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Re-design of Force Redundant Parallel Mechanisms by Introducing Kinematical Redundancy

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Abstract— **Force redundant mechanism is adopted to achieve high acceleration. However, utilizing the good velocity control performance of servo motor to achieve a high positional accuracy will arise excessive big internal forces. Therefore, an idea of introducing kinematical redundancy into force redundant parallel mechanism is proposed. In this idea, we do not add kinematical redundancy on the sub arm but introduce internal redundant motion into the top plate. The concept of this research is illustrated by improving a simple force redundant parallel example, and the merits of the improved mechanism are summarized. This theory is applied into modifying the real high speed parallel mechanism NINJA. Its kinematics and static force relationship are derived and compared respectively. The prototype of the new mechanism is proposed.**

I. INTRODUCTION

We aimed at designing a fast robot that achieves an acceleration performance over 100 [G] together with high precision. High speed and high precision positioning mechanisms are required in many applications, for example, for bonding semiconductor-tip terminals with a lead line. This procedure demands high speed and accuracy in a narrow workspace. Researches related to high speed motion and high precision can be found in [1][2][3][4].

Parallel mechanism have many excellent characteristics in terms of speed, accuracy, stiffness, and load carrying capacity compared to their serial counterparts [5][6]. Some of high speed mechanisms using the advantages of parallel manipulators, such as DELTA[7], HEXA[8], FALCON[9] and RP-lAH [10] have been developed , but these mechanisms did not achieve performances over 100 [G].

On the other hand, singularities are common in parallel mechanisms. They will bring disadvantages to parallel mechanisms [11][12]. Then force redundancy is introduced into parallel mechanism as it is an efficient way to reduce or even eliminate these singularities from parallel manipulator [12][13][14]. Besides, force redundant mechanism also has the nice features of increasing the reliability of the system, improving Cartesian stiffness, achieving more homogeneous output forces, increasing acceleration and minimizing internal loading torques of the actuators of the manipulator [15][16][17]. As a result, some high speed parallel mechanisms with force redundancy appeared. For example, Kock and Schumacher presented a planar parallel manipulator PA-R-MA, which is designed for high-speed pick-and-place application. And experimental results show that, for position tracking, tracking errors were less than 0.5 mm in X and Y directions with acceleration of 70 m/s^2 and velocity of 3 m/s [18]. By utilizing the good performance of parallel force redundant mechanism, we presented a fast parallel mechanism NINJA, which can reach the highest acceleration up to 100 [G], in [19].

However, force redundancy mechanism will make velocity of active joints not independent. As a result, it is hard to get a high positional accuracy by adopting velocity control scheme for servo motor because it has the risk of producing big excessive internal forces [16]. Therefore, we are introducing kinematical redundancy to redesign a force redundant parallel mechanism NINJA. In this designing, we do not add kinematical redundancy on any sub arm but between the sub arms and top plate. Hence, all the sub arms do not need any change but a new top plate is proposed. Due to introducing kinematical redundancy into the force redundant parallel mechanism, joint variable is no longer dependent and the accuracy performance in position control of the modified one is expected to be improved while keeping the high acceleration simultaneously.

This paper is organized as follows. In Section 2, a simple force redundant parallel mechanism is used to state the problem. The concept of the research is illustrated by finding the solution to improve it. In Section 3, basic equations of modifying general force redundant parallel mechanism by introducing internal kinematical redundancy are developed and the design conditions are given. In Section 4, this proposed theory is implemented to redesign the real mechanism NINJA. The new structure and kinematic equations are given. In Section 5, prototype of the new mechanism NINJA has been developed. Finally, conclusion and future work are given in Section 6.

II. CONCEPT ON INTRODUCING KINEMATICAL REDUNDANCY

A simple force redundant parallel manipulator is analyzed and the problem of force redundant mechanism is described. Then by finding the solution to this problem, the concept of adding internal kinematical redundancy between sub arms and the top plate is described.

A. Problem Statement

The simple force redundant parallel manipulator without internal kinematical redundancy is shown in Fig.1

It is a 1 DOF parallel manipulator with 1-dimensional workspace (linear motion) and 2 actuators in parallel. The nomenclatures are shown as follows:

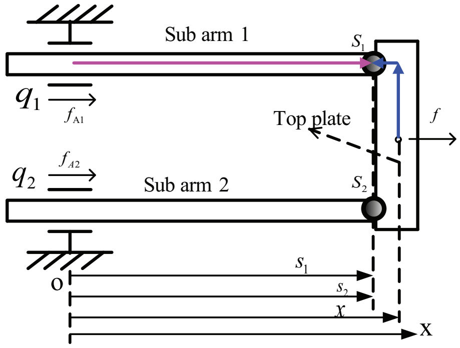

Fig. 1. 1 DOF force redundant parallel manipulator without internal kinematic redundancy

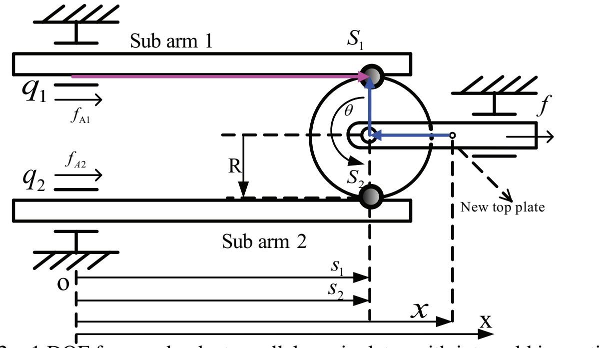

Fig. 2. 1 DOF force redundant parallel manipulator with internal kinematic redundancy

$\boldsymbol{q} = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^{\mathrm{T}}$: Joint variables. q_i is the active joint of the i-th sub arm ($i = 1,2$)

S_i: The connection point between the i-th sub arm and the top plate

$\boldsymbol{s} = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^{\mathrm{T}}$, s_i is Position of S_i along axis x with respect to O

x: Position of top plate along axis x with respect to O

$\boldsymbol{f}_A = \begin{bmatrix} f_{A1} & f_{A2} \end{bmatrix}^{\mathrm{T}}$: The set of actuator driving forces.

f: Generalized force at the top plate

As shown in Fig.1, there are two ways to calculate the connection point velocity. The first route is to set up the velocity relationship between connection point and joint variables, i.e.

$$\dot{s}_1 = \dot{q}_1, \qquad \dot{s}_2 = \dot{q}_2 \tag{1}$$

Using vector notation to write (1) is

$$\dot{\boldsymbol{s}} = J_S \dot{q}, \quad J_S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

J_S is the manipulator Jacobian matrix transforming from joint velocity to the velocity at the connection point.

The other route is the velocity relationship between connection point and the top plate, that is

$$\dot{s}_1 = \dot{x}, \qquad \dot{s}_2 = \dot{x} \tag{3}$$

Equation (3) is written compactly as

$$\dot{\boldsymbol{s}} = J_T \dot{x}, \quad J_T = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

J_T is the manipulator Jacobian matrix which relates top plate velocity to the velocity at the connection point.

Combining (2) and (4) reduces the kinematics as follow:

$$\dot{\boldsymbol{q}} = J\dot{x}, \quad J = J_S{}^{-1} J_T = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}} \tag{5}$$

J is the Jacobian matrix mapping top plate velocity to joint velocity.

Thus statics of this simple mechanism is:

$$f = J^{\mathrm{T}} \boldsymbol{f}_A = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} f_{A1} \\ f_{A2} \end{bmatrix} = f_{A1} + f_{A2} \tag{6}$$

In this force redundant mechanism, f_{A1} and f_{A2} can be different for a given f which is the sum of f_{A1} and f_{A2}, as shown in (6). However, $\dot{q}_1$ and $\dot{q}_2$ can not have different values as it is decided by $\dot{x}$, as shown in (5). As a result, the velocity control function of the actuators can not be applied. In other words, because of the particularity of force redundant mechanism, it is required that a set of joint variables must be dependent otherwise big internal forces will arise while pursuing high accuracy by using velocity control scheme for each actuator. Taking this mechanism shown in Fig.1 as an example, the velocity of each sub arm must be the same at any moment. If there is any small difference between them, the internal force between sub arm and the top plate will be very large and the mechanism may break down. This is a big problem. The requirement of no position difference existing between each sub arm makes that it is impossible to adopt velocity control scheme for servo controller.

Therefore, the concept of our solution is to find an improved force redundant mechanism which satisfies a set of design conditions as follow.

(A) Necessary conditions:

(A.1) The stated problem should be solved. It means the velocities of joints are independent, that is $\dot{q}_1$ and $\dot{q}_2$ are not required to be the same.

(A.2) When all the actuators stop, the mechanism including the top plate and the internal motion, should stop.

(B) Target conditions:

The improved force redundant mechanism keeps the novel performances of force redundant mechanism, that is the relationship, which holds between Cartesian space force and joint space force, should obey the form shown in (6).

B. *Problem Solution*

In order to find a mechanism which satisfies above design requirements, an idea of adding one internal kinematic redundant motion $\dot{\theta}$ between sub arms and End-effector comes into our mind, and the modified force redundant parallel mechanism is proposed in Fig.2. In this mechanism, required motion $\dot{x}$ is still the top plate output velocity along axis x. The procedure of calculating the kinematics is the same as the one shown in the previous subsection.

As shown in Fig.2, there are still two routes to calculate the connection point velocity. In this research concept, there are no change of sub arms but adding internal kinematical redundancy on the connection between the top plate and sub arms. Therefore, the relationship of the first route between connection point velocity and joint velocity is the same as (2). However, the relationship between connection point velocity and top plate velocity is

$$\dot{s}_1 = \dot{x} - R\dot{\theta} \tag{7}$$

$$\dot{s}_2 = \dot{x} + R\dot{\theta} \tag{8}$$

where R is the radius of the rotated round plate.

Equations (7) and (8) can be written compactly as:

$$\dot{\boldsymbol{s}} = \widetilde{J_T}\begin{bmatrix}\dot{x}\\ \dot{\theta}\end{bmatrix}, \quad \widetilde{J_T} = \begin{bmatrix} J_{To} | J_{TI}\end{bmatrix} \tag{9}$$

where $J_{To} = \begin{bmatrix} 1 & 1\end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{2\times 1}$ is the Jacobian matrix which relates Cartesian velocities of top plate $\dot{x}$ to the velocity at the connection point, and $J_{TI} = \begin{bmatrix} -R & R\end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{2\times 1}$ is the Jacobian matrix which relates internal motion velocity $\dot{\theta}$ to the velocity at the connection point respectively.

Combining (2) and (9) reduces the kinematics as follow:

$$\dot{\boldsymbol{q}} = \widetilde{J}\begin{bmatrix}\dot{x}\\ \dot{\theta}\end{bmatrix}, \quad \widetilde{J} = {J_S}^{-1}\widetilde{J_T} \tag{10}$$

where $\widetilde{J} = \begin{bmatrix} 1 & -R\\ 1 & R\end{bmatrix}$, is the Jacobian matrix transforming from top plate velocity and internal velocity to joint velocity.

Define J_o as the Jacobian matrix which relates Cartesian velocities of top plate $\dot{x}$ to the active joints velocity, and define J_I as the Jacobian matrix which relates internal motion velocity $\dot{\theta}$ to the active joints velocity. (10) can be written as

$$\dot{\boldsymbol{q}} = J_o\dot{x} + J_I\dot{\theta}, \quad \widetilde{J} = \begin{bmatrix} J_o | \; J_I\end{bmatrix} \tag{11}$$

where $J_o = {J_S}^{-1}J_{To} \in \mathrm{R}^{2\times 1}$, $J_I = {J_S}^{-1}J_{TI} \in \mathrm{R}^{2\times 1}$

Hence the static force relationship of the improved force redundant is as follow:

$$\begin{bmatrix} f\\ \tau_I\end{bmatrix} = (\widetilde{J})^{\mathrm{T}}\begin{bmatrix} f_{A1}\\ f_{A2}\end{bmatrix} \tag{12}$$

where τ_I is the internal force and f is generalized force at the top plate.

Generalized force f can be derived from (12), that is

$$f = {J_o}^{\mathrm{T}} f_A = \begin{bmatrix} 1 & 1\end{bmatrix}\begin{bmatrix} f_{A1}\\ f_{A2}\end{bmatrix} = f_{A1} + f_{A2} \tag{13}$$

Now, we evaluate specifications of the improved simple force redundant mechanism as below:

(a) Necessary conditions:

(a1) From (10), we know $\widetilde{J}$ is a regular matrix when $R \neq 0$, then $\dot{q}_1$ and $\dot{q}_2$ can have different values because of the introduced internal kinematic redundant motion $\dot{\theta}$. Then, (A.1) is solved.

(a2) And transposing both sides of (10), we know the mechanism stops ($\dot{\boldsymbol{x}} = \dot{\theta} = 0$) when $\dot{q}_1 = \dot{q}_2 = 0$. Then, (A.2) is solved.

What is more, the velocity difference among sub arms produces internal motion but has no influence on the required motion $\dot{x}$.

(b) Target conditions: By comparing (13) with (6), we get that the top plate driving force f of both of them is the summation of joint driving forces. This static force relationship shows that the novel performances of force redundant mechanism keeps. Then, (B) is satisfied.

Therefore, the force redundant mechanism with kinematical redundancy has the merits summarized as below:

1) Differences among the positions of the sub arms are allowed;

2) Each sub arms can be controlled without considering internal forces during control;

3) Fast motion can be expected.

III. Introducing Kinematical Redundancy into Force Redundant Manipulator

In this section, the general theory of introducing kinematical redundancy into force redundant manipulator is described, and basic equations are proposed.

Let us consider a general force redundant parallel manipulator with m dimensional task space and n_A actuations ($n_A > m$), shown in Fig.3. In the figure, Σ_B is the base coordinate frame. Σ_T is the object coordinate frame fixed on the top plate.

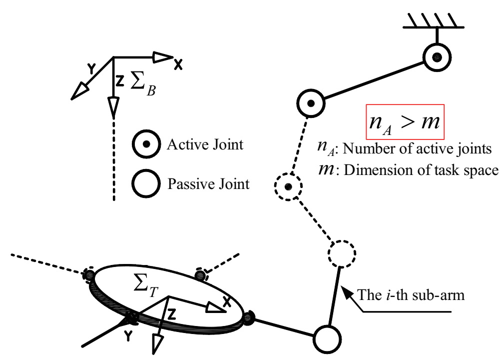

Fig. 3. A general force redundant parallel manipulator.

Let $\boldsymbol{r} \in \mathrm{R}^m$ be the position and orientation of top plate. Denote $\boldsymbol{q}_A \in \mathrm{R}^{n_A}$ as the active joint variable of the general force redundant parallel manipulator. Let n_r indicates the number of redundant actuators and it is given by

$$n_r = n_A - m \tag{14}$$

A. *Force Redundant Parallel Manipulator without Kinematic Redundancy*

The active joint velocity is related to the end-effector velocity in Cartesian space by the Jacobian matrix $J_A \in \mathrm{R}^{n_A\times m}$

$$\dot{\boldsymbol{q}}_A = J_A\dot{\boldsymbol{r}} \tag{15}$$

From this equation, we can clearly know that active joint velocities are linearly dependent by using the linear algebra theory because of the mathematical relationship $n_A > m$. Therefore, this parallel force redundant mechanism has the disadvantage of producing big internal forces if we adopt velocity control for servo motors when designing the control scheme for real mechanism.

Then the static force relationship is

$$\boldsymbol{F} = {J_A}^{\mathrm{T}} \tau \tag{16}$$

where $\tau \in \mathrm{R}^m$ is the force vector in joint space, and $\boldsymbol{F} \in \mathrm{R}^{n_A}$ is the force vector in cartesian space.

As ${J_A}^{\mathrm{T}} \in \mathrm{R}^{m \times n_A}$ and rank ${J_A}^{\mathrm{T}} \leq \min(m, n_A)$. Note that $n_A > m$. Hence, we can get the rank ${J_A}^{\mathrm{T}} \leq m$. On the other hand, the dimensions of τ is n_A. Therefore, by using null space of matrix ${J_A}^{\mathrm{T}}$, we proved and confirmed that this kind of parallel mechanism has force redundancy.

B. Parallel Force Redundant Manipulator with Kinematic Redundancy

Based on the concept of the idea, in order to avoid the problem in the force redundant mechanism when applying velocity control for servo motor to achieve high accuracy, introducing internal kinematic redundant motion is proposed. The number of added internal kinematical redundancy is assumed as n_I. The added internal redundant motion variable $\alpha \in \mathrm{R}^{n_I}$ (α is independent of top plate variables), is introduced to make:

$$\dot{\boldsymbol{q}}_A = \widetilde{J_A} \begin{bmatrix} \dot{\boldsymbol{r}} \\ \dot{\alpha} \end{bmatrix}, \quad \widetilde{J_A} = \begin{bmatrix} J_{Ao} \; J_{AI} \end{bmatrix} \in \mathrm{R}^{n_A \times (m+n_I)} \tag{17}$$

where $\widetilde{J_A}$ is the Jacobian matrix transforming from top plate velocity and internal velocity to active joint velocity. $J_{Ao} \in \mathrm{R}^{n_A \times m}$ is Jacobian matrix transforming from top plate motion velocity $\dot{\boldsymbol{r}}$ to active joint velocity. $J_{AI} \in \mathrm{R}^{n_A \times n_I}$ is the Jacobian matrix transforming from internal redundant motion velocity $\dot{\alpha}$ to active joint velocity. What is more, this improved force redundant mechanism should satisfy the following required conditions:

(A) Necessary conditions:

(A.1) The velocity incoincidence among sub arms is allowed. In other words, the problem of linearly dependent active joint velocities in (15) should be avoided.

(A.2) When all the actuators stop, the mechanism should stop ($\dot{\boldsymbol{r}} = \boldsymbol{0}$ and $\dot{\alpha} = \boldsymbol{0}$), that is $\widetilde{J_A}$ should be a full rank one, i.e.

$$\mathrm{rank}\, \widetilde{J_A} = n_A = m + n_I \tag{18}$$

Combining (18) and (14) yields:

$$n_I = n_r \tag{19}$$

Hence, from (19), another equivalent condition, to make the Jacobian matrix $\widetilde{J_A}$ to be a full rank square matrix, is to let the number of added kinematic redundancy n_I be equal to the number of redundant actuators n_r.

(B) Target condition:

The improved one keeps the novel force relationship of force redundant mechanism.

However, it should be noted that the design of the improved mechanism should be done carefully to avoid joint backlash .

IV. Structural Design of Mechanism

In this section, Mechanism NINJA, shown in Fig.4, will be modified by adding internal kinematical redundancy between sub arms and top plate.

Fig. 4. High speed mechanism NINJA.

A. Problem Statement of NINJA

The parallel mechanism NINJA, which is designed to achieve an acceleration rate over 100 [G] (G: gravitational acceleration), has four sub-arms, which are connected to a common top plate by boll joints. It is designed to be lightweight by arranging actuators on a base. Each sub-arm has two actuated joints and a passive joint. Therefore, NINJA has two degrees of force redundancy. The simplified structure of NINJA itself is shown in Fig.5.

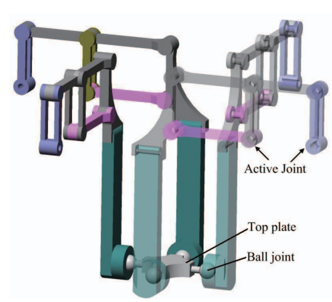

Fig. 5. Structure of NINJA.

Kinematic relationship of the ith sub-arm structure is shown Fig.6. The nomenclatures are shown as follows:
Σ_{B}: Base coordinate frame
Σ_{T}: top plate coordinate frame

C_i: The i-th ball joint
Σ_{SJi}: Sub arm coordinate frame whose origin is located at the ball joint connection between the i-th sub arm and top plate, i=1, 2, 3 , 4
Σ_{STJi}: top plate coordinate whose origin is the center of ball joint connection between the i-th sub arm and top plate
${}^{B}\boldsymbol{P}_{SJi} \in \mathrm{R}^3$: Position of Σ_{SJi} with respect to Σ_B
${}^{B}\boldsymbol{P}_{STJi} \in \mathrm{R}^3$: Position of Σ_{STJi} with respect to Σ_B
${}^{T}\boldsymbol{P}_{STJi} \in \mathrm{R}^3$: Position of Σ_{STJi} with respect to Σ_T
${}^{B}\boldsymbol{P}_T = \boldsymbol{r}_p = \begin{bmatrix} x & y & z \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^3$: Position of Σ_T with respect to Σ_B
$\phi = \begin{bmatrix} \varphi & \theta & \psi \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^3$: Orientation (roll-pitch-yaw) of Σ_T with respect to Σ_B
${}^{B}\boldsymbol{R}_T = \mathbf{RPY}(\varphi, \theta, \psi) = \mathbf{Rot}(\mathbf{z}, \varphi)\mathbf{Rot}(\mathbf{y}, \theta)\mathbf{Rot}(\mathbf{x}, \psi) \in \mathrm{R}^{3\times 3}$: Roll-pitch-yaw transformation from coordinates Σ_B to top plate coordinates Σ_T
$\omega = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^3$: Angular velocity of top plate
$\dot{\boldsymbol{r}} = \begin{bmatrix} \dot{\boldsymbol{r}}_p^{\mathrm{T}} & \omega^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^6$: Velocity of top plate
$\boldsymbol{q}_{iA} = \begin{bmatrix} q_{i1} & q_{i2} \end{bmatrix}^{\mathrm{T}}$: Active joint of the ith sub arm. (q_{i1} and q_{i2} are active joints)
$\boldsymbol{q}_{ip}$: Passive joint of the ith sub arm. (q_{i3} is the passive joint)
$\boldsymbol{q}_i = \begin{bmatrix} \boldsymbol{q}_{iA}{}^{\mathrm{T}} & \boldsymbol{q}_{ip} \end{bmatrix}^{\mathrm{T}}$: Joint variable of the ith sub arm
$\tau_i = \begin{bmatrix} \tau_{i1} & \tau_{i2} \end{bmatrix}^{\mathrm{T}}$: Joint driving force of the ith sub arm
$\tau = \begin{bmatrix} \tau_1^{\mathrm{T}} & \tau_2^{\mathrm{T}} & \tau_3^{\mathrm{T}} & \tau_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^8$: Joint driving force of mechanism
$\boldsymbol{F} \in \mathrm{R}^6$: Generalized vector of Cartesian force at top plate.

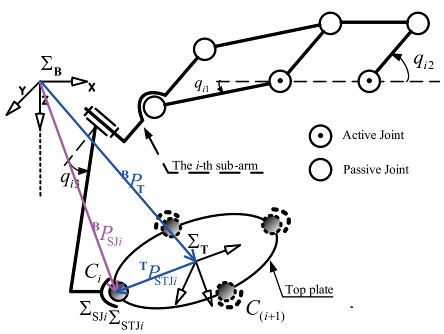

Fig. 6. Kinematic relationship of NINJA.

From the origin of base coordinates Σ_B to the origin of the ball joint connection point between the i-th sub arm and top plate, there are two equivalent position vectors, i.e.

$$ {}^{B}\boldsymbol{P}_{SJi} = {}^{B}\boldsymbol{P}_{STJi} \tag{20}$$

The vector of ${}^{B}\boldsymbol{P}_{SJi}$ is a function of joint variable, i.e.

$$ {}^{B}\boldsymbol{P}_{SJi} = f_i(\boldsymbol{q}_i) \tag{21}$$

While ${}^{B}\boldsymbol{P}_{STJi}$ is represented as by vector addition:

$$ {}^{B}\boldsymbol{P}_{STJi} = {}^{B}\boldsymbol{P}_T + {}^{B}\boldsymbol{R}_T\ {}^{T}\boldsymbol{P}_{STJi} = g_i(x, y, z, \varphi, \theta, \psi) \tag{22}$$

Here ${}^{T}\boldsymbol{P}_{STJi}$ is constant in the i-th closed kinematic chain.

Substituting (21) and (22) into (20), and differentiating both sides with respect to time:

$$ J_{Si}\dot{\boldsymbol{q}}_i = \hat{J}_{Ti} \begin{bmatrix} \dot{\boldsymbol{r}}_p^{\mathrm{T}} & \dot{\phi}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{23}$$

The angular velocity of the top plate can be computed as

$$ \omega = J_{RPY}\dot{\phi} \tag{24}$$

where $J_{RPY} = \begin{bmatrix} 0 & -\sin\varphi & \cos\varphi\cos\theta \\ 0 & \cos\varphi & \sin\varphi\cos\theta \\ 1 & 0 & -\sin\theta \end{bmatrix}$

Applying (24) to (23), we obtain

$$ J_{Si}\dot{\boldsymbol{q}}_i = J_{Ti}\dot{\boldsymbol{r}} \tag{25}$$

where $J_{Ti} = \hat{J}_{Ti} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & J_{RPY} \end{bmatrix}^{-1}$

Hence the relationship between $\dot{\boldsymbol{q}}_i$ and $\dot{\boldsymbol{r}}$ is

$$ \dot{\boldsymbol{q}}_i = J_i\dot{\boldsymbol{r}} \tag{26}$$

where $J_i = J_{Si}{}^{-1}J_{Ti} = \begin{bmatrix} J_{iA}^{\mathrm{T}} & J_{ip}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ and is a function of $\boldsymbol{r}_p$ and ϕ, $J_{iA} \in \mathrm{R}^{2\times 6}$ and $J_{ip} \in \mathrm{R}^{1\times 6}$ are the Jacobian matrix corresponding to active joint velocity $\dot{\boldsymbol{q}}_{iA}$ and passive joint velocity $\dot{q}_{ip}$ respectively.

Therefore, velocity relationship between active joints of the mechanism and the top plate is expressed compactly as

$$ \dot{\boldsymbol{q}}_A = J_A\dot{\boldsymbol{r}} \tag{27}$$

where $J_A = \begin{bmatrix} J_{1A}^{\mathrm{T}} & J_{2A}^{\mathrm{T}} & J_{3A}^{\mathrm{T}} & J_{4A}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{8\times 6}$

Thus the static relationship between the joint force τ and the generalized force $\boldsymbol{F}$ developed at the top plate is given by

$$ \boldsymbol{F} = J_A{}^{\mathrm{T}}\tau \tag{28}$$

Comparing (27) to (15) previously discussed, we can know that the velocities are not independent. Therefore, with the purpose of avoiding this problem, the theory of adding internal kinematic redundancy between sub-arms and top plate is applied into improving NINJA.

B. Solution of NINJA - - Structural Design of New Top Plate

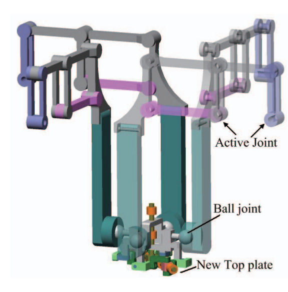

Fig. 7. Structure of the modified NINJA.

By following the design guidelines and taking account of the requirements discussed in the previous section, mechanism NINJA is to be modified by designing a new top plate without changing sub arms. It means we are not adding kinematical redundancy on sub arm but on the connection points between top plate and sub arms. According to (19), the new top plate should have 2DOF as NINJA has two degrees of force redundancy. What is more, new NINJA after adopting the new top plate should obey necessary conditions and target condition presented. On the basis of these requirements, the prototype of the modified NINJA is proposed. The simplified structure of the new prototype is shown in Fig.7.

Fig.8 shows the structure of the new top plate. These balls in the figure stand for the ball joints connecting with the sub-arms.

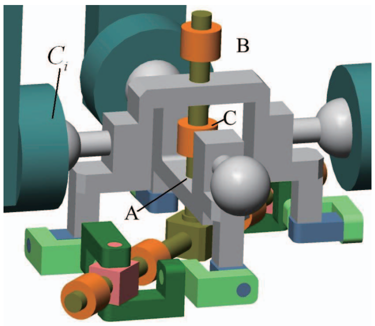

Fig. 8. Schematic drawing of the new top plate.

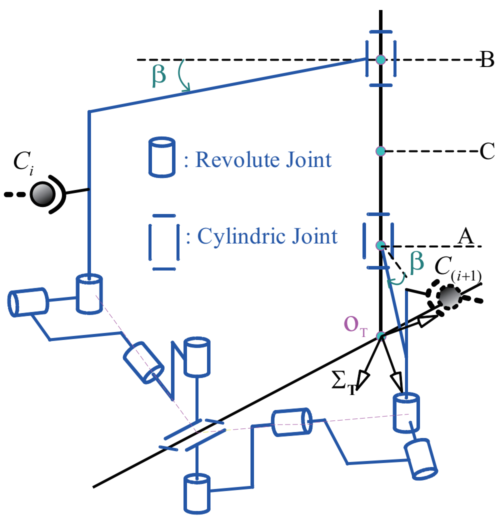

Fig. 9. Symmetrical and duality structure.

It is a symmetrical and duality mechanism as the structure simplified in Fig.9, and therefore it can average the position incoincidence. These proprieties are shown by e (or d, which is used to describe the relative translation motion along the vertical axis of this new top plate) and β. β denotes the relative rotation motion along the vertical axis of this new top plate. As a result, this top plate itself has two internal motions which are translation and rotation along the vertical axis of this new top plate. Therefore, the internal motion is $\alpha = \left[\beta \ d\right]^{\mathrm{T}}$.

The kinematic relationship of the i-th sub arm is described in Fig.10.

In the kinematic equation (20), the vector of ${}^{B}\boldsymbol{P}_{SJi}$ has no change as all the arms is not changed. However, the vector of ${}^{B}\boldsymbol{P}_{STJi}$ in (20) has to be modified as ${}^{T}\boldsymbol{P}_{STJi}$ is not a constant vector any more but a function of α after adopting this new top plate. Hence,

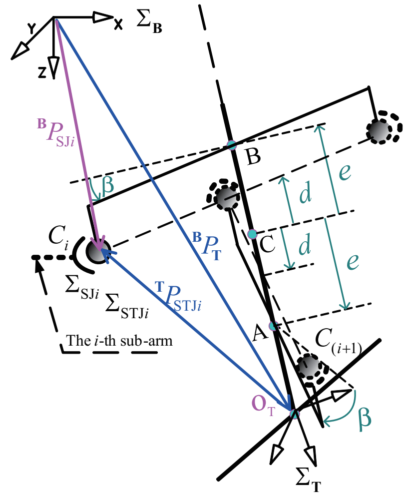

Fig. 10. Kinematic relationship of the modified NINJA.

$$ {}^{B}\boldsymbol{P}_{STJi} = {}^{B}\boldsymbol{P}_{T} + {}^{B}\boldsymbol{R}_{T}\ {}^{T}\boldsymbol{P}_{STJi} = g_i(x,y,z,\varphi,\theta,\psi,\beta,d) \quad (29) $$

Substituting (21) and (29) into (20), and differentiating both sides with respect to time:

$$ J_{Si}\dot{\boldsymbol{q}}_i = \widetilde{J_{Ti}}\left[\dot{\boldsymbol{r}}_p^{\mathrm{T}}\ \dot{\phi}^{\mathrm{T}}\ \dot{\alpha}^{\mathrm{T}}\right]^{\mathrm{T}} \quad (30) $$

and premultiply both sides by $J_{Si}{}^{-1}$, we have

$$ \dot{\boldsymbol{q}}_i = \widetilde{J_i}\begin{bmatrix}\dot{\boldsymbol{r}}\\ \dot{\alpha}\end{bmatrix} \quad (31) $$

where $\widetilde{J_i} = J_{Si}{}^{-1}\widetilde{J_{Ti}} = \left[\widetilde{J_{iA}}^{\mathrm{T}}\ \widetilde{J_{ip}}^{\mathrm{T}}\right]^{\mathrm{T}}$ and is a function of $\boldsymbol{r}_p$, ϕ and α. $\widetilde{J_{Ti}} = \widehat{J_{Ti}}[\mathrm{diag}(I_{3\times3}, J_{RPY}, I_{2\times2})]^{-1}$. $\widetilde{J_{iA}} \in \mathrm{R}^{2\times8}$ and $\widetilde{J_{ip}} \in \mathrm{R}^{1\times8}$ are the Jacobian matrix corresponding to active joint velocity $\dot{\boldsymbol{q}}_{iA}$ and passive joint velocity $\dot{q}_{ip}$ respectively.

Hence the active joint velocities of the ith sub arm is:

$$ \dot{\boldsymbol{q}}_{iA} = \widetilde{J_{iA}}\begin{bmatrix}\dot{\boldsymbol{r}}\\ \dot{\alpha}\end{bmatrix} \quad (32) $$

where $\widetilde{J_{iA}} = \left[J_{iAo}\ J_{iAI}\right]$. $J_{iAo} \in \mathrm{R}^{2\times6}$ is the Jacobian matrix which relates Cartesian velocities of top plate $\dot{\boldsymbol{r}}$ to the active joints velocity, and $J_{iAI} \in \mathrm{R}^{2\times2}$ relates internal motion velocity $\dot{\alpha}$ to the active joints velocity.

Combining (32) together, the active joint velocities of the modified NINJA is written compactly as

$$ \dot{\boldsymbol{q}}_A = \widetilde{J_A}\begin{bmatrix}\dot{\boldsymbol{r}}\\ \dot{\alpha}\end{bmatrix} \quad (33) $$

where $\widetilde{J_A} = \left[J_{Ao}\ J_{AI}\right] \in \mathrm{R}^{8\times8}$,

$J_{Ao} = \left[J_{1Ao}{}^{\mathrm{T}}\ J_{2Ao}{}^{\mathrm{T}}\ J_{3Ao}{}^{\mathrm{T}}\ J_{4Ao}{}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{8\times6}$,

$J_{AI} = \left[J_{1AI}{}^{\mathrm{T}}\ J_{2AI}{}^{\mathrm{T}}\ J_{3AI}{}^{\mathrm{T}}\ J_{4AI}{}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{8\times2}$.

Then the static relationship is given by

$$ \boldsymbol{F} = J_{Ao}{}^{\mathrm{T}}\tau \quad (34) $$

It is shown in (34) that the modified NINJA keeps the force relationship of force redundant parallel mechanism.

V. Prototype

The prototype of the new top plate is shown in Fig.11 (A is the old top plate and B is the new one).

After installing the new top plate, the prototype of the modified NINJA is shown in Fig.12. Designing the control scheme and experimental operation are the next works.

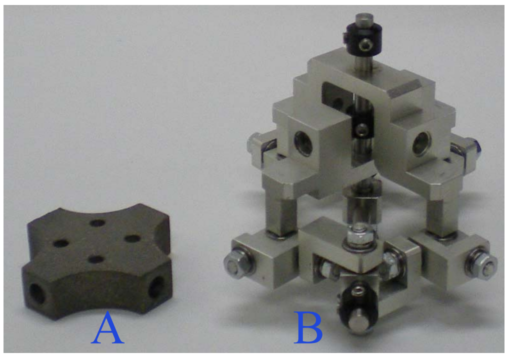

Fig. 11. Old top plate and Prototype of the new one.

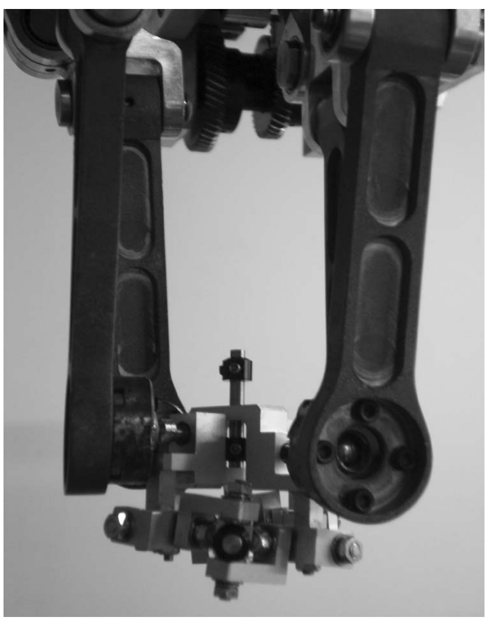

Fig. 12. Prototype of Parallel Mechanism, Modified NINJA.

VI. Conclusion

With the purpose of improving the accuracy performance in position control of force redundant parallel mechanisms, an idea of introducing kinematical redundancy into parallel mechanisms with force redundancy is adopted. The major results obtained in this paper are summarized as follows.

1) The basic concept on re-design of force redundant parallel mechanisms with kinematical redundancy is presented by using a simple example.

2) The design procedure to realize the proposed concept is developed.

3) The proposed design procedures is illustrated by implementing it for the re-design of the existing mechanism NINJA. A new prototype of NINJA has been designed.

Future work includes implementation of a control scheme using velocity controller for the new prototype and experimental evaluation of the performance of the proposed mechanism.

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