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Optimal Placement of a Two-Link Manipulator for Door Opening

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Abstract— **This paper presents a study on the optimal base location and arm motion of a mobile manipulator for door opening task. Numerical simulation results show that the base location where the manipulability of the two-link arm is almost degenerated at the start and end points of door opening is optimal. We show by analysis that the location has an advantage in supplying kinetic energy to the door by using torques at the joints of arm. In order to represent properly the arm motion near a singular point of manipulability, the rotational motion of the door is parameterized by piecewise fifth order polynomials of time, and the parameters of polynomials are optimized to minimize the joint torques.**

I. INTRODUCTION

The development of robots that help us at home currently attracts many researchers. Mobile manipulators would be useful at home, because they can move around and do various tasks by their manipulator[1]. The tasks that the mobile manipulators are expected to do include conveying something from one room to another one, taking something out of a refrigerator or a drawer, and so on. To accomplish such tasks, opening a door is an important sub-task and should be done as efficiently as possible.

The researches on door opening by a mobile manipulator have been performed from various points of view, because the task provides many challenging problems such as detection of the door knob by a vision system, grasp of the knob by a robot hand, and so on[2]. To pull or push a door, a mobile manipulator can utilize the motion of both the mobile base and the manipulator[3]. However, in this paper, we assume that the location of the mobile base is fixed during opening the door and the door is opened by using only the manipulator, because the mobile base is usually much heavier than the manipulator and the motion of the base may cause large energy consumption especially when the door is opened quickly. Even under the assumption, we can choose the base location before starting the door opening. Optimal motion planning of a manipulator with fixed or mobile base has also been extensively studied so far[4], [5], [6], [7]. Some studies use kinematic or geometric criteria for optimization to avoid obstacles and singular configurations of manipulability of the robot arm. Other studies use dynamic criteria to minimize the input forces and torques.

In this paper, we deal with the problem of finding the base location and arm motion for a mobile manipulator that minimize the integral of squared joint torques during opening a door. It is assumed that the mobile manipulator has a planar two-link arm and pulls a door to open it. Although this problem looks relatively simple, it is interesting when the singular configuration of the two-link arm and its effect on the optimization results are taken account of. For a planar two-link arm, the manipulability is degenerated when the arm is stretched out[8]. We call such configuration of the arm *singular configuration* in this paper. Since the velocity vector of the end effector is restricted to a certain direction at the singular configuration, motion planning of the manipulator through the singular configuration is difficult. Some methods to overcome the difficulty by using a time scale transformation have been proposed[9], [10]. In the case of door opening task, the end effector path is specified by the path of the door knob. Therefore, the end effector should follow a path called *degenerate path* in [10] when the arm is in a singular configuration at the start or end point of door opening. Then, the acceleration of the end effector along the path cannot be obtained by bounded joint torques, but the fourth time derivative of the end effector's position along the path can be generated. Therefore, we use fifth order splines of time to represent the rotational motion of the door. By using the parameters of the splines, the problem of finding the optimal arm motion at each base location becomes a parametric optimization problem. Numerical simulation results show that the location where the two-link arm is in the singular configuration at both the start and end points of door opening is optimal. We provide a theoretical explanation to the results by considering the energy supplied to the system by the joint torques. At the singular configuration, the energy can be generated most efficiently when the door is sufficiently heavier than the robot arm.

II. PROBLEM FORMULATION

We suppose that a mobile manipulator approaches a door, stops at a location near the door, grasps and opens it. The door is closed and at rest at the start time. The robot has a two-link arm, and the two joints of the arm are supposed to rotate about the perpendicular axis. Then, we can consider a planar problem of opening the door.

For simplicity, we make the following assumptions.

1) There are no frictions on the door such as frictional torque at the hinge and air resistance.
2) The connection between the door knob and the hand of the manipulator is modeled as a free joint.
3) The mobile base of the robot is fixed on the ground during opening the door.

Although there are many types of robot hands proposed so far, the torque generated at the hand is supposed to be small.

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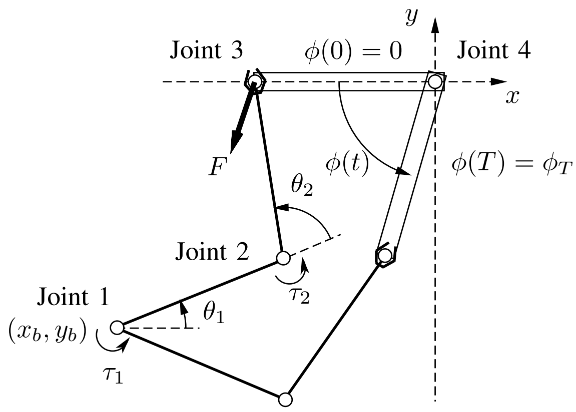

Fig. 1. Door opening by two-link manipulator

Therefore, the second assumption is made in this paper. And, when the door is opened quickly, the third assumption would be suitable because the robot's mobile base tends to be heavy.

Under the assumption 2), the system composed of the mobile manipulator and the door has four revolute joints (Fig. 1). The joint between the robot base and the first link of the arm is called Joint 1, and the joint between the first link and second link of the arm is called Joint 2. Joint 3 is the free joint that is described in the assumption 2), and Joint 4 is the hinge of the door. We choose a coordinate frame, (x, y), whose origin is placed on Joint 4, and the x axis is set to be along the door at the start time. The location of the robot base is denoted as (x_b, y_b), and the angles of Joint 1 and 2 are denoted as θ_1 and θ_2 respectively. The input torques at Joint 1 and 2 are expressed as τ_1 and τ_2 respectively. The angles and the torques are represented in vector forms as $\theta = [\theta_1, \theta_2]^T$ and $\tau = [\tau_1, \tau_2]^T$. The angle of the door with respect to x axis is expressed as ϕ, and the conditions on ϕ at the start time, $t = 0$, and the end time, $t = T$, are given as follows:

$$(\phi, \dot{\phi})|_{t=0} = (0, 0) \quad , \quad (\phi, \dot{\phi})|_{t=T} = (\phi_T, 0) \ . \tag{1}$$

We introduce the following cost function as a criterion for optimization.

$$J_c(\xi) = \int_{t=0}^{T} \tau_1^2 + \tau_2^2 dt \ , \tag{2}$$

where ξ represents the parameters for optimization and is chosen as the location of robot base and the trajectory of door angle:

$$(x_b, y_b) \ , \ \phi(t) \ . \tag{3}$$

Solving the optimal door opening problem is finding the parameters that minimize J_c:

$$\xi^* = \arg\min J_c \ . \tag{4}$$

Remark 1: The total mechanism shown in Fig. 1 can be regarded as a four-bar mechanism where Joint 1 and Joint 2 are actuated redundantly, and there are many researches on the singularity of closed chains such as [11]. However, in this paper, we will treat the system as the one composed of a two-link manipulator and the door, and will focus on the effect of the singular configuration of the two-link manipulator on the optimization problem.

III. Two-Link Manipulator Grasping a Door

A. Kinematics

The dimension of the configuration space of the system in Fig. 1 is only one, because the locations of Joint 1 and 4 are fixed. Therefore, if the angle of the door $\phi(t)$ is specified, the angles of the arm, $\theta(t)$, can be calculated from $\phi(t)$. Denoting the position of Joint 3 as $p_e = [x_e, y_e]^T$, we can express it by the following two ways:

$$p_e = f_1(\phi) = \begin{bmatrix} -l_3 \cos\phi \\ -l_3 \sin\phi \end{bmatrix} \ , \tag{5}$$

$$p_e = f_2(\theta) = \begin{bmatrix} x_b + l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ y_b + l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} \ , \tag{6}$$

where l_i is the length between Joint i and $i + 1$. From (5) and (6), the system has the following constraint between θ and ϕ:

$$\Psi(\theta, \phi) = f_2(\theta) - f_1(\phi) = 0 \ . \tag{7}$$

From the above equation, we can obtain two points of θ for each ϕ when $\theta_2 \neq 0$. To get a unique solution of θ, we choose one of them that satisfies $\theta_2 \geq 0$. From (6), we can obtain the following equations:

$$\dot{p}_e = J\dot{\theta} \ , \tag{8}$$

$$\ddot{p}_e = J\ddot{\theta} + \dot{J}\dot{\theta} \ , \tag{9}$$

where J is the Jacobian matrix of f_2 and can be written as

$$J = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \ . \tag{10}$$

By using (5), (8) and (9), we can obtain $\dot{\theta}$ and $\ddot{\theta}$ from $\phi(t)$ as long as $\det(J) \neq 0 \Leftrightarrow \theta_2 \neq 0$.

B. Dynamics

From the inverse kinematics in the above subsection, the equations of motion of the system can be written by using $\phi(t)$ as in the following form:

$$[\tau_1, \tau_2, 0]^T - EF = K(\phi, \dot{\phi}, \ddot{\phi}) \ , \tag{11}$$

where $F = [F_1, F_2]^T$ is the force applied to the door by the robot arm at Joint 3. The matrix E can be calculated as follow:

$$E^T = \left[\frac{\partial\Psi}{\partial\theta}, \frac{\partial\Psi}{\partial\phi}\right] = \left[J, \frac{\partial\Psi}{\partial\phi}\right] \ . \tag{12}$$

It is obvious that we cannot determine τ and F from (11), which means that the system is indeterminate. Moreover, the first and second rows of (11) can be rewritten as

$$\tau - J^T F = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) \ , \tag{13}$$

where the kinetic energy of the two-link arm can be expressed by using $M(\theta)$ as

$$E_a = (1/2)\dot{\theta}^T M(\theta)\dot{\theta} \ , \tag{14}$$

and h can be expressed as $h = \dot{M}(\theta)\dot{\theta} - \partial E_a/\partial\theta$.

IV. Optimization method

Since $\phi(t)$ in ξ is an infinite dimensional parameter, it is difficult to find the optimal solution ξ^* that minimizes the cost function (2) rigorously. Therefore, we approximate $\phi(t)$ as fifth order spline functions of time, and find the coefficients of splines that minimize the cost function. The location of the robot base (x_b, y_b) is discretized into a grid, and, at each grid point, the quasi-optimal motion of the door is calculated by using the spline functions.

A. Search for optimal motion of door

We divide the time interval $[0, T]$ by n and assume that the trajectory of $\phi(t)$ in each time interval $[t_i, t_{i+1}]$ ($i = 0, \cdots, n-1$ and $t_j = jT/n$ for $j = 0, \cdots, n$) is expressed by a fifth order polynomial function of time, $\varphi_i(t)$, as follows:

$$\begin{aligned} \varphi_i(t) &= \phi_i + b_i(t-t_i) + c_i(t-t_i)^2 + d_i(t-t_i)^3 \\ &\quad + e_i(t-t_i)^4 + f_i(t-t_i)^5 \ , \end{aligned} \quad (15)$$

where ϕ_i, b_i, c_i, d_i, e_i and f_i are scalar parameters. To make the input torque τ continuous, we choose the functions $\varphi_i(t)$ so that they satisfy

$$\begin{aligned} &\varphi_i(t_{i+1}) = \varphi_{i+1}(t_{i+1}) \ , \ \dot{\varphi}_i(t_{i+1}) = \dot{\varphi}_{i+1}(t_{i+1}) \ , \\ &\ddot{\varphi}_i(t_{i+1}) = \ddot{\varphi}_{i+1}(t_{i+1}) \ , \end{aligned} \quad (16)$$

where $i = 0, \cdots, n-2$. When the polynomials satisfy (1) and (16), there are $3n-1$ independent parameters, and they can be chosen as

$$\Phi = (\phi_1, \cdots, \phi_{n-1}, e_0, f_0, \cdots, e_{n-1}, f_{n-1}) \ . \quad (17)$$

Here, we assume that the angle $\phi(t)$ monotonically increases at $t = t_i$ and put a constraint that $0 \leq \phi_1 \leq \cdots \leq \phi_{n-1} \leq \phi_T$. At each location of the robot base, we search for the values of Φ that minimize the cost function (2) by the Quasi-Newton method.

B. Search for optimal location of robot base

The grid search method is used to find the optimal base location. The region of (x_b, y_b) defined by $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$ is divided into a grid, where each rectangle is given by $\Delta x \times \Delta y$. By calculating the cost function at each grid point by the method in IV-A, we can find the optimal location of the robot base.

C. Solution to indeterminate dynamics

A unique solution of joint torque τ cannot be obtained from (11). But, by minimizing $\|\tau\|$ at each time instance, we can determine τ uniquely. First, we can find a vector $g = [g_1, g_2, g_3]^T \in R^3$ such that $g^T E = 0$. There always exists g that satisfies $g^T E = 0$ because $E \in R^{3\times 2}$. Then, from (11), we can obtain

$$g_1\tau_1 + g_2\tau_2 = g^T K \ . \quad (18)$$

Any torque τ satisfying (18) can achieve a specified motion of the door $\phi(t)$. When $\|\tau\|$ is minimized, τ should be expressed as

$$\tau = k_\tau [g_1, g_2]^T \ , \quad (19)$$

where k_τ is a scalar parameter. By using (11) and (19), we can obtain τ and F uniquely.

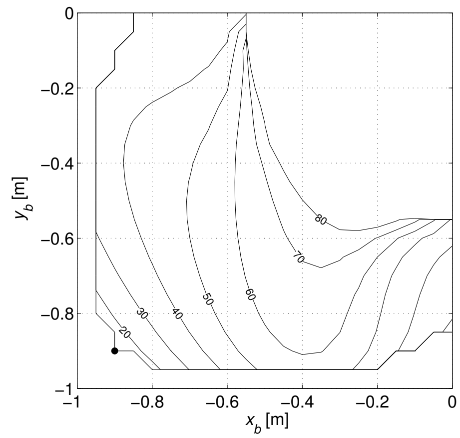

Fig. 2. Cost function for base location (x_b, y_b)

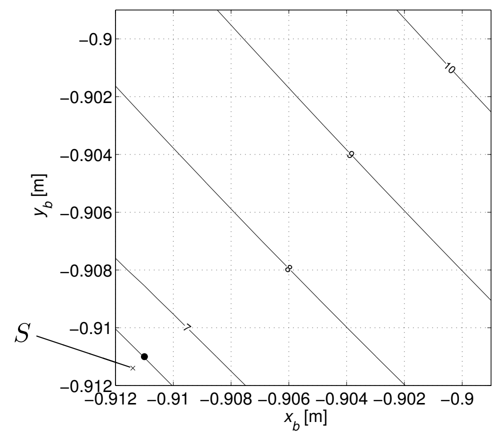

Fig. 3. Cost function for base location (x_b, y_b) (near the singular location S)

V. Numerical Simulations

In this section, we will show the optimal solution obtained by numerical simulations. To find the optimal values of Φ under the constraint that $0 \leq \phi_1 \leq \cdots \leq \phi_{n-1} \leq \phi_T$, the MATLAB function fmincon was used. The goal angle of the door, ϕ_T, is set to be $\pi/2$ at $T = 2.0$[s], and the time interval $[0, 2.0]$ is divided into four sub-intervals, that is, $n = 4$. The initial value of Φ is given so that the initial spline curves coincide with a single third order spline curve that satisfies (1). Then, for $\forall i$, the initial values of d_i are the same, and the initial values of e_i and f_i are zero. The lengths of door and two links of the arm are chosen as 0.5[m]. The mass and inertia of door are set to be 32.1[kg] and 2.51[kg·m^2]. The mass and inertia of the first and second links of the arm are set to be 6.19[kg], 1.30×10^{-1}[kg·m^2], 1.55[kg] and 3.23×10^{-2}[kg·m^2] respectively.

At first, the grid points of (x_b, y_b) were made by choosing Δx and Δy as 0.05[m], and the cost function J_c defined by (2) was calculated at each point. Figure 2 shows the contour plot of J_c, and the location that has the minimum of J_c in this figure is $(x_b, y_b) = (-0.9, -0.9)$. Next, to obtain more

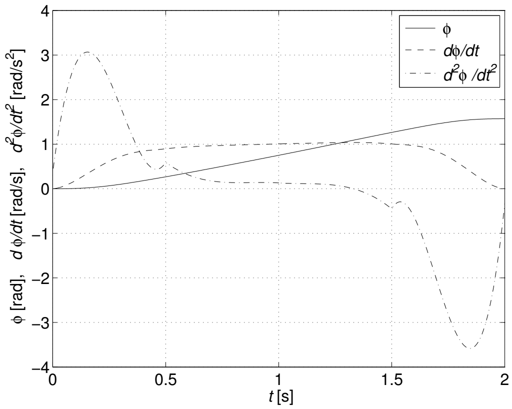

Fig. 4. Time history of door angle ϕ

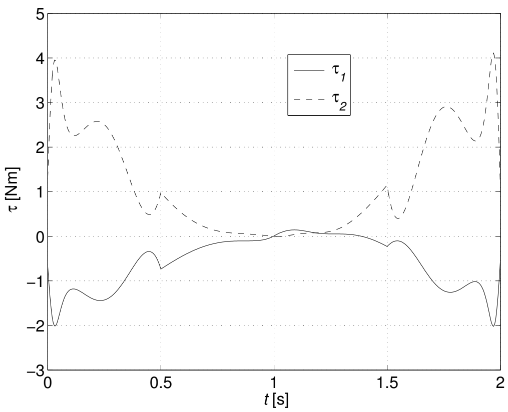

Fig. 5. Time history of joint torque τ

accurate solution of the optimal location, the grid points of (x_b, y_b) around $(-0.9, -0.9)$ were made by choosing Δx and Δy as 0.001[m], and J_c was calculated at each point. Figure 3 shows the contour plot of J_c, and the optimal location in this figure is $(x_b, y_b) = (-0.911, -0.911)$. The value of the cost function J is 6.58[$\mathrm{N}^2\mathrm{m}^2$s] at the location.

Here, we consider the location denoted as S in Fig. 3 where $(x_b, y_b) = (-0.9114, -0.9114)$. When the robot base is located at S, the matrix J defined by (10) is degenerated at the start and end points of time (Fig. 6). We call such a location as *singular location* in this paper. It should be noted that the location $(-0.911, -0.911)$ obtained in Fig. 3 is the closest to the singular location S among the grid points. Figures 4 and 5 show the time histories of $(\phi, \dot{\phi}, \ddot{\phi})$ and τ. Around the start and end points of door opening, the arm pulls the door strongly to accelerate or decelerate it, and the acceleration of the door angle is small between $t = 0.5$[s] and $t = 1.5$[s].

VI. Singularity Analysis

The results in V show that the location close to the point S has an advantage in minimizing the cost function J_c. We will show by analysis that the singular configuration of manipulability is useful for providing the energy to the door efficiently by joint torques.

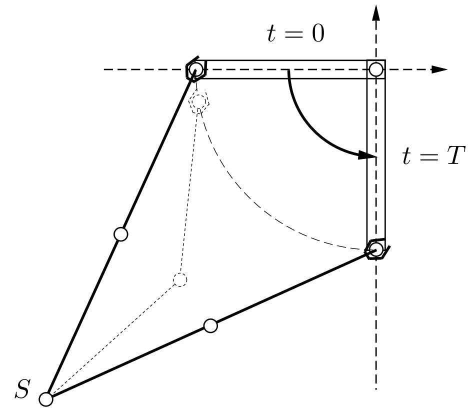

Fig. 6. Singular location S of robot base for $\phi_T = \pi/2$

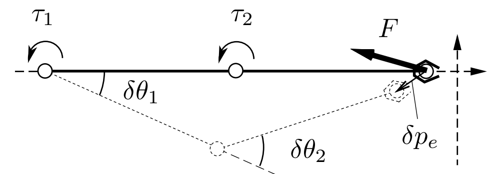

Fig. 7. Small displacements at the singular configuration

A. Removement of singularity by differentiation

From (8) and (9), $\dot{p}_e$ and $\ddot{p}_e$ are restricted to one dimensional space when $\theta_2 = 0$ and $\dot{\theta} = 0$. That is,

$$e_{\theta_1}^T \dot{p}_e = e_{\theta_1}^T \ddot{p}_e = 0 \ , \tag{20}$$

where $e_{\theta_1} = [\cos\theta_1, \sin\theta_1]^T$. On the other hand, the fourth order time derivative of p_e can be represented as

$$p_e^{(4)} = J\theta^{(4)} + 3\left(\frac{\partial J}{\partial \theta_1}\ddot{\theta}_1 + \frac{\partial J}{\partial \theta_2}\ddot{\theta}_2\right)\ddot{\theta} + R(\theta, \dot{\theta}, \ddot{\theta}, \theta^{(3)})\dot{\theta} \ , \tag{21}$$

where $R(\theta, \dot{\theta}, \ddot{\theta}, \theta^{(3)})$ is a 2×2 matrix and $(*)^{(i)}$ denotes the ith order time derivative of $(*)$ $(i = 3, 4)$. The matrices $\partial J/\partial\theta_1$ and $\partial J/\partial\theta_2$ can be calculated as

$$\frac{\partial J}{\partial \theta_1} = \begin{bmatrix} -l_1\cos\theta_1 - l_2\cos(\theta_1+\theta_2) & -l_2\cos(\theta_1+\theta_2) \\ -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) & -l_2\sin(\theta_1+\theta_2) \end{bmatrix} , \tag{22}$$

$$\frac{\partial J}{\partial \theta_2} = \begin{bmatrix} -l_2\cos(\theta_1+\theta_2) & -l_2\cos(\theta_1+\theta_2) \\ -l_2\sin(\theta_1+\theta_2) & -l_2\sin(\theta_1+\theta_2) \end{bmatrix} . \tag{23}$$

Therefore, even if $\theta_2 = 0$ and $\dot{\theta} = 0$, we can choose τ which satisfies $e_{\theta_1}^T p_e^{(4)} \neq 0$.

B. Energy supplied by joint torques

We consider the case where the base of the robot is located at the singular point S and the manipulator of the robot pulls the door which is at rest. By a suitable coordinate transformation, it can be assumed without loss of generality that the joint angles at the singular configuration satisfy $\theta = [0, 0]^T \equiv \theta_0$. Then, we consider small displacements of the joint angles and the position of the end effector, and denote them as $\delta\theta$ and δp_e (Fig. 7).

First, we derive the relationship between δp_e and $\delta\theta$. Although it is usual to use the first order approximation, $\delta p_e = J(\theta_0)\delta\theta$, it is not enough to represent the relationship at the singular configuration[12]. By using the higher order term, it can be represented as follows:

$$\delta p_e = J(\theta_0)\delta\theta + \begin{bmatrix} O(\delta\theta^2) \\ O(\delta\theta^3) \end{bmatrix} , \tag{24}$$

where

$$J(\theta_0) = \begin{bmatrix} 0 & 0 \\ l_1 + l_2 & l_2 \end{bmatrix} . \tag{25}$$

From (24), when $\delta p_e \neq k[O(\delta\theta), 1]^T$ for $\exists k$, the small displacements of joint angles should satisfy

$$\delta\theta_1 \approx -\frac{l_2}{l_1 + l_2}\delta\theta_2 . \tag{26}$$

Since $\delta p_e \neq k[O(\delta\theta), 1]^T$ along the path of the door knob, we obtain $\delta p_e = O(\delta\theta^2)$.

Next, we consider the statics of the system. Since all the work done by the joint torque τ is equal to the energy supplied to the door by the force at Joint 3, F, we obtain

$$W_J = \tau^T\delta\theta = \tau^T O(\sqrt{\delta p_e}) = F^T\delta p_e . \tag{27}$$

This equation means that an infinitesimal torque τ can cause a finite force F and supply W_J to the door in statics.

When we consider the dynamics of the system, we have to do more complicated analysis. To simplify the analysis, we make the following assumptions:

4) The end effector grasps a mass point whose mass is denoted as $\hat{m}_3$, and $\hat{m}_3$ is much larger than the mass and inertia of the arm.
5) At the start point, the arm and the mass point are at rest, and a bounded and constant torque τ is applied at the joints for $t \geq 0$.

Under the assumption 4), the force F can be expressed as

$$F = \hat{m}_3\ddot{p}_e = \hat{m}_3(J\ddot{\theta} + \dot{J}\dot{\theta}) . \tag{28}$$

From (13) and (28), we can obtain

$$(M + \hat{m}_3 J^T J)\ddot{\theta} = \tau - h - \hat{m}_3 J^T \dot{J}\dot{\theta} . \tag{29}$$

Since $\dot{\theta} = 0$ at the start point under the assumption 5), we can approximate $\ddot{\theta}$ as

$$\ddot{\theta} \approx (M + \hat{m}_3 J^T J)^{-1}\tau . \tag{30}$$

Therefore, the small displacement of θ at $t = \delta t$ can be represented as

$$\delta\theta \approx \frac{\delta t^2}{2}(M + \hat{m}_3 J^T J)^{-1}\tau . \tag{31}$$

Then, the work done by joint torque τ can be written as

$$W_J = \tau^T\delta\theta \approx \frac{\delta t^2}{2}\tau^T(M + \hat{m}_3 J^T J)^{-1}\tau . \tag{32}$$

From the assumption 4), when the matrix J is not degenerated, W_J can be approximated as

$$W_J \approx \frac{\delta t^2}{2}\tau^T(\hat{m}_3 J^T J)^{-1}\tau \equiv W_{J1} . \tag{33}$$

On the other hand, when the matrix J is degenerated, we can calculate the work W_J from (32) as follows:

$$\begin{aligned} W_J &\approx \frac{\delta t^2}{2}\frac{\hat{\tau}^T M\hat{\tau} + \hat{m}_3(c^T\tau)^2}{\det M + \hat{m}_3 c^T M c} \\ &\approx \frac{\delta t^2}{2}\frac{(c^T\tau)^2}{c^T M c} , \end{aligned} \tag{34}$$

where $c = [l_2, -(l_1 + l_2)]^T$, $\hat{\tau} = [\tau_2, -\tau_1]^T$ and we used the assumption 4). From (34), in order to maximize W_J, we can choose τ as

$$\tau \parallel c . \tag{35}$$

In other words, to minimize $\|\tau\|$ for $\exists W_J$, we can choose τ as in the above equation. Then,

$$W_J \approx \frac{\delta t^2}{2}\frac{\|c\|^2\|\tau\|^2}{c^T M c} \equiv W_{J2} . \tag{36}$$

From (33) and (36), we can see that $W_{J1} \ll W_{J2}$ because of the assumption 4). Therefore, at the singular configuration, the joint torque τ can generate the energy most efficiently when $\hat{m}_3$ is large enough.

It should be noted that the generated work W_J is not directly transmitted to the kinetic energy of the door. From (13), we obtain

$$\tau^T\delta\theta - F^T J\delta\theta = \delta E_a . \tag{37}$$

From (25) and (26), a small increase of the energy of the door, δE_d, can be approximated as

$$\delta E_d = F^T J\delta\theta \approx 0 . \tag{38}$$

Therefore, we obtain $W_J \approx \delta E_a$. Consequently, first, the work done by the joint torque, W_J, is stored in the arm as the energy of the arm, δE_a, and then, the energy δE_a is transmitted to the door through the joint between the arm and the door. That is to say, if once the system composed of three links, the two-link arm and the door, has an kinetic energy, the rotation of the door is necessarily caused by the energy because the angle ϕ is the only degree of freedom of the system.

Remark 2: Since $\delta E_a = 0$ in statics, we obtained (27) at the singular configuration. In dynamics, δE_a plays an important role to supply the energy to the system. At the singular configuration, δE_a is generated by joint torques independently of the mass $\hat{m}_3$ from (36).

Remark 3: In Fig. 5, we can see that the torque τ satisfies approximately (35) when the configuration of the manipulator is close to the singular one. Moreover, the torque is larger around $t = 0$ and T than at other time instances. The arm pulls the door strongly around the start and end times, and generates or eliminates the energy efficiently around the singular configuration.

Remark 4: We used the fifth order spline functions of time to represent the time history of the angle $\phi(t)$. Since $\ddot{p}_e = 0$ and $p_e^{(4)} \neq 0$ for a bounded torque τ at the start point, it is necessary to use more than fourth order splines. When the third order splines that are more common were used in

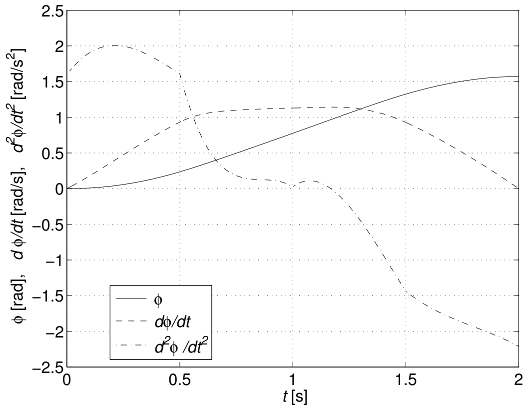

Fig. 8. Time history of door angle ϕ

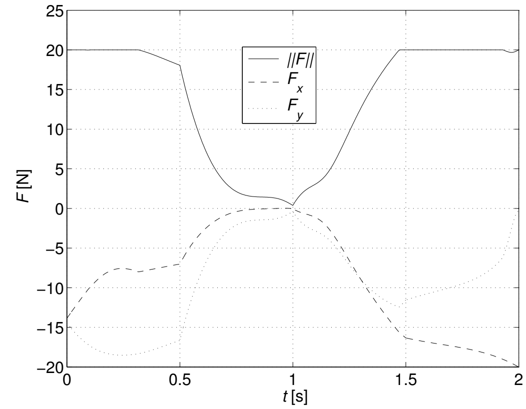

Fig. 9. Time history of joint force F

the numerical simulations presented in V, we obtained the optimal base location at $(x_b, y_b) = (-0.909, -0.909)$. The location is also very close to the singular location S, but the cost function increases as the location (x_b, y_b) becomes closer to S than $(x_b, y_b) = (-0.909, -0.909)$. It is because, as t approaches 0 or T, the torque τ becomes much larger than the one in Fig. 5.

Remark 5: In the above analysis, we assume that the mass of door is sufficiently larger than the one of the arm. When the mass of the door is set to 3.87×10^{-3} [kg], the optimal location is obtained at $(x_b, y_b) = (-0.79, -0.90)$. If the door is lighter than the arm, we cannot obtain $W_{J1} \ll W_{J2}$.

VII. Joint Force Limitation

From the above analysis and numerical results, the way of obtaining the minimum cost function is pulling the door strongly around the singular configuration. However, the strong pull of the door tends to cause a large joint force F, and the robot hand may not sustain the force. Therefore, we introduce the limitation of the force at Joint 3 as

$$\|F\| \leq F_{\max} \ , \tag{39}$$

where $F_{\max}$ is the maximum force sustainable by the hand. The third row of (11) is a constraint on F if the door angle $\phi(t)$ is given. When there exists F that satisfies both the third row and (39), we can find easily τ that minimize $\|\tau\|$ at each time instance. The details of this procedure are omitted because of limited space. When $F_{\max}$ is set to be 20.0[N], numerical simulation results show that the optimal location of the robot base is $(x_b, y_b) = (-0.909, -0.910)$ and the optimal cost is $J = 11.76[\mathrm{N}^2\mathrm{m}^2\mathrm{s}]$. Figures 8 and 9 show the time histories of $(\phi, \dot{\phi}, \ddot{\phi})$ and F. The limitation on F is satisfied as shown in Fig. 9, while the maximum of $\|F\|$ is about 35[N] in the results in V. The acceleration and deceleration of the door around the start and end points become smaller than in Fig. 4.

VIII. Conclusions

In this paper, we investigated the optimal base location and arm motion of a mobile manipulator for door opening task. When the cost function is set to be the integral of the squared norm of joint torques, numerical results show that the location where the manipulability of the arm is degenerated at the start and end point is optimal. We explained that the joint torques can do the work efficiently at the singular configuration of the arm when the door is sufficiently heavier than the arm. These results would be useful for motion planning of a mobile manipulator that is required to do various tasks.

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