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Bilateral Teleoperation of a Formation of Nonholonomic Mobile Robots Under Constant Time Delay

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Abstract— **We applied partial feedback linearization to the unicycle model to stabilize part of the state at a desired position on the plane and extend it to design formation control for teleoperation. Further, using output synchronization results we derive a Single Master Multiple Slave bilateral teleoperation system robust to constant unknown, possible different time delays between master and formation and among mobile robots and formation. We show our simulation's results to illustrate the performance of the derived control law.**

I. INTRODUCTION

Teleoperation has very promising values in real-world applications. Among the existing applications we can find: tele-manipulation in remote or inaccessible environments (i.e., deep sea and space exploration[9], [22] and micro- and nano-teleoperation [8]), tele-surgery [26], space construction, disaster zone exploration, handling of hazardous materials, surveillance sensor networks, and rescue [19].

The problem of teleoperation under constant time delay was solved within the realm of passivity based control in [1] using scattering transformation and later in [18] using wave variables. Those schemes are now known to be equivalent. The work done in [5] and [14] propose solutions to the problem of variable time delay in bilateral control for the case of Single Master Single Slave (SMSS) systems. SMSS systems have been studied in continuous [24], [11] and discrete time [2], [6], [23] in the past two decades.

The cost saving and maneuverability benefits of teleoperating a group of robots instead of a single one has led to the research on cooperative tele-manipulation [13], [16]. A common problem of telecommunication is the reliability of the communication channels. Often times, the channels are unreliable, subjecting to time delay and information loss. The use of such unreliable communications channels is attractive given their much lower cost compared to especially dedicated channels. Besides, the delay in transmission is inherent to the communications technology available today, given its finite transmission rate. In [21], the problems related to Internet based teleoperation are discussed. Some of these problems have been dealt with in [7]. We propose to extend the research by studying the case when the remote agents are non holonomic systems and there is constant time delay in the communications channel. To our knowledge, the only work on this relatively new direction, are [10], [16] considering a single mobile robot. The literature review in [15] points out the lack of research in this area.

The cooperative teleoperation of a group of robots is of importance given that in certain operations the use of several small robots to carry out a task is more convenient and cheaper than to use a single bigger robot, such as carrying an object in an unknown environment. Such systems can be applied in exploration, construction, recovery and rescue. Also, in these operations the task space is usually larger than the robot which makes it suitable for mobile robots.

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II. PROBLEM STATEMENT

We try to find a control law that allows us to abstract a formation of mobile robots to be teleoperated by a human operator through a communications channel with constant time delay. Such control law should allow the operator to drive the formation from A to B considering the nonholonomic constraints of differential drive mobile robots (DDMRs) and keeping them in formation at the same time. Figure 1 illustrates this problem.

III. CONTROL OF KINEMATIC MODEL

We start by studying a control for a single DDMR. We then extend it to multiple DDMRs in formation. The model of the DDMR we use here (w.r.t. Figure 2(a)) is the unicycle model (1), below. To save space, we will use the notation $S(*) := \sin(*)$ and $C(*) := \cos(*)$ from now on.

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} C(\phi) & 0 \\ S(\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}. \quad (1)$$

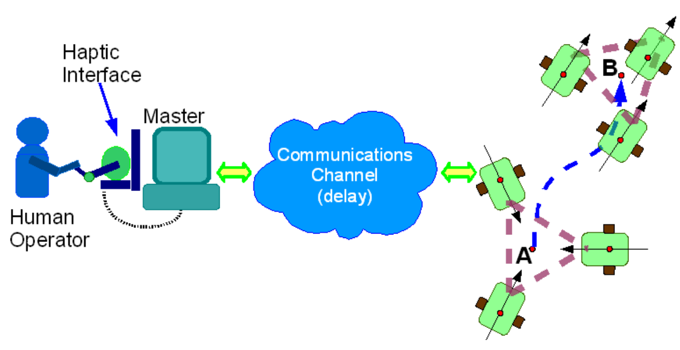


Fig. 1. Studied problem.

The problem we try to solve is: given the restrictions (1), how to design the inputs v and w such that the initial position of the robot (x_p, y_p) converges to a desired position (x_d, y_d).

By observing the caster wheels of a chair, we can see that the wheel follows any force applied to the chair without violating any velocity constraints. We can easily derive a model for this as in (2) below, with reference to Figure (2(a)),

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \end{bmatrix} + \begin{bmatrix} dC(\phi) \\ dS(\phi) \end{bmatrix}. \tag{2}$$

By differentiating (2) with respect to time and using (1) we obtain the Jacobian that maps angular and linear velocities of the robot, to velocities of the point at which the forces are applied,

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \underbrace{\begin{bmatrix} C(\phi) & -dS(\phi) \\ S(\phi) & dC(\phi) \end{bmatrix}}_{J} \begin{bmatrix} v \\ w \end{bmatrix} \tag{3}$$

We can see that the Jacobian J in (3) is full rank as long as $d \neq 0$, since $det(J) = d$.

Solving for (v, w) from (3) and substituting $(\dot{x}_p, \dot{y}_p)^T$ by desired velocities $(\dot{x}_p^d, \dot{y}_p^d)^T$ to be designed later, we obtain

$$\begin{bmatrix} v \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} C(\phi) & S(\phi) \\ -\frac{1}{d}S(\phi) & \frac{1}{d}C(\phi) \end{bmatrix}}_{J^{-1}} \begin{bmatrix} \dot{x}_p^d \\ \dot{y}_p^d \end{bmatrix}. \tag{4}$$

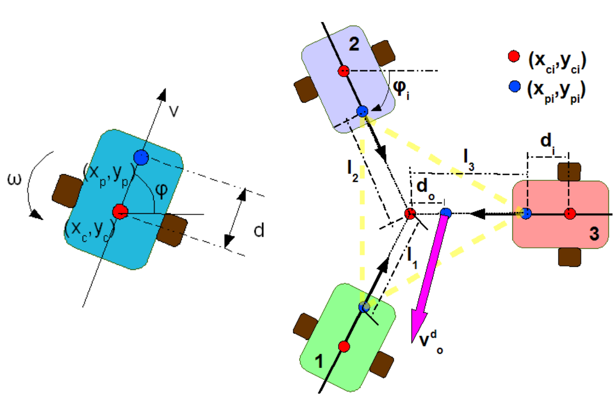
In this way the desired velocities in Cartesian space are taken to be the control inputs $(\dot{x}_p^d, \dot{y}_p^d)^T := (u_x, u_y)^T$ to system (3).

Now, we can define the inputs (u_x, u_y) as in (5), below.

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} k_x(x_d - x_c - dC\phi) \\ k_y(y_d - y_c - dS\phi) \end{bmatrix} \tag{5}$$

Substituting (4) in (3) we obtain the linear system (6), below, which is globally exponentially stable at the origin, $(x_d, y_d) = (0, 0)$, and can be made to converge arbitrarily fast.

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} k_x(x_d - x_p) \\ k_y(y_d - y_p) \end{bmatrix} \tag{6}$$



(a) (b)

Fig. 2. Variables definitions.

As $(x_p, y_p) \to (x^d, y^d)$ we get $(v, w) \to (0, 0)$. Since (x_p, y_p) converges to the desired position at (x^d, y^d) as $t \to \infty$, then (x_c, y_c) will converge to a point on a circle of radius d centered at (x^d, y^d).

We note that the state of the unicycle system is only guaranteed to be bounded, which is not surprising given that it has been shown that driftless systems do not satisfy all necessary conditions given in Brocket's theorem [3] for stabilizability of nonlinear systems using smooth feedback. Thus, we cannot stabilize the nonholonomic model using smooth feedback of the state. This does not represent a big obstacle for teleoperation applications since the human can apply the equivalent to switching references to get the formation to achieve the position and orientation desired while the orientation of individual robots is left to be decided by the inverse kinematics.

Proposition 3.1: The full state of (1), under control (5) is bounded and the limit as $t \to \infty$ of the state variables is

$$\begin{bmatrix} x_{c\infty} \\ y_{c\infty} \\ \phi_\infty \end{bmatrix} = \begin{bmatrix} x_p^d - dC(\phi_\infty) \\ y_p^d - dS(\phi_\infty) \\ \gamma_o + 2\arctan\left[e^{-r_o/d} \tan\left(\frac{\phi_o - \gamma_o}{2} \right) \right] \end{bmatrix} \tag{7}$$

Please see appendix for the proof. The authors became aware of a similar result in [20] after designing this control.

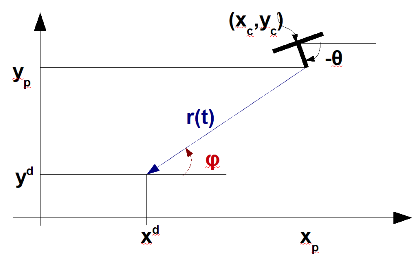


Fig. 3. Diagram of auxiliary variables used for the solution of $\phi(t)$.

A. Passivity of the Controlled System

It is easy to show that the system given in (3) is lossless with respect to position as output and with inputs defined as

$$u_x = -k_x(x_c + dC(\phi)) \tag{8}$$
$$u_y = -k_y(y_c + dS(\phi)) \tag{9}$$

First, using the Lyapunov function $V(x) = 0.5(x^2 + y^2)$ the system (3) can be found to be globally exponentially stable at the origin. Also, by using $V(x)$ as storage function we can show that $u^T y = \dot{V}(x)$, thus the systems is lossless.

B. Bounded Velocity

To consider bounded velocity restriction of the robots we use a varying scaling of the controls u_x, u_y (following [20]), such that the robots do not exceed an arbitrary velocity limit. The scaling factor used is given in (12) below. The new controls are:

$$u_x = k\eta(e_{xy})(x^d(t) - x_p) + \dot{x}^d(t) \tag{10}$$
$$u_y = k\eta(e_{xy})(y^d(t) - y_p) + \dot{y}^d(t) \tag{11}$$

where

$$\eta(e_{xy}) = \frac{k_1}{k_2 + \|e_{xy}\|}. \quad (12)$$

With $\|e_{xy}\| = \left((x^d(t) - x_p(t)^2 + (y^d(t) - y_p(t)^2)\right)^{1/2}$, we have that $0 \leq \eta \leq k_1/k_2$, and we choose $k_2 \geq k_1 > 0$.

Then, using $V = e_{xy}^T e_{xy}$, with $e_{xy} = (e_x, e_y) = (x^d - x, y^d - y)$ we can show the system is GES, since $\dot{V} = -k\eta(e_{xy})e_{xy}^T e_{xy} < 0$ and $\dot{V} = 0$ at $e_{xy} = 0$, because $1 \geq \eta(e_{xy}) > 0$, $\forall t$ by construction.

Since the scaled system $\dot{e}_{xy} = -k\eta(e_{xy})e_{xy}$ is a slower version of the original $\dot{e}_{xy} = -ke_{xy}$, the states (1) still approach the limits (7) for a finite desired set point.

C. Multiple Robots

The same procedure can be extended to control multiple robots. We model the formation as yet another differential mobile robot and using its Jacobian we design set points for each DDMR in the formation. From there, the Jacobian of robot i translates the references for its controlled point to velocities (v_i, w_i) using (4).

The desired velocities of the formation can be derived as in (4) if we want to move the formation from one point to another. Another advantage comes from being able to control velocities directly. In this way, it is straightforward for a human operator to control the direction and velocity of motion of the formation. The desired velocities for i^{th} robot are obtained using (4) with design inputs

$$u_{xi} = k_{xi}(x_i^d - x_i - d_i C(\phi_i)) \quad (13)$$
$$u_{yi} = k_{yi}(y_i^d - y_i - d_i S(\phi_i)), \quad (14)$$

and the desired positions (x_i^d, y_i^d) are different for each robot depending on their desired position within the formation and are given by

$$x_i^d = x_f^d + l_i C(\phi_f + \pi + \phi(0)_i) \quad (15)$$
$$y_i^d = y_f^d + l_i S(\phi_f + \pi + \phi(0)_i)), \quad (16)$$

where (x_f^d, y_f^d) is the desired position for the formation, l_i is the distance from the controlled point of the formation to the point $(x_{pf}, y_{pf})_i$ (the controlled point of i^{th} robot), $\phi(0)_i$ is the initial angle of the robot w. r. t. the angle of the formation. For each robot's controlled point we also have global asymptotic stability and arbitrary fast convergence. This enables open loop formation coordination. In Section IV, we will discuss how the bilateral connections between the formation and each agent produces closed loop formation control.

D. Collision Avoidance

To prevent collision against obstacles we define a potential field around the obstacles. The obstacle is detected by the formation but not by individual robots, in this way, the robots evade the obstacles without breaking the formation. Also, due to the radius reduction problem, which can cause the robots to collide with each other, we have to consider the limited velocities of the real robots for actual implementation.

For these purposes, following [25], we use the obstacle avoidance potential function defined by

$$U = \left(\min\left\{0,\ \frac{\eta^2 - R^2}{\eta^2 - r^2}\right\}\right)^2 \quad (17)$$

where

$$\eta = \sqrt{(x_f - x_{obs})^2 + (y_f - y_{obs})^2} \quad (18)$$

The gradient of (17) defines the repulsive velocity commands to be applied to the formation (19). This gradient is then subtracted from the control input of the formation to prevent collision with obstacles and to provide a way for the formation to interact with its environment.

$$\nabla U = \begin{bmatrix} \frac{4(R^2 - r^2)(\eta^2 - R^2)(x_f - x_{obs})}{(\eta^2 - r^2)^3} \\ \frac{4(R^2 - r^2)(\eta^2 - R^2)(y_f - y_{obs})}{(\eta^2 - r^2)^3} \end{bmatrix} \quad (19)$$

defined for $r < \eta \leq R$, undefined at $\eta = r$ (it becomes infinity) and zero every where else (i. e. $R < \eta < r$). (x_f, y_f) is the position of the formation and (x_{obs}, y_{obs}) is the position of the obstacle. Then the control for the formation becomes:

$$\begin{aligned} u_f(t) &= \sum_{i=1}^{N} K(q_i(t - T_{if}) - (q_f)) + \\ &\quad K(q_m(t - T_{mf}) - (q_f)) - \nabla U \end{aligned} \quad (20)$$

The same potential field is used in each robot to detect other robots and avoid collisions.

E. Local Master Control

We also consider a haptic interface as mater device, for which:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = J^T(F + F_h) \quad (21)$$

where F is the control force and F_h is the force applied by the human operator, both given in task (Cartesian) space. The reference velocity for kinematic control is designed to be

$$\dot{x}_m^d = K(q_f(t - T) - x_m) \quad (22)$$

where $x_m = (x, y)$ are the Cartesian coordinates of the master endpoint position projected on the horizontal plane.

We consider the kinematic control (23) for the master,

$$\dot{q}^d = J(q)^{-1} K(x_m^d - x_m) \quad (23)$$

In our case the haptic interface is mechanically constrained and thus it does not reach any singular configuration in its work space. However, for the sake of a more general discussion, we mention that we can use the Singularity Robust Inverse instead of $J(q)^{-1}$, (the reader is referred to [17], chapter 9 for detailed information) if it is needed to operate at or near singular points. Then we define a force in Cartesian space for the haptic master device as:

$$F = K_m(\dot{x}_m^d - \dot{x}_m) - K_o x_m \quad (24)$$

where: F_h is the input from the operator, K_o anchors the whole system to the origin to guarantee boundedness. The outputs used in (27) are the position of the controlled point given by (1) in Cartesian space.

IV. Bilateral Teleoperation

In this section we use the coordination control to design a bilateral teleoperation loop that is robust to constant unknown time delay between master and formation and among agents of the formation. We draw from the results in output synchronization found in [4]. Considering the following two assumptions:

Assumption 4.1: There is a unique path between any two subsystems.

Assumption 4.2: The subsystems are weakly connected point wise in time and form a balanced graph with respect to information exchange.

Our main result is stated as follows:

Proposition 4.3: Using assumptions (4.1) and (4.2), the system of N mobile robots, one virtual formation system and a master system simplified to (25)-(27), below, by controls (13) and (22)-(24), is globally asymptotically stable, the mobile robots and master synchronize their positions, and synchronization errors can be rendered as force feedback at the master subsystem:

$$\dot{q}_m = K\left(q_f(t-T_{mf}) - q_m(t)\right) - Kq_m(t) \quad (25)$$

$$\dot{q}_f = K\left(q_m(t-T_{mf}) - q_f(t)\right) + K\sum_{i=1}^{N}\left(q_i(t-T_{if}) - q_f(t)\right) \quad (26)$$

$$\dot{q}_i = K\left(q_f(t-T_{if}) - q_i(t)\right) \quad (27)$$

where $q_* := (x_p, y_p)_*^T$, $i = 1, 2..., N$, with subsystem output given by $q_*(t)$.

Proof. Let us define the index set $N_o = \{m, f, 1, 2, ..., n\}$, which stands for master, formation, robot 1, robot 2, up to robot n in the formation, and for any $k \in N_o$ let $N_k \subset N_o$ be the set of indices of subsystems connected to subsystem k. Consider the Lyapunov-Krasovskii functional

$$V(q) = K\sum_{k\in N_o}\sum_{j\in N_k}\int_{t-T_{kj}}^{t} q_j(\tau)q_j(\tau)d\tau + 2\left(V_m + V_f + V_1 + ... + V_n\right) \quad (28)$$

where $V_* = y_*^T y_*$ is the storage function of the corresponding subsystem. Because we use the linearized system (27), it is not difficult to see that $\dot{V}_* = q_*^T \dot{q}_* = y^T u_*$. Since $y_* = q_*$ and $\dot{q}_* = u_*$, each subsystem is lossless. Taking derivatives along trajectories of (27) (omitting the time dependency for variables that are not delayed) and simplifying we obtain

$$\dot{V} = -K\sum_{k\in N_o}\sum_{j\in N_k}\left(q_k - q_j^T(t-T_{kj})\right)^T \quad (29)$$

$$\left(q_k - q_j^T(t-T_{kj})\right) - 2Kq_m^T q_m \quad (30)$$

so that $\dot{V} \leq 0$, $\dot{V} = 0$ when $q_k = q_j = q_m = 0$, therefore the system is globally asymptotically stable. There is no disturbance rejection mechanism (only proportional controllers), so the disturbances in the slave environment are transmitted to the master system, where they are used to display force feedback.

Remarks. In the previous definition and proof, we used a gain K for simplicity, but note that, as long as $K_{ji} = K_{ij} > 0$, we can still complete the square to make the proof work. This provides more flexibility for control design and performance improvement.

A. Properties of the teleoperation system

For the proposed teleoperation system, we have that:

1) In free motion, the master's and slave formation's outputs converge to each other as $t \to \infty$.
2) The formation error converges to zero for free motion of the DDMRs.
3) In constrained motion, there is a control effort feedback proportional to the error in position between formation and master systems.

We demonstrate the properties enumerated above in the following sections.

Property (1) was proved in Section IV.

To show property (2) consider the formation error (stated without offsets for simplicity):

$$E = \begin{bmatrix} q_f(t-T_{f1}) - q_1(t) \\ q_f(t-T_{f2}) - q_2(t) \\ \vdots \\ q_f(t-T_{fn}) - q_n(t) \end{bmatrix} \quad (31)$$

where q_* is the position of system $* = f, 1, 2, ..., n$ and T is the time delay. In free motion $|E| \to 0$ as $t \to \infty$ by the output synchronization of the formation and each DDMR shown in (30). In constrained motion, if a single robot incurs in an error, such error is propagated to the formation by the bilateral connection between each robot and the formation. The error is also fed back to the master through the bilateral connection between master and formation since the master is at rest either when it synchronizes its position with the formation or when the human force equals the force due to any synchronization error for $K_o = 0$. If we have $K_o > 0$ then: $q_i = q_j = 0$ for all $i, j \in m, f, 1, 2, ..., n$, $i \neq j$ and $F_h = K_o x_m - K_m(q_f(t-T) - x_m)$.

For property (3), we have that by substituting (22) in (23), we obtain a PD position controller for the master:

$$F = K_m(q_f(t-T) - x_m) - K_m\dot{x}_m + F_h - K_o x_m \quad (32)$$

where as $\dot{x}_m \to 0$, we have $Fh \to K_m(x_m - q_f(t-T)) - K_o x_m$ which provides linear force feedback proportional to the error in the position between master and formation. In other words, $F_h \to (K_m(q_f(t-T) - x_m) - K_o x_m)$ if the operator wants to maintain the desired position of the master when the slave formation incurs in a synchronization error.

B. Communications graph

The communications graph of system (27) is shown in Figure 4. We can observe that assumptions A1 and A2 are met given that the graph is strongly connected and there is a unique path between any two nodes. Although the graph is undirected, arrows have been drawn to emphasize the bilateral connection between interconnected nodes. This is important because it enforces closed loop formation control.

V. Simulation

To verify our result we simulate a teleoperation system. The master is a unicycle bilaterally coupled to the formation through position output. The formation is bilaterally coupled to the mobile robots using position outputs as well. The delay is 0.5 seconds. All gains are set to 10. The initial conditions for the robots are: $(x_0, y_0, \theta_0)_1 = (3, -3, 5\pi/4)$, $(x_0, y_0, \theta_0)_2 = (-3, -3, 3 * pi/4)$ for robot 2; and $(x_0, y_0, \theta_0)_3 = (0, 2, 0)$ for robot 3. The master and formation start at the origin.

For this simulation, the obstacle avoidance potential function described in (III-D) is used. This function is subtracted from the control input of the formation. In this way, collisions with obstacles are prevented and the interaction of the formation with its environment is established. The error introduced due to (17) is reflected back to the master (most noticeable between $t = 40[s]$ and $t = 50[s]$). As soon as the formation passes the obstacle the synchronization of outputs is regained (observed at $t = 52[s]$).

The parameters used are detection radius $R = 6.5$ and avoidance radius $r = 2.5$. Two obstacles are placed in the workspace of the formation, one is fixed at $(0, -8)$ the other one follows a circular trajectory according to: $x(t) = 5S(0.1t)$ and $y(t) = -5C(0.1t)$. The results are shown in Figures 6 and 8.

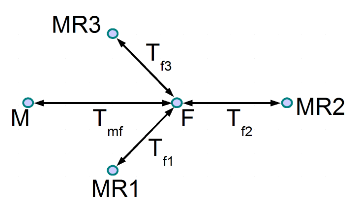


Fig. 4. Communications graph for a realization of the proposed teleoperated system. The variable definitions are: M stands for master; MR1 to MR3 for mobile robots 1 to 3, T for the time delay between corresponding adjacent nodes and F for formation.

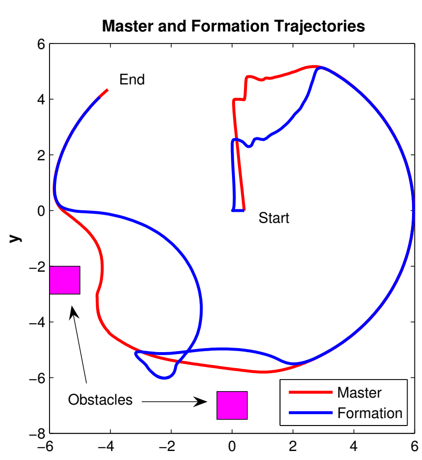


Fig. 5. Formation and master trajectories on the $x - y$ plane.

VI. Concluding Remarks

We applied partial feedback linearization approach to the kinematic model of unicycle to control a formation of robots that functions as the slave in a bilateral teleoperation system and proved asymptotic stability considering constant time delay in the communication links.

It is possible to extend this work include the dynamic model of the DDMR by using the Jacobian from angular and linear velocities to angular velocities of the left and right wheels. This is the topic of future research. Also extensions to variable time delay with quantization by using the results found in [7] is readily achievable.

A limitation of the current scheme comes from the bilateral connection between master and slave formation through position synchronization because the space on which the mobile robots can move is constrained to a scaled version of the master's work space. This problem can be solved using indexing or, perhaps more interesting, by using of position to velocity interconnection as in [16]. The authors very recently became aware of the work of [12] which also deals with bilateral teleoperation of multiple nonholonomic mobile robots. The advantage of our scheme, is that it can resolve arbitrary and different initial conditions for each robot in the formation.

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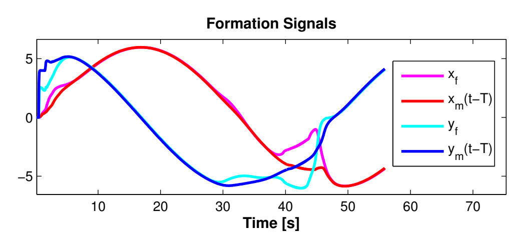

Fig. 6. Formation position (x_{cf}, y_{cf}) and master delayed position $(x_{cm}(t - T), y_{cm}(t - T))$.

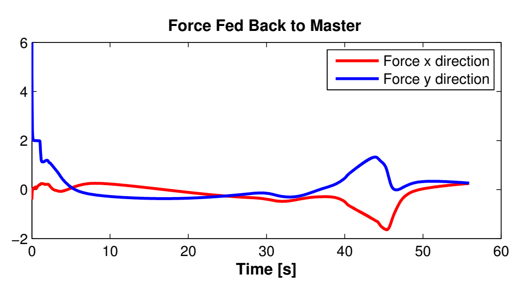

Fig. 7. Force feedback to Master.

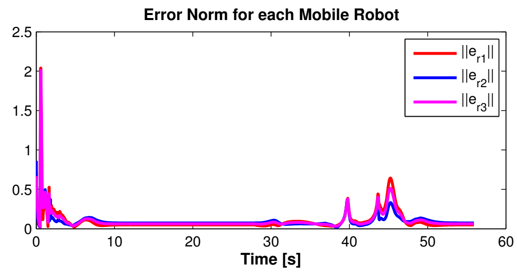

Fig. 8. Error norm for each robot.

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VII. Appendix

A. Proof of Proposition 3.1.

We can solve for the closed form solution of the heading angle, $\phi(t)$. The solution to (6) is $*_p(t) = x^d - (x^d - x_p(0))e^{-k(t-t_o)}$, where $*_p(t) = \{x_p(t), y_p(t)\}$. Equation (1) reduces to (33), below, by substituting control (5)

$$\begin{array}{rcl} \dot{x}_c & = & C(\phi)\left[k(x^d - x_p)C(\phi) + k(y^d - y_p)S(\phi)\right] \\ \dot{y}_c & = & S(\phi)\left[k(x^d - x_p)C(\phi) + k(y^d - y_p)S(\phi)\right] \\ \dot{\phi} & = & -\frac{k}{d}(x^d - x_p)S(\phi) + \frac{k}{d}(y^d - y_p)C(\phi) \end{array} \quad (33)$$

where (x^d, y^d) is the desired set point. W. r. t. Figure III, define $r(t)^2 := (x^d - x_p(t))^2 + (y^d - y_p(t))^2$ and $\gamma := \arctan((y^d - y_p(t))/(x^d - x_p(t)))$ and use the change of variables $(x^d - x_p(t)) = r(t)C(\gamma)$ and $(y^d - y_p(t)) = r(t)S(\gamma)$, the differential equation for $\phi(t)$ reduces to

$$\dot{\phi} = \frac{-k}{d} r(t) S(\phi - \gamma). \quad (34)$$

Note that $\gamma(t)$ can be found to be a constant, in fact $\gamma(t) = \gamma(0), \ \forall \ t$. Using the definition of Euclidean norm we find that $r(t) = r(0)exp(-kt)$. Thus we can separate variables in (34) to integrate, and after solving for $\phi(t)$ we obtain

$$\phi(t) = \gamma_0 + 2AT\left[\exp\left(-\frac{r_0}{d}(1 - e^{-kt})\right) T\left(\frac{\phi_0 - \gamma_0}{2}\right)\right]$$

where $AT(*) := \arctan(*)$, $T(*) := tan(*)$ and $*_0 = *(0)$. We can now solve the differential equations for $x_c(t)$ and $y_c(t)$ in (1) as follows:

$$\begin{array}{rcl} x_c(t) & = & x^d - \left(x^d - x_p(0)\right)e^{-kt} - dC(\phi(t)) \quad (35) \\ y_c(t) & = & y^d - \left(y^d - y_p(0)\right)e^{-kt} - dS(\phi(t)) \quad (36) \end{array}$$

where we have used (2) and the solution of (6), $*_c^d$ is the desired set point for $*_p(t)$. By taking the time limit of $(x_c(t), y_c(t), \phi(t))$ the result follows.