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Improved and Modified Geometric Formulation of POE Based Kinematic Calibration of Serial Robots

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Abstract— **We propose in this paper an improved geometric formulation of POE (Product Of Exponential) based kinematic calibration of serial robots, which is based on the work of [1]. We use both joint offset-free formulation and adjoint transformation errors of joint screws, and apply it to the calibration of an elbow manipulator. Our formulation explains why the original POE calibration always fails with the existence of joint offset errors; the adjoint formulation of joint screw errors eliminates joint screw constraints that was imposed in the original iterated least square calibration algorithm. The second contribution of this paper is the proposal of a modified POE formulation which adopts point measurement data instead of frame measurement data of the end-effector, which can be more realistic and convenient for practical implementation. Simulation results show that the proposed method is plausible and effective. An experiment is under preparation to verify the effectiveness of the proposed calibration method on an elbow manipulator built by Googol Technology.**

I. INTRODUCTION

Due to existence of manufacturing error, assembly error, and mechanical wear, the kinematics model of a serial robot will always deviate away from its nominal one, which creates problems in planning and control of the robot. For example, pure rotation or pure translation joint axis of a serial robot may have direction and/or position error, which can be modeled by the change of its joint screw coordinate; imperfect installation of joint encoders will also introduce errors in the reading of joint angles, known as the joint offset errors. A kinematic calibration is usually needed and implemented after the manipulator's assembly as an effective means to improve manipulator accuracy. Alternatively, kinematic calibration can be avoided through improved level of precision manufacturing, but at a rather higher cost. Judging from the scale of industrial robot production, kinematic calibration excels over quality control in terms of cost-effectiveness.

Kinematic calibration is a relatively old problem, which can often be categorized into two methods. The first one requires implementation of extra (redundant) sensors (self-calibration or auto-calibration)[2], [3], [4] or imposition of mechanical constraints of the end-effector or links[5]. Kinematic parameters and joint encoder offsets can be calibrated using the additional sensing information and constraint equations. The second one utilizes an external measuring device to measure poses of the end-effector. Kinematic parameters and joint offsets are identified via the manipulator's direct and/or inverse kinematics. Thus the first method is often referred to as implicit or intrinsic method, which utilizes the input-to-state geometric model of the manipulator; while the second one is often referred to as external method, which is based on input-to-output geometric model of the manipulator in discussion. A good overview of calibration formulations can be found in [6].

In this paper, we focus on the external calibration method for two major reasons: first, it is relatively difficult to add/assemble extra sensors to an assembled machine, which could also introduce additional error variables and complexify the design of calibration algorithm; second, recent development in computer vision has been widely applied to kinematic calibration for its preferably low cost and flexibility. A vision based measuring system could be setup within hours. Moreover, the auto-calibration of on-hand camera is also a relatively mature technique.

In [1], a method for kinematic calibration of open chain mechanisms based on the POE formula was presented. To be specific, the forward kinematics (input-output geometric model) of robots is modeled by POE, taking into consideration variation of geometric parameters. According to [1], unlike kinematic representations based on the Denavit-Hartenberg (D-H) parameters, kinematic parameters in the POE formula vary smoothly with changes of the joint screw coordinates. However, the geometric formulation and calibration algorithm proposed in [1] has several problems as we first implemented them ourselves: when the joint offset errors are to be calibrated alongside joint screw errors, the calibration result is not consistent with prescribed errors (the original paper [1] suggested that the calibration algorithm will also work under existence of joint offset errors, however the only example shown in that paper is one without joint offset errors). Several other issues also arise which brought our attention to a possible improvement of the underlying

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geometric formulation. For example, it is possible to use adjoint transformation to model the joint screw coordinate errors, which relieves of using quadratic constraints in the least square calibration algorithm, which turns the calibration problem into an unconstrained optimization problem. This is a considerable improvement no matter what the specific optimization algorithm is (aside from the iterated linear least square algorithm, it is also possible to use nonlinear optimization algorithms and/or any other equation solving algorithms). In addition, the practicability of measuring end-effector's frame transformation is doubtful. We reformulate the POE equation so that only point measurements are needed in the calibration process.

The paper is organized as follows: in Section II, we review the mathematical formulation of POE based calibration; in Section III, the improved geometric formulation is proposed and its advantages over the original are explained; in Section IV, the modified geometric formulation is proposed to further accommodate point measurements instead of frame measurements of the end-effector; in Section IV, we construct a simulation that apply both the improved and the modified formulation to the calibration of an elbow manipulator; at last, we draw several conclusion concerning the effectiveness of our calibration method.

II. Geometric formulation of POE based calibration

In this section, the geometric formulation of POE based kinematic calibration for serial manipulators is summarized. An improved and modified formulation is proposed accordingly.

A. *POE formulation of robot forward kinematics*

Given a n degree-of-freedom (DoF) non-redundant serial manipulator with only revolute or prismatic joints, its joint axis information is reflected by its corresponding joint twist (screw) coordinates $\{\xi_i\}_{i=1}^n$, which are elements of the Lie algebra $se(3)$ of the special Euclidean group $SE(3)$[7]. Recall that if ξ is a constant twist, the rigid motion associated with it, with respect to reference frame s and end-effector tool frame t,is given by:

$$g_{st} : \Gamma \mapsto SE(3), g_{st}(\theta) = e^{\hat{\xi}\theta} g_{st}(0) \tag{1}$$

where θ is the angle rotated about ξ. If ξ corresponds to a prismatic (infinite pitch) joint, then $\theta \in \mathbb{R}$ is the amount of translation along ξ; otherwise, $\theta \in S^1$ measures the angle of rotation about the rotational axis defined by ξ. Using the POE (Product of Exponential) formulation (or so called the zero reference formulation [8], or recently the active transformation [9]), the forward kinematics map for an open-chain manipulator with n degrees of freedom can be easily found

$$f : \Gamma^n \mapsto SE(3), f(\theta) = e^{\hat{\xi}_1\theta_1} \cdots e^{\hat{\xi}_n\theta_n} g_{st}(0) \tag{2}$$

(2) is known as the POE (product of exponential) formula [7], see Figure 1. Note that here $e^{\hat{\xi}_1\theta_1} \cdots e^{\hat{\xi}_6\theta_6}$ is the part of rigid body motion generated by the serial manipulator with

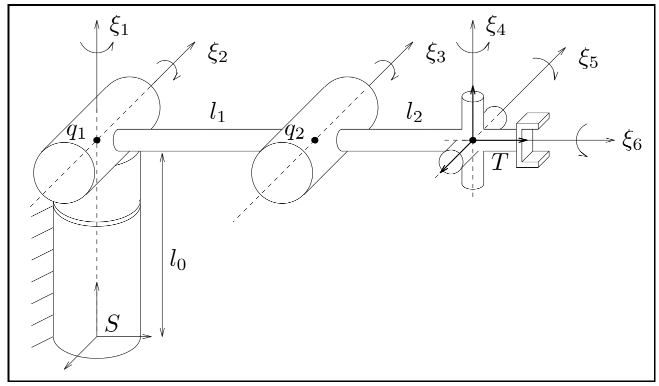


Fig. 1. Geometry of the elbow manipulator (Courtesy of Zexiang Li)

respect to the reference frame s, while $g_{st}(0)$ is the part of coordinate transformation of t with respect to s which does not count as rigid body motion. Both rigid body motion and initial coordinate transformation need be calibrated to produce an accurate output of a point on the end-effector.

B. *POE based calibration* [1]

Now that the forward kinematics of a serial manipulator is given by POE formula (2), we are also interested in the dependence of (2) on joint screw coordinates, then we have:

$$\begin{aligned} &f : \Gamma^n \times se(3)^n \mapsto SE(3), \\ &f(\theta_1, \ldots, \theta_n, \xi_1, \ldots, \xi_n) = e^{\hat{\xi}_1\theta_1} \cdots e^{\hat{\xi}_n\theta_n} M \end{aligned} \tag{3}$$

where $M = g_{st}(0)$. Using the exponential mapping, M is written as $M = e^{\Upsilon}$ for some constant $\Gamma \in se(3)$. Then the right pull back of the total differential of f is calculated by:

$$\begin{aligned} &\mathrm{d}f \cdot f^{-1} = \\ &\mathrm{d}(e^{\hat{\xi}_1\theta_1})e^{-\hat{\xi}_1\theta_1} + e^{\hat{\xi}_1\theta_1}\mathrm{d}(e^{\hat{\xi}_2\theta_2})e^{-\hat{\xi}_2\theta_2}e^{-\hat{\xi}_1\theta_1} + \\ &\cdots + e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots \mathrm{d}(e^{\hat{\xi}_n\theta_n})\cdot \\ &e^{-\hat{\xi}_n\theta_n}\cdots e^{-\hat{\xi}_2\theta_2}e^{-\hat{\xi}_1\theta_1} + e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_n\theta_n}\mathrm{d}M\cdot \\ &M^{-1}e^{-\hat{\xi}_n\theta_n}\cdots e^{-\hat{\xi}_2\theta_2}e^{-\hat{\xi}_1\theta_1} \end{aligned} \tag{4}$$

Each term $\mathrm{d}(e^{\hat{\xi}_i\theta_i})e^{-\hat{\xi}_i\theta_i}$in (4) can be expanded as:

$$\begin{aligned} \mathrm{d}(e^{\hat{\xi}_i\theta_i})e^{-\hat{\xi}_i\theta_i} &= \int_0^1 e^{\hat{\xi}_i\theta_i s}\mathrm{d}\hat{\xi}_i\theta_i e^{-\hat{\xi}_i\theta_i s}\mathrm{d}s \\ &+ \int_0^1 e^{\hat{\xi}_i\theta_i s}\hat{\xi}_i d\theta_i e^{-\hat{\xi}_i\theta_i s}\mathrm{d}s \\ &= \theta_i \int_0^1 e^{\hat{\xi}_i\theta_i s}\mathrm{d}\hat{\xi}_i e^{-\hat{\xi}_i\theta_i s}\mathrm{d}s + \hat{\xi}_i\mathrm{d}\theta_i \end{aligned} \tag{5}$$

Similarly, $\mathrm{d}M \cdot M^{-1}$in (4) becomes

$$\mathrm{d}M \cdot M^{-1} = \mathrm{d}(e^{\Upsilon})e^{-\Upsilon} = \int_0^1 e^{\Upsilon s}\mathrm{d}\Upsilon e^{-\Upsilon s}\mathrm{d}s$$

So a more compact form of (4) can be rewritten as:

$$\begin{aligned}
&\mathrm{d}f\cdot f^{-1}=\hat{\xi}_1\mathrm{d}\theta_1+Ad_{e^{\hat{\xi}_1\theta_1}}(\hat{\xi}_2)\mathrm{d}\theta_2\\
&+\cdots+Ad_{e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{n-1}\theta_{n-1}}}(\hat{\xi}_n)\mathrm{d}\theta_n\\
&+\theta_1\int_0^1 Ad_{e^{\hat{\xi}_1\theta_1 s}}(\mathrm{d}\hat{\xi}_1)\mathrm{d}s\\
&+\theta_2 Ad_{e^{\hat{\xi}_1\theta_1}}(\int_0^1 Ad_{e^{\hat{\xi}_2\theta_2 s}}(\mathrm{d}\hat{\xi}_2)\mathrm{d}s)\\
&+\cdots+\theta_n Ad_{e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{n-1}\theta_{n-1}}}\\
&\cdot(\int_0^1 Ad_{e^{\hat{\xi}_n\theta_n s}}(\mathrm{d}\hat{\xi}_n)\mathrm{d}s)\\
&+Ad_{e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_n\theta_n}}(\int_0^1 Ad_{e^{\Upsilon s}}(\mathrm{d}\Upsilon)\mathrm{d}s)
\end{aligned}\tag{6}$$

where Ad is the Adjoint transformation [7]. We collect the above linearized equation into a matrix form of $\mathbf{Ap}=\mathbf{y}$, where $\mathbf{p}\in\mathbb{R}^{7n+6}$ is the kinematic parameter vector

$$\mathbf{p}=[\mathrm{d}\theta_1\cdots\mathrm{d}\theta_n\ \mathrm{d}\xi_1^T\cdots\mathrm{d}\xi_n^T\ \mathrm{d}\Upsilon^T]^T\tag{7}$$

and $\mathbf{y}$ is the screw coordinate of $\mathrm{d}f\cdot f^{-1}$, or using the $\vee$ operator in [7], $\mathbf{y}=(\mathrm{d}f\cdot f^{-1})^{\vee}$.

We make the following notations,

$$e^{\hat{\xi}_i\theta_i}=\begin{bmatrix}R_i & p_i\\ 0 & 1\end{bmatrix},e^{\hat{\xi}_i\theta_i s}=\begin{bmatrix}R_i(s) & p_i(s)\\ 0 & 1\end{bmatrix}\tag{8a}$$

$$e^{\Upsilon}=\begin{bmatrix}R_M & p_M\\ 0 & 1\end{bmatrix},e^{\Upsilon s}=\begin{bmatrix}R_M(s) & p_M(s)\\ 0 & 1\end{bmatrix}\tag{8b}$$

$$\xi_i=[\mathrm{d}v_i^T\ \mathrm{d}\omega_i^T]^T\tag{8c}$$

$$r_k=\left(\prod_{i=0}^{k-1}\begin{bmatrix}R_i & \hat{p}_iR_i\\ 0 & R_i\end{bmatrix}\right)\begin{bmatrix}v_k\\ w_k\end{bmatrix}\tag{8d}$$

$$\begin{aligned}Q_k=&\left(\prod_{i=0}^{k-1}\begin{bmatrix}R_i & \hat{p}_iR_i\\ 0 & R_i\end{bmatrix}\right)x_k\\
&\cdot\int_0^1\begin{bmatrix}R_k(s) & \hat{p_k}(s)R_k(s)\\ 0 & R_k(s)\end{bmatrix}\mathrm{d}s\end{aligned}\tag{8e}$$

$$\begin{aligned}Q_M=&\left(\prod_{i=0}^{n}\begin{bmatrix}R_i & \hat{p}_iR_i\\ 0 & R_i\end{bmatrix}\right)x_k\\
&\cdot\int_0^1\begin{bmatrix}R_M(s) & \hat{p_M}(s)R_k(s)\\ 0 & R_k(s)\end{bmatrix}\mathrm{d}s\end{aligned}\tag{8f}$$

with $e^{\hat{\xi}_0\theta_0}$ defined to be the identity matrix. Then (6) can be expressed as :

$$\mathbf{y}=[r_1\cdots r_n\ Q_1\cdots Q_n\ Q_M]\mathbf{p}\triangleq\mathbf{Ap}\tag{9}$$

The original least square calibration is carried out in the following way: various instances of configurations of the manipulator in the workspace are measured. For each instance i, the error vectors $\mathbf{y}$ and Jacobian $\mathbf{A}$ are denoted by y_i and A_i, which shall be solved in a least square fashion. Collect all instances of (9) into a matrix form and we have:

$$y=\begin{bmatrix}y_1\\ \vdots\\ y_m\end{bmatrix}=\begin{bmatrix}A_1\\ \vdots\\ A_m\end{bmatrix}p$$

which we would like to denote by:

$$\mathscr{Y}=\mathscr{A}\mathbf{p}\tag{10}$$

So that the POE calibration problem is equivalent to iterations of the minimization problem of the least square error function

$$J(\mathbf{p})=\|\mathscr{A}p-\mathscr{Y}\|^2$$

subject to joint screw constraints. Each revolute joint screw $\hat{\xi}_i$ must satisfy the constraint $w_i^Tv_i=0$, $\|w_i\|=1$ while for prismatic joints only v_i need be identified and ω_i is always kept zero and thus left out of the calibration.

III. An improved geometric formulation for POE calibration

A. Joint offset-free calibration

When using the original POE formulation, we find out that joint offset errors can not be identified very well. In fact, there will always be some error in the joint offsets and joint screw errors. However it works well with joint offset-free assumption.

The following analysis can be made to explain such deficiency. Assume there exists no joint screw errors in the POE formula, but there exists joint offset errors for each joint. However, equivalently, we can ignore the joint offsets and regard them as deviations in the joint screw axis, such as shown in Figure 2. Denote the nominal value of a variable

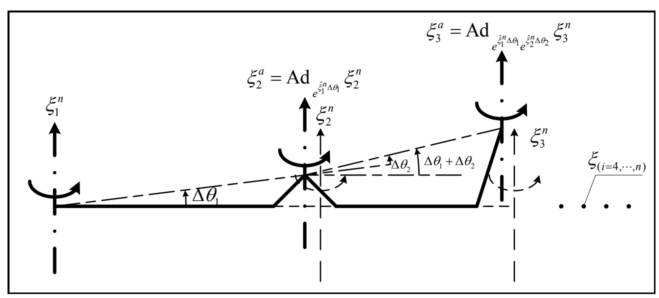


Fig. 2. Equivalent joint screw error due to joint offset errors

(either joint screw or joint variable) x by x^n and its actual value by x^a, then the screw motion of the i^{th} joint is:

$$e^{\hat{\xi}_i^n\theta_i^a}=e^{\hat{\xi}_i^n(\theta_i^n+\Delta\theta_i^n)}=e^{\hat{\xi}_i^n\theta_i^n}e^{\hat{\xi}_i^n\Delta\theta_i^n}\tag{11}$$

So the actual forward kinematic equation (2) becomes:

$$\begin{aligned}&f(\theta_1^a,\theta_2^a,....,\theta_n^a)\\
&=e^{\hat{\xi}_1^n(\theta_1^n+\Delta\theta_1^n)}\cdots e^{\hat{\xi}_n^n(\theta_n^n+\Delta\theta_n^n)}g_{st}^n(0)\\
&=e^{\hat{\xi}_1^n\theta_1^n}e^{\hat{\xi}_1^n\Delta\theta_1^n}\cdots e^{\hat{\xi}_n^n\theta_n^n}e^{\hat{\xi}_n^n\Delta\theta_n^n}g_{st}^n(0)\end{aligned}\tag{12}$$

Thus this equation can be simplified using the fact that if $M\in SE(3)$, then $M^{-1}e^pM=e^{M^{-1}pM}$. By repeatedly applying the identity $Me^p=e^{MpM^{-1}}M$, f can be rewritten as:

$$\begin{aligned}&f(\theta_1^a,\theta_2^a,\ldots,\theta_n^a)\\
&=e^{\hat{\xi}_1^a\theta_1^a}\cdots e^{\hat{\xi}_n^a\theta_n^a}\cdot e^{\hat{\xi}_1^n\Delta\theta_1^n}\cdots e^{\hat{\xi}_n^n\Delta\theta_n^n}g_{st}^n(0)\end{aligned}\tag{13}$$

where

$$
\begin{aligned}
&\hat{\xi}_1^a = \hat{\xi}_1^n \\
&\hat{\xi}_2^a = e^{\hat{\xi}_1^n \Delta\theta_1^n} \hat{\xi}_2^n e^{-\hat{\xi}_1^n \Delta\theta_1^n} = (Ad_{e^{\hat{\xi}_1^n \Delta\theta_1^n}} \xi_2^n)^\wedge \\
&\cdots \\
&\hat{\xi}_n^a = (Ad_{e^{\hat{\xi}_1^n \Delta\theta_1^n} \cdots e^{\hat{\xi}_{n-1}^n \Delta\theta_{n-1}^n}} \xi_6^n)^\wedge
\end{aligned} \tag{14}
$$

If we denote $e^{\hat{\xi}_1^n \Delta\theta_1^n} \cdots e^{\hat{\xi}_6^n \Delta\theta_6^n} g_{st}^n(0)$ by $g_{st}^a(0)$, the forward kinematic equation equals:

$$f(\theta_1^a, \ldots, \theta_6^a) = e^{\hat{\xi}_1^a \theta_1^n} \cdots e^{\hat{\xi}_6^a \theta_6^n} g_{st}^a(0) \tag{15}$$

hence showing the equivalence of joint offset error to adjoint errors of joint screws. In other words, a set of joint offset errors $\Delta\theta_i^n$ can always be separated into two parts $\Delta\theta_i^{n_1} + \Delta\theta_i^{n_2}$ so that $\Delta\theta_i^{n_1}$ is identified with the new joint offset errors while $\Delta\theta_i^{n_2}$ is identified with joint screw errors. Considering the infinite possibilities of $\Delta\theta_i^{n_1} + \Delta\theta_i^{n_2}$, the following conclusion can be drawn.

Proposition 1: In the above POE calibration settings, joint offset errors are not identifiable.

Proof: Trivial. ■

Yet once all joint offset errors are identified with certain joint screw errors, the gross joint screw errors thus caused can be identified by POE calibration. Thus the first contribution of this paper is the conclusion that there is no need to consider joint offset errors at all!

B. Eliminating the joint screw constraint

In the previous least-squares algorithm, after every iteration, the updated screw coordinate for each joint will no longer satisfy the joint screw constraints. Since linear least square optimization with quadratic equality constraints are difficult to solve analytically if not impossible [10], it is suggested that the normalization of $\|\omega_i\| = 1$ and orthogonalization of $v_i^T \cdot \omega_i = 0$ be taken right after each iteration of the least square algorithm. Such solution, though practically plausible, is not mathematically rigorous.

The second contribution of this paper is to o eliminate the joint screw constraints before the iterated linear least square algorithm take place, transforming the problem into an unconstrained one.

Intuitively, if the axis screw of any joint in a robot manipulator has some deviation, we can regard the offsets as a small rigid body motion. In robotics, the rigid transformation of joint screws is known as the Adjoint transformation [7], which is a special type of linear transformation on screw coordinates. Based on the notion of Adjoint transformation, we propose a geometrically meaningful formulation for the joint screw errors.

As show in Figure 3, the i^{th} axis (denoted by solid lines) is deviated from its nominal value (denoted by dashed lines). For an joint screw ξ_n going through small deviation, we can formulate its deviation by the Adjoint transformation $\xi^a = Ad_{e^{\hat{\eta}}} \cdot \xi^n$. If we denote $e^{\hat{\eta}}$ and ξ^n by:

$$e^{\hat{\eta}} = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}, \xi^n = \begin{bmatrix} v^n \\ \omega^n \end{bmatrix} \tag{16}$$

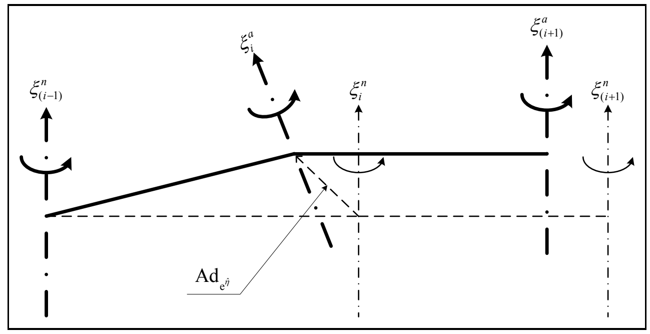


Fig. 3. Illustration of joint screw errors

Then

$$Ad_{e^{\hat{\eta}}} \xi^n = \begin{bmatrix} Rv^n + \hat{P}R\omega^n \\ R\omega^n \end{bmatrix}$$

Apparently, $\|R\omega^n\| = \|\omega^n\|$, $(R\omega^n)^T(Rv^n + \hat{P}R\omega^n) = 0$: after small rigid-body motion the joint screw ξ^n changes to a new screw $Ad_{e^{\hat{\eta}}} \xi^n$, which still satisfies joint screw constraints of the same type. If we regard the deviation of the axis as a small rigid motion, then the calibration task becomes to identify the Adjoint error η_i's instead of $\Delta\xi_i$'s. Infinitesimally,

$$e^{\hat{\eta}} \hat{\xi}^n e^{-\hat{\eta}} \approx \hat{\xi}^n + [\hat{\eta}, \hat{\xi}^n] \tag{17}$$

This is the equivalent form to the original update $\hat{\xi}^n + \Delta\hat{\xi}^n$, which means that the update $\Delta\hat{\xi}^n$ should in principle not be chosen arbitrarily but should be chosen as the Lie bracket $[\hat{\eta}, \hat{\xi}^n]$.

For the initial configuration error M, we simply apply a rigid motion of some error screw: Υ.

$$M^a = e^{\hat{\Upsilon}} M$$

Then the derivation of spatial residue $\mathrm{d}f \cdot f^{-1}$ boils down to derivation of $\mathrm{d}\xi_i = \mathrm{d}(Ad_{e^{\hat{\eta}_i}} \xi_i^n)$:

$$
\begin{aligned}
\mathrm{d}\hat{\xi}_i &= \mathrm{d}(Ad_{e^{\hat{\eta}_i}} \xi_i^n)^\wedge = \mathrm{d}(e^{\hat{\eta}_i} \hat{\xi}_i^n e^{-\hat{\eta}_i}) \\
&= \mathrm{d}(e^{\hat{\eta}_i}) e^{-\hat{\eta}_i} e^{\hat{\eta}_i} \hat{\xi}_i^n e^{-\hat{\eta}_i} + e^{\hat{\eta}_i} \hat{\xi}_i^n e^{-\hat{\eta}_i} e^{\hat{\eta}_i} \mathrm{d}(e^{-\hat{\eta}_i}) \\
&= \left(\left(\int_0^1 Ad_{e^{\hat{\eta}_i s}} \right) \mathrm{d}\eta_i \right)^\wedge (Ad_{e^{\hat{\eta}_i}} \xi_i^n)^\wedge \\
&\quad - (Ad_{e^{\hat{\eta}_i}} \xi_i^n)^\wedge \left(\left(\int_0^1 Ad_{e^{\hat{\eta}_i s}} \right) \mathrm{d}\eta_i \right)^\wedge \\
&= - \left[(Ad_{e^{\hat{\eta}_i}} \xi_i^n)^\wedge, \left(\left(\int_0^1 Ad_{e^{\hat{\eta}_i s}} \right) \mathrm{d}\eta_i \right)^\wedge \right]
\end{aligned}
$$

$\Longrightarrow$

$$\mathrm{d}\xi_i = -ad(Ad_{e^{\hat{\eta}_i}} \xi_i^n) \left(\int_0^1 Ad_{e^{\hat{\eta}_i s}} ds \right) \mathrm{d}\eta_i \tag{18}$$

Then the least square algorithm is formulated using $\mathrm{d}\eta_i$'s instead of $\mathrm{d}\xi_i$'s, and hence have achieved elimination of the joint screw constraints. From calculus's viewpoint, (18) is just a change of coordinate.

IV. MODIFIED GEOMETRIC FORMULATION OF POE CALIBRATION

In this section, we consider the calibration problem of a serial manipulator with only end-effector point measurements. Let P_c be a point on the end-effector, its position is calculated by:

$$\begin{aligned} P_c &= e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots e^{\hat{\xi}_n\theta_n}P_{c0} \\ &= f\cdot P_{c0} \end{aligned} \tag{19}$$

Where P_{c0} is its initial position with respect to the reference frame s. Take the differential on both sides of (19):

$$\begin{aligned} \mathrm{d}P_c &= \mathrm{d}f\cdot P_{c0} + f\cdot \mathrm{d}P_{c0} \\ &= (\mathrm{d}f\cdot f^{-1})fP_{c0} + f\cdot\mathrm{d}P_{c0} \\ &= (\mathrm{d}f\cdot f^{-1})P_c + f\cdot \mathrm{d}P_{c0} \end{aligned} \tag{20}$$

where $\mathrm{d}P_c \in \mathbb{R}^3$ is the deviation of point measurement P_c from its nominal value. Note that for the first term $(\mathrm{d}f\cdot f^{-1})fP_{c0}$, $\mathrm{d}\eta_i$ cannot be extracted on the right. So the original POE calibration algorithm cannot be directly applied. Now denote

$$s = \begin{bmatrix} \hat{w} & v \\ 0 & 0 \end{bmatrix} \in se(3) \tag{21}$$

and

$$P_c = \begin{bmatrix} P \\ 1 \end{bmatrix}_{4\times 1} \tag{22}$$

We observe that

$$\begin{aligned} sP_c &= \begin{bmatrix} \hat{w} & v \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P \\ 1 \end{bmatrix} = \begin{bmatrix} \hat{w}P+v \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} I & -\hat{P} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix} \\ &= \begin{bmatrix} I & -\hat{P} \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix} \end{aligned}$$

Since $\mathrm{d}f\cdot f^{-1}$ also in a twist form, (20) becomes:

$$\mathrm{d}P_c = [I \;\; -\hat{P}_c](\mathrm{d}f\cdot f^{-1})^{\wedge} + f\cdot\mathrm{d}P_{c0} \tag{23}$$

Combining (18) with 23 and we have:

$$\begin{aligned} &(Q_i d\xi_i)^{\wedge}P_c = \left[I \;\; -\hat{P}\right]Q_i\mathrm{d}\xi_i \\ &= -\left[I \;\; -\hat{P}\right]Q_i\cdot ad(Ad_{e^{\hat{\eta}_i}}\xi_i^n)\left(\int_0^1 Ad_{e^{\hat{\eta}_i s}}ds\right)\mathrm{d}\eta_i \end{aligned} \tag{24}$$

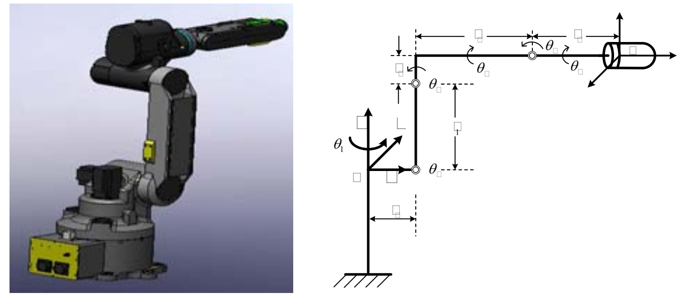
At the same time, suppose

$$e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots e^{\hat{\xi}_n\theta_n} = \begin{bmatrix} R_f & b_f \\ 0 & 1 \end{bmatrix}$$

$$dP_{c0} = \begin{bmatrix} dP_{c0} \\ 0 \end{bmatrix}$$

So

$$f\mathrm{d}P_{c0} = R_f\mathrm{d}P_{c0} \tag{25}$$



(a) (b)

Fig. 4. The elbow robot used in simulation: (a) a CAD model; (b) a schematic.

Parameters	l_0	150 mm
	l_1	570 mm
	l_2	150 mm
	l_3	634 mm
	l_4	50 mm
Joint reach	Joint 1	(-180,+180) deg
	Joint 2	(-105,+175) deg
	Joint 3	(-235,+85) deg
	Joint 4	(-180,+180) deg
	Joint 5	(-40,+220) deg
	Joint 6	(-360,+360) deg

TABLE I
KINEMATICS PARAMETERS AND JOINT REACHES OF AN ELBOW ROBOT

Then (20) becomes:

$$d\mathbf{P_c} = (df\cdot f^{-1})P_c + fdP_{c0} = \tilde{\mathbf{A}}\tilde{\mathbf{p}} \tag{26}$$

where

$$\begin{aligned} \tilde{Q}_i &= -[I \;\; -\hat{P}]Q_i\cdot ad(Ad_{e^{\hat{\eta}_i}}\xi_i^n)\left(\int_0^1 Ad_{e^{\hat{\eta}_i s}}\mathrm{d}s\right)\mathrm{d}\eta_i \\ \tilde{\mathbf{A}} &= \left[\; \tilde{Q}_1|\; \tilde{Q}_2| \;\cdots\; |\tilde{Q}_n|R_f \;\right] \\ \tilde{\mathbf{p}} &= \left[\; \mathrm{d}\eta_1{}^T \cdots \mathrm{d}\eta_n{}^T \;\mathrm{d}P_{c0} \;\right] \end{aligned}$$

V. SIMULATION RESULT

In the absence of calibration experiment, we verify the improved and modified geometric formulation for POE calibration by a simulation of an elbow robot. Fig. 4 shows a CAD model and a schematic of the robot. In Table I, its kinematic parameters and joint reaches are presented. The nominal and identified screw information is shown in Table II.

Two simulation were carried out. The first one is performed based on the complete information of end-effector posture and the second one is based on its position information. After 6 iterations, the kinematic parameters are successfully identified. In the second simulation, 10 iterations are required to achieve the same accuracy. The number of iteration nearly doubles for the position information case. This is predictable since only position information is used.

We also want to emphasize that the exponential mapping is only locally one-to-one. So when dealing with the initial position screw $\xi_{p_{c0}}$, there may exist more than one solution.

	Normal parameters	Identified parameters
ξ_1	(0 0 0 0 0 1)	(0.0222 -0.0144 0.0009 0.0194 -0.0317 0.9993)
ξ_2	(0 0 150 0 1 0)	(0.1764 2.8933 151.2188 0.0504 0.9985 -0.0192)
ξ_3	(570 0 -150 0 -1 0)	(583.8102 14.9619 -154.0245 0.0307 -0.9993 0.0195)
ξ_4	(0 -720 0 -1 0 0)	(60.3165 -690.8776 1.7673 -0.9958 -0.0870 -0.0290)
ξ_5	(-720 0 784 0 1 0)	(-733.2256 -29.1784 798.2115 -0.0143 0.9996 0.0234)
ξ_6	(0 -720 0 -1 0 0)	(-17.1142 -702.2271 -0.9022 -0.9984 0.0244 -0.0517)
$\xi_{p_{c0}}$	(834 0 720 0 0 0)	(839.0100 -5.0300 724.9740 0 0 0)

TABLE II

NOMINAL AND IDENTIFIED PARAMETERS FOR GOOGOL TECH ELBOW MANIPULATOR

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed an improved and modified geometric formulation of POE-based kinematic calibration of robot manipulators. Both joint offset errors and joint screw constraints are eliminated. This reduces the calibration problem to an unconstrained sequential linear least-square optimization problem. It is shown that the joint offset errors are equivalent to a set of joint screw errors. Hence they can be ignored without any danger. It is also shown that the joint screw error can be modeled using adjoint transformation by small magnitude screw motion.

In the modified formulation, Instead of measuring *complete* posture (position and orientation) of the end-effector with respect to a world reference frame, only position measurement is needed for calibration. This not only simplifies experiment implementation, but also alleviates requirement on measuring equipment since it is not an easy problem to measure orientations of an manipulator. By the modified formulation, the original constrained calibration problem immediately becomes an unconstrained optimization problem. The formulation is no more complex than the original one. In both cases, the speed of convergence is impressive, due to the robustness of sequential linear least-square algorithm.

The end-effector point measurement can be achieved by either a contacting coordinate measuring machine (CMM) or a non-contact vision based measuring system. A vision-based measuring system is flexible and relatively economical. In order to verify the improved geometric formulation, a vision-based calibration system is being set up to calibrate a 6-DoF serial robot.

VII. ACKNOWLEDGMENTS

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