

# Leader-following Formation Control Based on Pursuit Strategies

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**Abstract**—The paper studies formation control of multi-agent systems under a directed acyclic graph. In a directed acyclic graph, the agents without neighbors are leaders and the others are followers. Leaders move in a formation with a time-varying velocity and followers can access the relative positions of their neighbors and the leaders' velocity. A local formation control law is proposed in the paper based on pursuit strategies and necessary and sufficient conditions for stability and convergence are derived. Moreover, the results are extended to the case with arbitrary communication delays, for which the steady-state formation is presented according both the control parameters and time delays.

## I. INTRODUCTION

Increasing applications of multi-agent systems (e.g., microsatellite clusters, unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs) and sensor networks) have enabled researchers to pay more attention to the study of multi-agent systems [1], [2]. A fundamental problem in multi-agent systems is formation control [3], which is also related to emergent behaviors in nature such as flocks of birds, schools of fish, and colonies of bacteria [4]. Presently, three approaches are often used for formation control: leader-following [5], behavior-based approach [6], and virtual structure method [7]. Motivated by the effectiveness of leader-following approach in nature [8], it has attracted significant attention [9]–[15].

Using nearest neighbor rules, a group of agents are controlled to eventually agree on the common heading of the leaders so that they move in a formation with a constant speed [9]. In addition, artificial potential based control strategies are used for a group of agents to reach a desired formation with a constant velocity [11]. However, moving with a time-varying velocity and holding a formation pattern are more attractive. Shi et al. [16] extend the work of [11] by allowing time-varying velocity for the leaders, but it requires that the acceleration of the leaders should be available to all follower agents. In [13], under the condition that all agents know a common time-varying reference velocity, passivity is used as a design tool for formation control where the interaction graph is bidirectional.

In this paper, the interaction topology of agents is assumed to be a directed acyclic graph (DAG), in which the agents without neighbors are leaders and the others are followers. Leaders move with a prescribed time-varying velocity while

holding a formation pattern, and the followers are able to obtain the relative positions of their neighbors. Moreover, the leaders' velocity can be transmitted to every follower (possibly with some time delays) through wireless communication network. First, we consider the case without time delays. For this, a formation control strategy is presented, in which each follower pursues its neighbors with certain pursuit strength and pursuit angle. Then necessary and sufficient conditions are obtained to ensure that the group of agents can achieve a formation while moving. Moreover, the steady-state formation can be adjusted in terms of the pursuit strengths and pursuit angles. Second, we consider the case where multiple transmission delays exist. For that, we indicate that, when the leaders perform uniform velocity motion and/or uniformly accelerated motion, the agents are still able to achieve a formation under the same condition. Moreover, it is observed that small time-delays and acceleration parameters lead to a slight variation of steady-state formation compared with the one achieved without time delays. Our formation control strategy is extended from usual consensus algorithms [17], [18]. But unlike some work (e.g., [12]) which adds virtual displacements to solve the formation control problem, we add a rotation to each relative position vector so that any desired formation pattern can be achieved by designing suitable rotation angles. The controller obtained is very easy for implementation such as using image-based control. Moreover, under the interaction topology of directed acyclic graph we discuss in the paper, each follower's performance can be analyzed separately, so heterogeneous dynamics are allowed within the framework.

The paper is organized as follows. In Section II, we formulate the formation control problem. In Section III, the control strategy is proposed and necessary and sufficient condition for convergence is presented. Section IV discusses the impacts of communication time-delays and system switching on steady-state formations. Simulations are presented in Section V. Section VI concludes our work.

## II. PROBLEM SETUP

A *directed graph (digraph)*  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  consists of a non-empty node set  $\mathcal{V} = \{1, 2, \dots, N\}$  and an edge set  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ . An edge of  $\mathcal{G}$  is denoted by an ordered pair of nodes, e.g.,  $(i, j)$ , meaning that the edge leaves node  $i$  and enters node  $j$ . A *walk* is an ordered sequence of nodes such that any two consecutive nodes in the sequence correspond to an edge of the digraph. If the nodes in a walk are distinct, the walk is called a *path*. If a walk starts and ends at the same node and all other nodes on the walk are distinct, it is called

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a cycle. A digraph without cycles is a *directed acyclic graph* (DAG).

In this paper, the neighboring relationship of networked agents is schematically represented by a DAG, where each node represents an agent. In a DAG, the agents, which have no incoming edge, are called *leaders*, and the other agents are called *followers*. Suppose that the indices of the agents are arranged in such a way that agent  $i$  for  $i \in \{1, 2, \dots, N_l\}$  is a leader and agent  $i$  for  $i \in \{N_l + 1, N_l + 2, \dots, N\}$  is a follower. Follower  $i$ 's neighbor set is denoted by

$$\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}.$$

Two examples of DAG are given in Fig. 1.

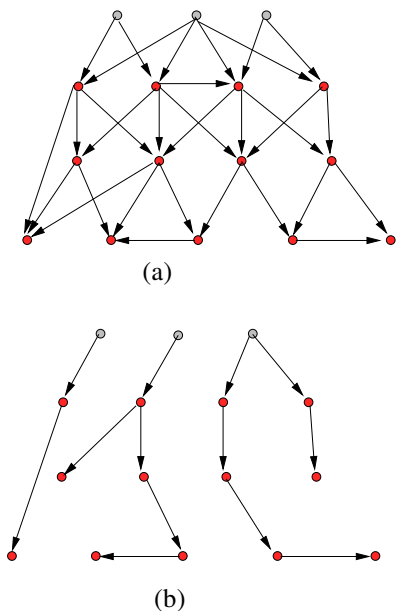


Fig. 1. Two examples of DAG. The gray nodes are leaders and the solid ones are followers.

Each follower knows relative positions of its neighbors by some onboard sensors (e.g., cameras). Furthermore, wireless communication devices are equipped on the agents so that the leader's velocity information can be transmitted to the followers. Our problem is to design a distributed control strategy for each follower using the local information of relative positions of its neighbors and the time-varying velocity obtained from leaders through wireless communication so that all agents make a collective motion while holding a formation pattern.

### III. COLLECTIVE MOTION WITHOUT TIME-DELAYS

Let  $z_i \in \mathbb{C}$  denote the position of agent  $i$  in the plane. Each agent has a single-integrator kinematics

$$\dot{z}_i = u_i,$$

where  $u_i \in \mathbb{C}$  is the control input.

The leaders perform collective motion with the time-varying velocity  $v_0(t)$  in a predefined formation. We consider

the following control for each follower  $i$

$$\dot{z}_i(t) = \sum_{j \in \mathcal{N}_i} k_{ij} e^{\iota \alpha_{ij}} (z_j(t) - z_i(t)) + v_0(t), \quad (1)$$

where the proportion  $k_{ij} > 0$  is called the pursuit strength, the offset angle  $\alpha_{ij} \in [-\pi, \pi)$  is called the pursuit angle, and  $\iota = \sqrt{-1}$  is the imaginary unit. Furthermore, we call  $w_{ij} = k_{ij} e^{\iota \alpha_{ij}}$  the pursuit weight from agent  $i$  to agent  $j$  and call

$$w_i = \sum_{j \in \mathcal{N}_i} w_{ij}$$

the pursuit degree of follower  $i$ .

Next, we introduce the description of a formation in the plane. A *formation* is defined by a set of  $N$  complex numbers, i.e.,  $[c_1, c_2, \dots, c_N]$ . It should be cleared up that this description is independent of the translation of complex plane. In other words, when we say a group of  $N$  agents is in a formation  $[c_1, c_2, \dots, c_N]$ , the trajectories of the agents in an inertia frame can be written as  $z_i(t) = c_i + \xi(t)$  ( $i = \{1, 2, \dots, N\}$ ) for some translation  $\xi(t)$ .

It is assumed that leaders in the group move with a time-varying velocity  $v_0(t)$  while holding a formation  $[c_1, c_2, \dots, c_{N_l}]$ . So each leader's trajectory can be written as

$$z_i(t) = c_i + \xi(t), \quad i = 1, \dots, N_l,$$

where  $\xi(t)$  satisfying  $\dot{\xi}(t) = v_0(t)$  is the translation of the group. Now we construct a moving frame whose origin trajectory is  $\xi(t)$  with respect to some inertial frame. We let

$$\hat{z}_i(t) = z_i(t) - \xi(t) \quad (2)$$

and rewrite the dynamics of each follower in the moving frame. Then we have

$$\dot{\hat{z}}_i(t) = \sum_{j \in \mathcal{N}_i} w_{ij} (\hat{z}_j(t) - \hat{z}_i(t)). \quad (3)$$

When  $w_{ij}$  is real, system (3) is just a consensus algorithm investigated in [18] using algebraic graph theory and matrix theory. For identical complex  $w_{ij}$ , Pavone and Frazzoli [19] use it to obtain rendezvous, circular motion, and/or logarithmic spiral motion under ring-coupled structure. Recently, Ren [20] extends the results of [19] to a digraph having a spanning tree so that the group of agents can still have similar circular motion or rendezvous behaviors. Instead, we consider different complex  $w_{ij}$  for each neighbor pair. Under the interaction structure of directed acyclic graph which does not need to satisfy the assumption in [20], we show that (3) can be used to achieve any desired formation pattern other than the motion in [19], [20].

Applying Laplace transform on both sides of (3), we have

$$s \hat{Z}_i(s) - \hat{z}_i(0) = \sum_{j \in \mathcal{N}_i} w_{ij} (\hat{Z}_j(s) - \hat{Z}_i(s)) \quad (4)$$

or

$$\hat{Z}_i(s) = \frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{w_{ij}}{s + w_i} \hat{Z}_j(s) \quad (5)$$

where  $\hat{z}_i(0)$  is the initial state of  $\hat{z}_i$ ,  $\hat{Z}_i(s)$  and  $\hat{Z}_j(s)$  are the Laplace transform of  $\hat{z}_i(t)$  and  $\hat{z}_j(t)$ , respectively.

First, we present a result on the response of system (3) with respect to exponential convergent inputs.

**Lemma 3.1:** For system (3), suppose that  $\Re(w_i)$  (the real part of  $w_i$ ) is positive. If  $\hat{z}_j(t)$  ( $j \in \mathcal{N}_i$ ) takes the form  $t^m e^{-\phi t}$  where  $m \geq 0$  is an integer and  $\Re(\phi) > 0$ , then  $\hat{z}_i(t)$  takes the same form and converges to 0.

**Proof:** Since  $\Re(w_i)$  is positive, we know from (5) that  $\hat{z}_i(t)$  always contains an exponentially convergent term  $\hat{z}_i(0)e^{-w_i t}$ . Due to the superposition principle, we can consider just one input  $\hat{z}_j(t) = t^m e^{-\phi t}$  for (3) while assuming others are zero. The Laplace transform of  $\hat{z}_j(t) = t^m e^{-\phi t}$  is

$$\hat{Z}_j(s) = \frac{m!}{(s + \phi)^{m+1}}.$$

Thus, we obtain the corresponding term in (5)

$$\frac{w_{ij}}{s + w_i} \hat{Z}_j(s) = \frac{w_{ij}}{s + w_i} \frac{m!}{(s + \phi)^{m+1}}.$$

If  $w_i = \phi$ ,

$$\mathcal{L}^{-1}\left(\frac{w_{ij}}{s + w_i} \hat{Z}_j(s)\right) = \frac{w_{ij}}{m+1} t^{m+1} e^{-w_i t}.$$

If  $w_i \neq \phi$ ,

$$\mathcal{L}^{-1}\left(\frac{w_{ij}}{s + w_i} \hat{Z}_j(s)\right) = a_0 e^{-w_i t} + \sum_{k=1}^{m+1} a_k t^{k-1} e^{-\phi t},$$

where  $\mathcal{L}^{-1}$  denotes the inverse Laplace transform and  $a_k \in \mathbb{C}$  ( $k = 0, 1, \dots, m+1$ ) is a constant depending on  $w_{ij}$ ,  $\phi$  and  $m$ . In both cases,  $\mathcal{L}^{-1}\left(\frac{w_{ij}}{s + w_i} \hat{Z}_j(s)\right)$  takes the form of  $t^m e^{-\phi t}$  and therefore it converges to zero. ■

Next, we present our main result on formations.

**Theorem 3.1:** Consider a group of  $N$  agents where leaders move with a time-varying velocity  $v_0(t)$  in a formation  $[c_1, c_2, \dots, c_{N_l}]$  and followers' dynamics are defined in (1). The group of agents achieves a formation

$$[c_1, \dots, c_{N_l}, c_{N_l+1}, \dots, c_N],$$

where

$$c_i = \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} \text{ for } i \in \{N_l + 1, \dots, N\}$$

if and only if the pursuit degree  $w_i$  of every follower  $i$  ( $i = N_l + 1, \dots, N$ ) has positive real part.

**Proof:** ( $\Leftarrow$ ) After the translation of (2), leaders are static in the moving frame, i.e.,

$$\hat{z}_i(t) = c_i \text{ for } i = 1, 2, \dots, N_l.$$

Firstly, we consider a set of followers whose neighbors are only leaders. Denote this set by  $\mathcal{A}_1$ . For any  $i \in \mathcal{A}_1$ , we have

$$\begin{aligned} \hat{Z}_i(s) &= \frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{w_{ij} c_j}{s(s + w_i)} \\ &= \frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{c_j w_{ij}}{w_i} \left( \frac{1}{s} - \frac{1}{s + w_i} \right). \end{aligned} \quad (6)$$

Applying the inverse Laplace transform, we obtain

$$\hat{z}_i(t) = \hat{z}_i(0) e^{-w_i t} + \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} (1 - e^{-w_i t}). \quad (7)$$

If  $\Re(w_i) > 0$ , the transient component exponentially converges to zero and the steady-state of  $\hat{z}_i(t)$  is

$$c_i = \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i}.$$

Next, consider a set of followers whose neighbors are in  $\mathcal{A}_1$  or just leaders. Denote this set by  $\mathcal{A}_2$ . According to Lemma 3.1, the transient component of any follower  $i \in \mathcal{A}_2$  converges to zero, too. Its steady state can be similarly obtained, which is

$$c_i = \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} \quad (8)$$

depending on its neighbors' steady states.

Repetitively, the steady-states of all agents can be calculated and constant values, which means the group of agents achieves a formation  $[c_1, \dots, c_{N_l}, c_{N_l+1}, \dots, c_N]$ . Moreover, the trajectory of each agent  $i$  is given by

$$z_i(t) = c_i + \xi(t).$$

They move as a whole with the same velocity  $v_0(t)$ .

( $\Rightarrow$ ) If there is a follower  $i$  such that  $\Re(w_i) \leq 0$ , then its transient component converges to infinity. Thus, no formation can be achieved. ■

**Remark 3.1:** From (8), we know that if a follower has only one neighbor, then it will eventually converge to the position of its unique neighbor. Therefore, to achieve a formation without overlapping agents, each follower requires at least two neighbors.

#### IV. TIME-DELAYS AND SYSTEM SWITCHING

In cooperative control of multi-vehicle systems, velocity information exchange is often necessary (e.g., [11], [12]). Usually, each vehicle's velocity information is measured by itself and then transmitted to others through wireless communication. Therefore, there exist transmission delays. Moreover, transmission delays may be different for different receiving agents.

Consider now that there is a constant time-delay  $\tau_i$  for agent  $i$  to receive the velocity of leaders. Then, the control strategy (1) becomes

$$\dot{z}_i(t) = \sum_{j \in \mathcal{N}_i} w_{ij} (z_j(t) - z_i(t)) + v_0(t - \tau_i). \quad (9)$$

In fact, time-delays also exist in measurement, which is exploited in [21], [22]. Generally, these measurement delays are much less than transmission delays. Thus, we only consider transmission delays in the paper.

Applying the coordinate transformation (2), we obtain the dynamics of follower  $i$  in the moving frame as

$$\dot{\hat{z}}_i(t) = \sum_{j \in \mathcal{N}_i} w_{ij} (\hat{z}_j(t) - \hat{z}_i(t)) + v_0(t - \tau_i) - v_0(t). \quad (10)$$

Applying the Laplace transform to (10) generates

$$\hat{Z}_i(s) = \frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{w_{ij}}{s + w_i} \hat{Z}_j(s) + \frac{V_0(s)}{s + w_i} (e^{-\tau_i s} - 1) \quad (11)$$

where  $V_0(s)$  is the Laplace transform of  $v_0(t)$ .

It is observed that  $\hat{Z}_i(s)$  consists of two parts: The first one  $\frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{w_{ij}}{s + w_i} \hat{Z}_j(s)$  has been discussed in Section III; The second one  $\frac{V_0(s)}{s + w_i} (e^{-\tau_i s} - 1)$  is introduced by the time-delays.

For the case with time-delays in communication, we suppose that leaders' time-varying velocity has the following form

$$v_0(t) = b_0 1(t) + b_1 t,$$

where  $b_0, b_1 \in \mathbb{C}$  and  $1(t)$  is the step function. That is, leaders are under uniform velocity motion or uniformly accelerated motion, which are much common in practice. The Laplace transform of  $v_0(t)$  is then

$$V_0(s) = \frac{b_0}{s} + \frac{b_1}{s^2}.$$

Thus,

$$\begin{aligned} & \frac{V_0(s)}{s + w_i} (e^{-\tau_i s} - 1) \\ &= \left( \left( \frac{b_1}{w_i^2} - \frac{b_0}{w_i} \right) \frac{1}{s + w_i} + \left( \frac{b_0}{w_i} - \frac{b_1}{w_i^2} \right) \frac{1}{s} + \frac{b_1}{w_i} \frac{1}{s^2} \right) \\ & \quad \times (e^{-\tau_i s} - 1) \end{aligned} \quad (12)$$

and

$$\begin{aligned} & \mathcal{L}^{-1} \left( \frac{V_0(s)}{s + w_i} (e^{-\tau_i s} - 1) \right) \\ &= \left( \frac{b_1}{w_i^2} - \frac{b_0}{w_i} \right) (e^{w_i \tau_i} - 1) e^{-w_i t} \\ & \quad + \left( \frac{b_0}{w_i} - \frac{b_1}{w_i^2} \right) (1(t - \tau_i) - 1(t)) - \frac{b_1}{w_i} \tau_i. \end{aligned} \quad (13)$$

Integrated with Theorem 3.1, we have the following result.

**Theorem 4.1:** Consider a group of  $N$  agents where leaders move with a time-varying velocity  $v_0(t) = b_0 1(t) + b_1 t$  in a formation  $[c_1, c_2, \dots, c_{N_l}]$  and followers' dynamics are defined in (9). The group of agents achieves a formation

$$[c_1, \dots, c_{N_l}, c_{N_l+1}, \dots, c_N],$$

where

$$c_i = \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} - \frac{b_1}{w_i} \tau_i \text{ for } i \in \{N_l + 1, \dots, N\}$$

if and only if the pursuit degree  $w_i$  of every follower  $i$  ( $i = N_l + 1, \dots, N$ ) has positive real part.

**Proof:** The proof is similar to the one for Theorem 3.1. The only difference is in the steady state of each follower, which is presented below. Given constant inputs for agent  $i$ 's neighbors in moving frame, then from (11), (6), and (12), we

have

$$\begin{aligned} \hat{Z}_i(s) &= \frac{\hat{z}_i(0)}{s + w_i} + \sum_{j \in \mathcal{N}_i} \frac{c_j w_{ij}}{w_i} \left( \frac{1}{s} - \frac{1}{s + w_i} \right) \\ & \quad + \left( \left( \frac{b_1}{w_i^2} - \frac{b_0}{w_i} \right) \frac{1}{s + w_i} + \left( \frac{b_0}{w_i} - \frac{b_1}{w_i^2} \right) \frac{1}{s} + \frac{b_1}{w_i} \frac{1}{s^2} \right) (e^{-\tau_i s} - 1) \end{aligned}$$

Applying the inverse Laplace transform gives

$$\begin{aligned} \hat{z}_i(t) &= \hat{z}_i(0) e^{-w_i t} + \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} (1 - e^{-w_i t}) \\ & \quad + \left( \frac{b_1}{w_i^2} - \frac{b_0}{w_i} \right) (e^{w_i \tau_i} - 1) e^{-w_i t} \\ & \quad + \left( \frac{b_0}{w_i} - \frac{b_1}{w_i^2} \right) (1(t - \tau_i) - 1(t)) - \frac{b_1}{w_i} \tau_i. \end{aligned}$$

Because  $\Re(w_i) > 0$  and  $\tau_i \ll \infty$ , the steady state of  $\hat{z}_i(t)$  is

$$c_i = \frac{\sum_{j \in \mathcal{N}_i} c_j w_{ij}}{w_i} - \frac{b_1}{w_i} \tau_i. \quad (14)$$

Thus, the conclusion follows.  $\blacksquare$

From the theorem, we can see that when the leaders' velocity is  $v_0(t) = b_0 + b_1 t$ , the agents under control law (1) are still able to achieve a formation (different from the desired one) despite the existence of transmission delays. In other words, time-delays do not affect the stability of the system.

suppose that leaders velocity  $v_0(t)$  may switch with respect to a switching signal  $\sigma(t)$ . According to Theorem 4.1, if each follower's pursuit degree has a positive real part, the agents still converge to form a steady formation as long as the time interval is long enough.

In a DAG, some edges may be disconnected because of obstacles or breakdown of sensors, and some edges may be added in. Both situations cause a switching topology. It can be observed that even for a switching topology, as long as each follower's pursuit degree in every interaction topology has positive real part, the system is still stable.

## V. SIMULATIONS

In this section, we illustrate how to design the parameters in control strategy of (1) to achieve the desired formations. Furthermore, we also present simulations when time-delays exist and the acceleration  $b_1$  switches.

We consider a group of eight agents, two of which are leaders. The DAG describing the relationship of the agents and the desired formation are shown in Fig. 2. Suppose that the leaders move in the formation of  $[0, 1]$  with the time-varying velocity  $v_0(t)$ . Then, we consider how to derive the pursuit strength and pursuit angle for each follower in (1) to achieve the desired formation.

We first design  $w_{41}$  and  $w_{42}$ . According to (8),  $-\iota = \frac{w_{42}}{w_{41} + w_{42}}$ . We set  $w_{41} = 1$ , and then  $w_{42} = \frac{\sqrt{2}}{2} e^{-\iota \frac{3\pi}{4}}$ . Obviously,  $\Re(w_{41} + w_{42}) > 0$ . Similarly, we get  $w_{31} = e^{\iota \frac{2\pi}{3}}$ ,  $w_{34} = 1$ ,  $w_{52} = 1$ ,  $w_{54} = \iota$ . Other pursuit weights can also be calculated similarly as follows:  $w_{63} = w_{31} = e^{\iota \frac{2\pi}{3}}$  and  $w_{64} = w_{34} = 1$ . Note that the pursuit weights are not unique. From the pursuit weights, we can obtain the pursuit strength and pursuit angles easily.

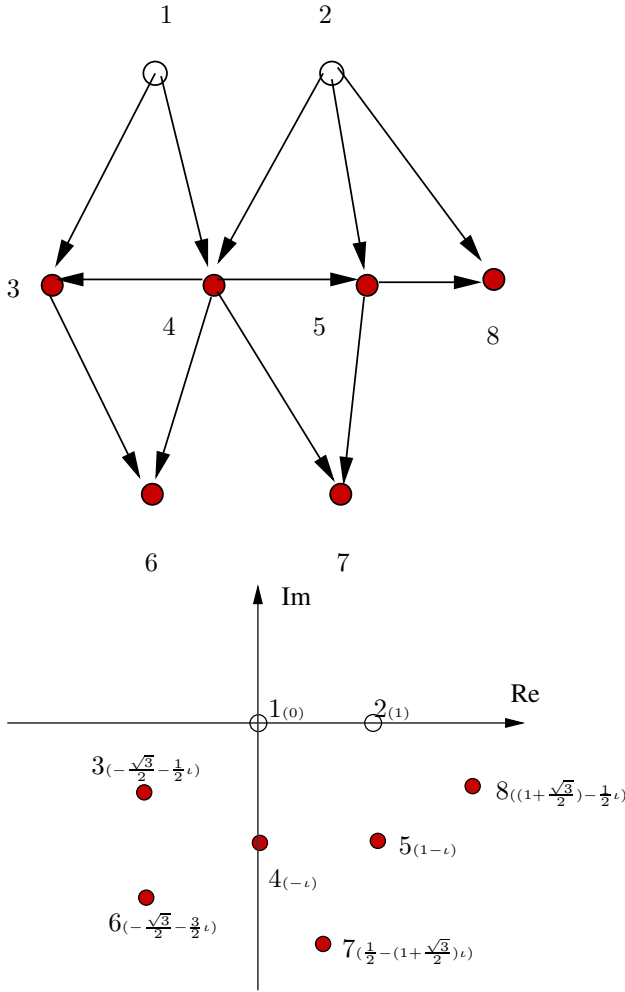


Fig. 2. A DAG and the desired formation (the complex number in bracket is the agent's coordinate).

The collective motion is simulated as shown in Fig. 3, in which  $v_0(t) = 0.01t + i0.02t \sin(0.2t)$ .

For the same DAG and the desired formation in Fig. 2, we consider the situation where time-delays exist. Suppose that the time-delay is defined by  $\tau_3 = \tau_4 = \tau_5 = 5$  and  $\tau_6 = \tau_7 = \tau_8 = 10$ . Because the acceleration is small numerically, the time-delays are supposed to be much longer than the general cases in order to illustrate the variations of formations explicitly. The switching procedure is as follows: at  $t = 0$ ,  $v_0 = 0.4 + 0.1i$  and the acceleration  $b_1$  switches to  $-0.02i$ ; at  $t = 30$ , the acceleration  $b_1$  switches to  $-0.02 + 0.1i$ ; at  $t = 50$ , the desired velocity  $1.5i$  is reached and the acceleration  $b_1$  switches to zero. We show the formations at  $t = 29$ ,  $t = 49$ , and  $t = 56$  in Fig 4, Fig 5, and Fig 6, respectively. Notice that, the formations in Fig 4 and Fig 5 are different from the desired formation due to the nonzero acceleration and time-delays. Furthermore, it is observed that the distinction is more obvious when  $\|b_1\|$  is bigger, and the formation recovers to the desired one while making uniform rectilinear motion.

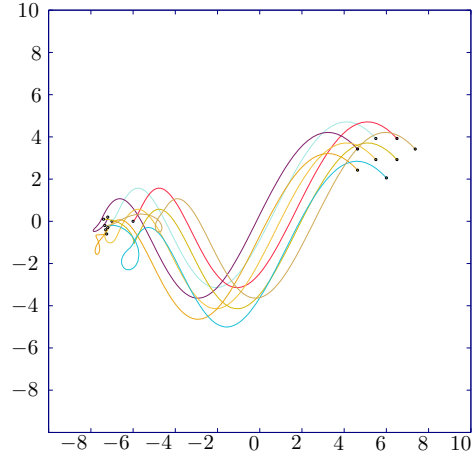


Fig. 3. Eight agents make a time-varying motion in the desired formation.

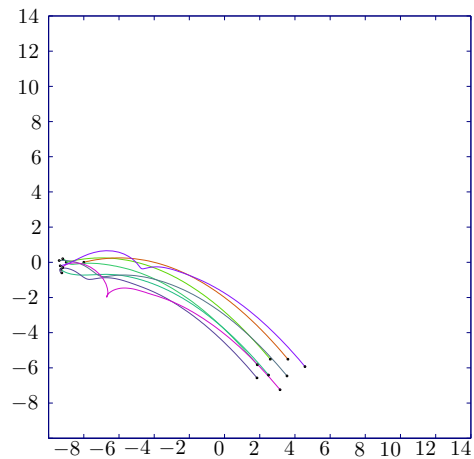


Fig. 4. Formation at  $t = 29$  and  $b_1 = -0.02i$ .

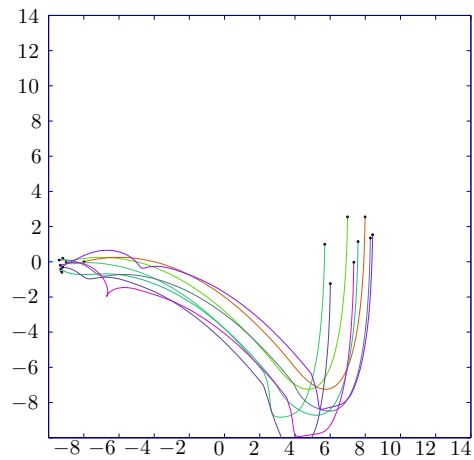


Fig. 5. Formation at  $t = 49$  and  $b_1 = -0.02 + 0.1i$ .

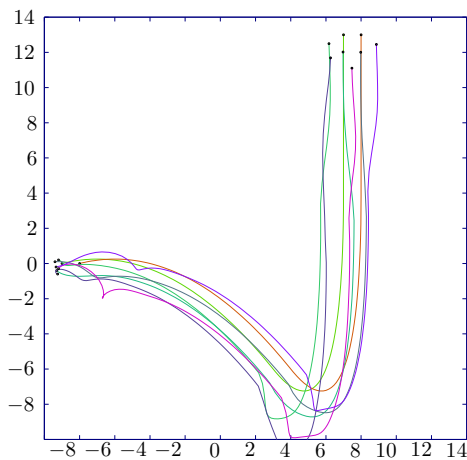


Fig. 6. Formation at  $t = 56$  and  $b_1 = 0$ .

## VI. CONCLUSIONS

This paper studies the formation control problem of multi-agent systems under directed acyclic graphs. We extend the existing pursuit algorithms by taking into account of leaders' velocity, and derive a formation control strategy for followers. Supposing that the leaders move in a formation with a time-varying velocity, necessary and sufficient conditions are presented for all the agents to reach desired formations. We also discuss the impacts on the formations when communication delays and system switching exist. Finally, through simulation, we specifically describe how to design pursuit strength and pursuit angles for each follower to reach a desired formation, and illustrate the variations of the formation caused by time-delays.

In the paper, we suppose that the directed acyclic graph, wireless communication network, pursuit strength, and pursuit angles are well established in advance, but it is more attractive if these can be constructed and optimized by the agents themselves. Collision avoidance is another issue to be considered in the future.

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