The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Swinging Up and Stabilization Control of Double Furuta Pendulums by Safe Manual Control

Keigo Noguchi, Masaki Izutsu, Norihiro Kamamichi, Tetsuo Shiotsuki, Jun Ishikawa and Katsuhisa Furuta

***Abstract*— In this paper, we propose a control method to swing up and stabilize a double Furuta pendulums (DFP).The DFP is a rotational type inverted pendulum system that has a base-link and two pendulums of different length at the both ends of the base-link. The proposed method consists of three controllers for the DFP, *i.e.*, a swinging up controller for a long pendulum, a controller to keep a long pendulum at the upright position and to swing up a short pendulum simultaneously, and a stabilization controller for both pendulums at the upright position. Since the second controller has two objectives, we use safe manual control[3] proposed by Åström and Åkesson.**

This paper proposes a novel application of the safe manual control. Swinging up and stabilization of the DFP was successfully achieved by appropriately switching three controllers. The effectiveness of the proposed method was verified by simulation and experiment.

I. INTRODUCTION

Inverted pendulums have been widely used as controlled plants to evaluate validities of control theories. The double furuta pendulums (DFP) has a rotational base-link and two pendulums at the both ends of the base-link. The DFP is controlled by a direct drive (DD) motor attached to the base-link. The lengths of the two pendulums are different from each other, and the characteristics of the DFP are changed depending on the lengths of the two pendulums. From this characteristics, the DFP is used as a benchmark to check the abilities of control theories for underactuated mechanical systems[2]. In this paper, we propose a control method to swing up and stabilize the DFP. The designed controller can stabilize both pendulums at the upright position, swinging them up from the pendant position. Dynamics of the DFP are changed fully depending on the states of the both pendulums. In order to achieve the control objective, the proposed method is consisted of three controllers. These controllers are switched by the states of the DFP. The purposes of the controllers respectively are as follows:

(1) swinging up a long pendulum,
(2) keeping a long pendulum around the upright position and swinging up a short pendulum, and
(3) stabilizing both pendulums at the upright position.

Then, the first controller is designed by energy control[6] for a long pendulum ignoring the motion of the short pendulum. The second controller is needed to control the pendulums so that two different objectives can be simultaneously achieved, *i.e.*, a swing-up control and stabilization control. For this purpose, we use safe manual control proposed by Åström, Åkesson. Usually, the safe manual control is used for a combination of manual control and automatic control, but it also can be used for combining two different automatic controls, *i.e.*, a stabilization control of a pendulum and a velocity control of the base-link of the inverted pendulum system. In this paper, we apply the safe manual control, replacing the manual control part with automatic swinging up for a short pendulum. The third controller is designed by common linear control theory because two pendulums can be linearized around the upright position. The effectiveness of the proposed controller is verified by simulation and experiment.

K. Noguchi, M. Izutsu, N. Kamamichi, T. Shiotuki and J. Ishikawa are with the Department of Robotics and Mechatronics, School of Science and Technology for Future Life, Tokyo Denki University, 2-2, Kanda Nishiki-cho, Chiyoda-ku, Tokyo 101-8457, Japan, phone: +81-3-5280-3915; fax: +81-3-5280-3793; ishikawa@fr.dendai.ac.jp.

K. Furuta is with Tokyo Denki University and is a Fellow of IEEE, a Distinguished Member of IEEE Control Systems Society

II. DOUBLE FURUTA PENDULUM SYSTEM

In this section, an equation of motion of the DFP is derived by Euler-Lagrange method. The schematic model of the DFP is shown in Fig. 1. The parameters of the DFP are listed in Table I. The equation of motion of the DFP is given by

$$M(\theta)\ddot{\theta} + H(\theta, \dot{\theta}) + G(\theta) = \tau \tag{1}$$

where $\theta = [\theta_1,\ \theta_2,\ \theta_3]^T$ and $\tau = [\tau_1,\ 0,\ 0]^T$, and $M(\theta)$ is inertia matrix, $H(\theta, \dot{\theta})$ is term of coriolis force and friction

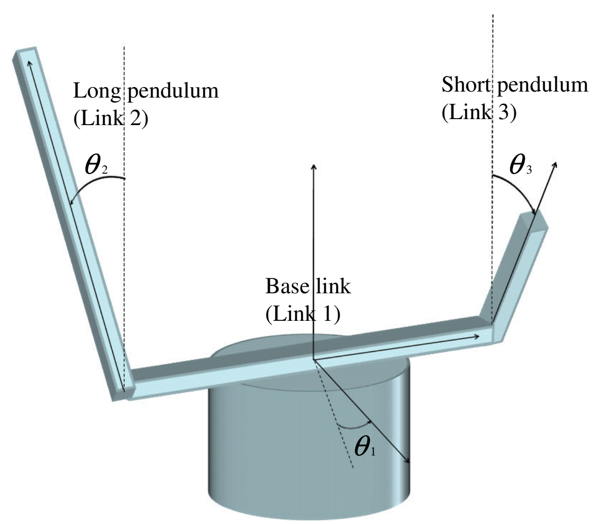

Fig. 1. Schematic diagram of DFP

TABLE I

PARAMETERS OF DFP(i=1, 2, 3)

m_i	Mass of link i (kg)
J_i	Moment of inertia of link i at COG (Nm·s^2/rad)
C_i	Friction coefficient of link i at pivot (Nm·s/rad)
l_i	Length from joint 1 to joint i (m)
r_i	Length from joint i to COG of link i (m)
τ_1	Input torque for base-link (Nm)
θ_i	Angle of link i (rad)

force and $G(\theta)$ is gravity term,

$$M(\theta) = \begin{bmatrix} \bar{m}_1 & \bar{m}_2 & \bar{m}_3 \\ \bar{m}_2 & \bar{J}_2 & 0 \\ \bar{m}_3 & 0 & \bar{J}_3 \end{bmatrix},$$

$$H(\theta,\dot{\theta}) = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, G(\theta) = \begin{bmatrix} 0 \\ g_2 \\ g_3 \end{bmatrix},$$

where

$$\begin{aligned}
\bar{m}_1 &= \bar{J}_1 + \bar{J}_2 \sin\theta_2^2 + \bar{J}_3 \sin\theta_3^2, \\
\bar{m}_2 &= -m_2 l_2 r_2 \cos\theta_2,\ \bar{m}_3 = -m_3 l_3 r_3 \cos\theta_3, \\
h_1 &= m_2 l_2 r_2 \dot{\theta}_2^2 \sin\theta_2 + m_3 l_3 r_3 \dot{\theta}_3^2 \sin\theta_3 \\
&\quad + \bar{J}_2 \dot{\theta}_1 \dot{\theta}_2 \sin 2\theta_2 + \bar{J}_3 \dot{\theta}_1 \dot{\theta}_3 \sin 2\theta_3 + C_1 \dot{\theta}_1, \\
h_2 &= C_2 \dot{\theta}_2 - \frac{1}{2} \bar{J}_2 \dot{\theta}_1^2 \sin 2\theta_2, \\
h_3 &= C_3 \dot{\theta}_3 - \frac{1}{2} \bar{J}_3 \dot{\theta}_1^2 \sin 2\theta_3, \\
g_2 &= -m_2 g r_2 \sin\theta_2,\ g_3 = -m_3 g r_3 \sin\theta_3, \\
\bar{J}_1 &= J_1 + m_1 r_1^2 + m_2 l_2^2 + m_3 l_3^2, \\
\bar{J}_2 &= J_2 + m_2 r_2^2 \text{ and } \bar{J}_3 = J_3 + m_3 r_3^2
\end{aligned}$$

Here, the state is divided into four condition:

- Up-Up condition: both pendulums stay around the upright position.
- Up-Down condition: a long pendulum stays around the upright position and a short pendulum is out of the range around the upright position.
- Down-Up condition: a short pendulum stays around the upright position and a long pendulum is out of the range around the upright position.
- Down-Down condition: both pendulums are outside of around the upright position.

Each condition is illustrated in Fig. 2.

III. CONTROLLER DESIGN

This section gives a control method to swing up and stabilize the DFP. The proposed method consists of three controllers.

- D-D controller: to transfer the states of the DFP from the Down-Down condition to the Up-Down condition, that is, for swinging up the long pendulum.
- U-D controller: to transfer the states of the DFP from the Up-Down condition to the Up-Up condition, that is, for keeping long pendulum near the upright position and swinging up the short pendulum.

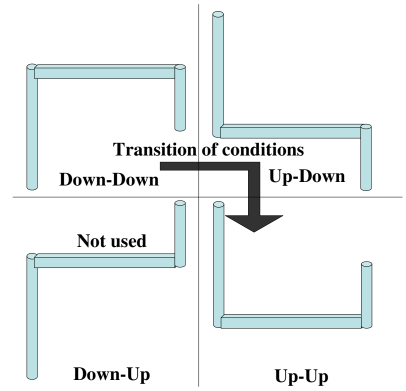

Fig. 2. Condition of DFP

- U-U controller: to stabilize both pendulums at the upright position

Swing-up and stabilization control achieves by switching three controllers.

A. D-D controller design

A D-D controller is designed to swing up the long pendulum by energy control[3]. The long pendulum is assumed to be a planar pendulum and have no influence of friction, for simplicity, the term h_2 of $H(\theta,\dot{\theta})$ in (1) is omitted. Under this assumption, the equation of motion of the long pendulum is given by

$$\bar{J}_2 \ddot{\theta}_2 = m_2 g r_2 \sin\theta_2 + m_2 r_2 \ddot{\theta}_1 \cos\theta_2. \quad (2)$$

The energy of the long pendulum is

$$E_2 = \frac{1}{2} \bar{J}_2 \dot{\theta}_2^2 + m_2 g r_2 (\cos\theta_2 - 1). \quad (3)$$

The energy E_2 is to be zero at the upright position. From (2) and (3), the time derivative of E_2 is calculated as

$$\dot{E}_2 = m_2 r_2 \ddot{\theta}_1 \dot{\theta}_2 \cos\theta_2. \quad (4)$$

Here, a candidate lyapunov function for the long pendulum energy is given by

$$V_2 = (E_{r2} - E_2)^2 / (2 m_2 r_2). \quad (5)$$

where E_{r2} is the reference energy. The time derivative of V_2 is

$$\dot{V}_2 = -(E_{r2} - E_2) \dot{\theta}_2 \ddot{\theta}_1 \cos\theta_2. \quad (6)$$

From (6), $\dot{V}_2$ can be negative semidefinite by using the angular acceleration of the base-link and the angular acceleration of the base-link can be controlled by the input torque. Let the acceleration of the base-link be a new control input. Then, the input acceleration is chosen to be

$$\ddot{\theta}_1 = u_{max} \cos\theta_3 \mathrm{sgn}((E_c - E)\dot{\theta}_3) \quad (7)$$

where $\mathrm{sgn}(\cdot)$ is

$$\mathrm{sgn}(u) = \begin{cases} 1 & u > 0 \\ 0 & u = 0 \\ -1 & u < 0 \end{cases},$$

and u_{max} (> 0) is the maximum input acceleration. Substituting (7) into (6), $\dot{V}_2$ becomes

$$\dot{V}_2 = -u_{max} \cos\theta_2^2 |(E_{r2} - E_2)\dot{\theta}_2| \ (\leq 0). \tag{8}$$

Equation (8) shows that function $\dot{V}_2$ is negative semi-definite. Thus, the long pendulum is swung up, because E_2 goes to zero.

From (1), the acceleration of the base-link $\ddot{\theta}_1$ is derived as

$$\ddot{\theta}_1 = \frac{1}{\det(M)} (\bar{J}_2 \bar{J}_3 \tau_1 + q) \tag{9}$$

where

$$\begin{aligned} q &= -h_1 \bar{J}_2 \bar{J}_3 + (h_2 + g_2)\bar{m}_2 \bar{J}_3 + (h_3 + g_3)\bar{m}_3 \bar{J}_2 \\ \det(M) &= \bar{J}_1 \bar{J}_2 \bar{J}_3 - \bar{J}_2 \bar{m}_3^2 - \bar{J}_3 \bar{m}_2^2 \end{aligned}$$

Therefore, the input torque to generate the desired acceleration of the base-link $\ddot{\theta}_1$ in (7) is given by

$$\tau_1 = \frac{\det(M)\ddot{\theta}_1 - q}{\bar{J}_2 \bar{J}_3}. \tag{10}$$

B. U-D controller design

The objective of the U-D controller is to keep the long pendulum around the upright position and to swing up the short pendulum simultaneously.

In order to achieve the control objective, the safe manual control[5] proposed by Åstöm is used. The safe manual control is a method that allows an operator to control the system manually with assistance of automatic control. In this paper, the automatic control part of the safe manual control is dedicated to stabilizing control for the long pendulum. On the other hand, the manual control part is for swinging up the short pendulum under an appropriate limitation for the control input by the safe manual control, but it is different from the original one in [5] from that the manual part in this paper is also an automatic controller. Designing the U-D controller is divided into four steps.

Step1. designing swinging up controller for the short pendulum.

Step2. developing the controllability set of the long pendulum around the upright position.

Step3. designing a stabilization controller for the base-link and the long pendulum.

Step4. combining the two controllers designed by Step 1 and Step 3.

In designing the U-D controller, the control input is derived based on the angular acceleration of the base-link, and then at the final step, the designed angular acceleration of the base-link is translated into the input torque. The details of the above mentioned four steps are as follows:

Step1

In order to swing up the short pendulum, energy control is used[6]. Under the same assumption as in designing the D-D controller, the energy control is applied to the short pendulum, omitting the term h_3 of $H(\theta, \dot{\theta})$ in (1). The equation of motion and the energy of the short pendulum are essentially the same as those in the previous section and are given by

$$\begin{aligned} \bar{J}_3 \ddot{\theta}_3 &= m_3 g r_3 \sin\theta_3 + m_3 r_3 u_E \cos\theta_3 \qquad (11) \\ E_3 &= \frac{1}{2} \bar{J}_3 \dot{\theta}_3^2 + m_3 g r_3 (\cos\theta_3 - 1). \qquad (12) \end{aligned}$$

where u_E ($= \ddot{\theta}_1$) is the input acceleration. From (11) and (12), the time derivative of E_3 is

$$\dot{E}_3 = m_3 r_3 u_E \dot{\theta}_3 \cos\theta_3. \tag{13}$$

Consider a candidate of the lyapunov function for the energy of the short pendulum, and E_3 is defined as

$$V_3 = (E_{r3} - E_3)^2 / (2 m_3 r_3) \tag{14}$$

where E_{r3} is the reference energy. The input acceleration is chosen to be

$$u_E = u_{max} \mathrm{sgn}((E_{r3} - E_3)\dot{\theta}_3 \cos\theta_3). \tag{15}$$

Then, $\dot{V}_3$ is calculated as

$$\dot{V}_3 = -u_{max} |(E_{r3} - E_3)\dot{\theta}_3 \cos\theta_3| \ (\leq 0). \tag{16}$$

Thus, $\dot{V}_3$ is proven to be negative semi-definite from (16). This means that the short pendulum can be swung up by the control input (15).

Step2

In this step, the angular velocity of the base-link is assumed to be a constant ($\dot{\theta}_1 = \omega$) and it is also assumed that there is no viscous friction. Then, the controllability set of the long pendulum on $\theta_2 - \dot{\theta}_2$ plane around the upright position is derived. From (1), when the input acceleration is $\pm u_{max}$, the equation of motion of the long pendulum can be rewritten as

$$\ddot{\theta}_2 = \omega^2 \sin 2\theta_2 / 2 + a \sin\theta_2 \pm b \cos\theta_2 u_{max} \tag{17}$$

where $a = (m_2 g r_2)/\bar{J}_2, b = (m_2 l_2 r_2)/\bar{J}_2$. The acceleration $\ddot{\theta}_2$ satisfies the following relation.

$$\ddot{\theta}_2 = \frac{d\dot{\theta}_2}{d\theta_2} \frac{d\theta_2}{dt} = \dot{\theta}_2 \frac{d\dot{\theta}_2}{d\theta_2}. \tag{18}$$

By substituting (18) into (17), (17) is rewritten as

$$\dot{\theta}_2 \frac{d\dot{\theta}_2}{d\theta_2} = \frac{\omega^2}{2} \sin 2\theta_2 + a \sin\theta_2 \pm b \cos\theta_2 u_{max}. \tag{19}$$

Integrating (19) with respect to θ_2 gives

$$\frac{1}{2}\dot{\theta}_2^2 = -\frac{1}{4}\omega^2 \cos 2\theta_2 - a \cos\theta_2 \mp b \sin\theta_2 u_{max} + C_I \tag{20}$$

where C_I is a integral constant that $\dot{\theta}_2 = 0$ at $\theta_2 = \theta_c$, that is,

$$C_I = \frac{1}{4}\omega^2 \cos 2\theta_c + a \cos\theta_c \pm b \sin\theta_c u_{max}. \tag{21}$$

θ_c in (21) is the angle of the long pendulum that satisfies (22).

$$0 = \frac{1}{2}\omega^2 \sin 2\theta_c + a \sin\theta_c \pm b\cos\theta_c u_{max}. \quad (22)$$

As mentioned above, because it is assumed that the angular velocity of the base-link is constant ($\dot{\theta}_1 = \omega$), the upper boundary $f^+(\theta_2)$ of the controllability set of the long pendulum on θ_2-$\dot{\theta}_2$ plane is

$$f^+(\theta_2) = \begin{cases} \sqrt{2\left(-\frac{\omega^2}{4}\cos 2\theta_2 - a\cos\theta_2 - b\sin\theta_2 u_{max} + C_I\right)} \\ (-\pi/2 \le \theta_2 \le \theta_c) \\ -\sqrt{2\left(-\frac{\omega^2}{4}\cos 2\theta_2 - a\cos\theta_2 - b\sin\theta_2 u_{max} + C_I\right)} \\ (\theta_c \le \theta_2 \le \pi/2) \end{cases} \quad (23)$$

On the other hand, the lower boundary $f^-(\theta_2)$ is $f^-(\theta_2) = -f^+(-\theta_2)$ because of the symmetrical property of the pendulum system.

Step3

A stabilization controller for the base-link and the long pendulum is designed as a linear quadratic regulator. By using a nonlinear feedback (10), the linearized state equation of linearized model of the base-link and the long pendulum at the upright position ($\theta_2 = 0$, $\dot{\theta}_1 = \dot{\theta}_2 = 0$) is given by

$$\dot{x}_u = A_u x_u + B_u u_u \quad (24)$$

$$A_u = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{m_2 g r_2}{J_2} & 0 & -\frac{C_2}{J_2} \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{m_2 l_2 r_2}{J_2} \end{bmatrix}$$

where the state vector is $x_u = [\theta_1,\ \theta_2,\ \dot{\theta}_1,\ \dot{\theta}_2]^T$ and u_u is the input acceleration. Then, consider a performance index

$$J_u = \int_0^\infty [x_u^T Q_u x_u + r_u u_u^2] dt \quad (25)$$

where Q_u (≥ 0) is a weighting matrix, and r_u (> 0) is scalar weight. The input minimizing (25) is given by

$$u_u = -r_u^{-1} B_u^T P_u x_u = -F_u x_u \quad (26)$$

where P_u is a symmetric positive definite solution of the Riccati equation

$$P_u A_u + A_u^T P_u - r_u^{-1} P_u B_u B_u^T P_u + Q_u = 0. \quad (27)$$

Step4

The control input which stabilize the base-link and the long pendulum and swings up the short pendulum simultaneously, is designed. Here, the input acceleration is given by

$$\ddot{\theta}_1 = \mathrm{sat}_{u_{max}}(-F_u x_u + u_E). \quad (28)$$

A saturation function $\mathrm{sat}_{u_{\max}}(\cdot)$ is defined as following

$$\mathrm{sat}_{u_{max}}(u) = \begin{cases} u_{max} & u \ge u_{max} \\ u & -u_{max} \le u \le u_{max} \\ -u_{max} & u \le -u_{max} \end{cases} \quad (29)$$

If two control inputs are simply combined as written in (28), the control objective are not achieved due to the interaction of each input. To keep the long pendulum inside the controllability set (23), all the inputs except for the input for stabilizing the long pendulum, that is, the feedback term $-f_1\theta_1 - f_3\dot{\theta}_3 + u_E$ is limited as

$$\begin{aligned} \ddot{\theta}_1 &= \mathrm{sat}_{u_{max}}(-f_2\theta_2 - f_4\dot{\theta}_2 + m) \\ m &= \mathrm{sat}_{m_a}(-f_1\theta_1 - f_3\dot{\theta}_1 + u_E) \end{aligned} \quad (30)$$

where $F_u = [f_1,\ f_2,\ f_3,\ f_4]$. From the controllability set of the long pendulum (23), the saturation limit m_a is given as

$$m_a = \begin{cases} m_a^+(\theta_2) = f_4(f^+(\theta_2) - d) - u_{max} + f_2\theta_2 \\ m_a^-(\theta_2) = f_4(f^-(\theta_2) + d) + u_{max} + f_2\theta_2 \end{cases} \quad (31)$$

where m_a^+ and m_a^- are the saturation limits. d (≤ 0) is a safety margin for robustness. Using the saturation limits m_a, the control input is limited to $\pm u_{max}$ on the $f^+(\theta_2) - d$ or $f^-(\theta_2) + d$. Under the assumption that the angular velocity of the base-link is constant ($\dot{\theta}_1 = \omega$), the set bounded $f^-(\theta_2) + d$ and $f^+(\theta_2) - d$ is positively a invariant set.

Finally, the designed acceleration input is converted to the torque input. The torque to generate the acceleration input (30) is given by (10).

C. U-U controller design

In this section, a stabilization controller for the DFP at Up-Up condition is designed as a linear quadratic regulator. The linearized model of the DFP at the Up-Up condition ($\theta_i = 0$, $\dot{\theta}_i = 0$) is

$$M_{uu}\ddot{\bar{\theta}} + C\dot{\bar{\theta}} + G_{uu}\bar{\theta} = \tau \quad (32)$$

$$\begin{aligned} M_{uu} &= \begin{bmatrix} \bar{J}_1 & -m_2 l_2 r_2 & -m_3 l_3 r_3 \\ -m_2 l_2 r_2 & \bar{J}_2 & 0 \\ -m_3 l_3 r_3 & 0 & \bar{J}_3 \end{bmatrix}, \\ C &= \mathrm{diag}(C_1, C_2, C_3), \\ G_{uu} &= \mathrm{diag}(0, -m_2 g r_2, -m_3 g r_3) \end{aligned}$$

where $\bar{\theta} = [\theta_1 - 2\pi n_1,\ \theta_2 - 2\pi n_2,\ \theta_3 - 2\pi n_3]^T$, and n_i is integer satisfying $\min_{n_i \in Z} |\theta_i - 2\pi n_i|$. The state equation of (32) is given by

$$\dot{x} = A_{uu} x + B_{uu} \tau_1 \quad (33)$$

$$A_{uu} = \begin{bmatrix} O_{3\times3} & I_{3\times3} \\ -M_{uu}^{-1} G_{uu} & -M_{uu}^{-1} C \end{bmatrix}, B_{uu} = \begin{bmatrix} O_{3\times1} \\ M_{uu}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

where $x = [\bar{\theta}^T\ \dot{\bar{\theta}}^T]^T$. Consider a following criterion function

$$J = \int_0^\infty [x^T Q_{uu} x + r_{uu} \tau_1^2] dt \quad (34)$$

where Q_{uu} (≥ 0) is weight matrix and r_{uu} (> 0) is scalar weight. The control input minimizing (25) is given by

$$\tau_1 = -r_{uu}^{-1} B_{uu}^T P_{uu} x = -F_{uu} x \quad (35)$$

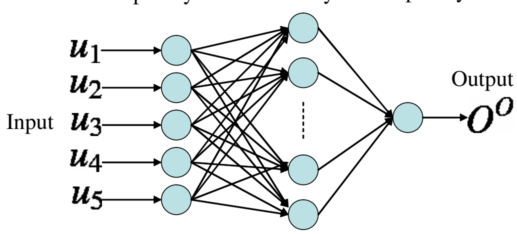

Fig. 3. Neural network model

where P_{uu} is a solution of the Riccati equation

$$P_{uu}A_{uu}+A_{uu}^TP_{uu}-r^{-1}P_{uu}B_{uu}B_{uu}^TP_{uu}+Q_{uu}=0. \quad (36)$$

In implementation, the feedback gain F_{uu} is redesigned as F_{duu} so that the discretized system has the same poles as the continuous system (33) in the sense of the matched z transformation.

D. Switching condition

In this section, switching conditions for each controller to smoothly make all the states of the DFP system relayed are explained. The timing to switch from the D-D controller to the U-D controller is decided by only angle of the long pendulum because the U-D controller has a wide effective region where the controllability set of the long pendulum is guaranteed to be (23). On the other hand, the effective region, *i.e.*, the stabilizable region for the U-U controller to be able to catch the short pendulum at the upright position is narrow, and five dimensions that consist of angular velocity of each joint and angle of the both pendulums. Thus, the switching condition should be decided on the five dimension space. The stabilizable region of the U-U controller is derived based on neural-network (NN). An acquired region is used to see whether the states of the DFP is inside or outside of the stabilizable region, and this result is used to determine the switching timing from the U-D controller to the U-U one.

1) Stabilizable region of U-U controller: The stabilizable region of the U-U controller for switching condition is acquired by a three-layer NN. Inputs for the NN are the five states of the DFP, *i.e.*, the angles of both pendulums and the angular velocity of each link. When these states are inside the stabilizable region of the U-U controller, the output of NN becomes larger than a threshold O^t. Hence, the numbers of neurons of the input layer and output layer are respectively five and one. The model of the NN is shown in Fig. 3. The parameters of the NN are listed in Table II. The number of neurons of hidden layer N_m is a design parameter. The range of the acquired region is $|\theta_i| \le \theta_{i\,\max}(i=2,3), |\dot{\theta}_i| \le \dot{\theta}_{i\,\max}(i=1,2,3)$ where $\theta_{i\max}$ and $\dot{\theta}_{i\,\max}$ are the maximum angle and angular velocity. The inputs for NN are given by

$$u_1=\frac{\theta_2+\theta_{2max}}{2\theta_{2max}},\quad u_2=\frac{\theta_3+\theta_{3max}}{2\theta_{3max}},\ u_3=\frac{\dot{\theta}_1+\dot{\theta}_{1max}}{2\dot{\theta}_{1max}},$$
$$u_4=\frac{\dot{\theta}_2+\dot{\theta}_{2max}}{2\dot{\theta}_{2max}},\quad \text{and } u_5=\frac{\dot{\theta}_3+\dot{\theta}_{3max}}{2\dot{\theta}_{3max}}.$$

The inputs for the hidden layer and output layer are

$$I_k=\sum_{i=1}^{5} w_{ik}^I O_i^I,\quad I^O=\sum_{i=1}^{N_M} w_i^O O_i^O. \quad (37)$$

An output function of each neuron is a sigmoid function given by

$$O_i^*(\beta)=\frac{1}{1+e^{-\epsilon\beta}},\ (*=I,O) \quad (38)$$

where ϵ is a positive constant. The synaptic weights are learned by error back propagation.

2) State transition of Controller: Switching conditions derived by the above are summarized as

$$\begin{cases} U-Ucontroller \text{ if } O^O > O^s,\ |\theta_i-2\pi n_i| < \theta_{i\,\max} \\ U-Dcontroller \text{ else if } |\theta_2| < \theta_{\max} \\ D-Dcontroller \text{ else where} \end{cases}$$

where θ_{max} is a positive constant.

IV. Simulation result

The proposed method is evaluated by a numerical simulation. Parameters of the DFP used in the simulation are listed in Table III.
The sampling time is 2 ms. The initial condition is

$$x(0)=\begin{bmatrix}0.0 & 3.124 & 3.14 & 0.0 & 0.0 & 0.0\end{bmatrix}^T$$

,the maximum input torque is ± 3 Nm. The design parameters are
U-D controller

$$Q_u=\text{diag}(5.0, 5, 0\times10^3, 0.1\times10^{-6}, 10.0),$$
$$r_u=0.1\times10^{-6}, E_c=0, d=30.0, \theta_{max}=\frac{\pi}{9}$$

TABLE II
PARAMETERS OF NEURAL NETWORK

O_i^I	i-th output of input layer
O_i^H	i-th output of hidden layer
O^O	Output of output layer
I_i^H	Input for i-th neuron of hidden layer
I^O	Input for neuron of output layer
w_{ik}^I	Synaptic weight from i-th neuron of input layer to k th neuron of hidden layer
w_i^O	Synaptic weight from i th neuron of hidden layer to neuron of output layer
t_s	Teacher signal
u_i	Input for i-th neuron of input layer

TABLE III
PHYSICS PARAMETERS OF DFP

i	1	2	3
m_i(kg)	3.01	1.11×10^{-1}	2.90×10^{-2}
J_i(Nm·s^2/rad)	5.00×10^{-1}	8.63×10^{-4}	2.81×10^{-5}
C_i(Nm· s/rad)	1.60×10^{-1}	2.08×10^{-3}	5.92×10^{-4}
l_i(m)	-	1.44×10^{-1}	1.44×10^{-1}
r_i(m)	0.0	2.0×10^{-1}	5.0×10^{-2}

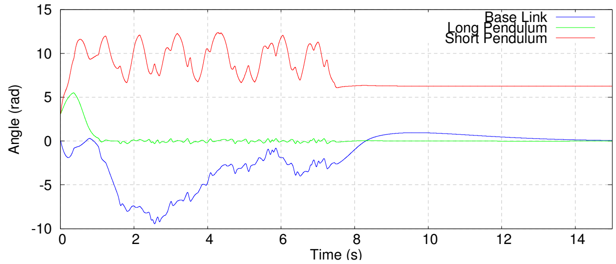

Fig. 4. Angle $\theta_1, \theta_2, \theta_3$(Simulation)

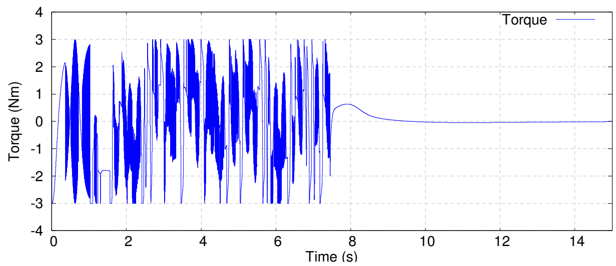

Fig. 5. Input torque τ_1(Simulation)

U-U controller

$$Q_{uu} = \text{diag}(0.1, 100.0, 100.0, 0.1 \times 10^{-6}, 0.1, 0.1),$$
$$r_{uu} = 1.0$$

The simulation result is shown in Figs. 4 and 5. From Figs. 4 and 5, it is found that the long pendulum is swung up and stabilized by switching the controller at 1.1(s) around the upright position ($\theta_1 = 0$). Then, the short pendulum is swung up after swinging up the long pendulum and stabilized by switching the controller at 7.4(s) around the upright position ($\theta_2 = 2\pi$). Although the input torque was saturated, the DFP was successfully swung up and stabilized. The simulation result shows that the proposed controller is effective to swing up and to stabilize the DFP.

V. Experimental result

In this section, experimental results using a DFP system are explained. Control experiments starts in the Down-Down condition of the DFP. An angular velocity is calculated by using the difference approximation. The design parameters are given as follows:

U-D controller

$$Q_u = \text{diag}(5.0, 5, 0 \times 10^3, 0.1 \times 10^{-6}, 10.0),$$
$$r_u = 0.1 \times 10^{-6}, E_c = 0, d = 50.0, \theta_{max} = \frac{\pi}{9}$$

U-U controller

$$Q_{uu} = \text{diag}(0.1, 100.0, 100.0, 0.1 \times 10^{-6}, 0.1, 0.1),$$
$$r_{uu} = 1.0$$

The allowable maximum torque and the time are the same as those of the previous simulation. The experimental results are shown in Figs. 6 and 7.

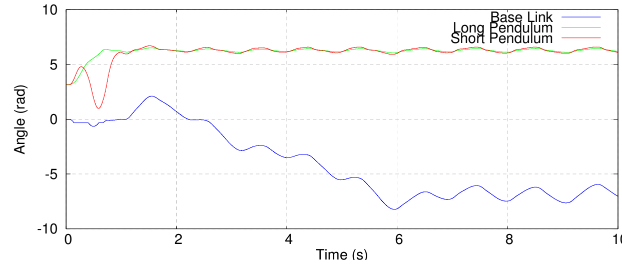

Fig. 6. Angle $\theta_1, \theta_2, \theta_3$(experiment)

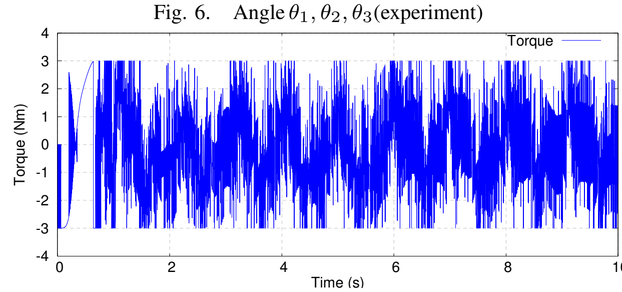

Fig. 7. Input torque τ_1(experiment)

From Figs. 6 and 7, it is found that the proposed controller swung up and stabilized the DFP ($\theta_1 = 2\pi, \theta_2 = 2\pi, \theta_3 = -2\pi$). Eventually, swinging up and stabilization control for the DFP has been achieved by the proposed controller. By using the safe manual control, during swinging up the short pendulum, the input was appropriately limited so that the long pendulum can be stayed at the upright position.

VI. Conclusions

In this paper, we have proposed control method to swing up and stabilize the DFP. The effectiveness was verified to through the numerical simulation and experiments for the DFP. A swing-up and stabilization simulation and experiment is a success. Hence, It is found that a proposed controller is effectiveness to swing up and stabilize the DFP.

References

[1] T. Sugie, M. Okada, "H^∞ Control of a Parallel Inverted Pendulum System," , *Proceedings of the Institute of Systems, Control and Information Engineers* vol. 6, No. 12, 1993, pp.543-551
[2] S. Hara, H. Iwadate, M. Yamakita, "Control Engineering Education through AUN/Seed-Net", , *SICE Annual Conference,* pp.499-502, 2007
[3] Johan Åkesson, Karl Johan Åström, "MANUAL CONTROL AND STABILIZATION OF AN INVERTED PENDULUM," if In Proc. 16th IFAC World Congress, Prague, 2005
[4] Johan Åkesson, Karl Johan Åström, "Safe Manual Control of the Furuta Pendulum," *Proceedings of the 2001 IEEE international Conference on Control Applications*, pp.499-502, 2001
[5] M. Iwase, K. J. Åström, K. Furuta and J. Åkesson "Analysis of safe manual control by using furuta pendulum," *Proceedings of the 2006 IEEE International Conference on Control Applications*, pp.568-572, 2006
[6] K. J. Åström, K. Furuta,"SWINGING UP A PENDULUM BY ENERGY CONTROL,"*IFAC 13th World Congress*, 1996
[7] Alamir. M, Murilo. A, "Swing-up and stabilization of a Twin-Pendulum under state and control constraints by fast NMPC scheme," *Automatica*, vol 44, 2008, pp. 1319-1324
[8] M. Yamakita, M.Iwashiro, Y.Sugahara, K. Furuta, "Robust Swing Up Control of Double Pendulum," *Proceedings of the American Control Conference*, pp.290-295, vol.1, 1995