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Compressive Mobile Sensing in Robotic Mapping

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Abstract— **This paper presents a novel approach, compressive mobile sensing, to use mobile sensors to sample and reconstruct sensing fields based on compressive sensing. Compressive sensing is an emerging research field based on the fact that a small number of linear measurements can recover a sparse signal without losing any useful information. Using compressive sensing, the signal can be recovered by a sampling rate that is much lower than the requirements from the well-known Shannon sampling theory. The proposed compressive mobile sensing approach has not only the merits of compressive sensing, but also the flexibility of different sampling densities for areas of different interests. A special measurement process makes it different from normal compressive sensing. Adopting importance sampling, compressive mobile sensing enables mobile sensors to move adaptively and acquire more samples from more important areas. A motion planning algorithm is designed based on the result of sparsity analysis to locate areas of more interests. At last, experimental results of 2-D mapping are presented as an implementation compressive mobile sensing.**

I. INTRODUCTION

Nowadays, mobile robotic sensors are playing an important role in sensor networks due to their locomotion capability [1], [2]. Unlike static sensors, which are deployed in advance with limited sensing capability, mobile sensors can move adaptively to specified areas of interests to acquire desired information under a controlling mechanism. This mobile robotic sensors are competent for quite a few fields, including environmental monitoring [3], surveillance [4], robotic mapping [5], etc. In this paper, an efficient robotic mapping method is proposed, compressive mobile sensing, which rebuilds a 2-D spatial map based on mobile sensors in unknown areas.

In robotic mapping, different areas of interests always require different mapping resolutions [2], that is, lower resolutions in simple-shaped areas, and higher resolutions for complicated areas, leading to different sampling densities. With the mobility and a careful designed motion planning, the proposed compressive mobile sensing approach enables mobile sensors to sample different areas with different sampling densities. They are flexible to collect more samples at areas of more interest, and less samples from areas of less interest. The difference in sampling densities takes best advantage of samples, so as to make the compressive mobile sensing more efficient to rebuilt a spatial map.

In addition to the motion flexibility, compressive mobile sensing also has merits of compressive sensing [6] on sampling and reconstructing information. Compressive sensing performs an under-sampling method to collect samples, unlike most Bayesian updating models, [7], [8], which lie on the prior information. Haupt [9] reconstructs sensing field without any prior knowledge using an emerging compressive sensing framework [10], [11]. The theory of compressive sensing demonstrates that a sparse signal can be recovered by a linear measurement process under some conditions by solving convex optimization problems, so that no prior information is needed in advance. Unlike the traditional "sample and process" way, this approach acquires signal in the condensed way directly.

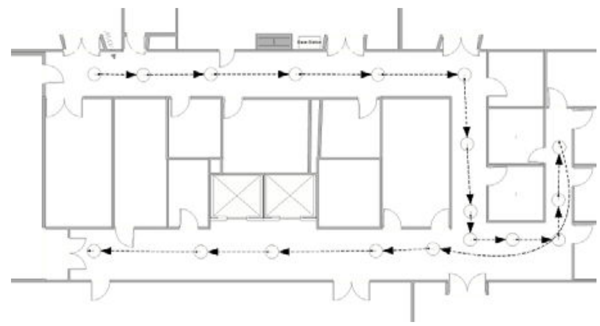

Fig. 1. A robotic mapping scenario using compressive mobile sensing: A mobile robot is driven to map the hallway on the 8th floor of the EERC Building at Michigan Tech. The circles represent sensing positions, and the dash lines with arrows show a possible trajectory of the mobile sensor.

Challenges of compressive mobile sensing include several respects. Since the proposed compressive mobile sensing is based on compressive sensing, sampling and reconstructing the spatial map is the first concern. In compressive sensing, signals are under sampled randomly and reconstructed by solving non-linear problems [12], [13]. In compressive mobile sensing, a special measurement model is established so that some important areas are emphasized by higher sampling densities. In this paper, an efficient method to sample the environment will be presented, where importance sampling is applied. Since encoders from mobile sensors cannot always provide accurate motion information due to some unexpected reasons, how to correct motion errors is another challenge, which is going to be addressed and solved in this paper.

Fig. 1 shows a typical application of compressive mobile sensing for robotic mapping. Adopting compressive mobile sensing approach, only a small amount of samples is required, and the map is reconstructed by solving non-linear problem. In this figure, corner and door areas are recognized as sparsity after a sparsity analysis, so they require higher sampling densities for better reconstructed result. A carefully designed motion planning is applied so that the mobile sensor can move adaptively to collect more samples

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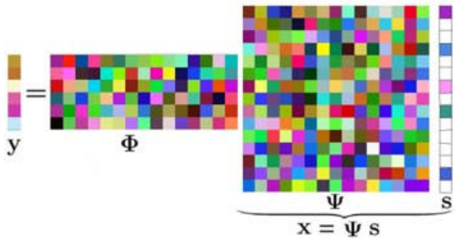


Fig. 2. Sparse representation of under sampling model: $y = \Phi x = \Phi\Psi s$.

(a) 1D signal.

(b) Target distribution.

(c) Even sampling density.

(d) Importance sampling.

(e) Compressive sensing.

(f) Compressive mobile sensing.

Fig. 3. One dimensional signal sampling and reconstruction.

from those areas. Finally, motion errors are corrected based on a correlation based method. In the following sections, compressive mobile sensing is illustrated in details, and an experimental result is shown at last.

II. COMPRESSIVE MOBILE SENSING FRAMEWORK

The proposed compressive mobile sensing provides an efficient way to sample and reconstruct signals. Compressive mobile sensing is developed based on compressive sensing, and enhanced by the importance sampling, so it outperforms compressive sensing. In compressive sensing, a signal is reconstructed from under-sampled data under a sparse domain. In compressive mobile sensing, additional samples are collected adaptively by adopting importance sampling. More samples are drawn from the non-zero entries in the sparse representation to improve the reconstruction, so that collected samples can be best used, and signals can be reconstructed efficiently.

In compressive mobile sensing, an under-sampling pattern is first established to collect samples. In this under-sampling process, signals are sampled at a much lower rate than the Nyquist frequency. Compressive sensing theory guarantees that signal can be recovered with the incomplete information. Fig. 2 shows the under-sampling pattern, where y is a measurement vector, and x is the original signal. A random measurement matrix Φ is chosen to project a signal from higher dimension to lower dimension. In Fig. 2, s is a sparse representation of x, where most entries are zero. Compressive sensing tells that s can be solved by solving a convex optimization problem. According to the linear relationship between x and s, the original signal x can be reconstructed. Up to here, we have exactly followed the steps of compressive sensing in sampling and reconstructing signals, [11].

The sparse representation s contains only a few non-zero entries, which contains most useful information, and more samples are collected adaptively from these areas in compressive mobile sensing, while in compressive sensing, signals are just under-sampled and reconstructed. We apply the importance sampling idea to gather more samples from these non-zeros entries. Importance sampling is developed in probabilistic field to draw samples according to the target distribution so that samples would finally converge to the target distribution. In compressive mobile sensing, importance sampling is applied in another way.

Note that the non-zero entries in the sparse signal s contain most useful information, and more samples from these areas would benefit the signal reconstruction, so it is very convincible to regard s as the target distribution, and non-zero entries as targets. Before applying importance sampling, we have two concerns. First, s is not any real signal that can be sampled directly. But from the linear relationship between x and s in Fig. 2, it is possible to sample x according to the sparsity of s. Second, a pre-requisite should be considered, how to acquire the sparsity of s. So a sparsity analysis is carefully designed in compressive mobile sensing. Once the sparsity information is acquired, importance sampling can be applied to guide mobile sensors to collect more sample from these sparsity locations.

What we have discussed above is focusing on 1-D situation, however, in a 2-D problem, which is always modeled as a matrix, the situation is much more complicated. The 2-D signal is vectorized into a long vector so that the model in Fig. 2 can be used. The sparsity of a 2-D signal is different from 1-D situation. It is not necessary for a 2-D signal to regard non-zero entries as sparsity like what we have discussed on 1-D case. Consider a 2-D indoor map in Fig. 1, the sparsity is defined in another way. Note that most areas are simple sharped like walls, only door and corner areas are relatively complicated. These door and corner areas are exactly areas requiring more samples, because a wall may be well described by only two points, but a corner deserve a lot more than that. So it is reasonable to define the sparsity as door and corner areas.

Compressive mobile sensing is formulated to sample and reconstruct a 1-D scenario shown in Fig. 3. In Fig. 3(a), most information concentrates at convex areas, so Fig. 3(b) is regarded as the target distribution. A normal sampling method samples the signal evenly in Fig. 3(c). However, it is not efficient, because most flat area in Fig. 3(b) contains useless information. According to importance sampling, the two convex areas deserve more samples to approximate the target distribution. Fig. 3(d) shows a sampling pattern with less samples from flat area and more sample from target areas.

Starting with compressive sensing, Fig. 3(e) shows an under-sampling pattern, where samples are thrown away at some random chosen areas. This condensed sampling way guarantees an efficient sampling way, but no emphasize on the target areas. Note that this 1-D signal in Fig. 3(a) appears sparse, so s and x in (2) are in the same domain. With the small amount of samples in compressive sensing, the sparsity analysis can be done to roughly locate targets. And more samples can be drawn adaptively from these areas, forming Fig. 3(f). In Fig. 3(f), flat areas are paid less attention with random samples, and target areas have been emphasized. In the method of compressive mobile sensing, samples are best used, that is, condensed samples with emphasize on important areas.

III. 2-D Spatial Map Reconstruction

In this section, compressive mobile sensing is discussed in details for 2-D map reconstruction. In the subsections followed, we will show why the compressive mobile sensing outperforms the normal compressive sensing in several aspects, including sampling and reconstructing signals, adaptive movement, etc. A mobile robot equipped with a laser scanner is used to implement compressive mobile sensing. The basic working process of compressive mobile sensing is shown in Algorithm. 1, in which each term is expanded into a corresponding subsection in the following parts.

Algorithm 1 Algorithm for mobile compressive sensing

A. Sampling and reconstructing signals
B. Sparsity analysis
C. Adaptive movement
D. Image Stitching
E. Repeat A, B, ,C and D until it is done

A. Sampling and Reconstruction

The first step of compressive mobile sensing is to apply compressive sensing to sample and reconstruct signals. In this subsection, compressive mobile sensing is compared with compressive sensing on how to sample and recover 2-D mapping signals. In compressive sensing, two domains of the signal to be reconstructed are involved, a sampling domain and a sparse domain. In the proposed compressive mobile sensing, sampling and recovering mapping signal is formulated in a similar way, however, the two domains, sampling and sparse domains, are combined into one domain in compressive mobile sensing.

An indoor environment is modeled as a 2D grid-based map, $\mathbf{m} = \left\{m_{x,y}\right\}$, where (x, y) denotes the coordinates of a grid and a binary variable $m_{x,y}$ denotes its occupancy, either the grid is occupied or it is free. We first adopt the normal approach of compressive sensing to sample and reconstruct the 2-D map, and then an improved method for sampling and reconstructing is discussed.

1) Reconstruction from Random Samples under Haar Wavelet Domain: The signal is sampled in spatial domain, and recovered under Haar Wavelet domain, under which most natural signals are sparse, including the spatial map signal. A random projection is involved to sample the signal in compressive sensing, so a random measurement matrix is generated as follows:

a). Vectorize $\mathbf{m}$ into a 1D vector, $\mathbf{m} = \{m_i\}$, $i = 1, 2, \ldots, N$, where N is the total number of points;
b). Draw M samples uniformly from $\mathbf{m}$ mutually excluded, denoted as $(s) = \{s_i = \mathbf{m}_{l_i}\}$, $i = 1, 2, \ldots, M$ where l_i is the ith samples index in $\mathbf{m}$;
c). Generate an $M \times N$ measurement matrix, Φ, where for each entry in Φ,

$$\begin{cases} \phi_{i,j} = 1, & j = l_i \\ \phi_{i,j} = 0. & j \neq l_i \end{cases} \tag{1}$$

Each of the measurements is a linear combination of the all elements in the sensing range with only one coefficient equal to one and others zero. The measurement matrix Φ is established by randomly sampling $\mathbf{m}$ under the spatial domain.

Once measurements are collected, a sparse basis should be selected, so that norm approach can be applied for recovery. Most natural signals are compressible under discrete wavelet basis, which describes the jumping rather the smoothness of the signal. Therefore, discrete Haar wavelet basis is selected as the sparse basis. Denote Ψ as the selected sparse basis, so $\mathbf{m}$ can be expressed under Ψ, that is $\mathbf{m} = \Psi\mathbf{z}$, where z is a sparse representation of $\mathbf{m}$. The compressive sensing measurements can be expressed by $\mathbf{y} = \Phi\mathbf{m} = \Phi\Psi^{-1}\mathbf{z}$, similar to Fig. 2. The sparse coefficient vector $\mathbf{z}$ is recovered from $\mathbf{y}$ by solving (2), where total variation(TV) minimization algorithm is applied to recover the grid-based map.

$$\hat{\mathbf{z}} = \arg\min_{\mathbf{z}} \|\mathbf{z}\|_{TV} \quad s.t. \quad y = \Phi\Psi^{-1}\mathbf{z} \tag{2}$$

Let $\|\mathbf{z}\|_{TV}$ be the TV norm of object $\mathbf{z}$. And the map signal can be solved by the linear projection between spatial domain and Haar Wavelet domain.

The aim to express the original signal in another domain is to make it sparse so that it can be reconstructed. However, we have some interesting observations against this domain transform. First, signal is sampled under a sparse domain. What we reconstruct is a binary map containing only 1 and 0 values. Each time the laser scanner is used to collect samples, there exists only one clear curve separating 1 value and 0 value parts of the map apart. Second, Haar Wavelet transition damages the sparsity actually. In spatial domain, the map contains only two values 1 and 0, and they have clear boundary. However, under Haar Wavelet domain, values of signal representation would vary in a relative big range, leading that when we consider the sparsity of signal, much information would be neglected. Moreover, when TV-norm method is applied to recover the map, which is to fill gaps of signals, the domain we are concerned is the spatial domain rather than the wavelet domain. Motivated by these observations, a unique sampling and reconstruction method is carefully designed in the following subsection.

2) Direct Reconstruction: 2-D TV-norm reflects the consistency of a figure. If a sparse basis is selected to transform the sensing data m into another domain for reconstruction, the 2-D TV-norm would account for consistency of the signal under that sparse domain rather than the original signal under spatial domain. Therefore, the signal should be reconstructed directly by minimizing the 2-D TV-norm of m in (3) to achieve better reconstruction performance.

$$\hat{\mathbf{x}} = arg\min_{\mathbf{x}} \|\mathbf{x}\|_{tv} \quad s.t. \quad \mathbf{y} = \Phi_1\mathbf{x} \tag{3}$$

where the measurement matrix Φ_1 is also different from the random measurement matrix in Equ. (1).

Note that the mobile sensor used to rebuild the spatial map is laser scanner, which is a quite special sensor that senses the environment by shooting a bunch of laser rays from the sensor with equal angle interval. So random samples are not easy to be covered even in its sensing range. A mobile sensor has to adjust several times to cover all the random preselected samples in its sensing range at a certain sensing position. That is a waste of time and energy. So we carefully design the measurement matrix according to the special type of sensor. For a sensing position, samples are drawn in the corresponding grids along the laser rays, so Φ_1 is designed to specify the star-shaped sampling pattern as follows:

a). Vectorize $\mathbf{m}$ into a 1D vector, $\mathbf{m} = \{m_i\}$, $i = 1, 2, \ldots, N$, where N is the total number of points;

b). Draw samples (grids covered) along rays emitted from the sensor, denote N as the total sample number, and project them in the 1D vector;

c). Generate an $M \times N$ measurement matrix, Φ_1 according to the sample locations and projections similar to Φ.

In description above, sampling and reconstructing are different from those in the normal compressive sensing method. Only one domain is selected in order to take better advantage of TV-norm method; a particular sampling pattern is used rather than random projection to suit to a particular sensor. So in compressive mobile sensing, a more efficient sampling and reconstructing process is applied.

B. Sparsity Analysis for Importance Sampling

After signals are sampled and reconstructed, more samples are about to be drawn to emphasize important areas. For a 1-D sparse signal, non-zeros entries contain most useful information. If more samples are collected from these areas, the signal would be better recovered. This is similar to the idea of importance sampling; the only difference is that one involves real signals, while the other is in probabilistic field. In the following paragraphs, we are focusing on how to define and analyze the sparsity of a 2-D mapping problem, and how to apply the probabilistic method, importance sampling, to real maps.

1) Sparsity Analysis: It is easy to define the sparsity of a 1-D signal as the non-zeros entries. However, the sparsity of a 2-D map is expressed in another way. For an indoor environment, most areas are smooth like walls, and some areas contain doors or at corner are relatively complicated. We denote these relatively complicated areas as the sparsity of the 2-D signal, which require more attention. However, the sparsity would be different from the point of view of a mobile sensor. Some areas, usually door or corner areas, would be blocked from mobile sensor at different sensing positions, and some other door or corner areas may be sensed very well. So the sparsity is redefined as blocked areas from mobile sensors.

Different methods can be applied to analyze the sparsity due to different types of mobile sensors used. One general way is to analyze the map contour. The contour of a map would change smoothly if the mapping information is collected properly. Sudden changes in a contour indicates something is missing, because mobile sensor is blocked by some objects in the map. So the sparsity features sudden changes in this 2-D mapping problem.

Wavelet transform is an effective method for edge detection, so it is proper to be applied to detect sudden changes. Equ. (4) is the wavelet transform for continuous functions, where a and b are shift and scale parameters separately in wavelet transform, $f(t)$ and $W_\psi(f(t))$ are the original function and wavelet transform based on the wavelet function $\psi(t)$. In our approach, the contour map can be expanded into a 1-D signal, so a discrete wavelet transform is used by simply changing the continuous form described in (4) to discrete form (5), i.e. changing the integral to sum, where $\psi(\theta)$ is defined in (6).

$$W_\psi(f(t)) = W(a, b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^*_{a,b} \left(\frac{t - b}{a} \right) dt \tag{4}$$

$$W_\psi(r(\theta)) = W(a, b) = \sum_\theta r(\theta) \psi_{a,b} \left(\frac{\theta - b}{a} \right) \tag{5}$$

$$\psi(\theta) = \begin{cases} -1, & \theta = 1 \\ 1, & \theta = 2 \\ 0. & \text{otherwise} \end{cases} \tag{6}$$

After this transform, the sparsity can be roughly located, which are denoted as important areas requiring more samples. Once the important areas are located, mobile robot moves there for more samples to approximate the target distribution in importance sampling.

2) Importance Sampling: Importance sampling is the theoretical basis to make compressive mobile sensing adaptive. In importance sampling, samples are drawn according to the target distribution so that sampled values can approximate target accurately. In our algorithm, a bunch of many samples in the sensing range of mobile sensor are drawn simultaneously each time according to the sparsity analysis from one of the so-called sparse areas, and this process repeat until no more samples are needed. In this sequential process, a bunch of sensed data is regarded as one sample in normal approach of importance sampling.

In Equ. 7, m_i denotes a set of samples collected each time, and $q(m)$ and $p(m)$ are sampling and target distribution separately. If $q(m) = p(m)$, which means the sampling distribution is the same as target distribution, samples are drawn according to target distribution corresponding to importance sampling. From the right term of Equ. 7, it can be seen that each sample has even weight. Since the target distribution is not even, different areas have been paid different attention, i.e. important areas have been emphasized.

$$\int mq(m)dm = \int m \left(\frac{q(m)}{p(m)} \right) p(m)dm = \frac{1}{n} \sum_{i=1}^{n} m_i \left(\frac{q(m_i)}{p(m_i)} \right) \tag{7}$$

The 2-D mapping has been simplified to a 1-D edge detection problem, in which only the map contour is concerned, when analyzing the sparsity. The sparsity exactly reflects areas we want to apply importance sampling. After new samples are collected, the 2-D spatial map is updated, and then a new round sparsity analysis and importance sampling would be applied. Since mobile sensors are being discussed, a motion planning to collect more samples at desired locations is required, which is designed in next section.

C. Motion Planning

Every sensor has limited sensing range, including the mobile sensor used to implement compressive mobile sensing method. When a mobile sensor is deployed in a indoor environment to reconstruct the 2-D spatial map, it can sample and rebuild a certain part of the whole map at each sensing position, and it also detects areas out of the sensing range, which would return some certain special values, like maximum. Denote $E_i = \{e^i_j, j = 1, 2, \cdots\}$ as the set of these areas out of the sensing range in the whole recovered map at sensing position p_i. Denote $L^k_i = \{l^i_k, k = 1, 2, \cdots\}$ as the sparsity defined in last section for position p_i. These two sets are kept updated to E_{i+1} and L_{i+1} as long as the 2-D map are updated.

For an indoor environment like Fig. 1, we propose a straight line moving strategy, trying to avoid unnecessary movement, and the mobile sensor makes turns at proper positions to cover some sparsity areas. Our basic moving algorithm for mobile sensor is to move to cover all the important areas L_i and then move to the out-of-range areas E_i for more exploration.

A virtual tree structure is established, and the first sensing is regarded as the root level. Suppose the mobile sensor is initially deployed at one known end of the indoor environment to collect mapping data, and E_1 and L_1 are generated. Each element in E_1 or L_1 would generate a new branch. An e_j is chosen, and mobile sensor move along a straight from the sensing position to the e_j area. When the robot is moving along the straight line, it may pass by some l_j. The motion planning requires the mobile sensor to take a turn to face a l_j when passing by. If this l_j is a dead end, the mobile sensor turns back to continue the straight line movement until the e_j is achieved; if not, a new branch will expand like the root level, and the mobile sensor would return to the root of this branch after this branch is explored. At a certain level, if no e_j is available, mobile sensor goes along the current direction to cover l_j as the same as above. Each element of L_i and E_i will generate a new branch, and the same moving strategy algorithm would apply to the new branch. Once the algorithm is done, the 2-D map is finished.

A special case should be paid more attention. The tree structure may be damaged, if a loop occurs, which is entirely possible to happen in a real map. Compared with the existing map, it can be detected whether the mobile moves to somewhere it has been. If so, the current branch would stop, and the mobile sensor returns to the branch root. The moving strategy would go as shown in Algorithm. 2 for each level in the tree structure.

Algorithm 2 Moving strategy

```
i = current
while E_i ∪ L_i ≠ ∅ do
  if Loop Detected then
    End Branch ; Update E_i&L_i
    Continue
  end if
  if Current Branch is Done then
    Return to Branch Root
    Continue
  end if
  if E_i ≠ ∅ then
    Go Straight to e_i^j and Cover Elements in L_i
    Update E_i&L_i
    Generate New Branch
  else
    Go Straight and Cover Elements in L_i
    Update E_i&L_i
    Generate New Branch
  end if
end while
```

D. Reconstructed Map Stitching

As the mobile sensor moves, pieces of reconstructed maps are generated, however, motion errors are involved. In this section, a stitching methods is used to correct motion errors, so that pieces of maps can be merged together.

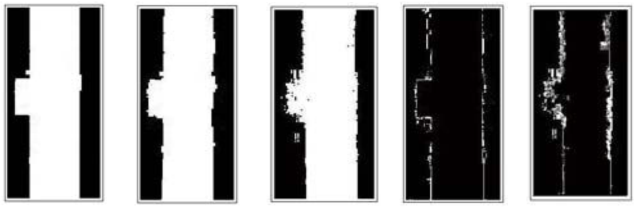
Encoders of a mobile robot do not always provide accurate distance and angle information due to uneven wheels, slippy floors, or some enforced movement. If maps are stitched according to the position information returned from robot, a blent or even massing map would show. We stitch pieces of maps together based on the encoder feedback, and correct motion errors using a correlation-based method [14].

Each time we have a new piece of map recovered, by comparing the overlap part between the new piece and the existing map, maps can be merged together. Define Ω as the whole existing map, and $\omega(\alpha,\beta,\theta)$ as a new piece of map, where α, β, and θ are the coordinates and orientation parameters from robot, telling where this piece should be in the whole map Ω. Let $\omega(\alpha + \Delta\alpha, \beta + \Delta\beta, \theta + \Delta\theta)$ be the corrected new piece, and S be the overlap part between $\omega(\alpha+\Delta\alpha, \beta+\Delta\beta, \theta+\Delta\theta)$ and Ω. The corrected error is obtained by

$$(\Delta a, \Delta b, \Delta\theta) = arg\min \sum_i (\Omega_i - \omega(a+\Delta a, b+\Delta b, \theta+\Delta\theta)_i)^2 \quad , i \in S \tag{8}$$

where i indicates every grid of the overlap part.

In summary, compressive mobile sensing performs as a sequential process to move adaptively, sample, reconstruct, and stitch spatial maps. Once all the maps are stitched together, the 2-D mapping for the indoor environment is done.



(a) (b) (c) (d) (e)

Fig. 4. Reconstructed results comparison. 4(a): original map. 4(b): direct reconstruction. 4(c): reconstruction under Haar Wavelet domain. 4(d): difference between 4(a) and 4(b). 4(e): difference between 4(a) and 4(c).

IV. EXPERIMENTAL RESULTS

In this section, the compressive mobile sensing is implemented in experiment. A Pioneer III mobile robot equipped with a LMS200 laser scanner is used to sample and reconstruct the indoor environment shown in Fig. 1. The laser scanner is configured to cover 180° for each scan with 181 readings and the sensing range is set to $8000mm$. The whole map is divided into many small grids corresponding to $62.5mm \times 62.5mm$ each. The final map is acquired by stitching them together.

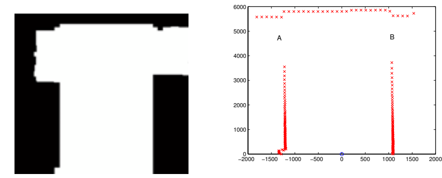
Two sensing and reconstructing methods are compared. Fig. 4(a) is the original map of a certain part in Fig. 1. Fig. 4(c) shows the reconstruction result from random samples under Haar Wavelet domain, while Fig. 4(b) shows recovered map from laser samples in star-sharp under spatial domain by 2-D TV-norm method. Fig. 4(d) and Fig. 4(e) shows the difference between the original one and reconstructed results in Fig. 4(b) and Fig. 4(c). Compared with Fig. 4(e), Fig. 4(d) shows a clearer map and less noise, indicating direct reconstruction in Fig.4(b) outperforms.

We also provide a set of typical experimental results of sparsity analysis in Fig. 5. Suppose one set of 181 laser readings is $r(\theta)$, $\theta = 1, 2, 3 \ldots 181$ in which the radius r is a function of angle θ. Fig. 5(a) shows a piece of real map at one corner of the indoor environment. Fig. 5(b) is the laser readings of Fig. 5(a). Fig. 5(c) illustrates the discrete function $r(\theta)$ of laser returns in Fig. 5(b). A Wavelet approach in Equ. 5 is applied to the sensing data for sparsity analysis, and Fig. 5(d) shows the the sparsity clearly.

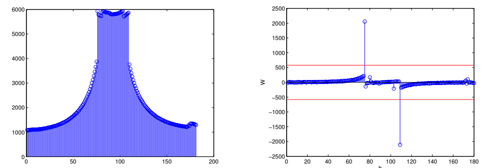
At last, a 2-D map is shown reconstructed by compressive mobile sensing. Fig. 6 shows the reconstructed map with about 27% samples. The important locations, including door areas and corners, are determined by sparsity analysis, and the mobile sensor is guided to move there and sense more accordingly. Compared with the original map in Fig. 1, most information has been retained by this small amount of samples, and the reconstructed map is good enough to represent the original one.

V. CONCLUSION

We have proposed compressive mobile sensing approach for robotic mapping, which use much fewer samples to reconstruct signals. Compared with compressive sensing, it features a more efficient way in sampling and reconstructing desired signals adaptively and sequentially. An unique measurement model to emphasize important areas is established, and motion errors are corrected. Based on a mobile platform, compressive mobile sensing technique guarantees an automatic map reconstruction. It proves to be an efficient approach in robotic mapping.



(a) Original map. (b) Red x: 181 laser readings. Blue o: laser scanner.



(c) Radius from Fig. 5(b). (d) Wavelet transform.

Fig. 5. Discrete wavelet transform.

Fig. 6. Rebuilt map.

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