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An Automated Method to Calibrate Industrial Robot Joint Offset Using Virtual Line-based Single-point Constraint Approach

Yong Liu, Ning Xi, George Zhang, Xiongzi Li, Heping Chen, Chi Zhang, Michael J. Jeffery and Thomas A. Fuhlbrigge

Abstract - **This paper describes an industrial robot joint offset calibration method called the virtual line-based single-point constraint approach. Previous methods such as using CMM, laser trackers or cameras are limited by the cost or the resolution. The proposed method relies mainly upon a laser pointer attached on the end-effector and single position-sensitive detector (PSD) arbitrarily located on the workcell. The automated calibration procedure (about three minutes) involves aiming the laser lines loaded by the robot towards the center of the PSD surface from various robot positions and orientations. The intersections of each pair of laser lines eventually should converge to the same point after compensating the joint offsets. An optimization model and algorithm have been formulated to identify the robot offset. For the highly precise feedback, a segmented PSD with a position resolution of better than 0.1 µm is employed. The mean accuracy of robot localization is up to 0.02** mm**, and the mean error of the parameter identification is less than 0.08 degrees. Both simulations and experiments implemented on an ABB industrial robot verify the feasibility of the proposed method and demonstrated the effectiveness of the developed calibration system. The goal of fast, automated, low-cost, and high precision offset calibration are achieved.**

Index Terms – **robot, offset calibration, virtual lines-based, single-point constraint**

I. INTRODUCTION

Researchers have been working to improve the accuracy of industrial robots. Though accuracy is not necessary in some point to point (PTP) applications, such as spot welding or pick and place, since a sequence of points were programmed by teach pendant and replay of these points relied only on repeatability. However, with the industrial robot widely used in the complicated tasks, eg., arc welding, offline programming and surgery etc., accuracy of the robot is more and more important.

Although there are many sources of inaccuracy, such as gear errors, thermal expansion and structural deformations, the main source of the inaccuracy lies in the parameter errors of robot kinematics model. Robot calibration is an efficient way to improve the accuracy. There has been considerable research in this field. Robot kinematics parameters calibration methodologies and systems have been developed [1]-[5]. One kind of method requires highly precise equipment measuring the robot end-effector pose, eg., coordinate measurement machines(CMM) [1] and laser tracking system [2]. However the process is time/manpower consuming and the device is expensive. Hence a fast, low-cost, and precise calibration system is essentially needed.

The other kind of method imposes some constraints on the end effector to form closed kinematic chains. Some researchers imposed physical contact constraints (either multi-plane constraints or a plane constraint [6][7][8]) on the end-effector. These methods suffer from inexact positioning and time consuming. Newman et al. [9] and Chen et al. [10] proposed a calibration method using laser line tracking. This approach relies upon constraining the point on the end-effector moving along a stationary laser beam. However, it is difficult to exactly and automatically fit the line constraint. Gatla et al. [11] described the virtual closed kinematic chain method. A laser tool attached to the end-effector aimed at two arbitrary but fixed points on some objects in order to create a virtual closed kinematics chain. The simulation results showed that the robot kinematics parameters should be calibrated. However, feedback system is only simulated using Simulink, and it suffers from the similar problems as the plane constraint method if laser aims at the points manually. Moreover, the system is complex and may be not accurate if two cameras are used for positioning.

In addition, after a calibration procedure using the previous methods in the robot factory, the kinematics parameter values are identified and thus forward and inverse kinematics model will use the updated values instead of the design values in order to improve the accuracy. Once the robot is shipped from the robot manufacture and installed for the user, some kinematics parameters, such as the link length, link twist and link offset, related to the mechanical structure of the robot itself, do not change too much, typically. However, some kinematics parameters such as joint offset might be changed

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more often because of the assembly or the replaced motors and encoders. What is more, the joint offset change only a little, then the positional accuracy is affected significantly. Joint offset refers to the error of joint home reference between the kninematic model and the encoder of the real robot. According to [9][10], more than 90% of the positional inaccuracy issues of the industrial robot are caused by the robot offset.

Thus it is essential to develop a practical industrial robot offset calibration device that can be used widely and frequently in the user factory, not only in the robot factory. That is to say, an offset calibration system that is fast, automated and highly precise, most important, more low-cost will have high demand in manufacturing using industrial robots. Although previous methods can be used to calibrate the robot offset, they are either very expensive or time-consuming to be used in the user factory.

A new parameter calibration approach called virtual lines-based single-point constraint (VLBSPC) is proposed and implemented to fit this requirement. Unlike previous calibration methods, this approach does not need any physical contact and the developed device is affordable, what is more, the calibration process is automated. The proposed method depends mainly on a laser pointer attached on the end-effector of a robot and only one position-sensitive detector (PSD). The coordinates of the PSD on the workcell are unknown. The automated calibration procedure (about three minutes) involves aiming the laser beams loaded by robot towards the center of the PSD surface from various robot positions and orientations. Once the precise positioning is done by PSD-based servo, all the laser lines will shoot on the same point at a very small range of error and a set of robot joint angles will be recorded. Based on the recorded joint angle and forward kinematics, a joint angle offset estimation method has been developed. Obviously if offset values of all joints are zero, the intersections of every pair laser-lines computed from the recorded joint angle and forward kinematics are the same point. However, if offset values of all joints are not zero, the intersections of every pair laser-lines are different points. In one word, the distribution of the intersections depends on the robot offset. An optimization model and algorithm have been formulated to identify the robot constant offset. A Levenberg-Marquardt (LM) algorithm is applied to solve the optimization problem and obtain the solution. For the purpose of precise positioning on the same point, the segmented PSD is employed for the high precision feedback with a resolution of better than 0.1 μm and a PSD-based controller is designed and implemented. Both simulation and real experiments implemented on an ABB industrial robot (IRB1600) verified the effectiveness of both the proposed method and the developed system. The price of one PSD is less than 70 dollars. This system fits the need for easy to set-up, totally automated, low-cost, and high precision robot offset calibration.

This paper is structured as follows: the calibration system is presented in Section II. The methodology of offset calibration is described in Section III. The simulation and experimental results are demonstrated in Section IV. Finally, we conclude the work.

II. CALIBRATION SYSTEM

A. Calibration Device

The schematic of robot offset calibration system is shown in Fig. 1. Accordingly the developed calibration device consists of laser and laser adapter, PSD and PSD fixture, signal process circuit board, and the PC-based controller, as shown in Fig. 2(a) and Fig. 2(b). In Fig. 2(b) there are sixteen PSDs mounted on the four planes of the fixture. The PSD fixture can be used for other calibration application such as calibration of the robot base frame and the workcell frame [12]. The calibration device is implemented and verified on an ABB robot as laboratory test-bed, Shown in Fig. 3, which includes ABB robot controller (IRC5) and 6-DOF manipulator (IRB1600).

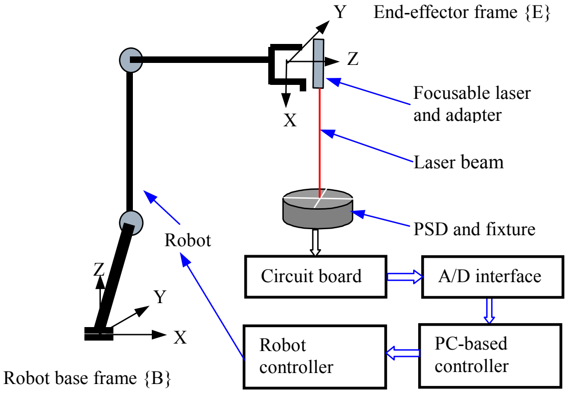


Fig.1. Schematic of offset calibration system

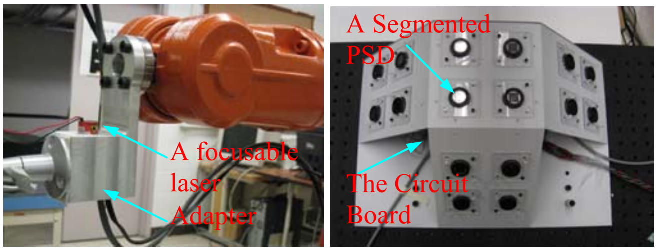
A focusable laser pointer with its adapter is fixed and rigidly attached on the end-effector of the robot. The laser beam is adjusted to align its orientation toward the X-axis of the end-effector frame. The robot loads the laser to shoot a beam onto the surface of the PSD. Once the laser pointer and the adapter is fixed, the laser line in the end-effector frame is given by

$$\frac{x_E - x_{0E}}{m_E} = \frac{y_E - y_{0E}}{n_E} = \frac{z_E - z_{0E}}{p_E} \tag{1}$$

where $\left(x_{0E}, y_{0E}, z_{0E}\right)$ is the position of one point of the laser line in the end-effector fame and $\left(m_E, n_E, p_E\right)$ is the unit vector of the laser line orientation in the end-effector fame.

The segmented PSD is employed and mounted on the fixture. The fixture is arbitrarily located on the workcell. The center point of the PSD is supposed to be the single-point constraint. The interface circuit is well designed and the signal tuning board can process the raw output of the laser spot on the PSD surface for two-dimension position feedback. PCI-DAS6025 is used to acquire the analogy signal from the processing board.

Through the network-based communication between robot controller and the computer, the PC-based controller can obtain the current robot position information and send the control command to the robot controller as well as update the target position in real-time, and thus control the robot manipulator for exact positioning.



(a) (b)

(a) The laser and adapter attached on the end-effector; (b) The segmented PSD mounted on the fixture

Fig.2. Developed calibration device

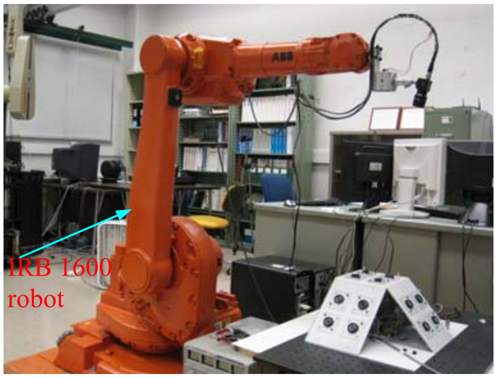


Fig.3. ABB robot test bed and calibration system

B. *Segmented PSD*

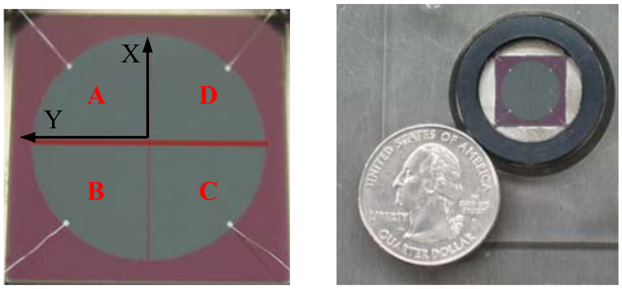
In general, there are two types of PSD. One is the lateral-effect PSD and the other is the segmented PSD, which is employed in the system. The active area and the coordinate frame are shown in Fig. 4(a). The sensor has excellent position resolution of better than 0.1μm and it is very suitable for the application such as machine tool alignment, position measurement and beam centering etc. The active surface consists of four separate areas (named A, B, C, D). The photo of PSD is given in Fig. 4(b).

The output voltage with respect to the laser beam positions on the active surface of segmented PSD can be expressed as:

$$U_X = K_X \frac{(U_A + U_D) - (U_B + U_C)}{U_A + U_B + U_C + U_D} \tag{2}$$

$$U_Y = K_Y \frac{(U_A + U_B) - (U_C + U_D)}{U_A + U_B + U_C + U_D} \tag{3}$$

where U_X : output voltage of relative beam position on X-axis [V]. U_Y : output voltage of relative beam position on Y-axis [V]. U_A, U_B, U_C, U_D : Voltage transferred from the four areas (seen in the Fig. 4a). K_X and K_Y are the gains for unit transformation from voltage to millimeter on X-axis and Y-axis, respectively.



(a) Active area of segmented PSD (b) The photo of the PSD

Fig.4. The segmented PSD

III. CALIBRATION METHODOLOGY

A. *Kinematics Error Model*

The Denavit-Hartenberg [13] is a widely used convention for frame of reference in the forward kinematics. A model of the IRB1600 robot according to DH conventions is built as shown in Fig. 5. Six coordinate frames from frame {1} to frame {6} of the system are defined, respectively. In the DH convention, each homogeneous transformation is represented as,

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where a_i, α_i, d_i and θ_i are generally named as link length, link twist, link offset, and joint angle, respectively [14]. $c\theta$ notates $\cos\theta$ and $s\theta$ notates $\sin\theta$.

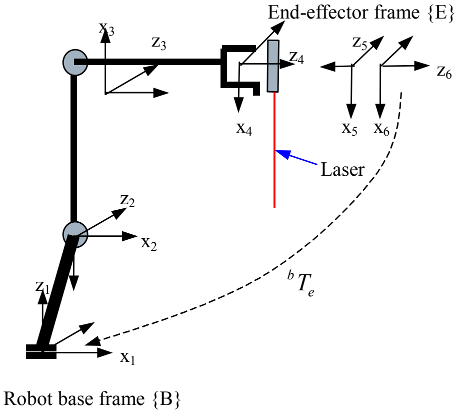


Fig.5. The D-H model

Consider the joint offset, let δ_i denote the offset value of the *i*th joint, Equation (4) is rewritten as,

$$\tilde{T}_i = \begin{bmatrix} c\tilde{\theta}_i & -s\tilde{\theta}_i c\alpha_i & s\tilde{\theta}_i s\alpha_i & a_i c\tilde{\theta}_i \\ s\tilde{\theta}_i & c\tilde{\theta}_i c\alpha_i & -c\tilde{\theta}_i s\alpha_i & a_i s\tilde{\theta}_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

where use notation $c\tilde{\theta}$ for $cos(\theta_i + \delta_i)$ and $s\tilde{\theta}$ for $\sin(\theta_i + \delta_i)$.

Combining the joint offset and the six coordinate frames, forward kinematics ${}^bT_e$ with the offset error is written as,

$$ {}^bT_e = \tilde{T}_1 \tilde{T}_2 \tilde{T}_3 \tilde{T}_4 \tilde{T}_5 \tilde{T}_6 \ . \tag{6} $$

B. *Offset Calibration*

A new parameter calibration approach called the VLBSPC, was developed to calibrate the joint offsets. The proposed method relies mainly upon a laser pointer attached on the end-effector of a robot and single position-sensitive detector (PSD). The calibration procedure, as shown in Fig. 6, is performed by shooting a laser beam from the laser pointer on the same point from various positions and orientations. The same point is the center point of the PSD and the coordinates of the point in the robot base frame are unknown. It is guaranteed that the laser beams shoot on the same point because the robot loads the laser pointer aiming the laser beam on the center of the PSD by PSD-based feedback and servo. Sets of robot joint angles are recorded during the localization. Substituting the recorded joint angle into the forward kinematics with offset error (Equation (6)), the homogeneous transformations of end-effector fame with regard to the robot base frame are given by

$$ \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{7} $$

Note the unknown parameters are the joint offset in the Equation (7).

Combing the Equation (1) and Equation (7), one of the laser lines translated from end-effector frame to robot base frame is described by

$$ \frac{x_B - x_{iB}}{m_{iB}} = \frac{y_B - y_{iB}}{n_{iB}} = \frac{z_B - z_{iB}}{p_{iB}} \tag{8} $$

where (x_{iB}, y_{iB}, z_{iB}) are the coordinates of one point of the laser line in the robot base fame and (m_{iB}, n_{iB}, p_{iB}) is the unit vector of the laser line direction in the robot base fame.

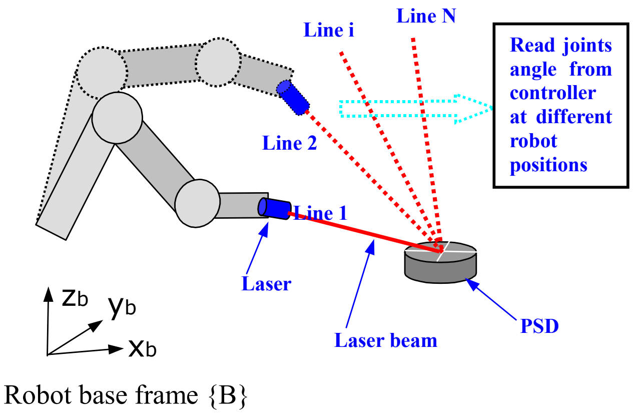


Fig.6. Schematic of new calibration method

Suppose N sets of joint angle are recorded after calibration. From Equation (8) N laser lines are obtained. Let Γ_{Li} denote the *i*th laser line, P_k denote the intersection or the center of the shortest distance between Γ_{Li} and Γ_{Lj} ($i \neq j, i, j \in N, k \in M$), and ${}^n\mathrm{P}_{Ave}$ denote the mean point of the total intersections P_k ($k = 1, \cdots, M$). The coordinate errors of the points between P_k and ${}^n\mathrm{P}_{Ave}$ are denoted as ${}^x\Psi_k, {}^y\Psi_k, {}^z\Psi_k$ in the x, y, z directions, respectively. The parameters δ of joint offset are identified by minimizing the total sum of the squares of the coordinate errors.

$$ \delta^* = \arg Min \sum_{k=1}^{M} \left({}^x\psi_k{}^2 + {}^y\psi_k{}^2 + {}^k\psi_k{}^2 \right) \tag{9} $$

where M is the number of the intersections between laser lines. Note ${}^n\mathrm{P}_{Ave}$ is updated during the minimization iteration process and P_k is the center of the line of the shortest distance from the lines between Γ_{Li} and Γ_{Lj} if the two lines do not have a real intersection.

C. *Minimization*

The method for the non-linear optimization is iterative algorithm. For this non-linear square problem, the Levenberg-Marquardt algorithm (LMA) [16, 17] is referenced and integrated by C++ code. The algorithm finds the minimization quickly, mostly after less than 10 iterations. The optimum algorithm is a damped Gauss-Newton method based on the Jacobian J and damping parameter μ. The step h_{lm} is defined by

$$ \left(J^T J + \mu I\right) h_{lm} = -g \tag{10} $$

where $g = J^T \Psi$ and $\mu \geq 0$.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. *Simulation Results with Precise Data*

The 3-D Virtual Reality (VR) of the industrial robotic manipulator was built in the computer. The DH parameters of the robot were used as the factory design, shown in Table I. The laser pointer was fixed on the end-effector toward the X-axis of the end-effector frame, as in the experimental design. A virtual PSD was built as a feedback to exactly locate the laser beam on the center of the PSD surface.

Simulation of the offset calibration was performed using precise joint angles and without regard to the robot inaccuracy. The experimental process was performed by locating laser beam on the same point at seven different positions and orientations.

The optimization process stopped due to small step size (the threshold is 10^{-6}) after four iterations. The residual error of the minimization was $0.0\,mm^2$. The result of calibration with perfect data is shown in Table II. Column 2 shows the actual offset parameters used by the simulation. Column 3 shows the initial parameters for the LMA. Column 4 shows the solution of the optimization and Column 5 shows the mean error by repeating the experiments. The result shows that solution was perfect. In theory it verified the effectiveness of the proposed method.

TABLE I
DH PARAMETERS OF THE ABB IRB1600 MANIPULATOR

Joints	$a(mm)$	$\alpha(rad)$	$d(mm)$	$\theta(rad)$
1	150	$-\pi/2$	486.5	0
2	700	0	0	$-\pi/2$
3	0	$\pi/2$	0	π
4	0	$-\pi/2$	600	0
5	0	$\pi/2$	0	0
6	0	0	0	0

TABLE II
SIMULATION RESULTS WITH PRECISE DATA (UNIT: DEGREE)

Parameters	Actual Value	Initial Values	Result	Mean Error
δ_2	1.20	0.0	1.20	0.0
δ_3	0.80	0.0	0.80	0.0
δ_4	-1.40	0.0	-1.40	0.0
δ_5	-0.60	0.0	-0.60	0.0
δ_6	-0.8	0.0	-0.8	0.0

B. *Simulation Results with Noisy Data*

However, a real robot joint has limited resolution and also is limited to accuracy of the robot localization even though the precise PSD-based feedback system is used. Therefore, we add noise to the joint and the localization in order to make the simulation more realistic.

TABLE III
SIMULATION RESULTS WITH NOISY DATA (UNIT: DEGREE)

Parameters	Actual Value	Initial Values	Result	Mean Error
δ_2	1.20	0.0	1.2015	0.021
δ_3	0.80	0.0	0.7884	0.042
δ_4	-1.40	0.0	-1.4389	0.020
δ_5	-0.60	0.0	-0.6329	0.025
δ_6	-1.0	0.0	-1.0087	0.011

Table III shows the result of the offset calibration with noisy data. The optimization process stopped due to small step size after six iterations. The residual error of the minimization was 0.046 mm^2 . The results show that the offset parameters were close to the actual parameters when the localization error is ±0.05 mm . However it depends on the level of noise.

The experiments were repeated with different noise levels. The results show that the algorithm can find the desired parameters if the localization accuracy is better than 0.08 mm . Both the accuracy of the robot repeatability (0.01 mm) and the resolution (0.1µm) of the PSD are far better than the requirement. The results are very useful for the designed experimental system and justified the proposed method feasibility.

C. *Experimental Results of IRB1600 Robot*

The calibration device was developed as descrbed in section II. The offset calibration experiment was implemented on the ABB manipulator IRB1600. The process of PSD-based robot localization is shown in Fig. 7. The actual offset parameters were calibrated from manufacture by the laser tracking method. The method of average value filtering (ten points) is applied to obtain the posiotion feedback from PSD sensor during the localization process.

Generally the whole experimental process takes couples of seconds. The mean error of positioning is less than 0.02 mm (it is much less than the industrial robot absolute accuracy), and the standard deviation is about 0.16mm. It is limited to the repeatability accuracy of the industrial robot. The experiment was repeated five times. Experimental results show the process of the PSD-based control is fast, stable and the localization is precise.

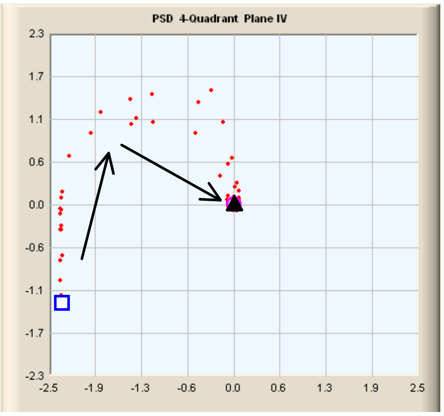


Fig.7. The localization of the laser beam on the center of PSD surface based on segmented PSD feedback (□: Initial position; ○: Desired position; ▲: Target position; ○ and ▲ overlay in the above diagram; Unit: mm)

TABLE IV
OFFSET CALIBRATION RESULTS OF IRB1600 ROBOT (UNIT: DEGREE)

Parameters	Actual Value	Initial Values	Result	Mean Error
δ_2	1.1	0.0	1.1834	0.062
δ_3	0.1	0.0	0.1221	0.051
δ_4	0.1	0.0	0.0798	0.026
δ_5	0.0	0.0	0.0414	0.038
δ_6	0.0	0.0	0.0353	0.026

Table IV shows one of the results of the offset calibration experiment implemented on the ABB manipulator IRB1600.

The results show that the offset parameters obtained from the optimum solution were close to the actual parameters. The mean error was small that verified the calibration method was feasible and demonstrated the calibration system was stable.

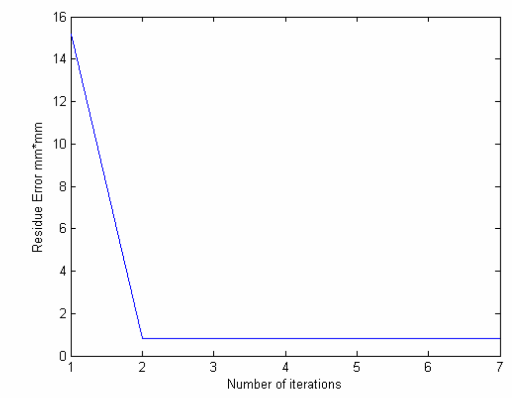


Fig.8. The residue error with the each iteration

The Fig. 8 shows the minimization residual error plotted with the number of iterations. The iterations stop after seven iterations. The error decreases in each iteration and the the offset parameters converge to the desired values with residue error of 0.807 mm^2.

Accuracy of robot localization is limited to the feedback error, control error, and robot accuracy. The resolution in the joint space is improved because the errors in joint space are magnified in PSD plane. However, the effect is related to the robot configuration. In addition, sensitivities of variation of object function to the variation of joint angle play a key role on the accuracy and efficiency of the solution, especially in the noise condition. The sensitivities also rely on the robot configuration and PSD position. These problems will be discussed in detail in the further research.

V. CONCLUSIONS

Robot calibration plays a significant role in the robot accuracy needs of the current complicated manufacturing processes. Robot joint offset has a much larger influence on robot positioning accuracy after leaving the robot factory for the end user. To address this issue, a virtual lines-based single-point constraint approach and well-developed offset calibration system for industrial robots were presented in this paper. Using a laser pointer attached on the end-effector of a robot and one position-sensitive detector (PSD), the calibration process aims a laser towards the center of the PSD surface from various robot positions and orientations. A PD controller for positioning has been designed based PSD feedback. The system has been shown to be fast, automated, and precise. The mean accuracy (about 0.02 mm) of robot localization fits the need of the calibration method. Both simulation and experimental results verify the feasibility of the proposed method and demonstrate the developed system can fit the need of offset calibration for the industrial robot user. The goal of achieving a fast, automated, low-cost, and precise robot offset calibration system is approaching. The work on letting the calibration equipment potable and wireless will be fulfilled. Further applications using the method for calibration of other kinematic parameters are ongoing.

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