synthesis. There exists several algorithms that follows classical approach where the description of an object is given and the algorithms plan grasping solution. Recommended survey can be found in [3], [4], [5]. Usually, a grasp synthesis problem is formulated as an optimization problem on a continuous domain, for example, as a linear programming or as a convex optimization problem. This approach works beautifully in a known environment where complete knowledge of an object is can be obtained. However, as there are more and more interests in introducing robot into everyday life where only minimal, if any, information of the surroundings are available, this classical approach which relies on complete knowledge faces greater challenge of object information acquisition.

Typically, sensors for object acquisition give information in the form of discrete representation of the boundary of an object, for example, as point cloud or discretized curves. Though it is possible to fit the acquired information into a particular model, the result is inaccurate. Thus, several recent works are explicitly designed for discrete description of the object (see [6], [7], [8], [9] for some examples).

Even though the nature of object representation are different, the underlying idea of the algorithms for the discrete representation are still the same. The problem is formulated as an optimization problem on a discrete domain where it is solved by discrete approach such as exhaustive search. These algorithms systematically search for an optimal grasping configuration in the finite discrete space. Each candidate configuration is tested for desirability by a grasp analysis module. Different methods under this scheme vary by applying different search policy. For example, a method based on hill climbing or branch and bound search is presented in [10]. Other optimizers such as evolutionary computation as in [11], [12], or generate-and-test approach as in [13] are also adopted.

An interesting property of these discrete algorithms is that a the search strategies and the grasp analysis modules of the algorithms are loosely integrated. Usually, we can conveniently incorporate any grasp quality criteria to the search. By developing a matching grasp analysis module, the user can compute a grasp that meets the requirement of their grasping task at hand without having to derive from scratch a new grasp synthesis method for the particular requirement. This advantage, however, arrives with the cost for assessing quality of every candidate grasp by the grasp analysis module. To maximize the benefit of this scheme, the grasp analysis module need to be computationally efficient.

In this work, we propose a method to speed up the grasp analysis module specifically in the situation where multiple grasp queries are to be performed. The proposed method can be easily applied to the aforementioned search approach.

For most grasp analysis methods, it is required that a grasp can securely hold the object. This property is formalized as the *force closure* property. Our proposed method utilizes a novel necessary condition for force closure. A grasp is tested by our method whether it satisfies our *filtering* criteria. If it does not, it is then rejected. However, if the grasp satisfies, it must then be analyzed by the grasp analysis module of the search approach.

At the surface, it seems that our method introduces additional workload to the process, since a satisfying grasp must undergoes two analysis modules: our criteria and the original grasp analysis. However, our method is derived such that it can be computed very efficiently compared to the original analysis module. Thus, the speed up is the time gained by quick rejection of non-satisfying grasps. This approach can be described as a *filtering approach* where our condition is considered as a filter of another grasp analysis module. This approach is proposed earlier in our work [1], [2]. In particular, this work extends the our previous works [1], [2] which considers the same problem in the special case of four finger grasp. In this work, we propose a more generalized algorithm which work on an arbitrary number of fingers.

Additionally, it should be noted that an interesting issue in force closure testing is the nonlinearity of the representation of the grasp. Unfortunately, majority of force closure tests

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All of the authors are with the Department of Computer Engineering, Faculty of Engineering, Chulalongkorn University, 10330, Thailand Email: nattee,phoka,attawith@cp.eng.chula.ac.th

do not directly deal with the issue but choose to avoid this hindrance by introducing linearity into the problem at the cost of some noticeable incompleteness. See for example the condition of θ-positively span in [14], [15] or several conditions based on the linearization of the friction model [16], [17]. It is the work of Han *et al.* [18] that tackles this nonlinearity problem directly using convex optimization method. However, it comes with the cost of running time since convex optimization takes considerable amount of computation power. Our proposed condition considers directly the nonlinear friction cone without linearization thus providing additional completeness, i.e., its necessity is indeed complete.

the main contribution of the paper is a new necessary condition of 3D n-finger force closure which considers directly the nonlinear friction cone without simplification into m-sided pyramid. The condition, geometrically derived from the nonlinear friction model, can be reduced to the existing problem of 2D n-finger force closure where several existing algorithms have been proposed. It is based on the fact that a force closure grasp must be able to exert wrenches that positively span the torque space (the reverse is not necessarily true).

The rest of the paper is organized as follows. Section II briefly describes past background of grasping In Section III, we describe the novel necessary condition that is used as the heuristic for our force closure test. The implementation of the condition is discussed in Section IV. In Section V we present numerical examples comparing efficiency gained by our approach. Finally, Section VI concludes our work.

II. GRASPING BACKGROUND

A grasp is defined by a set of contact points, each of which is represented by a position and an inward normal vector. Force closure is a binary property for a grasp that is able to counter any external disturbance to the grasped object. Effect of contact points or external disturbance on the object is represented by a force vector and a torque vector. To represent a force and a torque simultaneously, a force $\boldsymbol{f} = (f_x, f_y, f_z)$ and a torque $\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)$ are combined into an entity called *wrench* $\boldsymbol{w} = (f_x, f_y, f_z, \tau_x, \tau_y, \tau_z) \in \mathbb{R}^6$.

We associate each contact point with a set of wrenches exertable by the contact point. A grasp is said to achieve force closure when its contact points are able to produce any wrench in the wrench space $\mathbb{R}^6$. Usually, it is assume that a contact point can exert arbitrary magnitude of force. Thus, a set of wrenches are said to achieve force closure when their positive linear combination can produce a wrench in every direction in the space. The term $\mathbb{R}^n$*-positive span* is used to represent such property.

Definition 2.1: A set of n wrenches $\{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n\}$ *positively spans* $\mathbb{R}^n$ if and only if, for any vector $\boldsymbol{v}$ in $\mathbb{R}^n$, there exists nonnegative constants $\alpha_1, \ldots, \alpha_n$ such that $\boldsymbol{v} = \alpha_1 \boldsymbol{w}_1 + \ldots + \alpha_n \boldsymbol{w}_n$.

This work assumes hard contact with Coulomb friction model. A torque from a hard contact must be the result of the applied force only. As a result, a contact point at $\boldsymbol{p}$ that exerts a force $\boldsymbol{f}$ can be represented by a wrench $\boldsymbol{w} = (\boldsymbol{f}, \boldsymbol{p} \times \boldsymbol{f})$. Coulomb friction model indicates that a contact point can exert some tangential force without slippage. The maximum ratio between the magnitude of tangential force and the magnitude of the force in the normal direction is indicated by the frictional coefficient μ between the object and the contact point of the grasping finger. In other words, the net force exerted by a non-slipping contact must lie in a cone at the contact point whose axis lies in the normal direction and its half angle [1] is given by $\theta = \tan^{-1}(\mu)$. This force cone is referred to as *friction cone*.

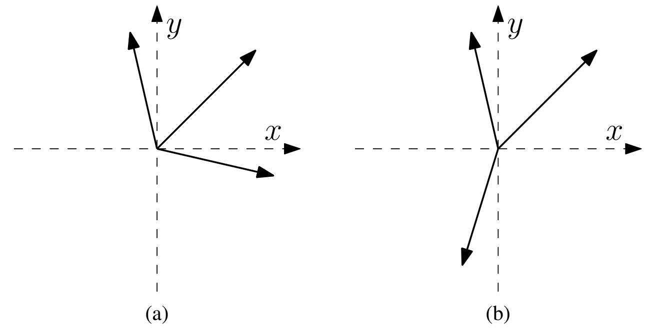

Fig. 1: Example of vectors that satisfied Proposition 3.1. (a) the vectors do not positively span the space. (b) the vectors positively span the space.

A. Preliminaries and Notations

We denote by $\mathrm{int}(\cdot)$ the interior function. Let P be an arbitrary plane through the origin in $\mathbb{R}^3$. The plane P can be described by its normal vector $\boldsymbol{n}$. Formally, $P = \{\boldsymbol{x} | \boldsymbol{x} \cdot \boldsymbol{n} = 0\}$. We say that a vector $\boldsymbol{x}$ is on the positive (resp. negative) side of P when the sign of $\boldsymbol{x} \cdot \boldsymbol{n}$ is positive (resp. negative). Two vectors are said to be on different sides of P when one of them is on the positive side and the other is on the negative side. Additionally, let us refer to the set of all positive combinations of members in a vector set W (i.e., $\{\sum \alpha_i \boldsymbol{v}_i | \alpha_i \geq 0, \boldsymbol{v}_i \in W\}$) as a *positive span* of W.

III. NECESSARY CONDITION FOR N-FINGER FORCE CLOSURE GRASP

In this section, we present a necessary condition of force closure for an n-finger grasp. First, we introduce a proposition which is the basis of our condition. This proposition considers the property of positively span.

Proposition 3.1: A necessary condition for a set of vectors to positively span $\mathbb{R}^n$ is that the projection of the vectors on any subspace $\mathbb{R}^{k<n}$ must positively span the subspace.

It is clear that the condition is necessary. Figure 1 illustrates two examples that satisfy the condition in Proposition 3.1 where the first one actually does not achieve force closure while the second one does.

Our condition for n-finger is the application of Proposition 3.1 on the wrench space. One interesting subspace of the

[1] the angle between the normal and the vector on the boundary of the cone

wrench space is the torque space which is $\mathbb{R}^3$. Our condition checks whether the set of wrenches associated with the contact points positively span the torque space. When the wrenches fail to positively span the torque space, Proposition 3.1 the wrenches definitely fail to achieve force closure.

It should be noted that any subspace can be used in Proposition 3.1. Intuitively, a larger subspace more closely represent the actual wrenches set thus there is a strong preference on a large subspace. However, larger subspace requires more computational effort. One must select an appropriate trade-off between the choice of the subspace and the time required to determine the positively spanning of the projection of the wrenches. In this work, the torque subspace is chosen mainly because the torques of force cones exhibit a special property which allow us to efficiently test whether they positively span the torque space. The property of the torque and the test are described in Section IV.

An interesting property of a torque is that its varies according to the choice of the origin, although the force closure property as a whole is invariant to the relocation of the origin. It is possible that for some choice of the origin, a non force closure grasp may have its wrenches positively span the torque space. See for example the planar case in Figure 2. In the figure, it is clear that the contact points generate only counterclockwise torques when the origin is located at A. However, when the origin is located at B, the contact points generate both clockwise and counterclockwise torques, thus positively span the torque space. When there exist any choice of the origin such that the respective torque set does not positively span the torque space, the grasp definitely does not achieve force closure.

This implies that a torque should be tested respect to several choices of origins to improve the chance that the non-positively spanning of torque space is detected. However, any additional choice of origin requires additional computational effort. In our work, we limit the test to two choices of origin which are the points coinciding with two contact points. By making the origin coincide with a contact point, the torque associated with that contact point is null. This further increase the chance that we can detect non force closure grasp and also reduce the number of vector we have to considered. The two contact points chosen as the origin are the point lying closest to and the point lying furthest from point $\boldsymbol{q}$. The point $\boldsymbol{q}$ is an arbitrary point lying outside the convex hull of all contact points. The point $\boldsymbol{q}$ can be simply computed by taking the coordinate of a contact point with minimal x coordinate and minus that coordinate with $(1, 0, 0)$.

Here we describe our necessary condition for force closure. Let the contact point be located at $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n$ and let T_i be the set of torques associated with the contact point at $\boldsymbol{p}_i$. Let $\boldsymbol{q}$ be the point lying outside the convex hull of all contact points. We define $\boldsymbol{p}_{near}$ and $\boldsymbol{p}_{far}$ to be the contact point being closest and furthest to $\boldsymbol{q}$. Our necessary condition for n-finger force closure grasp is as follows.

1) Let the origin be located at $\boldsymbol{p}_{near}$; $\{T_1, \ldots, T_n\} \backslash T_{near}$ must positively span the torque space.

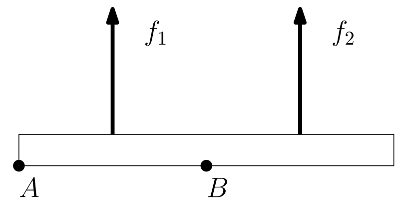

Fig. 2: Choices of origin resulting in different torque.

2) Let the origin be located at $\boldsymbol{p}_{far}$; $\{T_1, \ldots, T_n\} \backslash T_{far}$ must positively span the torque space.

IV. $\mathbb{R}^3$-POSITIVE SPAN OF TORQUE COMPONENTS

This section examines the geometric relationship between the torque space and the friction cones. We also introduce a novel method to test whether torque sets of an arbitrary number of force cones positively span the torque space. This is the implementation of our condition given at the end of the previous section.

A. Torque Sets of Force Cones

The content in this section simply retells the essential result from our previous work [1].

Let us denote by T_i the set of all torques generated by all forces in F_i (which is the friction cone of $\boldsymbol{p}_i$), i.e., $T_i = \{\boldsymbol{p}_i \times \boldsymbol{f} | \boldsymbol{f} \in F_i\}$. Since any torque $\boldsymbol{p}_i \times \boldsymbol{f}$ is obviously perpendicular to $\boldsymbol{p}_i$, T_i must lie on the plane through the origin and perpendicular to $\boldsymbol{p}_i$. Let us call this plane P_i.

To describe how T_i occupies P_i, let us consider a plane P_f through the origin that contains $\boldsymbol{p}_i$ and intersects with F_i. Observe that a torque generated by any force in $P_f \cap F_i$ (a slice of F_i on P_f) must lie in the direction parallel to the normal of P_f. The idea is to consider all possible planes P_f so that all forces in F_i can be taken into account (see Figure 3a). With this idea, it can be shown that T_i lies on P_i in two different ways: 1) $\boldsymbol{p}_i$ is not in $\operatorname{int}(F_i)$. As the plane P_f rotates around $\boldsymbol{p}_i$ and continuously sweeps through F_i, correspondingly generated torques continuously sweep P_i. As a result, the resulting torques, T_i, form a fan of torques, i.e., the set of all positive combinations of two boundary torques. 2) When $\boldsymbol{p}_i$ is in $\operatorname{int}(F_i)$, T_i covers the entire plane P_i. This is the case because for each possible P_f, resulting torques span two opposite directions on P_i. Since P_f in this case intersects with F_i in all orientations around $\boldsymbol{p}_i$, resulting torques cover all directions in P_i.

To compute the resulting fan of case 1, it is necessary to identify the two boundary torques of the fan. Since each of these torques is generated when P_f touches F_i, let us describe how to compute the corresponding forces $\boldsymbol{f}_a, \boldsymbol{f}_b \in F_i$ at which this event occurs. Let Π be the plane lying perpendicular to $\boldsymbol{n}_i$ at the distance $\boldsymbol{p}_i \cdot \boldsymbol{n}_i$ from the origin. Consider the intersection of Π with F_i and the lines through the vectors $\boldsymbol{n}_i, \boldsymbol{p}_i, \boldsymbol{f}_a, \boldsymbol{f}_b$. Figure 3b illustrates this intersection as observed on Π. From the figure, $\boldsymbol{f}_a$ and $\boldsymbol{f}_b$ can be determined from the angle ϕ. Let A and B be the intersection on Π of the line through $\boldsymbol{p}_i$ and the line through

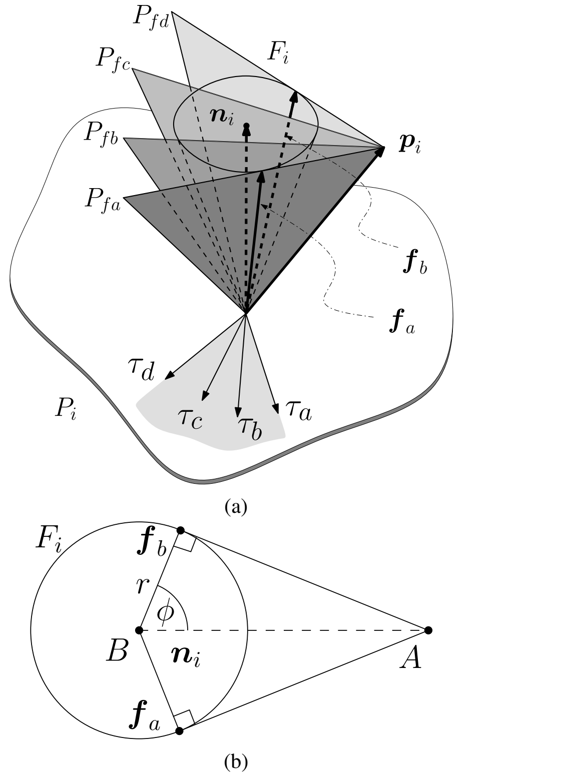

Fig. 3: (a) A force cone F_i and its position vector $\boldsymbol{p}_i$. The curved line represents plane P_i which is perpendicular to $\boldsymbol{p}_i$. The plane $P_{fa}, \ldots, P_{fd}$ are planes through the origin that contains $\boldsymbol{p}_i$ and intersects F_i. P_{fa} and P_{fd} tangentially touch F_i and the intersection of which is $\boldsymbol{f}_a$ and $\boldsymbol{f}_b$, respectively. Vector $\tau_a, \ldots, \tau_d$ represents the direction of torque generated from $P_{fa}, \ldots, P_{fd}$, respectively. (b) The plane Π, lying perpendicular to $\boldsymbol{n}_f$ at the distance $\boldsymbol{p}_i \cdot \boldsymbol{n}_i$ from the origin. The radius of the boundary of the cone is $r = \tan(\theta)(\boldsymbol{p}_i \cdot \boldsymbol{n}_f)$. The angle ϕ equals to $\arccos(r/|\overline{AB}|)$.

$\boldsymbol{n}_i$, respectively. The angle ϕ is equal to $\arccos(r/|\overline{AB}|)$ where r is the radius of the circle from the intersection of F_i and Π, i.e., $r = \tan(\theta)(\boldsymbol{p}_i \cdot \boldsymbol{n}_i)$.

B. Positively Span of Torque Sets

In section IV-A, it is established that a torque set of a force cone is either a fan (a positive span of two vectors) or a plane, both in 3D torque space. This finding is very crucial because it indicates that, in the case that all torque sets are fans, there are finite number of vectors that describes the torque set of the grasp. Even though the original force cone is a quadratic cone, the associated torque set is simply a linear fan.

When all torque sets are fans, the problem is simply to test whether a finite number of boundary vectors of the fans positively span the space. It should be noted that this problem is exactly the same as the problem of 2D n-Finger force closure test. This means that we can utilize several methods that are proposed for positively spanning test. For example, the GJK algorithm [19] which was suggested for force closure test in [20], the Q-Distance method [16], the ray shooting method of Liu [17] or the Quick Hull algorithm [21], all can be used to solve this problem. From our preliminary study, the ray shooting algorithm yields best performance for our framework.

The remaining case is when some torque sets are planes. When a torque set is a plane, if there are at least one other torque lying on each side of the plane, they positively span the space. Hence, for each torque set that is a plane, we simply test whether the other torque sets which are fan have their boundary vectors lying on the different side of the plane. and whether the other torque set which a plane . For the other torque sets that are also planes, we simply test whether they are not parallel to the original plane.

As a final note to this section, we provide analysis on the computational complexity of this implementation. It should be noted that computing the torque set requires constant time for each finger. In the case of torque fans, each fan is described by two vectors. So, the time to compute the vectors of torque fans is $O(n)$, where n is the number of finger. To actually test these vectors, the time depends on the plug-in method. For the case that some torque set is a plane. Let $k \leq n$ denote the number of a torque plane. We have to test every other torque set with each plane using a constant time. Hence, the time used in this case is $O(kn)$. Notice that, usually, the time used to perform the plug-in method dominates this small time complexity.

V. NUMERICAL EXAMPLE

Our presented condition and implementation are introduced as a filtering criteria which guarantees that a non-satisfying grasp does not achieve force closure while the force closure property of a satisfying grasp is undetermined. An additional complete method is therefore needed to test these satisfying grasps. We will refer to a satisfying grasp that turns out to be non-force closure grasp as a false positive. Our condition sacrifices completeness in favor of an efficiency in rejection. To benefit from the condition, the time additionally taken by our condition in the case of false positive must be offset by the time saved from the reduced number of performed complete methods.

To help justify the benefit of our approach, we compare two grasp analysis frameworks: a canonical framework which uses a complete method to assert force closure and our filtering approach which uses the same complete method together with our presented condition as a filtering criteria. We select a complete test for force closure presented by Han *et al.* [18] for comparison. The method in [18] is selected because it considers directly the quadratic friction cone without linearization, yielding most theoretical accuracy. Figure 4 illustrates the flowchart of both methods.

A. Method for Force Closure Test by Han et al. [18]

Briefly speaking, the method of Han *et al.* formulates the problem as an *LMI feasibility problem.* A grasping configuration is described by a vector $\boldsymbol{x} = \{x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{nm}\}$ and a mapping matrix G. The component x_{ij} of vector $\boldsymbol{x} \in \mathbb{R}^{mn}$ indicates the magnitude

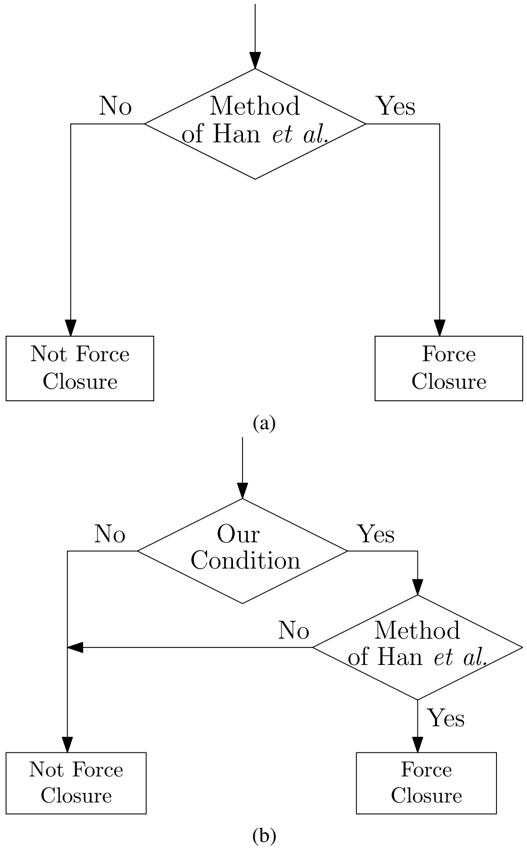

Fig. 4: Flowchart of force closure testing method. (a) The method of Han *et al.* (b)The method of Han *et al.* with our condition (described at the end of Section III) as a filtering criteria.

of jth component of intensity vector at ith contact point and n, m indicates the number of contact point and the number of components of intensity vectors. In the case of a hard contact with friction, the intensity vector is the force vector which has three components. The matrix $G \in \mathbb{R}^{6 \times mn}$ transforms $\boldsymbol{x}$ into a wrench. The resulting wrench is equal to $G\boldsymbol{x}$. A nontrivial solution to $G\boldsymbol{x} = 0$ indicates an equilibrium grasp. We let V be a matrix whose columns are basis vectors of the null space of G. Hence, an equilibrium grasp can be written as $\boldsymbol{x} = V\boldsymbol{z}$ where $\boldsymbol{z}$ is a free variable.

Based on the work of Buss *et al.* [22], the Coulomb Friction model can be represented in the form of LMI as $P(\boldsymbol{x}) \succeq 0$ where $\succeq 0$ denotes semi-positive definiteness. When the inequality is written as positive definite condition, i.e., $P(\boldsymbol{x}) \succ 0$, the force is restricted to lies in the interior of the friction cone. Since non-marginal equilibrium implies force closure, force closure test can be asserted from whether $P(V\boldsymbol{z}) \succ 0$ has an admissible solution. This inequalities can be solved by a traditional convex optimization technique.

B. Comparison and Result

We provide an empirical comparison in the scenario of force closure grasp identification: given an object described by a set of discrete contact points, the task is to identify

TABLE I: Result of the Experiment

Objects	Time (seconds)		
	Unfiltered	Filtered	Speedup Factor
(a)	1,174.06	395.82	2.97
(b)	1,342.35	262.37	5.12
(c)	1,365.37	451.52	3.02
(d)	466.73	98.18	4.75
(e)	1,065.49	278.31	3.83
(f)	442.11	82.98	5.33
Avg	976.02	261.53	4.17

force closure grasps randomly generated from the object.

The comparison is conducted on the six test objects shown in Figure 5. For each object, we randomly generate 100,000 configurations of 6 fingered grasp. The friction coefficient is assumed to be $\tan(10°)$. Both methods are implemented in C++ using the convex optimization package *maxdet* [23] and the linear algebra package *LAPACK* [24]. The comparison is run on Intel Core 2 Quad Q6600 machine with 2GB of memory.

The result of the comparison is shown in Table I. The second and the third column show the actual running time of the method in [18] and the filtered version, respectively. Speedup factor is given in the forth column. It can be seen that by using our method as a filtering criteria, we could speed up the running time by the factor of approximately four on average.

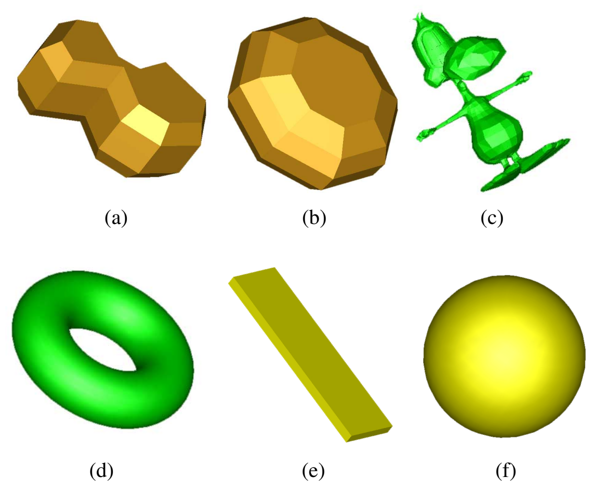

Fig. 5: Test Objects.

Analysis shows that the benefit of our method on filtering approach is affected by three subjects. The first is the difference of the time used for our filtering method and that of the complete analysis method. This difference is the time saved for each rejected non force closure grasp. The second subject is the ratio between the number of grasps not satisfying the condition and the total number non force closure grasps, i.e., the *specificity* [25] of our method. A high specificity indicates that a large fraction of non force closure grasps are correctly identified by the filtering criteria, and the computational effort is saved by the difference between that of the criteria and that of the complete method. Finally,

TABLE II: Analysis of the Experiment

Objects	#Solutions	#False Positives	Specificity
(a)	25,164	28,132	0.727
(b)	16,584	19,112	0.814
(c)	14,038	30,273	0.740
(d)	35,665	40,720	0.612
(e)	23,613	28,605	0.728
(f)	36,778	39,713	0.614
Avg	25,307	31,093	0.706

the number of non force closure grasps being tested. This number varies according to the situation that the condition is integrated into and the nature of the object. The more non force closure grasps, the more chance that our method can reduce the running time. To further analyze the method, these subjects are measured and shown in Table II.

The time used per query of our condition and of the method of Han *et al.* is approximately 0.05ms and 2.62ms, respectively. This indicates that, for each true negative solution, a running time is reduced to approximately 1.89%. In the case of false positive, the running time is increased to 101.89%.

The second column of Table II indicates the number of force closure grasps from the method in [18]. The third column shows the number of false positives while the specificity of our method are given in the forth column. From the average value of specificity, approximately 70.6% of the negative solutions is correctly identified by our condition. In other words, the time used for 70.6% of non force closure grasps are reduced to 1.89% while the time for the 29.4% remaining non force closure grasps are increased to 101.89%. These empirical data at least suggest that our method exhibits the favorable properties of a good filtering criteria.

VI. CONCLUSIONS

We extend the filtering approach presented in [1], [2] which originally consider only the four finger force closure grasp to be applicable for an arbitrary number of fingers. This filtering approach should find its use in several grasp synthesis algorithm especially when dealing with a large number of discrete contact point representation.

The underlying idea of the filter is similar to our earlier work with the additional modification to cover the case of n-finger grasping. The essential idea of this work is that a grasp does not achieve force closure when the projection of its associated wrenches does not positively span the torque space. It has been shown that the torque associated with a 3D frictional hard contact is either a fan or a plane thus allowing us to utilize several methods to test whether they positively span the torque space. We also present numerical example which confirms that by incorporating our method, we can significantly improve the time used to compute multiple queries of n-finger force closure test.

REFERENCES

[1] Nattee Niparnan and Attawith Sudsang. Positive span of force and torque components of four-fingered three-dimensional force-closure grasps. In *IEEE Int. Conf. on Robotics and Automation*, pages 4701–4706, Roma, Italy, April 2007.

[2] Nattee Niparnan, Attawith Sudsang, and Prabhas Chongstitvatana. Positive span of force and torque components in three dimensional four finger force closure grasps. *Advanced Robotics*, In Press, 2008.

[3] A. Bicchi and V. Kumar. Robotic grasping and contact: A review. In *IEEE Int. Conf. on Robotics and Automation*, pages 348–353, San Francisco, CA, USA, 2000.

[4] Antonio Bicchi. Hands for dexterous manipulation and robust grasping: A difficult road toward simplicity. *IEEE Trans. Robot. Autom.*, 16(16):652–662, December 2000.

[5] K. B. Shimoga. Robot grasp synthesis algorithms: a survey. *Int. J. Robot. Research*, 15(3):203–266, 1996.

[6] R.C. Brost and K. Goldberg. A complete algorithm for designing planar fixtures using modular components. *IEEE Trans. Robot. Autom.*, 12(1):31–46, February 1996.

[7] Michael Yu Wang. An optimum design for 3-D fixture synthesis in a point set domain. *IEEE Trans. Robot. Autom.*, 16(6):839–846, December 2000.

[8] Yun-Hui Liu, Miu-Ling Lam, and Dan Ding. A complete and efficient algorithm for searching 3-D form-closure grasps in the discrete domain. *IEEE Trans. Robot.*, 20(5):805–816, October 2004.

[9] Jordi Cornella and Raul Suarez. A new framework for planning three-finger grasps of 2D irregular objects. In *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pages 5688–5694, Beijing, Chaina, October 2006.

[10] Xiangyang Zhu and Han Ding. Computation of force-closure grasps: An iterative algorithm. *IEEE Trans. Robot.*, 22(1):172–179, February 2006.

[11] Yasuhisa Hasegawa, Junya Matsuno, and Toshio Fukuda. Regrasping behavior and object transition generated by ep. In *Proceedings of the Congress on Evolutionary Computation*, pages 746–752, San Diego, CA, USA, 2000.

[12] Yoshiaki Katada, Mikhail Svinin, Kazuhiro Ohkura, and Kanji Ueda. Stable grasp planning by evolutionary programming. *IEEE Trans. Robot. Autom.*, 48(4):749–756, August 2001.

[13] Ch. Borst, M. Fischer, and G. Hirzinger. Grasping the dice by dicing the grasp. In *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pages 3692–3697, Las Vegas, Nevada, USA, 2003.

[14] J. Ponce and B. Faverjon. On computing three-finger force-closure grasps of polygonal objects. *IEEE Trans. Robot. Autom.*, 11(6):868–881, December 1995.

[15] J. Ponce, A. Sudsang, S. Sullivan, B. Faverjon, J.-D. Boissonnat, and J.-P. Merlet. On computing four-finger equilibrium and force-closure grasps of polyhedral objects. *Int. J. Robot. Research*, 16(1):11–35, February 1997.

[16] Xiangyang Zhu and Jun Wang. Synthesis of force-closure grasps on 3-D objects based on the q distance. *IEEE Trans. Robot. Autom.*, 19(4):669–679, August 2003.

[17] Yun-Hui Liu. Qualitative test and force optimization of 3-D frictional form-closure grasps using linear programming. *IEEE Trans. Robot. Autom.*, 15(1):163–173, February 1999.

[18] Li Han, Jeff C. Trinkle, and Zexiang X. Li. Grasp analysis as linear matrix inequality problems. *IEEE Trans. Robot. Autom.*, 16(6):663–674, December 2000.

[19] E. G. Gilbert, D. W. Johnson, and S. S. Keerthi. A fast procedure for computing the distance between complex objects in three-dimentional space. *IEEE Trans. Robot. Autom.*, 4:193–203, April 1988.

[20] Xiangyang Zhu, Han Ding, and S. K. Tso. A pseudodistance function and its applications. *IEEE Trans. Robot. Autom.*, 20(2):344–352, April 2004.

[21] C. Bradford Barber, David P. Dobkin, and Hannu Huhdanpaa. The quickhull algorithm for convex hulls. *ACM Trans. Math. Soft.*, 22(4):469–483, 1996.

[22] H. Hashimoto M. Buss and J. Moore. Dexterous hand grasping force optimization. *IEEE Trans. Robot. Autom.*, 12(3):406–418, 1996.

[23] Shao-Po Wu, Lieven Vandenberghe, and Stephen Boyd. *MAXDET Software for Determinant Maximization Problems User Guide*. Stanford University, alpha version edition, 1996.

[24] E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. *LAPACK Users' Guide*. Society for Industrial and Applied Mathematics, Philadelphia, PA, third edition, 1999.

[25] J. M. Bland D. G. Altman. Statistics notes: Diagnostic tests 1: sensitivity and specificity. *BMJ*, 308:1552, 1994.