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Real-Time Estimation Algorithm for the Center of Mass of a Bipedal Robot with Flexible Inverted Pendulum Model

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Abstract— **A closed-loop observer to extract the center of mass (CoM) of a bipedal robot is suggested. Comparing with the simple conversion equation of using just joint angle measurements, it enables to get more reliable estimates by using both joint angle measurements and F/T sensor outputs at the ankle joint. First, a nonlinear type observer is constructed in the extended Kalman filter framework to estimate the flexible rotational motion of biped. It is based on the inverted pendulum model with flexible beam which is to simply address the flexible behavior of a biped, specifically in the single support phase. Then, the predicted estimates of CoM by the flexible motion observer are combined with the outputs of the CoM conversion equation and the final estimates will be determined according to the weighting value which penalizes the flexible motion model and the CoM conversion equation. Simulation results are followed to show the effectiveness of the proposed scheme.**

I. Introduction

BOTH reliable sensory information and stable control algorithm are indispensable to achieve stable walking of a bipedal robot. The zero moment point (ZMP) [1, 2] is popularly used as a stability index of bipedal robot and the walking controller is usually designed so that the robot follows a ZMP pattern which has been generated to guarantee a stable walking [3-5]. Recently, the notion of whole body coordination is widely adopted, where all the limbs of arms and legs are involved in performing specific tasks or arbitrary gestures [6-9]. Also, a dual-loop walking controller was suggested to raise the walking stability by controlling the center of mass (CoM) trajectory together with the ZMP trajectory. In authors' experience, it was actually hard to realize a stable bipedal walking without implementing a CoM controller even in the flat plane.

In all these cases, we need real-time information on the center of mass of robot as well as ZMP during the bipedal motion. The ZMP history can be obtained with fair exactness by using F/T sensor at ankle joints, but still there is no suitable way to directly measure the global CoM position of the whole robot body. A possible way is to utilize the CoM conversion equation, the kinematic relationship between the global CoM and the local linkage parameters with joint angles. However, owing to the intrinsic uncertainties such as manufacturing tolerance, play at joints, and sensory errors, a big difference may happen between the real and the estimated. Furthermore, due to the structural characteristic of bipeds consisting of tens of links and joints, it is inevitable to experience flexible motions which cannot be detected by internal sensors.

Hence, in order to implement the whole body coordinated control through the CoM manipulation [7, 8] and also to ensure the stability margin of ZMP trajectory which is dynamically coupled with the CoM behavior in the ZMP/CoM dual-loop control [8, 9], it is earnestly desired to develop a CoM observer which enables to output more reliable estimates than currently available CoM conversion equation and integrate it to the robot controller.

As an extended version of [10], we are to suggest a dynamical estimation algorithm for the CoM position. We adopt an inverted pendulum model with flexible beam and a nonlinear observer is designed to estimate the flexible rotational motion of a biped by using the discrete Kalman filter. Then, the CoM predictions from the flexible motion observer and the outputs of CoM conversion equation are balanced to determine final estimates. The suggested CoM observer requires measurements of ankle joint torque as well as all the joint angles.

II. CoM Jacobian Resolution of Bipedal Robot

Bipedal robot is a kind of multi-body system with a lot of limbs (typically two arms, two legs and upper body), each of which consist of a number of links and joints. Also, it is a floating-body system where the origin of the body coordinates system (BCS) is moving along the walking motion. In order to implement a stable walking of bipedal robot, it is necessary to determine joint trajectories corresponding to the predefined ZMP/CoM trajectory for all sampling times. Then, we need a proper joint resolution scheme. The CoM Jacobian resolution method [7, 8] is a reasonable way to generate the joint trajectories to follow a given CoM trajectory while maintaining embedded motions.

In Fig. 1, the CoM position and the end-point position of each limb are respectively given by

$$c = r_0 + R_0\,{}^0c = r_0 + \sum\nolimits_{i=1}^{n} R_0\,{}^0c_i \qquad (1)$$

$$r_i = r_0 + R_0\,{}^0r_i \quad \text{with } i = 1,\cdots,n \qquad (2)$$

in the fixed world coordinate system (WCS), where r_0 is the

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origin of BCS, R_0 the rotational matrix indicating the orientation of BCS with respect to WCS, n the number of limbs, and the superscript 0 means "with respect to BCS." Then, ${}^0c$ denotes the position of the whole body CoM, ${}^0c_i$ the center of mass position of the i-th limb, and ${}^0r_i$ the end-point position of the i-th limb in BCS.

It we let $\dot{x}_0 = [\dot{r}_0 \quad \omega_0]^T$ the velocity of the body center, $\dot{x}_i = [\dot{r}_i \quad \omega_i]^T$ the end-point velocity of the i-th limb, $\dot{q}_i$ the joint velocity of i-th limb, and J_i the Jacobian of i-th limb represented on WCS, the following relationship is given.

$$\dot{x}_0 = \begin{bmatrix} \dot{r}_0 \\ \omega_0 \end{bmatrix} = X_i(\dot{x}_i - J_i\dot{q}_i) \text{ with } X_i = \begin{bmatrix} I_3 & [R_0{}^0r_i\times] \\ 0_3 & I_3 \end{bmatrix} \quad (3)$$

where $[(\cdot)\times]$ is a skew-symmetric matrix for the cross product. By differentiating (1), we have

$$\begin{aligned} \dot{c} &= \dot{r}_0 + \omega_0 \times \sum\nolimits_{i=1}^{n} R_0{}^0c_i + \sum\nolimits_{i=1}^{n} R_0{}^0\dot{c}_i \\ &= \dot{r}_0 + \omega_0 \times (c - r_0) + \sum\nolimits_{i=1}^{n} R_0{}^0J_{ci}\dot{q}_i \end{aligned} \quad (4)$$

where ${}^0J_{ci}$ is the CoM position Jacobian of the i-th limb. Again, the above equation can be rewritten as

$$\begin{aligned} \dot{c} &= \dot{r}_1 + \omega_1 \times (c - r_1) - J_{v1}\dot{q}_1 + (c - r_0) \times J_{\omega1}\dot{q}_1 + J_{c1}\dot{q}_1 \\ &\quad + \sum\nolimits_{i=2}^{n} J_{ci}J_i^+(\dot{x}_i - X_{i1}\dot{x}_1) + \sum\nolimits_{i=2}^{n} J_{ci}J_i^+X_{i1}J_1\dot{q}_1, \end{aligned} \quad (5)$$

where $X_{i1} = X_i^{-1}X_1$, $J_{ci} = R_0{}^0J_{ci}$, and the subscript i = 1 indicates the supporting leg and also i = 2 the shifting leg during the successive alternating gaits.

By letting the end-point of the supporting limb stationary during the gaits, that is $\dot{x}_1 = 0$ ($\dot{r}_1 = \omega_1 = 0$) in (5), we have the following linear relationship between the CoM velocity and the joint velocity of the supporting limb.

$$\begin{aligned} &\underbrace{\dot{c} - \sum\nolimits_{i=2}^{n} J_{ci}J_i^+\dot{x}_i}_{\dot{c}_{\text{fsem}}} \\ &= \underbrace{(-J_{v1} + (c - r_0) \times J_{\omega1} + J_{c1} + \sum\nolimits_{i=2}^{n} J_{ci}J_i^+X_{i1}J_1)}_{J_{\text{fsem}}}\dot{q}_1 \quad (6) \\ &\rightarrow \dot{c}_{\text{fsem}} = J_{\text{fsem}}\dot{q}_1 \end{aligned}$$

while $J_1 = [J_{v1} \; J_{\omega1}]^T$ and $\dot{x}_1 = [\dot{r}_1 \quad \omega_1]^T$.

When $\dot{x}_1 = 0$, we know that $\omega_0 = -J_{\omega1}\dot{q}_1$ from (3). By augmenting this to (6), we have

$$\begin{bmatrix} \dot{c}_{\text{fsem}} \\ \omega_0 \end{bmatrix} = \begin{bmatrix} J_{\text{fsem}} \\ -J_{\omega1} \end{bmatrix}\dot{q}_1 \quad (7)$$

Finally, when the CoM trajectory, end-point trajectory of each limb, and the angular velocity of the body center are given, the desired joint velocity of the supporting limb can be determined as

$$\dot{q}_{1d} = \begin{bmatrix} J_{\text{fsem}} \\ -J_{\omega1} \end{bmatrix}^+ \begin{bmatrix} \dot{c}_{\text{fsem,d}} \\ \omega_{0,d} \end{bmatrix} \quad (8)$$

Since the velocity of the body center is the same with respect to all the limbs, from (3) we have

$$\dot{x}_0 = X_i(\dot{x}_i - J_i\dot{q}_i) = X_1(\dot{x}_1 - J_1\dot{q}_1) \quad (9)$$

Then, noting that $\dot{x}_1 = 0$, the joint velocity of the other limbs can be determined as

$$\dot{q}_{id} = J_i^+(\dot{x}_{id} - X_{i1}J_1\dot{q}_{1d}), \; i = 2, \cdots, n \quad (10)$$

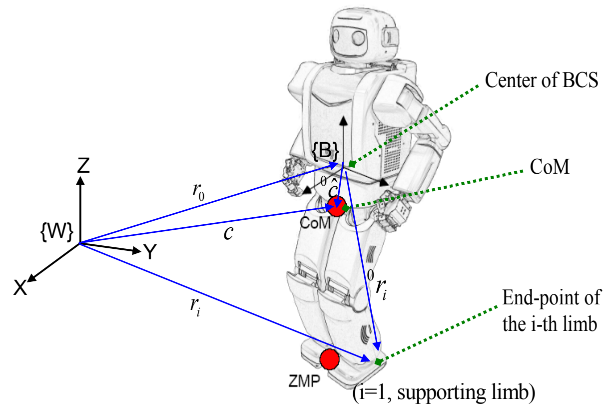

Fig. 1. Position of the BCS origin, CoM, and end-point of the i-th limb.

III. Modeling of Walking Motion

A. Rigid Inverted Pendulum Model

In generating walking pattern of a bipedal robot, an inverted pendulum model has been widely adopted, where the total mass is considered to be concentrated at the center of mass and the pendulum length is varied so that the CoM position in Z-direction is fixed [3-8].

The zero moment point (ZMP) of a robot is defined as the point where the reaction moment about any axis on the walking plane is zero. It is well-known that the ZMP should lie inside the polygon of footprint boundaries in order for a biped not to be tumbled. In Fig. 2, if it is assumed that the Z-directional CoM position (c_z) is fixed in the sagittal plane, we have the following relationship between ZMP and CoM.

$$\begin{aligned} &p_i(t) = c_i(t) - \frac{c_z}{g}\ddot{c}_i(t) \\ &\rightarrow \ddot{c}_i(t) - \omega_n^2 c_i(t) = -\omega_n^2 p_i(t) \;\text{ for }\; i = x, y \end{aligned} \quad (11)$$

where $\omega_n^2 = g / c_z$ and the ZMP $\bar{p} = (p_x, p_y)$ in X and Y directions can be obtained by converting the torque measurements at the ankle joints. In terms of the above equation, it is possible to plan the reference trajectory of CoM which is dynamically matched with the ZMP pattern and utilize it in the bipedal robot control.

In Fig. 2, the center of mass of each linkage (${}^oc_{ik}$) can be determined by using given linkage parameters and joint sensor measurements and then the position of the whole-body CoM with respect to BCS is given by

$${}^oc = \sum_i \sum_k \mu_{ik}{}^oc_{ik} \;\text{ with }\; \mu_{ik} = \frac{m_{ik}}{m} \quad (12)$$

where m_{ik} and ${}^{o}c_{ik}$ are the mass and center of mass of the k-th link in the i-th limb, respectively. Hence, the CoM position equation in (1) can be rewritten as

$$c = r_0 + R_0 \sum_i \sum_k \mu_{ik} \, {}^{o}c_{ik} \tag{13}$$

The above is a currently available equation to get the CoM position of a bipedal robot and it could be a complete equation for any rigid multi-body system not considering joint flexibility. However, it may invoke a large error due to the play at joints and transmissions, the flexibility of links, and the uncertainty of the body center position.

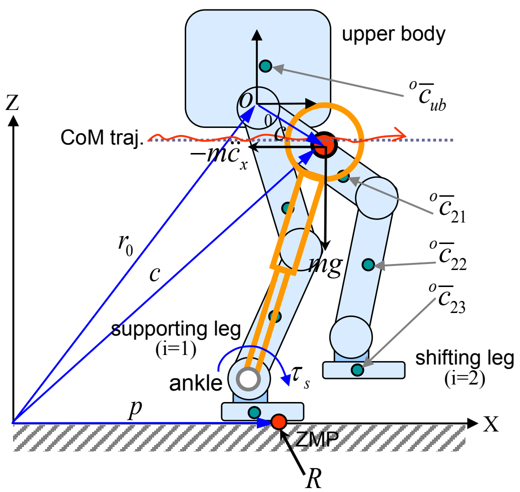

Fig. 2. Rigid inverted pendulum model for a biped.

B. Flexible Inverted Pendulum Model

Considering the characteristics of a bipedal robot which consists of a lot of joints, it inevitably shows flexible motion during the walking motion. Actually, most of the uncertainties in determining the CoM position results from the flexibility of robot mechanism which cannot be detected by joint sensors. The flexible motion will become severer in the single support phase when one leg is swing than the double support phase and in XZ sagittal plane rather than in YZ frontal plane. In case of a human-sized bipedal robot, it denotes a lightly damped flexible mode which keeps residual vibration quite a long time [11].

In Fig. 3, the walking motion of a biped can be assumed to be a rotational motion of the flexible inverted pendulum about the ankle joint. It is assumed that the flexible beam is massless and the total mass is concentrated at the center of mass. And the beam length is varied according to the CoM position. In the sagittal plane, the sum of moments with respect to the origin of the toque sensor is given by

$$\sum M_s = mgl\sin\theta - c_b(\dot{\theta}-\dot{u}) - k_b(\theta-u) = ml^2\ddot{\theta} \tag{14}$$

where u is the rotational angle of the CoM of rigid body when the flexibility of mechanism is not considered at all and θ denotes that of flexible body. Also, k_b and c_b are the effective stiffness and damping coefficient of the flexible beam model, which must be identified through a proper experimental technique.

By neglecting the small magnitude of the damping term in (14), a simple nonlinear model for the flexible motion of a biped is given by

$$ml^2\ddot{\theta}(t) + k_b\theta(t) - mgl\sin\theta(t) = ku(t) \tag{15}$$

which expresses a dynamical relationship between the input of rigid body rotation and the output of flexible body rotational motion.

In general, the torque sensor detects only the difference between the external force and the inertial force. Hence, in (14), the torque sensor at the ankle joint is to produce outputs as much as the sum of spring and damping force due to the relative motions between the rigid and flexible mode of the biped. And we have the measurement model of the torque sensor:

$$\tau_s = k_b(\theta-u) + c_b(\dot{\theta}-\dot{u}) \tag{16}$$

Similarly, in the frontal YZ plane, it is possible to obtain the same forms of equations as (15) and (16) but with different stiffness and damping coefficient.

In the next sections, the flexible motion model (15) will be applied to the design of a CoM estimation algorithm, while the rigid inverted pendulum model (11) is used to generate a CoM reference trajectory in the walking simulation.

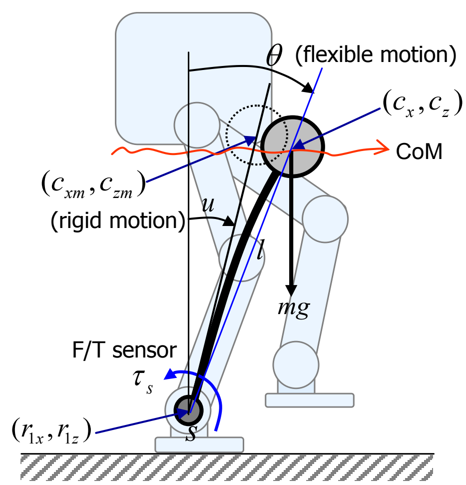

Fig. 3. Flexible inverted pendulum model for the single support phase of bipedal walking in the sagittal plane.

IV. CoM Estimation Algorithm Based on the Flexible Motion Observer

First of all, a flexible motion observer is constructed to estimate the rotational angle of CoM in the extended Kalman filter framework. It uses the motion model (15) of the flexible inverted pendulum and the measurement model (16) for the ankle torque. Then, by combining the predicted CoM position using the flexible motion observer with the output of the CoM conversion equation in (11), we propose a real-time algorithm to track the change of CoM position.

A. Flexible Motion Observer

If a state vector $\mathbf{x} = [x_1 \quad x_2]^T = [\theta \quad \dot{\theta}]^T$ is defined, (15) can be transformed into the state equation:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} x_2 \\ \frac{g}{l}\sin x_1 - \frac{k_b}{ml^2} x_1 \end{bmatrix} + \frac{k_b}{ml^2}\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \mathbf{w}(t) \qquad (17)$$

where $\mathbf{w} \sim N(0,Q)$ denotes the model uncertainty.

By approximating the above continuous-time equation with discrete sampling time, it can be rewritten as a discrete state equation:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{G}u_k + \mathbf{w}_k \qquad (18)$$

where $\mathbf{x}_k = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $\mathbf{G} = \frac{k_b}{ml^2}\begin{bmatrix} 0 \\ h \end{bmatrix}$, h is the sampling time, and $\mathbf{f}(\mathbf{x}_k) = \begin{bmatrix} x_1(k) + hx_2(k) \\ x_2(k) + h\left(\frac{g}{l}\sin x_1(k) - \frac{k_b}{ml^2} x_1(k)\right) \end{bmatrix}$.

If the small damping term is neglected in (16), we have the following measurement model for the flexible rotational angle.

$$\theta_m(k) = \frac{1}{k_b}\tau_s(k) + u(k) + v(k) \qquad (19)$$

where $v(k) \sim N(0,R)$ is the sensor noise.

Given the motion model (18) and measurement one (19), a recursive algorithm can be constructed to determine the estimates of flexible rotational angle by adopting the extended Kalman filter [12]. In Table I, the predictor produces prior estimate in terms of the flexible motion model and the posterior estimate by the corrector corresponds to best estimates in the respect of the minimum variance of the estimation error.

In order to implement the above flexible motion observer, ankle torque τ_k, rotational angle of rigid pendulum u_k, and the time-varying length of pendulum l_k must be updated at every sampling time. Also, the mass m and the spring constant k of the flexible beam should be given through an extra identification process.

In this paper, only the sagittal plane walking is assumed. Then, the center of mass $\mathbf{c}_m = (c_{xm}, c_{zm})$ of the rigid pendulum in Fig. 3 can be determined according to (13) using joint sensor measurements. If the origin of torque sensor is assumed to be identical to the end-point $r_1 = (r_{1x}, r_{1z})$ of the supporting leg, we have the rigid body rotational angle:

$$u(k) = \tan^{-1}\left(\frac{c_{xm}(k) - r_{1x}(k)}{c_{zm}(k) - r_{1z}(k)}\right) \qquad (20)$$

Also, when the ankle joint and CoM positions are given, the straight length of pendulum can be determined by

$$l_k^2 = \left(c_x(k) - r_{1x}(k)\right)^2 + \left(c_z(k) - r_{1z}(k)\right)^2 \qquad (21)$$

where the CoM position (c_x, c_z) can be substituted with the real-time estimates.

It is certain that the effect of the above flexible motion observer is largely dependant upon the reliability of model parameters. The total mass of robot can be readily obtained by using a scale and the length of pendulum can be evaluated with (21). However, the stiffness value k_b could be greatly changed according to the variation of robot posture. Bipedal walking is a repetition of single support and double support by alternating supporting leg and shifting one. In fact, dominant flexible modes of a bipedal robot are invoked during the swing motion of the single support phase, where the stiffness of robot structure is relatively much weaker than the double support phase. Actually, the suggested model in Fig. 3 is appropriate for the single support phase. Related to the experimental identification for the bipedal robot stiffness, an effective way was suggested in [11].

TABLE I. FLEXIBLE MOTION OBSERVER.

Motion model: $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{G}u_k + \mathbf{w}_k$, $\mathbf{w}_k \sim N(0,Q)$
Measurement model: $z_k = \frac{1}{k}\tau_k + u_k + v_k$, $v_k \sim N(0,R)$
Predictor : $\hat{\mathbf{x}}_k^- = \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+) + \mathbf{G}u_{k-1}$
Covariance of the prior estimation error: $P_k^- = A_{k-1}P_{k-1}^+A_{k-1}^T + Q$
Covariance of the posterior estimation error $P_k^+ = (I - K_kH)P_k^-$
Kalman gain : $K_k = P_k^-H^T\left(HP_k^-H^T + R\right)^{-1}$
Corrector: $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + K_k\left((\frac{1}{k_b}\tau_k + u_k) - \hat{x}_{1,k}^-\right)$
System Jacobian: $A_k = \begin{bmatrix} 1 & h \\ h(\frac{g}{l_k}\cos\hat{x}_{1,k} - \frac{k_b}{ml_k^2}) & 1 \end{bmatrix}$,
Measurement Jacobian: $H = [1 \quad 0]$

B. CoM Estimation Algorithm

The overall flow of the CoM estimation algorithm is denoted in Fig. 4, where the CoM predictor determines the prediction values of CoM by using the real-time output of the flexible motion observer as follows.

$$\hat{\mathbf{c}}_k^- = \begin{bmatrix} \hat{c}_x^-(k) \\ \hat{c}_z^-(k) \end{bmatrix} = \begin{bmatrix} l(k)\sin\hat{\theta}(k) + r_{1x}(k) \\ l(k)\cos\hat{\theta}(k) + r_{1z}(k) \end{bmatrix} \qquad (22)$$

Then, the CoM corrector produces final CoM estimates by combining the prediction values in (22) and the measurements in (13) as

$$\hat{\mathbf{c}}_k^+ = \hat{\mathbf{c}}_k^- + K_g(\mathbf{c}_m - \hat{\mathbf{c}}_k^-) \qquad (23)$$

where the gain matrix consists of weighting values. Here, if we assume σ_1^2 and σ_2^2 the variances of uncertainties of predicted values and measurements, respectively, the gain matrix can be determined as

$$K_g = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (24)$$

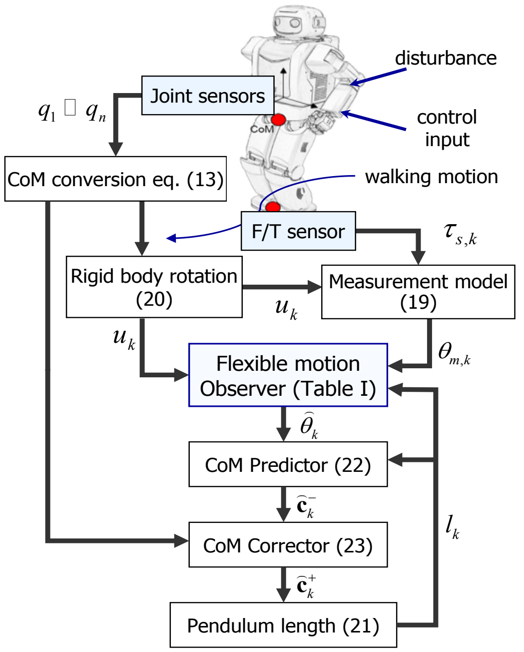

Fig. 4. Overall flow of the CoM estimation algorithm.

V. SIMULATION

A. Walking Pattern Generation

For the 6-DOF bipedal robot shown in Fig. 2, the parameter s of two legs and upper body are given in Table II and the center of mass of each linkage is assumed to be located at its geometric center. The walking pattern for the sagittal plane motion is given in Fig. 5, which includes the reference trajectories of ZMP and CoM in X-direction and the end-point trajectory of the swing leg (r_{2x}, r_{2z}) in X and Z directions. The CoM trajectory is the solution of (11) for the specific ZMP pattern. In Fig. 5, the supporting phase is changed as double support (0~0.5 sec), single support (0.5~1.5 sec), and double support (1.5~2 sec). Also, the joint trajectories for the walking pattern were generated according to the resolution scheme (8) and (10). As a result, the two legs of a biped carry out the joint angle configuration change like Fig. 6 during a single gait.

B. CoM Estimation Performance

The flexible rotational equation of motion (14) was used as the real inverted pendulum model with the parameter values of m = 26 kg, k = 1000 Nm/rad, c = 7 Nm/rad, and l = 0.8 m. And the model parameters of the flexible motion observer in Table I were assumed to contain 15% errors with respect to the real values. Fig. 7 denotes the simulation results of the flexible motion observer, where the input u denotes the rigid body rotation, $(\theta, \dot{\theta})$ are the time responses of flexible inverted pendulum according to the model (14), and $(\hat{\theta}, \hat{\dot{\theta}})$ are the observer outputs. It shows that the observer well reconstructs the real states in spite of the large parametric errors.

Now, Fig. 8 is the result of the CoM estimation algorithm for the reference trajectories in Fig. 5, where the input of rigid body rotation will be naturally determined by (20) in the closed-loop observer. As shown, the proposed algorithm ('estimaor') enables to reduce the estimation error when compared with the open-loop style of CoM conversion equation ('encoder').

TABLE II. LINKAGE PARAMETERS.

leg	mass (kg)	length (m)
left	$m_{11}=4$ (thigh), $m_{12}=3$ (calf), $m_{13}=1$(foot)	$l_{11}=0.5$ (thigh), $l_{12}=0.5$ (calf), $l_{13}=0.1$ (foot)
right	$m_{21}=4$ (thigh), $m_{22}=3$ (calf)), $m_{13}=1$ (foot)	$l_{21}=0.5$ (thigh), $l_{22}=0.5$ (calf), $l_{23}=0.1$ (foot)
upper body	$m_{ub}=10$	$l_{ub}=0.5$

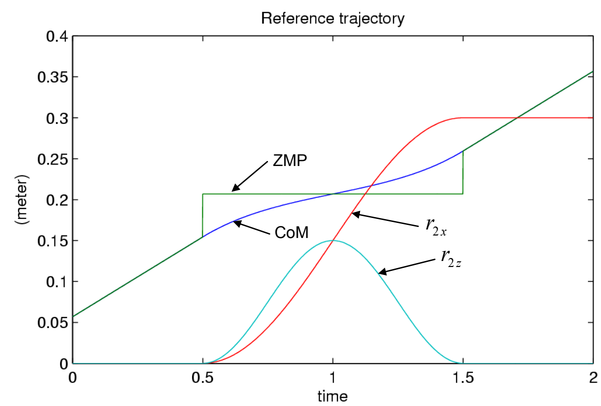

Fig. 5. Reference trajectories of ZMP, CoM, and the end-point of swing leg during 30 cm single stride.

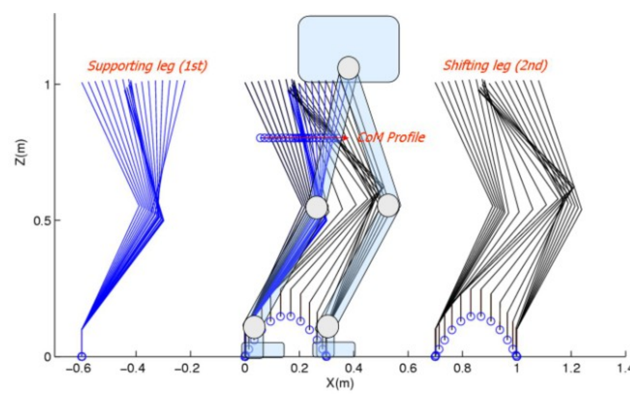

Fig. 6. Change of joint angle configurations during a single gait.

VI. CONCLUSION

Comparing with the CoM conversion equation in (13) which depends on joint measurements only, the CoM estimation algorithm in Fig. 4 enables to produce more reliable estimates with the aid of torque sensor outputs at the ankle joint and the simple inverted pendulum model. In the simulations, we have used the same values for model parameters. However, in applying the algorithm to the real biped, the following works must be considered: i) sensivity of the CoM estimation performance to the change of model parametric errors, ii) identification of stiffness parameters according to the supporting phase transitions.

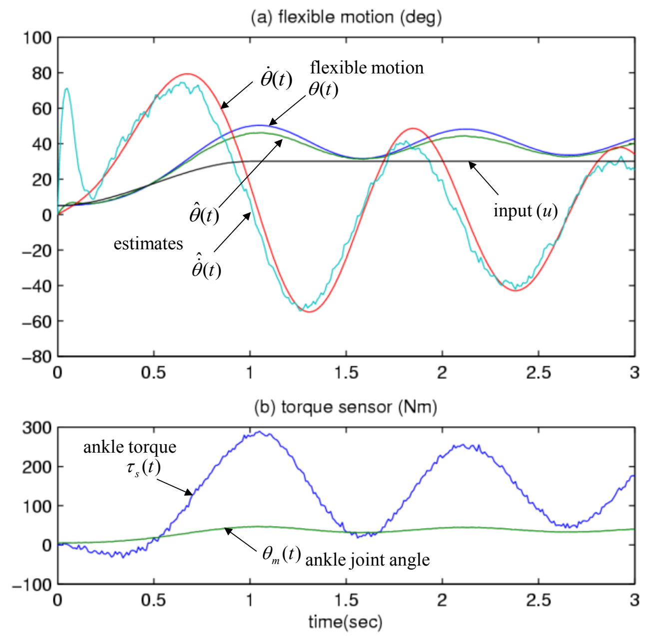

Fig. 7. Performance simulation of the flexible motion observer. (a) flexible motion and the estimates (b) ankle torque and the conversion value (19) of angle joint angle.

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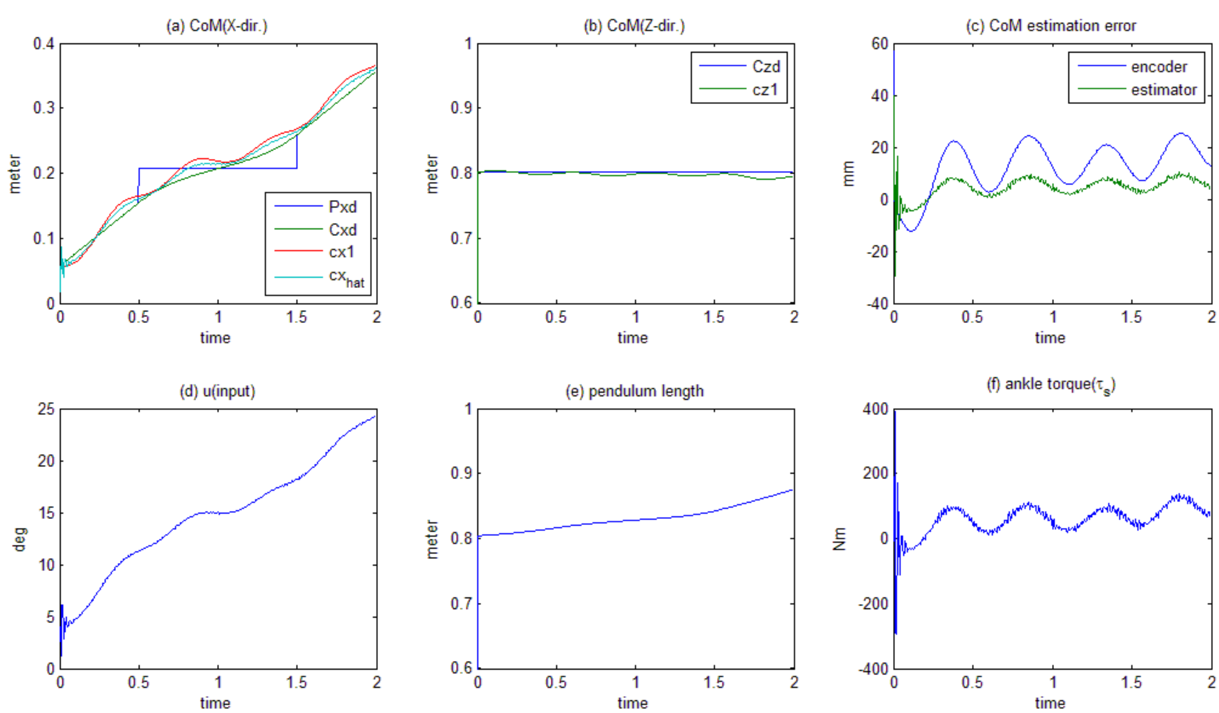

Fig. 8. Simulation results of the proposed CoM estimation algorithm. Pxd: ZMP reference in X-direction, (Cxd, Czd): CoM reference in (X, Z) directions, (cx1, cz1): real CoM, cxhat: estimated CoM.