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Decentralized Adaptive Control of a Class of Discrete-Time Multi-Agent Systems for Hidden Leader Following Problem

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Abstract— **In this paper, adaptive control is investigated for a class of discrete-time nonlinear multi-agent systems (MAS). Each agent is of uncertain dynamics and is affected by other agents in its neighborhood. An agent is able to sense the outputs of the agents inside its neighborhood but is unable to sense those outside its neighborhood. Among all the agents, there is a hidden leader, which knows the desired tracking trajectory, but it is affected by and can only affect those agents inside its neighborhood while all other agents are not aware of its leadership. The decentralized adaptive control is designed for each agent by using the information of its neighbors. Under the proposed decentralized adaptive controls, both rigid mathematical proof and simulation studies are provided to show that all the agents are guaranteed to reach their common goal, i.e., following the desired reference.**

I. Introduction

The research on MAS has a wide range of background, which is motivated by the physical discoveries on some nonlinear phenomena, such as chaos, fractals, turbulence, cellular automata, etc. Applications of MAS control include scheduling of automated highway systems, formation control of satellite clusters, and distributed optimization of multiple mobile robotic systems, etc. Several examples can be found in [1], [2].

Various control strategies developed for MAS can be roughly assorted into two architectures: centralized and decentralized. In the decentralized control, local control for each agent is designed only using locally available information so it requires less computational effort and is relatively more scalable with respect to the swarm size. But up to now, there are relatively few results obtained for the decentralized control of complex systems. In [3], a discrete-time model (Vicsek model) of n agents has been proposed, in which all the agents moved in the plane with the same speed but with different headings. By exploring matrix and graph properties, a theoretical explanation for the consensus behavior of the Vicsek model has been provided in [4]. In [5], a discrete-time MAS model has been studied with fixed undirected topology and all the agents are assumed to transmit their state information in turn. In [6], some sufficient conditions for the solvability of consensus problems for discrete-time MAS with switching topology and time-varying delays have been presented by using matrix theories. In [7], a discrete-time network model of agents interacting via time-dependent communication links has been investigated. The result in [7] has been extended to the case with time-varying delays by set-value Lyapunov theory in [8].

In this paper, we are going to study the decentralized control for a class of MAS, in the framework of leader following problem. For the MAS control problem, sometimes it is desired that there is a leader agent in the whole system. For example, in the natural world, each bird in the flock follows a leader bird at front such that the whole flock is synchronized to a desired formation in order to resist aggression during the migration. Many researches are dedicated to the leader-follower control of MAS in order to solve the target (leader) tracking. In [9], the decentralized control is proposed to ensure that a swarm of mobile agents with limited sensing ranges converge into a moving target region. The coordinated motion of a group of motile particles with a leader has been analyzed in [10]. To avoid the problem with disturbance rejection inherent in the leader-follower approach, several works, such as in [11], [12], [13], utilized a virtual leader architecture. This approach needed to synthesize the virtual leader and communicate its position in time so it required high communication and computation capability, as indicated in [14].

In most of the works mentioned above, the leader agent is assumed to be known to and can be sensed by all the other agents. In this paper, we will consider a hidden leader following problem, in which the leader agent knows the target trajectory to follow but the leadership of itself is unknown to all the others, and the leader can only affect its neighbors who can sense its outputs. In fact, this sort of problems may be found in many real applications. For example, a capper in the casino lures the players to follow his action but at the same time he has to keep not recognized. For another example, the plainclothes policeman can handle the crowd guide work very well in a crowd of people although he may only affect people around him. The objective of hidden leader following problem for the MAS is to make each agent eventually follow the hidden leader such that the whole system is in order. It is obvious that the hidden leader following problem is more complicated than the conventional leader following problem and investigations of this problem are of significance in both theory and practice.

On the other hand, most of the recent research works are concerned with the consensus of agents with simple first- or second-order dynamics, as indicated in [15]. However,

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for real-world applications, practical systems usually have complicated nonlinear dynamics and there are usually uncertainties in the dynamics. Therefore, it is meaningful to study adaptive control of MAS, on which some attempts can be found in [16], [17]. In particular, adaptive control was studied for both consensus problem and leader following problem for a class of MAS in [17], where there were noise and one unknown parameter in the dynamics of each agent. To further illustrate the demands for adaptive control of MAS, let us consider an example, in which many cars running on a crowded road are considered as a MAS with each car as an agent and the driver as the controller. Drivers of their cars take actions to avoid possible collision and keep cars running normally. Considering interactions among cars and the time-varying environment, driving a car is actually a typical problem of decentralized adaptive control.

Based on the above discussion, the main contributions of this paper lie in:

(i) The decentralized control has been studied for a class of MAS with a hidden leader, whose leadership is unknown to others and whose outputs can only be sensed by its neighbors.

(ii) Local adaptive control is proposed for each agent to compensate parametric uncertainties in its dynamics and couplings with other agents in its neighborhood.

(iii) Under the proposed decentralized adaptive control, each agent is made to track the average of outputs of its neighbors. By establishment of relationship among various error signals, it is proved that all the agents eventually follow the desired reference.

II. Preliminaries and Problem Formulation

A. Preliminaries

Definition 1: A sub-stochastic matrix is a square matrix each of whose rows consists of nonnegative real numbers, with at least one row summing strictly less than 1 and other rows summing to 1.

Definition 2: [18] Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals, $\forall k \in Z_t^+$, for any t, where Z_t^+ is the set of all integers not less than a given integer t.

- We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \leq m_1 \max_{k'\leq k} \|x_2(k')\| + m_2, \ \ \forall k > k_0$.
- We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty} \alpha(k) \to 0$ and a constant k_0 such that $\|x_1(k)\| \leq \alpha(k) \max_{k'\leq k} \|x_2(k')\|, \ \ \forall k > k_0$.
- We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.

Remark 1: For the convenience, we use $O[1]$ and $o[1]$ to denote a bounded sequence and a sequence converging to zero, respectively.

The following lemma is a special case of Lemma 4.4 in [17].

Lemma 1: Consider the following iterative system

$$X(k+1) = A(k)X(k) + W(k) \tag{1}$$

where$\|W(k)\| = O[1]$, and $A(k) \to A$ as $k \to \infty$. Assume $\rho(A)$ is the spectral radius of A, i.e. $\rho(A) = \max\{|\lambda(A)|\}$ and $\rho(A) < 1$, then we can obtain

$$X(k+1) = O[1] \tag{2}$$

Proof. For arbitrary $\epsilon > 0$, by the definition of $\rho(A)$, there exists a matrix norm (denoted by $\|\cdot\|_p$) such that $\|A\|_p < \rho + \frac{\epsilon}{2}$. We can also get $\|A(k)\|_p \to \|A\|_p$ from $A(k) \to A$ as $t \to \infty$. Hence for sufficiently large k,

$$\|A(k)\|_p < \|A\|_p + \tfrac{\epsilon}{2} < \rho + \epsilon \tag{3}$$

According to the equivalence among norms, $\|W(k)\|_p = O[\|W(k)\|] = O[1]$, therefore for sufficiently large k,

$$\begin{aligned} \|X(k+1)\|_p &\leq \|A(k)\|_p \|X(k)\|_p + \|W(k)\|_p \\ &\leq (\rho+\epsilon)\|X(k)\|_p + C_p \end{aligned} \tag{4}$$

Iterating the inequality above, we have

$$\|X(k)\|_p \leq C_p \sum_{k'=1}^{k-m} (\rho+\epsilon)^{k'-1} + (\rho+\epsilon)^{k-m} \|X(m)\|_p$$

where m is a constant depending on ϵ and p. Then it is obvious to obtain

$$\|X(k)\|_p = O[(\rho+\epsilon)^k + O[1]] + O[(\rho+\epsilon)^k] = O[1]$$

■

B. Problem Formulation

In the MAS under study, each agent is affected by other agents in its neighborhood and it is able to sense the output information of its neighbors. It can be represented by a directed graph based on the graph theory, which can be found in many texts, see [14], [19], [20]. An example is given in Figure 1, where every vertex stands for an agent and all the agents with an edge directing at the jth agent are inside the neighborhood of the jth agent.

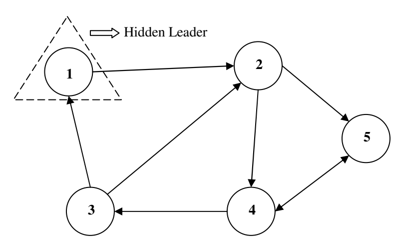

Fig. 1. An example of MAS

The directed graph can be further represented by an adjacent matrix. For example, the graph in Figure 1 can be described by the following adjacent matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Definition 3: [19] A directed graph is called strongly connected if and only if any two distinct nodes of the graph can be connected via a path that follows the direction of the edges of the graph.

Assumption 1: The graph of the MAS under study is strongly connected such that its adjacent matrix G is irreducible.

Hidden Leader following problem: Let us consider a MAS in which there are n dynamic agents. The control objective is to synthesize a local control input for each agent such that all the agents follow a desired reference trajectory $y^*(k)$, which is only available to a leader agent and unknown to other agents. The leader agent is hidden, that is, all the other agents are not aware of its leadership.

Define the error between the the output of the jth agent and the reference trajectory as

$$e_j(k) = y_j(k) - y^*(k) \tag{5}$$

then the control objective is to make the average tracking error to zero, i.e.,

$$\lim_{k\to\infty} \frac{1}{k} \sum_{k'=1}^{k} |e_j(k')| = 0,\ j = 1, 2, \ldots, n \tag{6}$$

Assumption 2: The desired reference $y^*(k)$ for the MAS is a bounded sequence and satisfies $y^*(k+1)-y^*(k) = o[1]$.

C. System Representation

The dynamics of the MAS under study is in the following manner:

$$y_j(k+1) = \Theta_j^T \Phi_j(\underline{y}_j(k), Y_j^T(k)) + g_j u_j(k) \tag{7}$$

where $u_j(k)$ and $y_j(k)$, $j = 1, 2, \ldots, n$, are input and output of the jth agent and $Y_j(k)$ is a vector of the outputs from the neighbors of the jth agent. The function $\Phi_j(\cdot)$ in the dynamics of the jth agent is known but parameters $\Theta_j^T \in R^{p_j}$ and $g_j \in R$ are unknown. In particular,

$$\begin{aligned} \underline{y}_j(k) &= [y_j(k), \cdots, y_j(k-n_j)]^T && (8)\\ N_j &= \{s_{j,1}, s_{j,2}, \ldots, s_{j,m_j}\} && (9)\\ Y_j(k) &= [y_{s_{j,1}}(k), y_{s_{j,2}}(k), \ldots, y_{s_{j,m_j}}(k)] && (10) \end{aligned}$$

where n_j is the jth agent's order and N_j is the set of all agents in the jth agent's neighborhood (excluding the jth agent itself) with m_j being the number of agents in the jth agent's neighborhood. It is noted that each agent is affected by its neighbors by the term of $Y_j(k)$ in its dynamics.

Remark 2: From (7) we can find that there is no information to indicate which agent is the leader in the system representation.

Assumption 3: Without loss of generality, it is assumed that the first agent is a hidden leader who knows the desired reference $y^*(k)$ while other agents are unaware of either the desired reference or which agent is the leader.

Assumption 4: The system functions, $\Phi_j(\cdot)$, $1 \le j \le n$, are Lipschitz functions with Lipschitz coefficients L_j.

Assumption 5: The sign of control gain g_j, $1 \le j \le n$, is known and satisfies $|g_j| \ge \underline{g}_j > 0$. Without loss of generality, it is assumed that g_j is positive.

III. Adaptive Control Design

For convenience, we denote $\hat{\Theta}_j^T(k)$ and $\hat{g}_j(k)$ as the estimates of Θ_j^T and g_j at the kth step, respectively, and use $\Phi_j(k)$ as the abbreviation of $\Phi_j(\underline{y}_j(k), Y_j^T(k))$, $j = 1, 2, \ldots, n$, without cause of confusion.

According to Assumption 3, the first agent knows the reference signal $y^*(k)$, so the control for the first agent can be directly designed by using the certainty equivalence principal to track $y^*(k)$. The controller is given as follows:

$$u_1(k) = \frac{1}{\hat{g}_1(k)}[-\hat{\Theta}_1^T(k)\Phi_1(k) + y^*(k+1)] \tag{11}$$

As for the other agents, because they are unaware of either the reference trajectory or the existence of the leader and the outputs of their neighbors are the only external information available for them, we consider the following adaptive controller:

$$u_j(k) = \frac{1}{\hat{g}_j(k)}[-\hat{\Theta}_j^T(k)\Phi_j(k) + z_j(k)] \tag{12}$$

where $z_j(k)$ is the average value of the outputs of the jth agent' neighbors, defined as

$$z_j(k) = \frac{1}{m_j} \sum_{l\in N_j} y_l(k),\ j = 2, 3, \ldots, n \tag{13}$$

Substituting the controls in (11) and (12) into the MAS (7), we have the following error signals dynamics for $j = 2, 3, \ldots, n$:

$$\begin{aligned} \tilde{y}_1(k+1) &= y_1(k+1) - y^*(k+1)\\ &= -\tilde{\Theta}_1(k)\Phi_1(k) - \tilde{g}_1(k)u_1(k) && (14)\\ \tilde{y}_j(k+1) &= y_j(k+1) - z_j(k)\\ &= -\tilde{\Theta}_j(k)\Phi_j(k) - \tilde{g}_j(k)u_j(k) && (15) \end{aligned}$$

where $\tilde{\Theta}_j(k) = \hat{\Theta}_j(k) - \Theta_j,\ \tilde{g}_j(k) = \hat{g}_j(k) - g_j$. Using the error dynamics defined above, the update law for the estimated parameters in the adaptive control (11) and (12) is given below:

$$\begin{aligned} &\hat{\Theta}_j(k) = \hat{\Theta}_j(k-1) + \frac{\gamma_j \tilde{y}_j(k)\Phi_j(k-1)}{D_j(k-1)}\\ &\hat{g}_j(k) = \begin{cases} \hat{g}'_j(k) & \text{if } \hat{g}'_j(k) > \underline{g}_j\\ \underline{g}_j & \text{otherwise} \end{cases}\\ &\hat{g}'_j(k) = \hat{g}'_j(k-1) + \frac{\gamma_j \tilde{y}_j(k)u_j(k-1)}{D_j(k-1)}\\ &D_j(k) = 1 + \|\Phi_j(k)\|^2 + u_j^2(k),\ j = 1, 2, \ldots, n \end{aligned} \tag{16}$$

where $0 < \gamma_j < 2$.

Remark 3: It is noted that $\hat{g}_j(k)$ is guaranteed to be bounded away from zero such that the adaptive control is free of control singularity problem.

Let us define

$$\begin{aligned} Y(k) &= [y_1(k), y_2(k), \ldots, y_n(k)]^T && (17)\\ \tilde{Y}(k) &= [\tilde{y}_1(k), \tilde{y}_2(k), \ldots, \tilde{y}_n(k)]^T && (18)\\ H &= [1, 0, \ldots, 0]^T \in R^n && (19) \end{aligned}$$

such that the closed-loop MAS can be written in the following compact form by using equality $[0, z_2(k), \ldots, z_n(k)] = \Lambda G Y(k)$:

$$Y(k+1) = \Lambda G Y(k) + H y^*(k+1) + \tilde{Y}(k+1) \tag{20}$$

where G is an adjacent matrix of the MAS system (7) and

$$\Lambda = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{m_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{m_n} \end{bmatrix} \tag{21}$$

Lemma 2: According to Assumption 1, the product matrix ΛG is a substochastic matrix (refer to Definition 1) such that $\rho(\Lambda G) < 1$ [21], where $\rho(A)$ stands for the spectral radius of a matrix A.

IV. Control Performance Analysis

Under the proposed decentralized adaptive control, the control performance for the MAS is summarized as the following theorem.

Theorem 1: Considering the closed-loop MAS consisting of open loop system in (7) under Assumptions 2-5, adaptive control inputs defined in (11) and (12), parameter estimates update law in (16), the control objective given by (6) is achieved.

In the following, the proof of mathematic rigor is presented in two steps. In the first step, we prove that $\tilde{y}_j(k) \to 0$, which leads to $y_1(k) - y^*(k) \to 0$ such that the hidden leader follows the reference trajectory. In the second step, we further prove that the output of each agent can track the output of hidden leader such that the control objective is achieved.

Proof. Step 1: Consider a Lyapunov candidate

$$V_j(k) = \|\tilde{\Theta}_j(k)\|^2 + \hat{g}_j^2(k) \tag{22}$$

Noting that

$$\begin{aligned} \|\hat{\Theta}_j(k) - \hat{\Theta}_j(k-1)\| &= \|\tilde{\Theta}_j(k) - \tilde{\Theta}_j(k-1)\| \\ \hat{g}_j(k) - \hat{g}_j(k-1) &= \tilde{g}_j(k) - \tilde{g}_j(k-1) \\ \tilde{g}_j^2(k) &\le \tilde{g}_j'^2(k) \end{aligned} \tag{23}$$

and according to the update law (16), error dynamics (14) and (15), the difference of Lyapunov function $V_j(k)$ can be written as

$$\begin{aligned} &\Delta V_j(k) = V_j(k) - V_j(k-1) \\ &= \|\tilde{\Theta}_j(k)\|^2 - \|\tilde{\Theta}_j(k-1)\|^2 + \hat{g}_j^2(k) - \hat{g}_j^2(k-1) \\ &= \|\hat{\Theta}_j(k) - \hat{\Theta}_j(k-1)\|^2 + 2\|\tilde{\Theta}_j(k-1)\| \\ &\|\hat{\Theta}_j(k) - \hat{\Theta}_j(k-1)\| + (\hat{g}_j(k) - \hat{g}_j(k-1))^2 \\ &+2\tilde{g}_j(k-1)(\hat{g}_j(k) - \hat{g}_j(k-1)) \\ &\le \frac{\gamma_j^2 \tilde{y}_j^2(k)}{D_j(k-1)} - \frac{2\gamma_j \tilde{y}_j^2(k)}{D_j(k-1)} = -\frac{\gamma_j(2-\gamma_j)\tilde{y}_j^2(k)}{D_j(k-1)} \end{aligned}$$

Noting $0 < \gamma_j < 2$, we see that $\Delta V_j(k)$ is guaranteed to be non-positive such that the boundedness of $V_j(k)$ is obvious, and immediately the boundedness of $\hat{\Theta}_j(k)$ and $\hat{g}_j(k)$ is guaranteed. Taking summation on both sides of the above equation, we obtain

$$\sum_{k=0}^{\infty} \gamma_j(2-\gamma_j)\frac{\tilde{y}_j^2(k)}{D_j(k-1)} \le V_j(0) \tag{24}$$

which implies

$$\lim_{k\to\infty} \frac{\tilde{y}_j^2(k)}{D_j(k-1)} = 0, \text{ or } \tilde{y}_j(k) = \alpha_j(k) D_j^{\frac{1}{2}}(k-1) \tag{25}$$

with $\alpha_j(k) \in L^2[0,\infty)$.

Define

$$\bar{Y}_j(k) = [y_j(k), Y_j^T(k)]^T \tag{26}$$

Because of the Lipschitz condition of $\Phi_j(k)$, we have

$$\begin{aligned} u_j(k) &= \frac{1}{g_j}(y_j(k+1) - \Theta_j^T \Phi_j(k)) = O[\bar{Y}_j(k+1)] \\ \Phi_j(k) &= O[\bar{Y}_j(k)] \end{aligned} \tag{27}$$

then it is obvious that

$$\begin{aligned} D_j^{\frac{1}{2}}(k-1) &\le 1 + \|\Phi_j(k-1)\| + |u_j(k-1)| \\ &= 1 + O[\bar{Y}_j(k)] \end{aligned} \tag{28}$$

From (25) we obtain

$$\tilde{y}_j(k) = o[1] + o[\bar{Y}_j(k)], \ j = 1, 2, \ldots, n \tag{29}$$

Using $o[Y(k)] \sim o[y_1(k)] + o[y_2(k)] + \ldots + o[y_n(k)]$, we rewrite the above equation as

$$\begin{aligned} \tilde{Y}(k) \;\sim\; & \operatorname{diag}(o[1], \ldots, o[1])(G+I)Y(k) \\ & +[o[1], \ldots, o[1]]^T \end{aligned} \tag{30}$$

where I is the $n \times n$ identity matrix. Substituting the above equation into equation (20), we obtain

$$\begin{aligned} Y(k+1) &= (\Lambda G + \operatorname{diag}(o[1], \ldots, o[1])(G+I))Y(k) \\ &\quad +[y^*(k+1) + o[1], o[1], \ldots, o[1]]^T \end{aligned}$$

Since

$$(\Lambda G + \operatorname{diag}(o[1], \ldots, o[1])(G+I))Y(k) \to \Lambda G \tag{31}$$

as $k \to \infty$, and $\rho(\Lambda G) < 1$ according to Lemma 2 and

$$[y^*(k+1) + o[1], o[1], \ldots, o[1]]^T = O[1] \tag{32}$$

from Lemma 1, we have

$$Y(k+1) = O[1] \tag{33}$$

Then, together with equation (30), we have $\tilde{Y}(k) = [o[1], \ldots, o[1]]^T$, which implies

$$\tilde{y}_j(k) \to 0 \text{ as } k \to \infty, j = 1, 2, \ldots, n \tag{34}$$

which leads to $y_1(k) - y^*(k) \to 0$.

Step 2: Next, we define a vector of the errors between each agent's output and the hidden leader's output as follows

$$\begin{aligned} E(k) &= Y(k) - [0,\ 1, \ldots,\ 1]^T y_1(k) \\ &= [e_{11}(k), e_{21}(k), \ldots, e_{n1}(k)]^T \end{aligned}$$

where $e_{j1}(k)$ satisfies

$$\begin{aligned} &e_{11}(k+1) = y_1(k+1) - y_1(k+1) = 0, \\ &e_{j1}(k+1) = y_j(k+1) - y_1(k+1) \\ &= z_j(k) - y_1(k+1) + \tilde{y}_j(k+1), \\ &j = 2, 3, \ldots, n \end{aligned} \tag{35}$$

Noting that except the first row, the summations of the other rows in the sub-stochastic matrix ΛG are 1, we have

$$[0, 1, \ldots, 1]^T = \Lambda G[0, 1, \ldots, 1]^T$$

such that equations in (35) can be written as

$$\begin{aligned} E(k+1) = \Lambda GY(k) - \Lambda G[0, 1, \ldots, 1]^T y_1(k+1) \\ + \operatorname{diag}(0, 1, \ldots, 1)\tilde{Y}(k) \end{aligned} \quad (36)$$

According to Assumption 2, we obtain

$$\begin{aligned} &E(k+1) = \Lambda G(Y(k) - [0, 1, \ldots, 1]^T y_1(k)) \\ &+[0, 1, \ldots, 1]^T (y_1(k) - y_1(k+1)) \\ &+[o[1], \ldots, o[1]]^T \\ &= \Lambda GE(k) + [o[1], \ldots, o[1]]^T \end{aligned} \quad (37)$$

Assume ρ' is the spectral radius of ΛG, then there exists a matrix norm, which is denoted as $\|\cdot\|_p$, such that

$$\|E(k+1)\|_p \le \rho' \|E(k)\|_p + o[1] \quad (38)$$

where $\rho' < 1$. Then, it is straightforward to show that

$$\sum_{k'=1}^{k+1} \|E(k')\|_p \le \rho' \sum_{k'=1}^{k} \|E(k')\|_p + o[k] + \|E(1)\|_p \quad (39)$$

By defining

$$S(k) = \sum_{k'=1}^{k} \|E(k')\|_p \quad (40)$$

we can obtain

$$S(k+1) \le \rho' S(k) + o[k] + C_1 \quad (41)$$

where $C_1 = \|E(1)\|_p$ is a constant. From the above equation, we have

$$\begin{aligned} S(2) &\le \rho' S(1) + \alpha(1) + C_1 \\ S(3) &\le \rho' S(2) + \alpha(2) + C_1 \\ &\le \rho'^2 S(1) + (\rho'\alpha(1) + \alpha(2)) + (\rho' + 1)C_1 \\ &\vdots \\ S(k) &\le \rho'^{k-1} S(1) + \sum_{k'=0}^{k-2} (\rho'^{k'} \alpha(k - k' - 1)) \\ &+ \tfrac{1-\rho'^{k-1}}{1-\rho'} C_1 \end{aligned}$$

where $\alpha(k) \to 0$ as $k \to \infty$. According to Schwartz's inequality,

$$\begin{aligned} &\sum_{k'=0}^{k-2} (\rho'^{k'} \alpha(k - k' - 1)) \\ &\le (\sum_{k'=0}^{k-2} (\rho'^{2k'}))^{\frac{1}{2}} (\sum_{k'=0}^{k-2} (\alpha^2(k - k' - 1)))^{\frac{1}{2}} \end{aligned}$$

then we obtain

$$\begin{aligned} S(k) \le \tfrac{1}{(1-\rho'^2)^{\frac{1}{2}}} (\sum_{k'=0}^{k-2} (\alpha^2(k - k' - 1)))^{\frac{1}{2}} \\ + \tfrac{1}{1-\rho'} C_1 \text{ as } k \to \infty \end{aligned} \quad (42)$$

from which it is easy to obtain

$$S(k) = o[k], \quad \frac{S(k)}{k} = o[1] \quad (43)$$

which implies

$$\frac{1}{k} \sum_{k'=1}^{k} \|E(k')\|_p \to 0 \text{ as } k \to \infty \quad (44)$$

Then it is straightforward to obtain

$$\tfrac{1}{k} \sum_{k'=1}^{k} |e_{j1}(k')| \to 0 \text{ as } k \to \infty \quad (45)$$

Together with (34), we have

$$\frac{1}{k} \sum_{k'=1}^{k} |e_j(k')| \quad = \quad \frac{1}{k} \sum_{k'=1}^{k} |e_{j1}(k') + \tilde{y}_1(k')| \to 0 \quad (46)$$

as $k \to \infty$. It completes the proof. ∎

V. Simulation Results

In the simulation, we utilize the MAS example shown in Figure 1, of which the dynamics is given as follows:

$$y_j(k+1) = \Theta_j^T \Phi_j(\underline{y}_j(k), Y_j^T(k)) + g_j u_j(k) \quad (47)$$

where

$$\begin{aligned} &\Theta_j^T = [1, 1], \ g_j = 1, \ j = 1, \ldots, 5 \\ &\Phi_1^T(\cdot) = [y_1(k) + 0.1e^{-y_1(k-1)}, 0.1\sin(y_3(k))], \\ &\Phi_2^T(\cdot) = [y_2(k), 0.1\cos(y_1(k) + y_3(k))], \\ &\Phi_3^T(\cdot) = [y_3(k), 0.6\cos(y_4(k))], \\ &\Phi_4^T(\cdot) = [0.9y_4(k) + 0.1y_4(k-1), 0.01e^{-(y_2(k)+y_5(k))}], \\ &\Phi_5^T(\cdot) = [y_5(k), \sin(y_2(k)y_4(k))] \end{aligned}$$

It is easy to check that this system satisfies Assumption 1, i.e., it is strongly connected. In particular, these five agents affect their corresponding neighbors with nonlinear couplings.

As mentioned above, the control objective is to make the output of each agent track the desired reference trajectory. To satisfy Assumption 2, the desired reference trajectory for the first agent is set as a step signal with amplitude 10. The initial outputs are set as $[0, 0, 0, 0, 0]^T$ and $T = 0.01$.

The results are presented in Figures 2 and 3. From Figure 2, we can find that the output of each agent tracks the desired reference trajectory as $k \to \infty$. In particular, the output of the first agent, which is the hidden leader, tracks the desired reference faster than the other agents. It is because the hidden leader tracks the reference directly. Although the second and third agents are connected with the leader agent, they actually do not have any idea about its leadership. The other agents, i.e., the fourth and fifth agents, are in the worse case since they are not connected with the leader agent directly. However, all of them track the desired trajectory by tracking the average output of their own neighbors, which has been indicated in the control design.

Besides, Figure 2 shows that the system control inputs $u_j(k)$ are bounded. On the other hand, Figure 3 indicates the boundedness of the estimated parameters and control gains, which tend to converge as the steps increase.

In summary, although there are nonlinear couplings between neighboring agents and there is not any information about the leadership of the leader agent, the local adaptive control for each agent can guarantee the control objective, which is defined for the whole MAS.

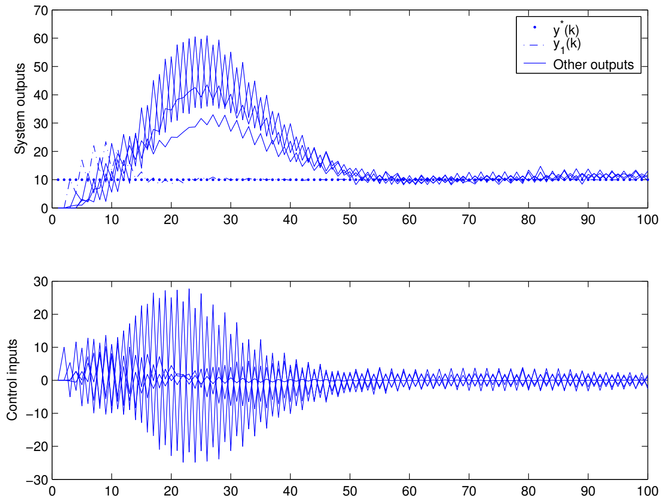

Fig. 2. System signals

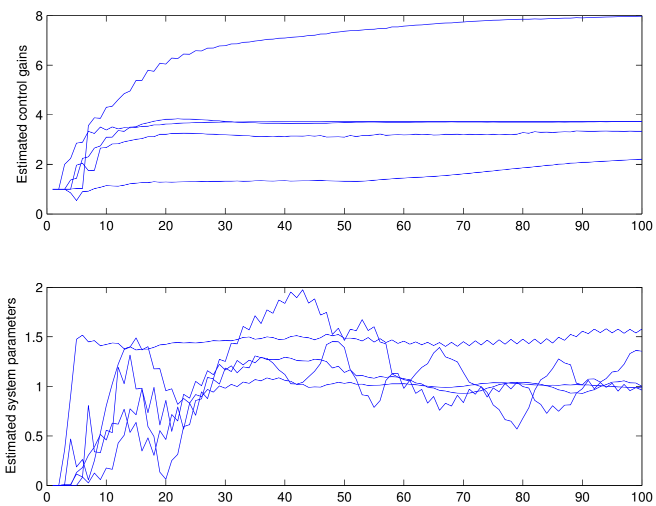

Fig. 3. Estimated parameters and control gains

VI. Conclusion

In this paper, adaptive control has been investigated for a class of discrete-time nonlinear MAS with uncertain interactions between each agent and its neighborhoods. For each agent, a parameter update law is presented to estimate the unknown parameters in its dynamics. Using the estimated parameters, decentralized adaptive control is presented for each agent by exploration of the relationship among each agent and its neighbors. The local adaptive control compensates the parametric uncertainties for each agent and makes the agent output track the average of the outputs of its neighbors. Under the proposed decentralized adaptive control, the outputs of all the agents eventually follow the output of a hidden leader agent, although other agents including those in the leader's neighborhood are unaware of the leader agent's leadership. At the same time, all the signals in the whole closed-loop MAS are guaranteed to be bounded.

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