

Biologically Inspired Adaptive Mobile Robot Search With and Without Gradient Sensing

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Abstract – Many biologically inspired approaches have been investigated in relation with researches on mobile robot(s) that can effectively locate targets that induce gradient information. Here, we concentrate on realizing an adaptive searching behavior in mobile robot that is simple, yet effective with and without sensing the gradient information. We are interested in two searching behaviors found in biological creatures: bacterial chemotaxis, probably the simplest yet effective gradient sources searching behavior found in living creatures; and Levy walk, specialized random walks with fractal movement trajectories that optimize random search for sparsely and randomly distributed target(s). Our approach is to implement and combine the two searching behaviors based on “yuragi”, a Japanese word for biological fluctuation.

I. INTRODUCTION

BIOLOGICAL creatures provide many examples on how to realize adaptive behaviors in robots or artificial agents. Chemotaxis behavior, a motion of organism in response to the presence of chemical concentration gradients [1], has been exploited in many literatures as an effort to realize mobile robot(s) that can effectively search and locate targets that induce gradient information. Some examples include biomimetic robot lobster built to investigate the way lobster localize and track odor plumes [2], insect-size mobile robot inspired by silkworm moth [3], moth-inspired behavior-based AUV [4], as well as a multi robot system inspired by swarm intelligence of ants [5].

The simplest organism in which its behavior has been imitated to perform such task is bacteria. The motion of bacteria in response to chemical concentration gradients is called bacterial chemotaxis [1, 6], which has been adopted in certain ways to realize simple yet effective searching behavior for gradient-inducing targets performed by robot or artificial agent [1, 8].

Indeed, while such robots can be instrumental in tasks such as locating hazardous chemical leaks, oil spill in the water and environmental monitoring, problems such as local minima and absence of gradient are commonly faced. Furthermore, when the targets are sparse, it is likely that most of the time the robot won't be able to sense the induced gradient information.

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In fact, ranging predators actually have to make foraging, searching for food, decisions with little, if any, knowledge of targets distribution and availability [9]. Therefore, as a result, the creatures do a random search. In relation with this, a special kind of random walk, called Levy flight, or to be more exact Levy walk, receives many attention in the literatures [9][10][11][12][13].

Our aim is to realize a searching behavior in mobile robot that is simple, yet effective with and without sensing the gradient information, by imitating certain searching behaviors found in biological creatures. We are interested in two of them: bacterial chemotaxis and Levy walk. Our approach is to implement and combine these two searching behaviors based on “yuragi”, or biological fluctuation [14].

The organization of the paper is as follow. First, we will explain about the two mentioned searching behaviors: bacterial chemotaxis and Levy walk. Then, the principle and realization of yuragi-based searching behavior in mobile robot that implement and combine them will be explained. In order to verify the approach, we perform simulation experiments, with sensory model from real robot. The experiment setup, the aim of each experiment condition, and the results will be described. At the end, we will discuss some conclusions as well as future works.

II. SEARCHING BEHAVIORS IN BIOLOGICAL CREATURES

A. Bacterial Chemotaxis

In a bacterial chemotaxis, such as performed by *Escherichia coli*, the motion can be characterized as a sequence of smooth-swimming runs, punctuated by intermittent tumbles that effectively randomize the direction of the next run [6]. These two motions can be called the “swimming” and the “tumbling” mode. The probability that a smooth swimming *E. coli* cell will stop its run and tumble is dictated by measurement of attractant chemical gradient in the environment. As *E. coli* are only a few microns long, they are unable to measure the gradient by comparing head-to-tail concentration differences, but use a kind of memory to compare current and past concentration.

When the bacterium perceives conditions to be worsening, the tendency to tumble is enhanced. Conversely, when it detects that the condition, i.e. the attractant chemical concentration, is improving, tumbling is suppressed and it keeps running. As a result, when the bacterium runs up a gradient of attractant, it will do chemotaxis, as it tends to continue on course and do a biased random walk toward the source of the attractant. However, in the absence of this gradient, the bacterium will simply do random walk.

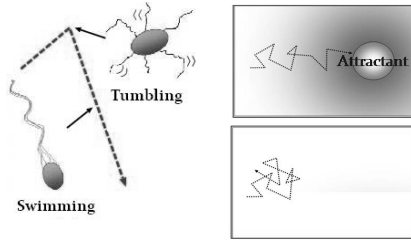


Fig. 1 Bacterial chemotaxis. left: swimming and tumbling mode, top right: “chemotaxis”, biased random walk toward the source of attractant, bottom right: random walk in the absence of a gradient of attractant

This bacterial chemotaxis behavior with and without the gradient information can be explained by Fig. 1. Here, an interesting question is whether the random walk part is really effective.

B. Levy Walk

As many creatures do random search, in [10], a general question of what is the best statistical strategy to optimize a random search has been addressed. It is shown that the search efficiency depends on the probability distribution of the flight lengths taken by the forager, and when the target sites are sparsely and randomly distributed, the optimum strategy is a specialized random walks movement so called Levy flight. By performing Levy flight, creatures can optimize the number of targets encountered versus the traveled distance. This Levy flight motion has been found among various organisms, such as marine predators [9], fruit flies [12], even humans [13].

Levy flight is specialized random walks comprising ‘walk clusters’ of short move step lengths (distance moved per unit time) with longer reorientation jumps between them. This pattern is repeated across all scales, with the resultant scale-invariant clusters creating trajectories with fractal patterns [12]. Levy flight move steps are drawn from a probability distribution with a power-law tail:

$$p(l) \approx l^{-\mu-1} \quad (1)$$

with $0 < \mu < 2$, and l is the flight length, that is the length before changing direction randomly. Compare to Brownian motion, where the flight distribution are drawn from Gaussian distribution, the probability of Levy flight for returning to previously visited site is smaller, and therefore advantageous when target sites are sparsely and randomly distributed [10].

However, to be more exact, a more technically correct term is actually Levy walk. While Levy flight’s expression only considers the flight length, and does not consider a time cost which depends on this length, Levy walk does [11].

A creature performing a Levy flight jumps between sites, no matter distant, which leads to a divergence of the mean-squared displacement. Levy walk is a stochastic process which visits the same sites as in the Levy flight, but with a time “cost” that depends on the flight length or distance, and therefore has a finite mean-squared displacement [11]. In other word, the sites visited in Levy flight are the turning points in Levy walk [9]. While in Levy flight we only need to specify

$p(l)$, in a framework of continuous time random walk, we can write $p(l, t)$, the probability to move a distance l in time t in a single motion event and to stop at t for initiating a new motion event at random. It is given by:

$$p(l, t) = \delta(|l| - t) p(t) \quad (2)$$

The delta function accounts for the motion at a constant velocity, while length l and time t are given in dimensionless units. Therefore, (2) describes Levy walk with constant velocity, where $p(t)$ follows a probability distribution with a power-law tail with $0 < \mu < 2$:

$$p(t) \approx t^{-\mu-1} \quad (3)$$

III. YURAGI-BASED SEARCHING BEHAVIORS IN MOBILE ROBOT

A. The Principle

“Yuragi” is a Japanese word for biological fluctuation. It is used by Kashiwagi et al [14] to explain bacteria adaptation to environmental changes by altering their gene expression. This gene expression is controlled by a dynamical system with some attractors, and the model can be represented by Langevin equation as:

$$\dot{x} = f(x) \times A + \varepsilon \quad (4)$$

where x and $f(x)$ are the state and the dynamics of the attractor selection model, with $f(x)$ can be designed to have some attractors. ε is the noise term, which will be called the “yuragi” noise, because we could also have noises from the environment. A is a variable called “activity” which indicates the fitness of the state to the environment. From the equation, $f(x) \times A$ becomes dominant when the activity is large, and the state transition approaches deterministic. When the activity is small, ε becomes dominant, and the state transition becomes more stochastic. The activity is therefore designed to be large when the state is suited to the environment and vice versa. This framework actually introduces many design possibilities, with one explained in [15].

In order to apply (4) to robot control, we interpret the state x as the posture of the robot, thus \dot{x} as the motion of the robot. $f(x)$ can be designed to have some attractors which correspond to particular motions. Therefore, the state of the system entrained into a particular attractor when the activity is large and let the robot tends to keep taking the same motion. Here, we want the robot to keep going forward when the value of the activity is large, and change direction randomly when the value is small.

While we could construct more complex equations by having some attractors in $f(x)$ as shown in [15], this behavior can simply be seen as having an “imaginary” attractor in front of the robot. Therefore, if the orientation of the robot is defined as α , we can construct a Langevin equation based on (4), shown in (5). Here $x(t)$ and $y(t)$ are position of the robot at time t in Cartesian coordinates, k is a constant, $A(t)$ is the activity, and $\varepsilon_\omega(t)$ is the “yuragi” noise term.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\alpha}(t) \end{bmatrix} = A(t) \begin{bmatrix} k \cos \alpha(t) \\ k \sin \alpha(t) \\ 0 \end{bmatrix} + (1 - A(t)) \begin{bmatrix} 0 \\ 0 \\ \varepsilon_\omega(t) \end{bmatrix} \quad (5)$$

In the simplest scenario, one can easily see that we can let the activity has a binary value that causes the robot to switch behavior between purely moving straight forward in α direction when the activity is “1”, and purely change direction randomly when the activity is “0”. This also explains the addition of term $(1 - A(t))$ and why we only let the yuragi noise term exists for the orientation state. In this paper, $\varepsilon_\omega(t)$ is set as zero-mean Gaussian noise.

As the kinematic equation of a (nonholonomic) mobile robot is as shown in (6) [16], then (5) is actually the same as (7), where $v(t)$ is the linear velocity, with a constant value of k , and $\omega(t)$ is the angular velocity.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\alpha}(t) \end{bmatrix} = \begin{bmatrix} v(t) \cos \alpha(t) \\ v(t) \sin \alpha(t) \\ \omega(t) \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = A(t) \begin{bmatrix} k \\ 0 \end{bmatrix} + (1 - A(t)) \begin{bmatrix} 0 \\ \varepsilon_\omega(t) \end{bmatrix} \quad (7)$$

By observing (7), we see that different type of searching behaviors can be realized by designing a rule on how the value of the activity changes. We call it “the activity rule”. Here, we are interested in designing the activity rule to realize three types of searching behaviors: bacterial chemotaxis type, Levy walk type and the combination of the two. As for the target, we define it as a particular signal source that induces gradient.

B. The Realized Searching Behavior Types

1) *Bacterial chemotaxis type.* For this type, the activity must depend on sensory input, as it is the searching behavior type that will help the robot to find the target by following the induced gradient. We define sensory input as the change of detected signal amplitude emitted from the target:

$$\Delta S(t) = S(t) - S(t-1) \quad (8)$$

We implement the simplest relationship between the activity and sensory input as a step function shown in (9):

$$A(t) = \begin{cases} 0, & \text{if } \Delta S(t) \leq \theta \\ 1, & \text{if } \Delta S(t) > \theta \end{cases} \quad (9)$$

where θ is the “activity threshold”, the sensory input value that will change the activity value to become “1” or “0”, corresponds to the “swimming” and “tumbling” mode in bacterial chemotaxis behavior.

As can be seen from (9), when the sensory input only contains noise from the environment, the sequence of swimming and tumbling will only be random, which corresponds to random walk part in bacterial chemotaxis. However, when there is a gradient, the robot will do “chemotaxis”, a biased random walk toward the source.

2) *Levy walk type.* For this type, the activity does not depend on sensory input, as it is the searching behavior type that will help the robot to do an effective random search when the targets are sparse, and therefore most of the time there is no useful gradient information.

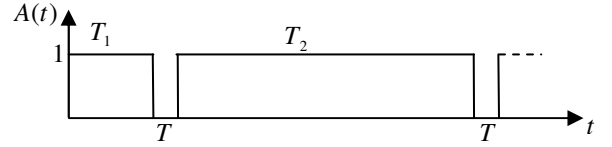


Fig. 2 Internal oscillations of the activity in Levy walk type. The durations of moving forward, T_1, T_2, \dots , follows power law-tail (Levy) distribution, while durations of changing direction, T , is constant

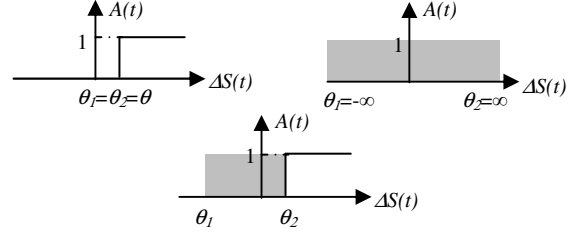


Fig. 3 The comparison of how the activity changes in the bacterial chemotaxis type (top left), the Levy walk type (top right) and the combination type (bottom)

Instead, here the activity is a function of an “internal oscillation”, similar to the concept of central pattern generator that drives spontaneous behavior in organism [12]. By letting $A(t)=1$ follows the probability distribution function with a power-law tail as $p(t)$ in (3), punctuated by constant short periods of $A(t)=0$, the robot will do Levy walk with constant velocity like the model explained in (2). As sequence of the activity is now driven by a stochastic, Levy, process, the whole behavior therefore becomes stochastic. This “internal activity oscillation” is illustrated in Fig.2.

3) *Combination type.* In combining the two searching behavior types, the activity depends on both sensory input and the internal oscillation. Fig. 3 explains how the activity, $A(t)$, changes due to sensory input, $\Delta S(t)$, sensed by the robot. In order to define the range where the value of the activity is a function of internal oscillation instead of sensory input, two activity threshold parameters, θ_1 and θ_2 , are used. This range is shown by the gray area. As can be seen, in bacterial chemotaxis type, $\theta_f = \theta_t = \theta$ in (9). Levy walk type means $\theta_f = -\infty$ and $\theta_t = \infty$. In the combination type, θ_1 and θ_2 have particular different values.

IV. SIMULATION EXPERIMENTS

A. Experiment Setup and Conditions

We verify the effectiveness of the searching behavior types by building a simulation. The sensory model is obtained from a real robot by attaching a microphone to the robot and collecting sound amplitude data versus the distance and the angle between the robot and a speaker.

The movement of the robot is modeled by using (8). The constant linear velocity of the robot is chosen as 15 [cm/sec], while the random angular velocity is limited to ± 90 [deg/sec]. The simulated area is 30 x 30 [m], while the robot diameter is set as 25 [cm].

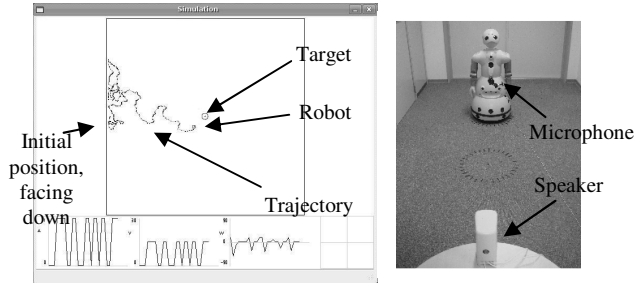


Fig. 4 Simulation screenshot and data collection from real robot

Fig. 4 (left) shows screenshot of the simulation. The goal is put at the center of the area. The initial position of the center of the robot is 1450 [cm] left from the goal, facing down. The small dot in the middle is the signal source, and it shows the robot searches for the goal and moves towards it. The intensity of the trajectory indicates the activity. When it is 0, the color is black, and when it is 1, the color is gray. From the left, the graphs at the bottom of the screenshot show $A(t)$, $v(t)$, and $\omega(t)$. An obstacle avoidance algorithm works if the robot faces the wall to orient it perpendicularly.

The simulated time length for the simulation is 1000 [s]. For the bacterial chemotaxis type, a new value of the activity based on the change of sound amplitude is calculated every simulated 1 [s], and therefore so is the new linear and angular velocity. For the Levy walk, the $A(t)=1$ periods follows power law distribution with resolution of 0.02 [s], with the random numbers for the period produced using stable random number generators encoded in Matlab by J. H. McCulloch at Ohio State University [17] and is based on the method described in [18]. We use the characteristic exponent $\mu=0.75$, the skewness parameter $\beta=1$, scale $c=0.5$, and location parameter $\tau=0$. The punctuating periods of $A(t)=0$ is 1 [s]. Every simulated 0.02 [s], a new position and orientation for the robot is shown.

As for the sensory model, it follows certain gradient pattern that depends on the distance and relative orientation between the robot and the speaker, plus sensory noises. The overall mathematical model is shown in (10):

$$S(t) = \begin{cases} \frac{h}{d(t)} e^{-p\phi(t)^2} + \eta(t), & \text{if } |\phi(t)| \leq 90 \\ \eta(t), & \text{if } |\phi(t)| > 90 \end{cases} \quad (10)$$

where $S(t)$ is the detected sound amplitude at time t , $d(t)$ is the distance of the robot to the sound source, $\phi(t)$ is the angle between the sound source and the robot, which equals to 0 when the robot perfectly facing toward the sound source, $\eta(t)$ is the sensory noise, h and p are constants. Based on the collected data, we choose $h=950$ and $p=2$. $\eta(t)$ is modeled as zero mean Gaussian noise with standard deviation 0.7. Fig. 4 (right) shows the data collection from real robot.

We want to confirm the effectiveness of each searching behavior type with and without sensing the sound gradient, and see how the related parameters affect the performance. Therefore, three experiment conditions are chosen as:

1) *Bacterial chemotaxis type with sound gradient.* The sound source is turned on, gradient information exists everywhere in the environment. We expect the bacterial chemotaxis type to do chemotaxis toward the target.

2) *Comparison of random walks.* The sound source is turned off, sensory input contains only noises and the bacterial chemotaxis type will simply become random walk. The random walk behavior is compared with the Levy walk type.

3) *Comparison of searching behavior types with limited sound gradient.* The sound gradient is limited within certain area. The three searching behavior types are compared.

B. Experiment Results

1) *Bacterial chemotaxis type with sound gradient.* Fig. 5 shows the trajectory examples of the first condition. The first row of the pictures show trajectory of the robot with the same activity threshold θ and different size of yuragi noise $\varepsilon_d(t)$, defined by the standard deviation. The second row shows those with the same size of yuragi noise and different value for the activity threshold. We confirm that the robot can reach the goal, and the next interesting question is how the parameters affect the behavior and performance.

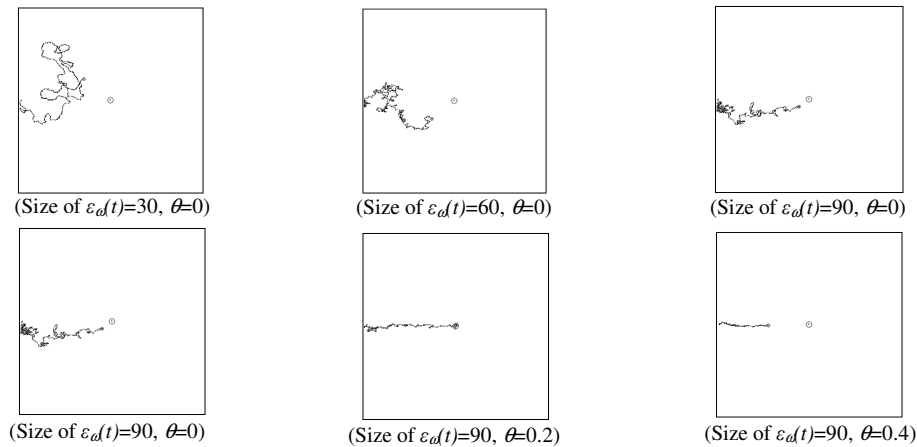


Fig. 5 Trajectory examples of bacterial chemotaxis searching behavior type within sound gradient, top: with different size of yuragi noise $\varepsilon_d(t)$, bottom: with different value for the activity threshold θ

TABLE I
PERFORMANCE OF THE BACTERIAL CHEMOTAXIS SEARCHING TYPE
WITH SOUND GRADIENT

Searching behavior type	Size of $\varepsilon_{\omega}(t)$	θ	Success rate
Bacterial chemotaxis	30	0	18
	30	0.2	26
	30	0.4	0
	60	0	30
	60	0.2	66
	60	0.4	0
	90	0	70
	90	0.2	96
	90	0.4	4

The performance is measured from the success rate, the percentage of reaching the target in 50 trials. Reaching target is defined as reaching 50 [cm] area from the sound source. The parameters are the size of yuragi noise $\varepsilon_{\omega}(t)$, and the value for activity threshold θ . We simply choose some reasonable values to reveal the general principle, paying attention to the parameter value of sensory model and the robot limitation. The result is shown in Table I, and can be explained as follow.

As shown from the trajectory intensity in Fig. 5, the activity keeps alternating between “1” and “0”. For the first value, the robot will do the swimming mode, or moving forward. For the second value, the robot will do the tumbling mode, or change direction randomly. From the first row of the figures, it can be seen that when the size of the yuragi noise is too low, the tendency to change direction significantly when the activity equals to “0” is lower. Therefore, it will take a longer time before the robot can find the correct orientation toward the goal, and the performance is lower. The top-left figure shows the lowest noise size that causes the longest time to find the correct orientation.

From the second row of the figure, it can be seen that when the activity threshold is too high, the tendency to easily go forward is too low. When the activity threshold is high, it will take a large change of detected signal amplitude, $\Delta S(t)$, before activity becomes “1” and let the robot moves forward. Therefore, for example, in the bottom-right figure, the trajectory is quite straight, but the robot cannot reach the target within the simulation time.

For the used setup (e.g.: initial condition, target’s place, size of the sensory noise), the best combination for the size of yuragi noise and the activity threshold seems to be 90 [deg/s] and 0.2.

2) *Comparison of random walks.* As for this second experiment condition there is merely noise and no sound gradient in the environment, the bacterial chemotaxis type simply becomes random walk. Table II confirms that the Levy walk type is the better random walk, regardless of the size of the yuragi noise $\varepsilon_{\omega}(t)$. Fig. 6 shows the trajectory comparison with particular size of the yuragi noise. Even the change of the parameters can affect the random walk behavior of the bacterial chemotaxis type, the Levy walk type still brings the robot nearer to target.

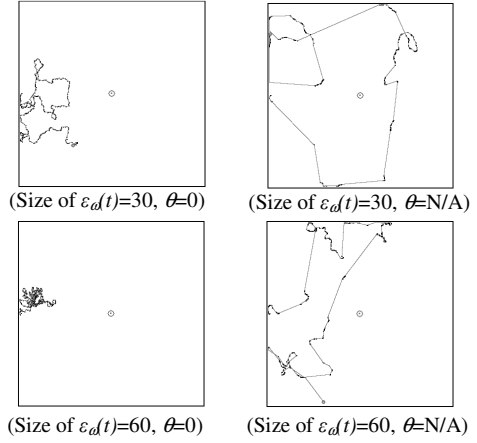


Fig. 6 Trajectory examples of random walk in bacterial chemotaxis type (left) versus the Levy walk type (right) with particular size of yuragi noise $\varepsilon_{\omega}(t)$

TABLE II
PERFORMANCE OF THE RANDOM WALK IN BACTERIAL CHEMOTAXIS TYPE
VERSUS THE LEVY WALK TYPE

Searching behavior type	Size of $\varepsilon_{\omega}(t)$	θ	Success rate
Bacterial chemotaxis	30	0	2
	30	0.2	0
	30	0.4	0
Levy walk	30	N/A	10
Bacterial chemotaxis	60	0	0
	60	0.2	0
	60	0.4	0
Levy walk	60	N/A	4
Bacterial chemotaxis	90	0	0
	90	0.2	0
	90	0.4	0
Levy walk	90	N/A	6

It can also be seen that the size of yuragi noise affects the behavior of the Levy walk type. When it is smaller, the tendency to thoroughly search in a particular area before “jump” to another area is lower. Among the tried size of the yuragi noise with this experiment setup, it seems that standard deviation of 30 [deg/s] results in the best performance.

3) *Comparison of searching behavior types with limited sound gradient.* For the third condition, the sound gradient is limited within 500 [cm] radius from the source. Outside it, only sensory noise exists. We compare all the three searching behavior types. We choose the size of yuragi noise $\varepsilon_{\omega}(t)$ as 90 [deg/s] for all the types. For the bacterial chemotaxis type, we choose $\theta_f = \theta_z = \theta = 0.2$. Levy walk type obviously means $\theta_f = -\infty$ and $\theta_z = \infty$. For the combination, we choose $\theta_f = -\infty$ and $\theta_z = 0.2$.

The trajectory examples are shown in Fig. 7. While in both the chemotaxis and Levy walk type the trajectory intensity indicates the activity value, for the combination, the grey color means the robot performs Levy walk, and the black one means it performs chemotaxis. It can be seen that the chemotaxis behavior type simply becomes random walk as it cannot reach the area where the sound gradient exists, and the Levy walk type is the better random walk. It is interesting to notice that when the sound gradient exists anywhere inside the

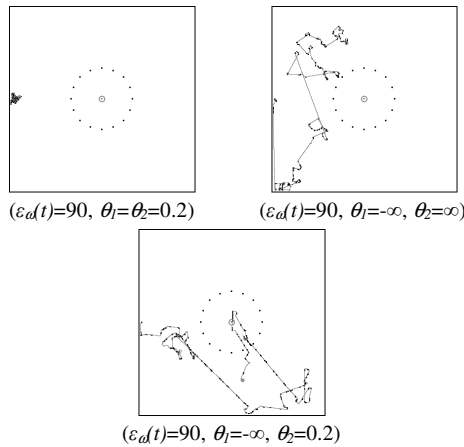


Fig. 7 Trajectory examples of the bacterial chemotaxis (top left), the Levy walk (top right) and the combination type (bottom) with limited area of sound gradient

TABLE III
PERFORMANCE OF ALL THE SEARCHING BEHAVIOR TYPES WITH LIMITED AREA OF SOUND GRADIENT

Searching behavior type	Size of $\epsilon_d(t)$	θ_1	θ_2	Success Rate
Bacterial chemotaxis	90	0.2	0.2	0
Levy walk	90	$-\infty$	∞	6
Combination	90	$-\infty$	0.2	20

30x30[m] area, the chosen parameters actually gives the best result for bacterial chemotaxis type.

Table III shows that the performance comparison. It can be seen that the combination type is the best one. From the trajectory intensity in Fig. 7 (bottom) it can be seen that the Levy walk behavior can bring the robot to the necessary area. However, in order to actually reach the target once the gradient information exists, the chemotaxis behavior is beneficial.

V. CONCLUSION AND FUTURE WORK

Here we propose a biologically inspired approach to realize an adaptive searching behavior in mobile robot, that is effective with and without sensing the gradient induced by the target. The proposed approach is to implement and combine two already known effective searching behavior: bacterial chemotaxis and Levy walk, based on yuragi, or biological fluctuation. The yuragi-based framework itself is simple, utilizes noise to keep the robot searching for the target, and can switch elegantly between stochastic and deterministic behavior. It also does not need any model of the environment, as already modeled through the activity.

While the framework actually introduces many design possibilities, we show that bacterial chemotaxis and Levy walk searching behavior type can be implemented and combined based on this framework. In bacterial chemotaxis type, when there is gradient information, the robot will do a biased random walk toward the source. In the absence of gradient, it

will simply do random walk, and we have shown that the robot can also perform a Levy walk type. We have also shown how the parameters affect the behavior and performance in each type, and how the combination of the two can be beneficial.

While in this paper we simply choose some reasonable parameter values to reveal the general principle, a more thorough analysis is an interesting next step. Aside from that, a more complex scenario, real robot experiments, other designs based on yuragi framework, and comparison with other methods remain as our future work.

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