The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

A Walking Pattern Generation Method with Feedback and Feedforward Control for Humanoid Robots

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Abstract— **This paper proposes a new walking pattern generation method for humanoid robots. The proposed method consists of feedforward control and feedback control for walking pattern generation. The pole placement method as a feedback controller changes the poles of system in order to generate more stable and smoother walking pattern. The advanced pole-zero cancelation by series approximation(PZCSA) as a feedforward controller plays a role of reducing the inherent property of linear inverted pendulum model (LIPM), that is, non-minimum phase property due to an unstable zero of LIPM and tracking efficiently the desired zero moment point (ZMP). The efficiency of the proposed method is verified by three simulations such as arbitrary walking step length, arbitrary walking phase time and sudden change of walking path.**

I. INTRODUCTION

Humanoid robots have been developed for the convenience of human beings. For example, these robots will play an important role in the assistance of the elderly people in aging societies and emergencies like the scene of a fire and construction site with humans in the future. Until now, many humanoid robots have been developed successfully [1], [2], [9]. And there has been a lot of research on humanoid robots relating to balancing, walking pattern generation, motion generation, and so on. This paper deals with the walking pattern generation method which is an essential part for stable walking.

Much research on walking pattern generation has been proposed in order to generate stable walking patterns based on zero moment point(ZMP)[2], [3], [5], [6], [7], [8], [13], [14], [15]. Yamaguchi *et al.* proposed a set of particles to describe the humanoid robot and used Fourier series to generate stable walking [2]. However, their method needs an iterative procedure for proper upper motion which guaranteed stability, which made it difficult to implement in real-time. Yang *et al.* implemented a walking motion with the neural oscillator which imitates the nervous system of human body [13]. The humanoid robot has walked successfully on flat and sloped planes using this method. However this method has a complex and difficult process to set proper parameters for walking successfully. Harada *et al.* introduced an analytical method on real-time gait planning for humanoid robots [6]. It made use of cubic polynomials of ZMP trajectory to satisfy initial and terminal conditions. Although it makes the real-time walking pattern possible, it is difficult to generate various walking paths due to cubic polynomials of ZMP. Oh *et al.* also proposed an analytical method using linear polynomials. Their method utilized the non-minimum phase property of the linear inverted pendulum model(LIPM) [5]. However, it is also insufficient to satisfy the desired ZMP which has been changed largely. Kajita *et al.* proposed a walking pattern generation method using preview control [3]. This optimal method obtained a good result using forthcoming values of the desired ZMP. However, since it utilizes high gain of a preview control, it is very sensitive to sudden changes of desired ZMP trajectory. To overcome sudden changes, they additionally introduced the auxiliary ZMP control, recently [4].

This paper introduces an omni-directional and real-time walking pattern generation method. Basically the proposed method utilizes LIPM consisted of ZMP and center of mass(CoM). To generate omni-directional walking patterns for humanoid robots, a feedback and a feedforward control concept are employed.

This paper is organized as follows: Section II describes the linear inverted pendulum model as the simplified model for humanoid robots. And section III introduces the overall system for generating more effective walking patterns of the ZMP and the CoM. Section IV shows the results of the simulations to verify the efficiency of the proposed method. Finally we conclude this paper in section V.

II. SIMPLIFIED HUMANOID ROBOT MODEL

The humanoid robot requires at least 12-DOFs in order to walk on the human environment. If we take the full dynamics of humanoid robots into consideration, more accurate results can be obtained. However, the full dynamics demands complicated and tedious computations and requires a lot of time to compute the proper walking pattern. Therefore we use the simplified humanoid robot model for walking pattern.

Many researchers have utilized LIPM as a simplified humanoid robot model. LIPM consists of the CoM and the ZMP as shown in Fig. 1. In the figure, $p(t)$ and $c(t)$ denote the ZMP and the CoM, respectively. The ZMP is utilized as the criterion of stability of the robot. The CoM stands for the entire motion of the humanoid robot instead of the full dynamics of the multi-body.

Here, let us assume that CoM can move around the specific z-axis and the time derivative of angular momentum can be neglected because the effect of angular momentum is slight in walking. If we do not use two assumptions, LIPM

This work was supported in part by Ministry Knowledge Economy(MKE) & Institute for Information Technology Advancement(IITA) through IT Leading R&D support Project.

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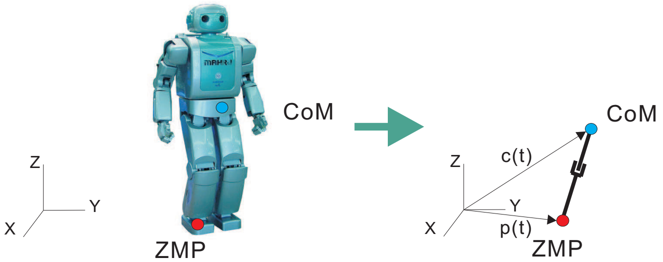

Fig. 1. Simplified humanoid robot model

demands complicated computations because the relation between the ZMP and the CoM on one axis is actually coupled by the components of other axis.

The relation between the ZMP and the CoM can be represented as follows.

$$p(t) = c(t) - \frac{1}{\omega^2}\ddot{c}(t) \tag{1}$$

where ω is $\sqrt{g/(Z_{CoM} - Z_{ZMP})}$. Z_{CoM},Z_{ZMP} and g are the value of the CoM and the ZMP on the z-axis and the gravity acceleration separately. According to (1), we can control the CoM and the ZMP. This paper designs the desired ZMP trajectory according to footprints. Then reference values of the ZMP and the CoM are generated to track the desired ZMP trajectory using the proposed tracking control.

To generate walking pattern, we transfer (1) into the state equation[3]. For the state equation, we define u, $\mathbf{z}$ as follows.

$$u = \frac{d}{dt}\ddot{c}, \quad \mathbf{x} = \{c \quad \dot{c} \quad \ddot{c}\}^T$$

where u is defined to guarantee the continuity of CoM up to the acceleration level.

Then the state space model of LIPM can be represented by

$$\dot{\mathbf{x}}(t) = \mathbf{A}^*\mathbf{x}(t) + \mathbf{B}^*u(t) \tag{2}$$
$$y(t) = \mathbf{C}^*\mathbf{z}(t) \tag{3}$$

where

$$\mathbf{A}^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \; \mathbf{B}^* = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}, \; \mathbf{C}^* = \begin{bmatrix} 1 & 0 & -\frac{1}{\omega^2} \end{bmatrix},$$

and the output $y(t)$ denotes the ZMP, *i.e.,* $y(t) = p(t)$.

For real-time implementation, it is more convenient to convert a continuous system into a discrete one. To this end, (2) and (3) are discretized into the following form.

$$\mathbf{x}(k+1) = \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,u(k) \tag{4}$$
$$y(k) = \mathbf{C}\,\mathbf{x}(k) \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{Bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{Bmatrix}, \quad \mathbf{C} = \mathbf{C}^*,$$

and T is the sampling time.

In the next section, the walking pattern generation method will be explained in detail

III. WALKING PATTERN GENERATION METHOD

A. Pole and Zero Characteristics of LIPM

We can analyze the poles and zeros of LIPM because the parameters of LIPM is given by the humanoid robot's mechanic property. LIPM has three poles and two zeros in discrete frequency domain. Three poles which are placed on unit circle make the LIPM to be marginally stable. These poles have the influence on the settle time and overshoot of the system. With our method, the feedback controller moves these poles in order that the system becomes more stable. In case of zeros, one is out of the unit circle and the other is inside the unit circle. The zero out of the unit circle is called unstable zero. The unstable zero makes the unwanted undershoot behavior in initial parts of the system response. When we generate the walking pattern with (1), the unstable zero forces the reference ZMP, *i.e.,* $y(k)$ to move the opposite direction of the desired ZMP, *i.e.,* $y_d(k)$. This undershoot behavior is produced by odd unstable zeros[12]. The effect can not be eliminated by the feedback controller only. However, by use of a proper feedforward control method, the effect can be reduced sufficiently.

Fig. 2 shows the conceptual block diagram of the overall system to generate the reference ZMP and CoM for walking. The proposed method makes use of the feedback controller for stabilizing the system of LIPM and improving the system property like settling time and overshoot. And this method utilizes the feedforward controller for the reference ZMP $y(k)$ to track the desired ZMP $y_d(k)$ effectively. The reference CoM $x(t)$ is obtained by LIPM state equation. In the following subsections, we will describe a feedback controller and a feedforward controller in detail.

B. Feedback Control Method

In order to stabilize the system, the pole placement method is used for a feedback controller as shown in Fig. 3. The pole placement method locates the poles of the closed loop system properly to ensure the satisfactory transient response and improve stability. For satisfying the design purpose, the state feedback gain of pole placement method can be obtained by Ackermann's formula. The purpose of the feedback controller is to guarantee stability and improve the transient response like settling time and overshoot of LIPM. When we design three designed poles for this purposes, one

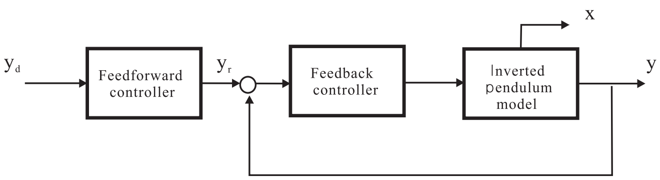

Fig. 2. Conceptual block diagram of the overall system for walking pattern generation

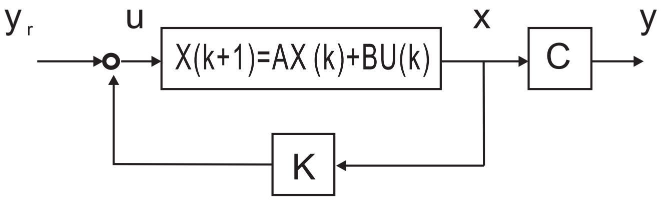

Fig. 3. Block diagram of feedback control

designed pole is decided with the same value of a zero which is inside the unit circle. Since the designed pole removes the stable zero of LIPM, the overall system can be transferred into the system with two poles and one zero. And the other two poles of designed poles are determined considering the proper settling time and overshoot. This transient property has greatly influences on the system response in case of a sudden change of the desired ZMP. The sudden change of the desired ZMP happens in walking pattern generation when the humanoid robot avoids the external object which appears unexpectedly.

C. Feedforward Control

The purpose of the feedforward controller is to minimize the tracking error between input and output. If the system transfer function is known, the feedforward controller can utilize the pole-zero cancelation method to reduce the tracking error. When the closed loop system is minimum phase system, perfect tracking control can be obtained[10]. But when it is non-minimum phase system, the feedforward controller which consists of the inverse of the non-minimum phase system becomes unstable.

The non-minimum phase system is due to odd unstable zeros which is out of the unit circle in discrete domain. As mentioned in previous subsection, the LIPM has an unstable zero. The effect of the unstable zero makes the initial response of the system to move the opposite direction of the desired response[12]. The research on reducing the effect of unstable zeros on tracking performances has been done by many researchers. Zero error phase tracking control(ZPETC) algorithm was introduced how to reduce the effect of unstable zero[10]. Phase delay and the gain error of the low frequency are reduced using ZPETC, but ZPETC has the large gain error of high frequency. To compensate the gain error of the ZPETC, many researches have been proposed. This paper makes use of pole-zero cancelation by series approximation(PZCSA) control as an alternative to the ZPETC [11]. This method is compensated by feedforward terms composed of truncated power series.

PZCSA consists of two parts as shown in Fig. 4. One part reduces the effect of the unstable zero with series approximation of the inverse form of the unstable zero. The other part eliminates the dynamics of the closed loop system except the unstable zero by use of the pole-zero cancelation.

The closed loop system including LIPM and feedback controller is expressed as the following equation.

$$G_{closed}(z^{-1}) = \frac{z^{-d}N_s(z^{-1})N_u(z^{-1})}{D(z^{-1})} \qquad (6)$$

where z^{-d} represents a d-step delay, and $N_s(z^{-1}),\ N_u(z^{-1})$ describe stable zeros, unstable zeros, respectively.

The ideal feedforward controller is the inverse of (6). But since the inverse of $N_u(z^{-1})$ becomes the unstable poles, the implementation of inverse form $N_u(z^{-1})$ is impossible. Therefore PZCSA approximates $N_u(z^{-1})$ with series expansion about the inverse form of $N_u(z^{-1})$.

For example,

$$\begin{aligned}\frac{1}{z-z_u} &= z^{-1}+z_u z^{-2}+z_u^2 z^{-2}+\cdots \quad |z|>|z_u| &(7)\\ &= -\frac{1}{z_u}-\frac{1}{z_u^2}z-\frac{1}{z_u^3}z^2+\cdots \quad |z|<|z_u| &(8)\end{aligned}$$

Since $|z_u| > 1$, (8) approximates the unstable zero in this paper.

Equation (8) shows that the compensation is composed of the noncausal parts. In other words, PZCSA is implemented with the forthcoming values of input so as to reduce the effect of the unstable zero.

Equation (8) points out another fact that if we use the infinite approximation, we can remove the effect of the unstable zero. However we can only deal with the finite summation of (8).

$$\frac{1}{z-z_u} \simeq \sum_{n=0}^{N_{order}-1} -(\frac{1}{z_u}z)^n \qquad (9)$$

where z_u represents the unstable zero and N_{order} is the number of forthcoming information.

Thus the overall system response has tracking errors because of using the finite approximation. The overall system response can be expressed as follows:

$$y(k) = y_d(k) - \frac{1}{{z_u}^{N_{order}}}\, y_d(k+N_{order}) \qquad (10)$$

The second term on the right-hand side of (10) represents the tracking error due to the finite approximation. When the z_u is known, the tracking error depends on N_{order}.

Fig. 5 and 6 show the results that PZCSA approximates the inverse form of the unstable zero according to the number of forthcoming values. In Fig. 5 and 6, the phase difference between $1/(z-z_u)$ and PZCSA seems large. But since the phase difference is 2π times, it is not a huge problem despite the large phase difference.

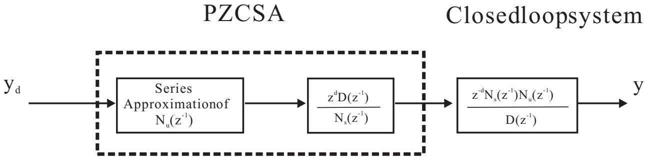

Fig. 4. Block diagram of PZCSA and the closed loop system

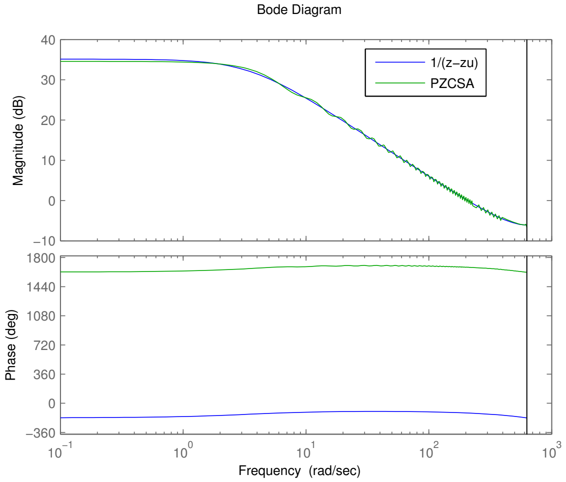

Fig. 5. Bode plot of $1/(z - z_u)$ and PZCSA with 160 forthcoming information on the desired ZMP($z_u = 1.0175$).

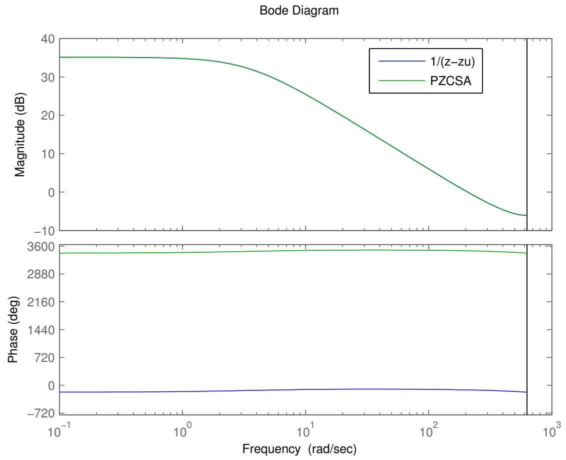

Fig. 6. Bode plot of $1/(z - z_u)$ and PZCSA with 320 forthcoming information on the desirede ZMP($z_u = 1.0175$).

Here, the larger N_{order} obtains more accurate results. However, the large N_{order} requires that the N_{order} number of the desired ZMP are known for the walking pattern generation in advance.

This paper proposes the advanced PZCSA to reduce the tracking error due to the finite approximation as shown in (10). The second term on the right-hand side of (10) as the tracking error may increase if $y_d(k + N_{order})$ increases in long-term walking. In the advanced PZCSA, we adds some value to input y_d in order to reduce the tracking error.

$$\hat{y}_d(k) = y_d(k) + \sum_{n=1}^{M-1} \frac{1}{{z_u}^{nN_{order}}} y_d(k + nN_{order}) \tag{11}$$

$$y(k) = y_d(k) - \frac{1}{{z_u}^{MN_{order}}} y_d(k + MN_{order}) \tag{12}$$

Although the tracking error can be reduced in (12), the added values in (11) demand the forthcoming ZMP from $k+N_{order}$ to $k+(M-1)N_{order}$. For improving this problem, the advanced PZCSA assumes that the values of y_d are same value after discrete time k. Therefore the input of the advanced PZCSA is represented as follows.

$$\hat{y}_d(k) = y_d(k) + \frac{\frac{1}{{z_u}^{N_{order}}}}{1 - \frac{1}{{z_u}^{N_{order}}}} y_d(k) \tag{13}$$

The system output $y(k)$ is described as follows

$$\begin{aligned} y(k) &= \hat{y}_d(k) - \frac{1}{{z_u}^{N_{order}}} \hat{y}_d(k) \\ &= \frac{{z_u}^{N_{order}}}{{z_u}^{N_{order}} - 1} y_d(k) \\ &\quad - \frac{1}{{z_u}^{N_{order}} - 1} y_d(k + N_{order}) \end{aligned}$$

Now the tracking error between input $y_d(k)$ and output $y(k)$ is

$$y_d(k) - y(k) = \frac{1}{{z_u}^{N_{order}} - 1} (-y_d(k) + y_d(k + N_{order})) \tag{14}$$

The tracking error becomes zero at the static state as shown in (14). The advanced PZCSA can be simply implemented as shown in Fig. 7.

D. Property of the Proposed Method

There are three advantages of the proposed method. The first is that the proposed method guarantees system stability because the feedback controller transfers the pole of LIPM into more stable poles by the pole placement method.

The second is related with tracking error boundary. The tracking error are bounded by N_{order} and difference of ZMP represented by (14). Particularly, N_{order} has an important role on tracking errors. If we want to obtain smaller tracking errors, more accurate results can be achieved using simply larger N_{order}.

The last property is that it is easily to implement realtime walking pattern generation because the proposed control system simply consists of a feedback and a feedforward controller. Moreover the procedure of walking pattern generation does not demand complex computations for the stable walking pattern. However, the proposed method has a time delay due to compensating the unstable zero of LIPM.

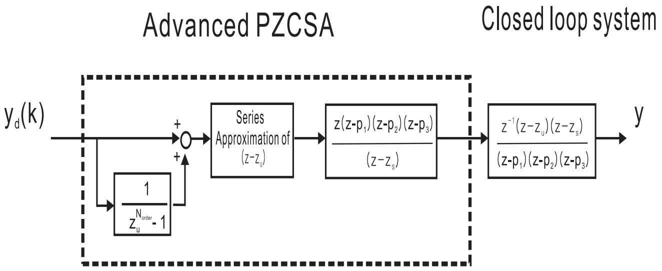
Advanced PZCSA
Closed loop system
$y_d(k)$
+
+
Series Approximation of $(z-z_u)$
$\frac{z(z-p_1)(z-p_2)(z-p_3)}{(z-z_s)}$
$\frac{z^{-1}(z-z_u)(z-z_s)}{(z-p_1)(z-p_2)(z-p_3)}$
$\frac{1}{{z_u}^{N_{order}}-1}$
y

Fig. 7. Block diagram of the advanced PZCSA and the closed loop system

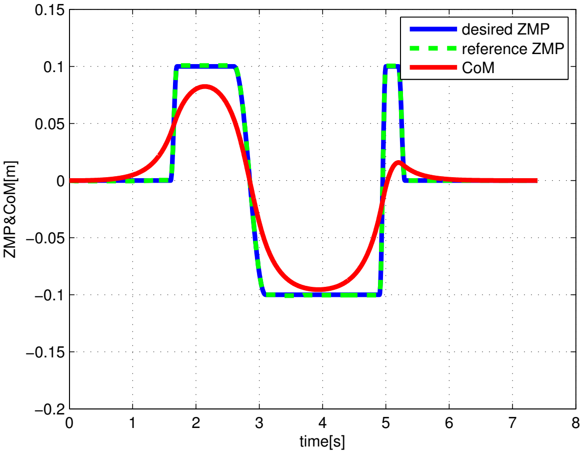

Fig. 8. ZMP and CoM trajectory according to various walking phase time

IV. SIMULATION

This section shows the effectiveness of the proposed approach. We do three simulations such as arbitrary walking phase time, arbitrary desired ZMP position and sudden change of the desired ZMP position.

Since there is little difference of the walking pattern generation procedure of the frontal and the sagittal planes with the proposed method, the simulations are limited in the frontal plane. And we take the tracking error of the single support phase into consideration because the tracking error in the single support phase in which the humanoid robot is supported by one leg has an important influence on stability of the robot.

The parameter condition for walking pattern generation is set as follows.

$$\begin{aligned} z_{CoM} &= 0.814[m] \\ T &= 0.005[s] \\ N_{order} &= 320 \end{aligned}$$

Considering the sampling time T and forthcoming value length N_{order}, the time interval of forthcoming ZMP used for walking pattern is 1.6 second. In simulations, it means 1.6 second is the time delay for walking pattern generation.

Fig. 8 shows the result according to various walking phase time. We changes the double support phase time(T_d) and the single support phase time(T_s) in this simulation. The time intervals of the double support phase time change to 0.1, 0.5, 0.1 second at 1.6, 2.6, 4.9 second, respectively. The time intervals of the single support phase time also change to 0.9, 1.8, 0.2 second at 1.7, 3.1, 5.1 second, respectively. The ZMP and the CoM trajectory are generated stably irrespective of varying the walking phase time. The tracking errors in the single support phase are less than 1 mm.

Next simulation is related to arbitrary desired ZMP position. The desired ZMP positions in Fig. 9 change from 0.1m, through -0.5m, to 0.3m. The tracking errors are less than 3.2 mm in the single support phase.

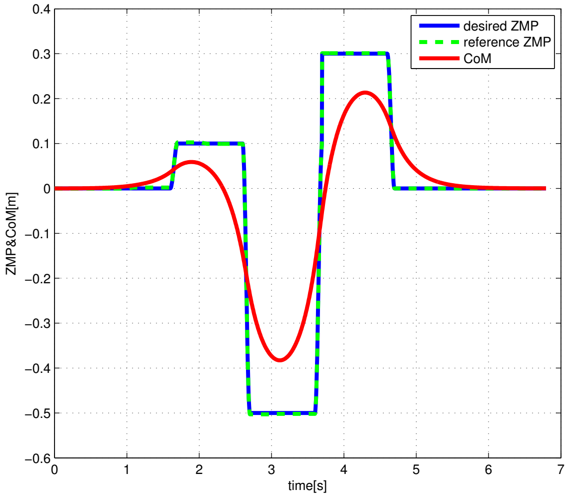

Fig. 9. ZMP and CoM trajectory according to various desired ZMP position

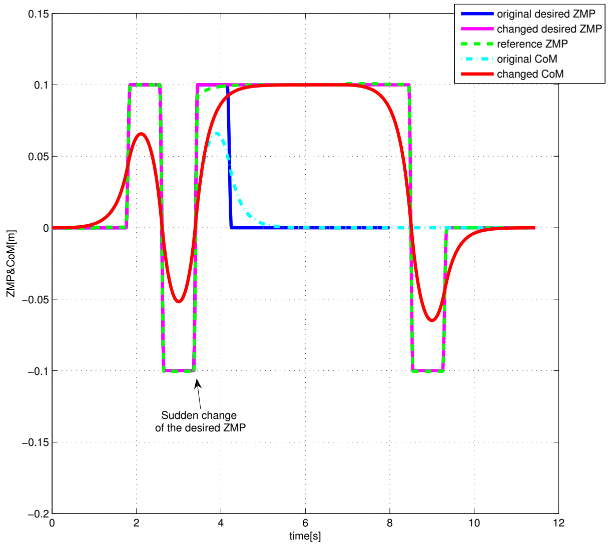

Fig. 10. ZMP and CoM trajectory according to sudden change of the desire ZMP position

The final simulation is a sudden changing in walking path to avoid the external object and keep the robot balance. The desired ZMP is changed suddenly at 3.4 second as shown in Fig. 10. However the proposed method generates a stable reference ZMP trajectory to the satisfaction of robot's stability. The fluctuation at 3.5second is due to non-minimum phase property according to the sudden desired ZMP change. The tracking error of reference ZMP at 3.5 second is within 1 cm. The error is not a significant problem because the robot can keep stability considering supporting zone of the robot.

V. CONCLUSION

This paper presented a new walking pattern method for the humanoid robot. This proposed method is based on the linear inverted pendulum model. Stable and smooth walking pattern is generated by the pole placement method as a feedback controller and advanced PZCSA as a feedforward controller. The feedback controller improves the system stability. And the feedforward controller makes the reference ZMP to track the desired ZMP efficiently. The efficiency and usefulness of the proposed approach is illustrated by three simulations.

Current, we are programming the proposed method as a real-time walking pattern generator of a humanoid robot MAHRU-R. We will implement the proposed method with a humanoid robot MAHRU-R in the near future.

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